1. Suppose there are 10 drawers, each containing two coins. Drawers 1 and 2 contain only gold coins, while in drawers 3, 4 and 5 one is a gold coin and the other is a silver coin, and rest of the drawers contain only silver coins. Suppose you pick a drawer \( D \) at random, and then a coin at random from it. Given that you choose a gold coin find the conditional probability that the other coin is also gold. Find the conditional distribution of the random drawer \( D \).

\[6 + 6\]

2. An urn contains 3 tickets labeled 1, 2 and 3. The tickets 1 and 3 are green and the ticket 2 is red. Two tickets are drawn at random without replacement from the urn. Let \( X \) be the number of green tickets in the sample and \( Y \) be the total of the two numbers selected.

(a) Write the joint distribution of \( X \) and \( Y \) as form of a table. Are \( X \) and \( Y \) independent? \[4 + 1\]

(b) Name the distribution of \( X \) and specify the parameter values. \[1\]

(c) Calculate the expected value and variance of \( Y \). \[3 + 3\]

3. Suppose \( X_1, X_2, \cdots, X_n \) are independent and identically distributed random variables. Each \( X_i \) takes only two values namely \( \pm 1 \) with equal probabilities. Let \( S_n := X_1 + X_2 + \cdots + X_n \).

(a) Find the distribution of \( S_n \). \[5\]

(b) Suppose \( n = 2m \) then find \( \lim_{m \to \infty} \sqrt{m} P(S_{2m} = 0) \). \[3\]

(c) If \( n = 100 \) then find approximate numerical value for \( P(|S_n| < 10) \). \[4\]

4. Roll a standard six sided fair die till a 6 appears. Let \( X \) be the total number of rolls and \( Y \) be the number of times 1 has appeared.

(a) What is the distribution of \( X \)? \[1\]

(b) Find the conditional distribution of \( Y \) given \( X = x \). \[5\]
(c) Find $\mathbb{E}[Y]$ and $\text{Var}(Y)$. $[3+3]$

5. There are 10 empty boxes numbered 1, 2, . . . , 10 placed sequentially on a circular table. We perform 100 independent trials. At each trial, a box is selected at random and one ball is added in the two neighboring boxes of the selected box. Let $X_k$ be the number of balls in the $k^{th}$ box at the end of 100 trials.

(a) Is the sequence of random variables $(X_1, X_2, \ldots, X_{10})$ exchangeable? Explain your answer. $[4]$
(b) Are they independent? Explain your answer. $[4]$
(c) Find $\mathbb{E}[X_k]$ for $1 \leq k \leq 10$. $[4]$