Indian Statistical Institute, Delhi Centre

Branching Processes

Fall 2009

Assignment # 9

Date Given: December 17, 2009 **Total Points: 20**
Date Due: December 21, 2009

1. Let $\mathcal{V} := \{\emptyset\} \cup \left(\bigcup_{d=1}^{\infty} \mathbb{N}^d \right)$ be the set of all finite words including the empty word using $\mathbb{N}$ as the set of alphabets. Let $\mathbb{T}$ be graph with vertex set $\mathcal{V}$ and edge set $\mathcal{E} := \left\{ (u, v) \;\middle|\; v = us \text{ for some } s \in \mathbb{N} \text{ and } u, v \in \mathcal{V} \right\}$. Here we write $u \in \mathcal{V}$ as $u = x_1 x_2 \cdots x_d$ if $u \in \mathbb{N}^d$ and for any $u \in \mathcal{V}$ and $s \in \mathbb{N}$ we write us as an element in $\mathbb{N}^{d+1}$. Let $\mathcal{T}$ be the set of all finite sub-graphs of $\mathbb{T}$ rooted at $\emptyset$. $[3 + 2 + 5 + 10 = 20]$

   (a) Show that $\mathbb{T}$ is a tree.

   (b) Show that $\mathcal{T}$ is a countably infinite set.

   (c) Show that any sub-critical or critical Galton-Watson branching process starting with one individual puts a discrete probability measure μ on $\mathcal{T}$.

   (d) Find the probability mass function for μ if the progeny distribution is Poisson (λ) with $\lambda \leq 1$.