Solutions to Assignment 12
Stat 155: Game Theory

Question 1

First we try to simplify the definition of symmetric game for two-player games. The only permutations of $\{1, 2\}$ are $\{1, 2\}$ and $\{2, 1\}$. Then, the condition given in the question simplifies to $U_2(x_2, x_1) = U_1(x_1, x_2)$ for every $x_1, x_2 \in X$.

a) For 2-person 0-sum games, $U_2 = -U_1$. Thus, writing in terms of payoff matrix instead of utility, $a_{ij} = U_1(i, j) = U_2(j, i)$ by symmetry $= -U_1(j, i) = -a_{ji}$, implying skew symmetry of the payoff matrix.

b) Similarly, in 2-person general sum games, $a_{ij} = U_1(i, j) = U_2(j, i)$ by symmetry $= B_{ji} = B^T_{ij}$. Thus, $A = B^T$.

c) No its not. Consider a 2-person game, where the utility is defined on the strategy space $\{-3, -2, -1, -, 1, 2, 3\}$ with utility $U_1(x_1, x_2) = U_2(x_1, x_2) = Sign(x_1)$. Then $U_1(2, -2) = 1$. If the game were symmetric, then $U_2(-2, 2)$ should have been the same as $U_1(2, -2)$. But $U_2(-2, 2) = U_1(-2, 2) = -1 \neq 1$. Thus, this game is not symmetric.

Question 2

This game is an instance of *graph coloring* that we have discussed in class and understood as a potential game, hence there is a unique pure NE. The pure NE is given by the following configuration: Red on each vertex of V_1 and Blue on each vertex of V_2. First, notice that this is a proper coloring as no two neighbors have the same color (in every pair of neighbors, there is one member of V_1 and one member of V_2). To show that this is a pure NE, it is enough to show that this configuration maximizes the potential. Consider any vertex in V_1, all of its non-neighbors share the same color which is the maximum that its utility can be, and similarly for any vertex in V_2. Thus, this is the NE. Hence, the chromatic number of this graph is 2.