UNIVERSITY OF CALIFORNIA, BERKELEY

DEPARTMENT OF STATISTICS

STAT-155: Game Theory

Fall 2013

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Practice Midterm

Date Given: October 16, 2013 **Total Points: 30**

Total Time: 50 minutes

1. Consider the following Combinatorial Game

   - The game starts with a 8×8 standard *chess board* with 14 bishops placed on each of the 14 squares which are on the bottom and left most of the board. The bottom-left corner of the board is left empty.
   - There are two players, namely, Players I and II who alternate their moves.
   - At each move a player selects one of the 14 bishops and moves it along the top-right direction diagonally any number of squares. Note in a move only one bishop is moved and bishops always move diagonally towards top-right direction. No bishop is allowed to move out of the board.
   - The game ends when all the 14 bishops are at 14 squares which are on the top and right most the of the board. Note that the top-right corner of the board remains empty. The last player to make a move is the winner.

   (a) For the above game analyze who will be the winner. Give reasons for your answer.

   (b) What is the Sprague-Grundy Function value of the above game at the given starting position?

   (c) Suppose there is another bishop placed at the bottom-left cornet which can also move only to the top-right direction along the diagonal. For this new game who do you think will be the winner and what is a winning move? What is the Sprague-Grundy Function value of this new game at this new starting position?

2. Suppose two players I and II call two numbers simultaneously from the set $\{1, 2, 3\}$. If the numbers Player I and II call are a and b respectively then Player I receives an amount of $\$\,(a - b)$ from Player II.

   Find the pay-off matrix for this game. What is the *value of the game*? Can you find *optimal strategies* for Players I & II?

   Do think the value will change if the two players call two numbers simultaneously from the set $\{1, 2, 3, \cdots, 10\}$ instead?