Indian Statistical Institute, Delhi Centre
Linear Models and GLM
Spring 2008
Answers to the Quiz # 1

Date: February 8, 2008 (Friday) Total Points: 2 × 5 = 10

1. Consider the following models:
   (a) \( y_i = \alpha + \beta x_i^2 + \varepsilon_i^2, 1 \leq i \leq n. \)
   (b) \( y_i^2 = \theta + \varepsilon_i, 1 \leq i \leq n. \)
   (c) \( \log p_{i,j} = \mu + \alpha_i + \beta_j + \gamma_{i,j} + \varepsilon_{i,j}, 1 \leq i, j \leq N. \)
   (d) \( y_{i,j} = \mu + \alpha_i + \beta_j + \alpha_i\beta_j + \varepsilon_{i,j}, 1 \leq i, j \leq N. \)

   In all of above the errors are i.i.d. Normal (0,1) random variables.

   State which of the above can be described as a Linear Model and which can not be.

   \textbf{Ans.} (a) \textbf{A Linear Model} \quad (b) \textbf{A Linear Model} \quad (c) \textbf{A Linear Model} \quad (d) \textbf{Not a Linear Model}

2. Fill in the following table:

<table>
<thead>
<tr>
<th>Symmetric Matrix</th>
<th>Is it p.d./p.s.d./n.d./n.s.d./indefinite ?</th>
</tr>
</thead>
</table>
   | \[
   \begin{pmatrix}
   1 & 2 \\
   2 & 1 \\
   \end{pmatrix}
   \] | Indefinite |
   | \[
   \begin{pmatrix}
   0 & 1 \\
   1 & 0 \\
   \end{pmatrix}
   \] | Indefinite |
   | \[
   \begin{pmatrix}
   1 & 1 & 1 \\
   1 & 1 & 1 \\
   1 & 1 & 1 \\
   \end{pmatrix}
   \] | p.s.d. |
   | \[
   \begin{pmatrix}
   1 & 1/2 & 1/4 \\
   1/2 & 1 & 1/2 \\
   1/4 & 1/2 & 1 \\
   \end{pmatrix}
   \] | p.d. |
3. Suppose $A_{n \times n}$ be a real matrix and consider its Singular Value Decomposition given by:

$$A = P \left( \begin{array}{ccc} \Delta & 0 \\ 0 & 0 \end{array} \right) Q,$$

where $P$ and $Q$ are two orthogonal matrices and $\Delta$ is a diagonal matrix with strictly positive diagonal entries.

Indicate if the following statements are True or False:

(a) $\Delta$ has all the non-zero eigen-values of $A$. False
(b) If $A = A^T$ then $\Delta$ has all the non-zero eigen-values of $A$. False
(c) $PQ = QP$. False
(d) $\exists$ an orthogonal matrix $B_{n \times n}$ such that $P = BQ$. True

4. Consider the following model:

$$y_{i} = \mu_{i} + \varepsilon_{i}, \ 1 \leq i \leq n,$$

where $\{\varepsilon_{i}\}_{i \geq 1}$ are i.i.d. with Normal $(0,1)$ random variables.

Indicate if the following statements are True or False:

(a) This is not a Linear Model. False
(b) For each $1 \leq i \leq n$ the parameter $\mu_{i}$ is estimable. True
(c) $R_{0}^{2} > 0$. False
(d) We can test $H_{0} : \mu_{1} = \mu_{2} = \cdots = \mu_{n}$. True

5. Consider the following linear model:

$$y_{1,j} = \mu + \nu + \varepsilon_{1,j} \text{ for } 1 \leq j \leq n_{1}$$
$$y_{2,j} = \mu + \varepsilon_{2,j} \text{ for } 1 \leq j \leq n_{2},$$

where $\mu, \nu \in \mathbb{R}$ are unknown parameters, and $\{\varepsilon_{i,j} | 1 \leq i \leq 2, 1 \leq j \leq n_{i}\}$ are i.i.d. Normal $(0, \sigma^{2})$, where $\sigma^{2}$ is not known.

Indicate if the following statements are True or False:

(a) The MLE of $\mu^{2}$ is $\left( \frac{1}{n_{2}} \sum_{j=1}^{n_{2}} y_{2,j} \right)^2$. True
(b) The MLE of $\nu$ is not unique. False
(c) The MLE of $\sigma^{2}$ exists no matter what $n_{1}$ and $n_{2}$ are. False
(d) Under a hypothesis $H_{0} : \mu = 0$ a MLE of $\sigma^{2}$ exists no matter what $n_{1}$ and $n_{2}$ are. True