1. Suppose $X$ is a random variable with the following density:

$$f(x) = \frac{1}{\pi(1 + x^2)}, \quad -\infty < x < \infty.$$  

(a) Find the CDF of $|X|$.  

(b) Find the density of $X^2$.  

2. Let $T_1 \leq T_2$ be the times of 1st and 2nd arrivals in a Poisson arrival process of rate $\lambda$ on $(0, \infty)$.

(a) Find $E[T_1 | T_2 = 10]$.  

(b) Find $E[T_1 T_2]$.  

3. There are 90 students in a statistics class. Suppose each student has a standard deck of 52 cards of his/her own, and each of them selects 13 cards at random without replacement from his/her own deck independent of the others. What is the chance that there are at least 50 students who got 2 or more aces?  

4. Suppose you and me are tossing two fair coins independently, and we will stop as soon as each one of us gets a head.

(a) Find the chance that we stop simultaneously.  

(b) Find the conditional distribution of the number of coin tosses given that we stop simultaneously.  

5. Suppose $(X, Y)$ have the following joint density:

$$f(x, y) = \begin{cases} \frac{1}{2} & \text{if } |X| + |Y| \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$
(a) Find the marginal distribution of \( X \). \[5\]
(b) Find the conditional distribution of \( Y \) given \( X = 1/2 \). \[5\]

6. Suppose a box contains 10 green, 10 red and 10 black balls. We draw 10 balls from the box by sampling with replacement. Let \( X \) be the number of green balls, and \( Y \) be the number of black balls in the sample.

- Find \( E[XY] \). \[8\]
- Are \( X \) and \( Y \) independent? Explain. \[2\]

7. Julia wants to catch a flight from Oakland Airport at 10:30 AM. In reality the flight leaves at a time uniformly distributed between 10:30 AM and 10:45 AM. Julia also knows that because of bad traffic, if she plans to reach to the airport by time \( t \), then she can only be able to make it by a time which is uniformly distributed between \( t \) and \( t + 15 \) minutes. What time Julia should plan to reach the airport so that she will have exactly 90% chance of catching the flight? \[10\]

8. Let \( X \) and \( Y \) be two independent random variables such that \( X \sim \text{Normal}(\mu, 1) \) and \( Y \sim \text{Normal}(0, 1) \).

(a) Find the density of \( Z = \min(X, Y) \). \[5\]
(b) For each \( t \in \mathbb{R} \), calculate \( \mathbb{P} (\max(X, Y) - \min(X, Y) > t) \). \[5\]

9. There are \( n \) balls labeled 1, 2, 3, \ldots, \( n \), and \( n \) boxes also labeled 1, 2, 3, \ldots, \( n \). Balls are being placed in the boxes at random such that each box can contain only one ball. Say that there is a matching at the \( i \)th position if the \( i \)th ball goes into the \( i \)th box. Let \( X \) be the number of matchings. Find \( E[X] \) and \( \text{Var}(X) \). \[4 + 6\]

10. Let \( Y \) be a random variable with a density \( f_Y \) given by:

\[
 f_Y(y) = \begin{cases} 
 \frac{\alpha - 1}{y^\alpha} & y > 1 \\
 0 & \text{otherwise}, 
\end{cases}
\]

where \( \alpha > 1 \). Given \( Y = y \), let \( X \) be a random variable which is Uniformly distributed on \((0, y)\).

(a) Find the marginal distribution of \( X \). \[4\]
(b) Calculate \( E[Y | X = x] \), for every \( x > 0 \). \[6\]