1. Consider the following model: \[ Y = X\beta + C\gamma + \varepsilon, \]
where co-ordinates of \( \varepsilon \) are i.i.d. Normal (0, 1).
Indicate if the following statements are True or False:
(a) If \( C^T C \) invertible and \( PXC = 0 \) then \( \gamma \) is estimable. True
(b) If \( \gamma \) is estimable with LSE \( \hat{\gamma} \) then \( \hat{\gamma} \) has a multivariate normal distribution which is singular. False
(c) Suppose \( \gamma \) is estimable, put \( Z = Y - C\hat{\gamma} \) then \( E[Z] = X\beta \). True
(d) Two-way classification model with no interaction and one observation per cell can be written as a special case of the above model. True

2. Consider the following model:
\[ y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ijk} \quad 1 \leq k \leq K, 1 \leq j \leq J, 1 \leq i \leq I, \]
where \( \varepsilon_{ijk} \)'s are i.i.d. Normal (0, \( \sigma^2 \)).
Fill in the blanks:
(a) It is called the Three-way classification model with no interaction.
(b) The degrees of freedom for the residual sum of square is \( IJK - I - J - K + 2 \).
(c) The maximum likelihood estimate of \( \sigma^2 \) is given by \( \frac{1}{IJK} \sum_{i,j,k} (y_{ijk} - y_{i-} - y_{-j} - y_{-k} + 2y_{-..}) \).
(d) The linear parametric functions \( \alpha_i - \alpha_i' \) for \( 1 \leq i \neq i' \leq I \) are estimable.

3. Consider the following model:
\[ y_{ij} = \mu + \alpha_i + \gamma_{ij} + \varepsilon_{ij} \quad 1 \leq j \leq k_i, 1 \leq i \leq I, \]
where \( \varepsilon_{ij} \)'s are i.i.d. Normal (0, 1).
Indicate if the following statements are True or False:
(a) \( \alpha_1 - \alpha_2 \) is estimable. False
(b) If we fix an \( i \) then the observations indexed by \( j \) form an one-way classification model. True
(c) Suppose \( k_1 = 10 \) then we can do multiple comparison using Tukey’s Honest Significant Difference to test for \( \gamma_{11} - \gamma_{12} = 0 \) and \( \gamma_{12} - \gamma_{13} = 0 \). True
(d) For the multiple comparison in (c) above if we use Bonferroni’s method then we should do the one-degrees of freedom testing at a level 0.025 to achieve an experimental error rate of 5%. True

4. Indicate if the following statements are True or False:

(a) A log-linear model is a linear model. False
(b) A two-way classification data represented as a \( I \times J \) table can be modeled by a log-linear model. False
(c) The estimates obtained in logistic regression are MLEs under appropriate model. True
(d) The following log-linear model is a saturated model False

\[ \log m_{ijk} = u_0 + u_1(i) + u_2(j) + u_3(k) + u_{12}(ij) + u_{23}(jk) + u_{31}(ki) . \]

5. Fill in the blanks:

(a) For a linear model the residuals are always uncorrelated (independent) of the LSEs.
(b) One-way classification model is a sub-model of two-way classification model.
(c) The degrees of freedom for the residual sum of square from a four-way classification model with no interaction and one observation per cell is \( K^4 - 4K - 3 \) where each classification has \( K \) categories.
(d) Tukey’s one degrees of freedom test is a test of non-additivity.