Solutions to Assignment 1
Stat 155: Game Theory

September 15, 2013

Question 1

Here we are trying to enumerate the P-positions and N-positions of a subtraction game with subtraction set $\{1, 2, 4, 5\}$. As we are playing under normal winning rule, 0 is a P-position.

Using the technique 'backward induction' discussed in class we can discover the following pattern in the position of N- and P- positions.

0	1	2	3	4	5	6	7	8	9	10	...
P	N	N	P	N	N	P	N	N	P	N	...

a) We notice that the pattern PNN of length 3 emerges from our calculations and hence make the following observation, any position of the form,

$$3k \text{ is a P-position, } 3k \pm 1 \text{ is a N-position}$$

where k is an integer.

b) As $31 = 3 \times 10 + 1$, it is a N-position, which means that Player I has an winning strategy.

c) As $347 = 3 \times 115 + 2$, it is a N-position, which means that Player I has an winning strategy.

Question 2

Here we do exactly the same thing as Question 1, with the only exception that as we are playing under the *misere* rule, 0 is a N-position.

The updated table is as follows,

0	1	2	3	4	5	6	7	8	9	10	...
N	P	N	N	P	N	N	P	N	N	P	...

a) We notice that the pattern NPN of length 3 emerges from our calculations and hence make the following observation, any position of the form,

$$3k + 1 \text{ is a P-position, } 3k \text{ or } 3k + 2 \text{ is a N-position}$$

where k is an integer.

b) As $31 = 3 \times 10 + 1$, it is a P-position, which means that Player II has an winning strategy.

c) As $347 = 3 \times 115 + 2$, it is a N-position, which means that Player I has an winning strategy.

Notice that the pattern under *misere* rule is not the exact reversal of the pattern under normal play.