Solutions to Assignment 4
Stat 155: Game Theory

Question 1

This game has a mind-boggling number of symmetries which can be exploited to analyse the game. The simplest one is detailed here. Observe that, at each step a move is to select a rook and then move it as many steps as desired towards the right, at least one step. Then it is clear, that a rook placed in the i^{th} column from the right, is equivalent to a Nim pile with $i-1$ chips. In particular, the rooks on the north-east and south-east corners do not move at all.

Then, the game can be written as a 14 pile Nim with starting position $(1, 2, \ldots, 7, 1, 2, \ldots 7)$, which is a P position as the Nim Sum is $1 \oplus 2 \oplus \cdots \oplus 7 \oplus 1 \oplus 2 \oplus \cdots \oplus 7 = 1 \oplus 1 \oplus 2 \oplus 2 \oplus \cdots \oplus 7 \oplus 7 = 0 \oplus 0 \oplus \cdots \oplus 0 = 0$.

It can also be analyzed as sums of Nims or Nimbles etc.

Question 2

a) Lets fix an arbitrary game (G_5, g_5). Then, we need to show that for all such (G_5, g_5), $(G_1+G_3+G_5, (g_1, g_3, g_5))$ is a P position iff $(G_2+G_3+G_5, (g_2, g_3, g_5))$ is a P position. Denote the game $(G_3 + G_5, (g_3, g_5))$ as (H, h), thus we need to show that $(G_1 + H, (g_1, h))$ is a P position iff $(G_2 + H, (g_2, h))$ is a P position.

But this follows from the definition of G_1 and G_2 being equivalent, as (H, h) is a combinatorial game.

b) This follows by applying part a) twice. $(G_1 + G_3, (g_1, g_3)) \sim (G_2 + G_3, (g_2, g_3))$ by applying part a). $(G_2+G_3, (g_2, g_3)) \sim (G_2+G_4, (g_2, g_4))$ by again applying part a). As equivalence is a transitive relation, $(G_1 + G_3, (g_1, g_3)) \sim (G_2 + G_4, (g_2, g_4))$.