Aggregation in Area-Yield Crop Insurance: The Linear Additive Model

Bharat Ramaswami and Terry L. Roe

Previous analyses of area yield crop insurance have used a linear additive model (LAM) to express the relationship between individual and area yield. However, the theoretical foundations of the LAM are unknown. This shortcoming is addressed by establishing two conditions linking microvariables and LAM parameters. The conditions relate to the interaction of risks in individual technologies and the extent of aggregation. If systemic and individual risks are additive in individual yields, and if the law of large numbers hold, then the LAM obtains. This article also shows how departures from these conditions affect the results derived from a LAM analysis.

Key words: area yield, beta, crop insurance, systemic risks.

A classic issue in agricultural economics is the design of schemes that would offer insurance against risks in agriculture. The experience with conventional crop insurance has been disappointing, as insurers have struggled to obtain reliable actuarial data on individual yields (Skees, Black, and Barnett). The primary attraction of area-yield insurance schemes is that insurers do not have to contend with the informational problems of moral hazard and adverse selection (Halcrow). These problems can be dismissed because indemnities and premiums are based not on a producer's individual yield but rather on the aggregate yield of a surrounding geographical area. However, the key question is: How adequate are aggregate yield instruments for reducing the risks faced by producers?

To address this question, previous studies have expressed individual yields as a linear stochastic function of area yield (Mahul, Miranda, Vercammen). The approach has been to use the form of a linear regression model where the dependent variable is the yield of an individual producer, the only independent variable is area yield and the additive random error term measures omitted individual-specific factors uncorrelated with area yield. Thus, the model decomposes variations in individual yield in to variations in area yield that represent systemic risk and variations in the error term that represent individual-specific or nonsystemic risk. The key parameter of the model is the so-called beta coefficient, which is the slope coefficient in the relation. It has been shown that the beta determines the extent of risk reduction as well as the form of the optimal insurance. As the model combines linearity and additivity (of the error term to area yield), it can be called the linear additive model (LAM). The literature assumes the LAM and does not derive it. In principle, the LAM can be useful in any kind of risk analysis where it is important to distinguish between systemic and nonsystemic risk, and thus, it is important to better understand its underlying conceptual fundamentals.

This article investigates the theoretical foundations of the LAM. On surface, the LAM bears a striking similarity to the capital asset pricing model (CAPM) of finance. The CAPM postulates returns on individual assets to be a linear stochastic function of the returns on the market portfolio. The CAPM beta—the slope coefficient in the model—measures the sensitivity of asset returns to the returns on the market portfolio. Variations in asset returns are the sum of variations in systemic risk (as measured by the variation in the returns to the market portfolio), and variations in individual-specific risk (as denoted by the random term in the CAPM).

The theoretical basis of the CAPM is well known. It lies in mean variance utility functions, optimizing investor behavior, two-fund separation results, and the efficiency of a market portfolio (Merton). However, there is no

Bharat Ramaswami is senior economist in the Indian Statistical Institute, New Delhi, India. Terry L. Roe is professor, Department of Applied Economics, University of Minnesota.

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meaningful way of transferring these arguments to the context of area-yield crop insurance. Clearly, the LAM of area-yield crop insurance is the consequence of aggregation of individual producer technologies and is not the outcome of optimization. CAPM type arguments are, therefore, inapplicable.

This article derives the precise conditions under which the LAM is valid. The conditions are two fold, applying to individual technologies and on the extent of aggregation. We show that if systemic and individual risks are additive in individual yields, and if the aggregation is such that the law of large numbers holds, then the LAM obtains. These are sufficient conditions. The additivity property of systemic and individual risks is a necessary condition. Interestingly, the LAM is otherwise independent of assumptions about the functional form of the production function. Moreover, the LAM does not require assumptions about the functional form of the density function of the random variables.

These results are important for two reasons. First, they extend the applicability of LAM to new questions. For instance, what are the underlying factors determining the magnitude of the individual betas and the additive disturbance term of the LAM? These factors can include producer actions as well as features of insurance design. In this article, we use this understanding to analyze how the level of aggregation matters to risk reduction and insurance demand. There are other potential uses as well. The LAM can be used to analyze the "nexus between the producer's insurance choice and his farm-level production decisions," which otherwise would not be possible (Chambers and Quiggin). Second, our results shed light on the circumstances in which the LAM is *not* valid. To develop these extensions we begin with a discussion of the basic model.

The Basic Model

The LAM is of the following form:

$$(1) \qquad y_i = \mu_i + \beta_i(y - \mu) + \varepsilon_i$$

where y_i is producer i's yield, μ_i is the unconditional mean of y_i, that is, $E(y_i)$, y is area yield, β_i is the slope parameter satisfying $\beta_i = \mathrm{Cov}(y_i, y)/\mathrm{Var}(y)$, μ is the unconditional mean of y and ε_i is a mean zero random variable uncorrelated with area yield. Equation (1) decomposes individual yield variation into a systemic component $\beta_i(y - \mu)$ perfectly correlated with area yield (since $\mathrm{Cov}(\beta_i(y-\mu), y)^2/\beta^2{}_i\mathrm{Var}(y)^2 = 1$) and a nonsystemic or individual-specific component ε_i uncorrelated with area yield.

Suppose the indemnity schedule is $I(y) = \max(y_c - y, 0)$ where y_c is a yield trigger fixed exogenously. Then Miranda showed that the extent of variance reduction is proportional to β_i (and other exogenous parameters that do not vary across producers). It follows that the more highly correlated a producer's yield is to the area yield, the greater is the risk reduction.

Mahul considered the choice of an optimal contract $I(y)$. If insurance is actuarially fair then the optimal contract is characterized by $I(y) = \beta_i(y_m - y)$, where y_m, the yield trigger, is the maximum possible value of y.[1] Hence the slope of the optimal indemnity schedule is $-\beta_i$. An aspect of this result, not noted by Mahul but relevant for us, is that the optimal indemnity schedule is independent of the nonsystemic risk and its moments (such as $\mathrm{Var}(\varepsilon_i)$).

Another implication is that optimal area-yield insurance completely eliminates the systemic risk. To see this, note that a producer's revenue with insurance (denoted π) is

$$(2) \qquad \pi = y_i + I(y) - P$$

where P is the premium. When a producer chooses the optimal area-yield insurance, (2) becomes

$$(3) \qquad \pi = \mu_i + \beta_i(y - \mu) + \varepsilon_i + \beta_i(y_m - y) - P$$

where we have used (1). But when insurance is actuarially fair, $P = \beta_i(y_m - \mu)$. Substituting in (3), we see that the producer bears only the nonsystemic risk, that is,

$$(4) \qquad \pi = \mu_i + \varepsilon_i.$$

Thus optimal area-yield insurance fully insures against the systemic risk under actuarially fair premiums. Since the optimal insurance is independent of the riskiness of the nonsystemic risk ε_i, we have the result that the optimal area-yield insurance delivers full insurance against the insured (systemic) risk, whatever be the riskiness of the uninsured (nonsystemic) risk.

[1] For the LAM in (1), Vercammen considers the optimal design of an area-yield crop insurance contract when the yield trigger is constrained, for institutional reasons, to be below the maximum possible value of area yield.

Conventional individual-yield crop insurance offers insurance against both systemic and nonsystemic risks; however, because of moral hazard, such insurance comes with a deductible. By contrast, optimal area-yield crop insurance does not contain a deductible but insures only against systemic risk. If the deductible in the individual-yield insurance is large enough, area-yield insurance would reduce risk more effectively than individual-yield insurance. Miranda demonstrates this possibility empirically.

The LAM is tractable and delivers clear predictions about the design of optimal insurance and its effectiveness in reducing producer risk. However, several fundamental questions remain. Although the properties of optimal insurance depend on the LAM betas, the LAM itself says nothing about how the betas are determined. In terms of their individual characteristics, why might some producers have higher betas than others? An even more basic question is: Why should individual yields be related to area yields linearly as in the LAM?

Chambers and Quiggin have criticized the LAM because it models yield as a stochastic variable not subject to control by the producer. As they correctly point out, this makes the LAM inappropriate for investigating producer response to area-yield insurance. However, we will show that if the LAM is derived from the aggregation of individual technologies, then its parameters can be seen to be functions of individual choice variables. The criticism of Chambers and Quiggin will then no longer apply.

A Structural Model of Systemic and Nonsystemic Risks

In this section, we derive the LAM from a description of individual production technologies. As these are the primitives, the specification of production environments constitutes a structural model.

Consider a region R where there are n producers. Producer i's yield y_i, is given by

$$(5) \qquad y_i = \mu_i \eta_i$$

where μ_i is producer i's mean yield and η_i is a unit mean random variable capturing the risks of farming. Equation (5) is a standard specification of stochastic technologies where risks are multiplicative to mean yields. The mean yield μ_i is a function of inputs controlled by the producer. However, we purposely leave the functional form of this relationship unspecified. η_i is a linear combination of two independent shocks and is given by

$$(6) \qquad \eta_i = \alpha e_i + \gamma\theta$$

where e_i is a shock specific to i and θ is a shock common to all producers in region R. We, therefore, refer to e_i as the nonsystemic or individual risk and θ as the systemic or aggregate risk. The individual and aggregate risks satisfy the following properties: $E(\theta) = 1$, $E(e_i) = 1$, $\mathrm{Cov}(e_i\,\theta) = 0$ for all i, and $\mathrm{Cov}(e_i e_j) = 0$ for all $i \neq j$. To ensure the composite risk η_i has unit mean, we impose the restriction $(\alpha + \gamma) = 1$. Individual yields are, therefore,

$$(7) \qquad y_i = \mu_i(\gamma\theta + \alpha e_i).$$

We also assume that individual risks are independent of mean yields, that is, $E(e_i|\mu_i) = E(e_i)$.

This completes the description of the structural model. In this model, the composite risk is multiplicative to mean yields and its components are additive. We, therefore, call it a model with multiplicative risks and additive components (MRAC). Our goal is to discover whether the MRAC model can be represented as a LAM. If so, how do the parameters of the LAM (β_i and $\mathrm{Var}(\varepsilon_i)$) depend on the microparameters of the structural model? The answers are not obvious.

The area yield for the region R is

$$y = \sum_i w_i y_i = \left[\gamma\theta\left(\sum_i w_i\mu_i\right) + \alpha\sum_i (w_i\mu_i e_i)\right]$$

where w_i denotes the area share of the ith producer. Let μ denote the mean area yield (i.e., average of the mean yields of producers). Then, $\mu = \sum_i w_i\mu_i$ and

$$(8) \qquad y = \gamma\theta\mu + \alpha\sum_i (\mu_i w_i e_i).$$

Now decompose $\sum_i (\mu_i w_i e_i)$ as

$$(9) \qquad \sum_i (\mu_i w_i e_i) = \sum_i w_i(\mu_i - \mu)(e_i - \bar{e}) + \mu\bar{e}$$

where $\bar{e} = \sum_i w_i e_i$ is the area average of individual risks. Note that the first term on the right-hand side of (9) is the (weighted) sample covariance between mean yields and individual risk. If the region contains a large number of producers, and if the law of large numbers applies, the sample covariance will approach (in probability) the population covariance (assumed to be zero). Similarly, $\bar{e}$ in large samples will be close to $E(e_i)$.

When $w_i = (1/n)$, it is straightforward to use the law of large numbers to obtain large sample results. In the case of weighted averages, however, a restriction on the weights is necessary. Essentially, we need to assume that the average yield is not dominated by the yield of any single producer. This requirement is automatically satisfied by the unweighted sum but needs to be explicitly assumed in the case of weighted sums.[2] Assuming this condition to be satisfied, we use large sample approximations to obtain

$$(10) \quad \sum_i (\mu_i w_i e_i) = \mathrm{Cov}(\mu_i, e_i) + \mu E(e_i) = \mu.$$

Substituting in (8), area yield is

$$(11) \quad y = [\gamma\theta + \alpha]\mu.$$

Thus, area yield is random, only because of aggregate systemic shocks as individual risks cancel out in the aggregate. Since area yield is a monotonic function of θ, the inverse function exists and is given by

$$\theta = [y - \mu\alpha]/\mu\gamma.$$

Substituting for θ in (7), we obtain producer yield as a function of area yield, that is,

$$(12) \quad y_i = (\mu_i/\mu)(y - \mu\alpha) + \mu_i\alpha e_i \quad \text{or}$$
$$y_i = \mu_i + (\mu_i/\mu)(y - \mu) + \mu_i\alpha(e_i - 1)$$

which is identical to the LAM in (1) if we denote $(\mu_i/\mu) = \beta_i$ and $\mu_i\alpha(e_i - 1) = \varepsilon_i$. Hence we have the following result.

PROPOSITION 1. *In the MRAC model described by equations (5)–(7), the relationship between individual yield and area yield follows a LAM. The LAM parameters are related to the structural parameters in the following manner: (a)* $\beta_i = (\mu_i/\mu)$ *and (b)* $\varepsilon_i = \mu_i\alpha(e_i - 1)$.

From part (a), we see that for any individual producer the β parameter is the ratio of a producer's mean yield to the mean of area yield. It follows immediately that $\sum_i w_i\beta_i = 1$. Miranda noted this result earlier. From part (b), we see that the error term in the linear projection of individual yield on area yield is heteroscedastic. In particular, $\mathrm{Var}(\varepsilon_i) = \mu_i^2\alpha^2\sigma_e^2$, which varies across producers even if the nonsystemic risk in the structural model is homoscedastic.

In an empirical analysis of 102 cotton farms in Kentucky, Miranda observed that the distribution of the empirical betas possesses a regular, bell shape centered on one. We now know the conditions under which this result obtains. Proposition 1 says that this property is inherited from the distribution of average yields. Since the distribution of average yields depends on the dispersion of soil and climatic conditions in the region, Proposition 1 provides the formal basis for Miranda's conjecture that "…the more homogenous are the soil and climatic conditions faced by producers in a given area, the more closely the β_is will cluster around one." (p. 236). To this, we can also add that the dispersion of betas will depend on the heterogeneity in the other factors that determine yield, such as management practices, farming skills and capital assets. In the extreme when all farmers have the same mean yield, they will also have betas identically equal to 1. As mean yield depends on input application and technology, production decisions affect the beta parameter and the disturbance term of the LAM. Proposition 1 thus provides the basis for using the LAM to investigate producer behavior in the presence of area-yield insurance.[3]

Recall that the structural model leaves unspecified (*a*) the functional form of the relationship between input application and mean yield and (*b*) the probability density of the systemic and nonsystemic risks. Interestingly, the LAM is surprisingly general, as its parameters are, therefore, independent of assumptions about them.

Systemic Risks, Nonsystemic Risks, and Aggregation

A design problem is the selection of the area that should be used as the basis for computing

[2] Consider $\sum_i a_i x_i$ where x_i is i.i.d. with mean μ and $\sum_i a_i = 1$. Then $E(\sum_i a_i x_i) = \mu$. By Chebychev's inequality, given any $\delta > 0$, $\mathrm{Prob}[|\sum_i a_i x_i - \mu| > \delta] \le (\mathrm{Var}(x_i)/\delta^2)\sum_i a_i^2$, the limit of which tends to zero as long as for every n, there exists a bound c such that $a_i \le c$ and $c(n) \to 0$ for large n.

[3] This issue was brought to the fore by Chambers and Quiggin who examined it within a state-contingent model.

area yields. To maximize correlation of producer yield with area yield, it has been suggested that "the area or zone boundaries for an area yield contract should be selected so as to group together the largest possible number of farms with similar soils and climate" (Skees, Black, and Barnett). Can this recommendation be evaluated using the LAM, or do we need to turn to an underlying structural model?

Suppose producer yields can be averaged at two levels of aggregation.[4] For convenience, call the smaller aggregation a cluster and the larger aggregation a county. Under the LAM, producer yields are related to cluster yields in the following manner:

$$(13) \quad y_{ick} = \mu_{ick} + \beta^1_{ick}(y_{ck} - \mu_{ck}) + \varepsilon^1_{ick}$$

where y_{ick} and μ_{ick} are the individual yield and its expected value of producer i in cluster c of county k, y_{ck} and μ_{ck} are the area yield and its expected value of cluster c in county k, β^1_{ick} is the slope coefficient and ε^1_{ick} is a shock specific to producer i in cluster c of county k. Similarly, by applying the LAM to aggregation at the county level we obtain

$$(14) \quad y_{ick} = \mu_{ick} + \beta^2_{ick}(y_k - \mu_k) + \varepsilon^2_{ick}$$

where y_k and μ_k are the area yield and its expected value of county k, β^2_{ick} is the slope coefficient and ε^2_{ick} is a shock specific to producer i in cluster c of county k. Note that the slope coefficient, as well as the individual-specific risk are shown to vary with the level of aggregation.

From (4), we know that when area-yield insurance is optimal, the producer bears only the nonsystemic risk. Thus, in the case of cluster-yield insurance, the variability of producer's profit is $\mathrm{Var}(\varepsilon^1_{ick})$ while it is $\mathrm{Var}(\varepsilon^2_{ick})$ in the case of county-yield insurance. When is variability lower? The LAM cannot answer this question because it does not show how the nonsystemic risk is determined. For this, we have to turn to the underlying structural model.

Consider a variant of the MRAC model of the previous section. Yield of producer i in cluster c of county k is given by

$$(15) \quad y_{ick} = \mu_{ick}\eta_{ick} \quad \text{where}$$
$$\eta_{ick} = \alpha_1 e_{ick} + \alpha_2\theta_{1ck} + \alpha_3\theta_{2k}$$

where e_{ick} is a shock specific to i, θ_{1ck} is a shock specific to all producers in cluster c of county k, and θ_{2k} is a shock common to all producers in county k. In other words, e_{ick} is the individual risk, θ_{1ck} is the cluster-specific risk and θ_{2k} is the county-specific risk. The risks have unit means and constant variances, and are stochastically independent. Also assume $\sum \alpha_i = 1$. This ensures the mean of y_{ick} is μ_{ick}. The individual risk e_{ick} is distributed independently of the individual mean yield μ_{ick}.

The average yield of cluster c in county k can be calculated as

$$\sum_{i\in c} w_{ick}y_{ick} = \alpha_1 \sum_{i\in c} w_{ick}\mu_{ick}e_{ick} + (\alpha_2\theta_{1ck} + \alpha_3\theta_{2k})\sum_{i\in c} w_{ick}\mu_{ick}$$

where w_{ick} is the share of the ith producer in the area of cluster c. Denote cluster c's yield as y_{ck} and its mean as μ_{ck}. By arguments similar to that in the preceding section, substitute $\sum_i w_{ick}\mu_{ick}e_{ick}$ by its large sample approximation μ_{ck}. Hence

$$(15) \quad y_{ck} = [\alpha_1 + \alpha_2\theta_{1ck} + \alpha_3\theta_{2k}]\mu_{ck}.$$

Thus, cluster yields are random because of cluster-specific risk and county-specific risk. Area-yield insurance schemes at the cluster level would, therefore, offer protection against both these risks. Write $\theta_k = (\alpha_2\theta_{1ck} + \alpha_3\theta_{2k})$. θ_k denotes the systemic risk at the cluster level. Hence, for the cluster-yield insurance scheme, we can write the equations of the structural model as

$$(16) \quad y_{ick} = \mu_{ick}\eta_{ick} = \mu_{ick}(\alpha_1 e_{ick} + \theta_k) \quad \text{and}$$

$$(17) \quad y_{ck} = (\alpha_1 + \theta_k)\mu_{ck}.$$

By Proposition 1, the relationship between individual and cluster yields follows a LAM as in (13). Furthermore, the beta of an individual producer can be computed as $\beta^1_{ick} = \mu_{ick}/\mu_{ck}$. By the same proposition, the disturbance term in the LAM model is $\varepsilon^1_{ick} = \alpha_1\mu_{ick}(e_{ick} - 1)$. Hence, for a producer with cluster-yield insurance, the variance of profits is $\mathrm{Var}(\varepsilon^1_{ick}) = (\alpha_1\mu_{ick})^2\mathrm{Var}(e_{ick})$. The reduction in variance due to cluster-yield insurance is, therefore, $\mu^2_{ick}\mathrm{Var}(\eta_{ick}) - \mathrm{Var}(\varepsilon^1_{ick}) = \mathrm{Var}(\theta_k)$.

Consider next area-yield insurance schemes where the indemnity is contingent on county yield rather than cluster yield. The average yield of county k can be calculated by using

[4] Extension to many levels is straightforward.

(15) to average across clusters within the county. Hence

$$\sum_c w_{ck} y_{ck} = \alpha_1 \sum_c w_{ck} \mu_{ck} + \alpha_2 \sum_c w_{ck} \theta_{1ck} \mu_{ck} + \alpha_3 \theta_{2k} \sum_c w_{ck} \mu_{ck}$$

where w_{ck} is the share of cluster c in area of county k. Denote y_k to be county yield and μ_k to be its mean. Because θ_{1ck} is a cluster-specific risk, averaging across clusters should lead this risk to be approximately equal to its expected value. Using this approximation and arguments similar to that in equations (8)–(10), $\sum_c w_{ck} \theta_{1ck} \mu_{ck} = \mu_k$. Substituting this,

$$y_k = (\alpha_1 + \alpha_2 + \alpha_3 \theta_{2k}) \mu_k.$$

Denoting $\alpha_1 + \alpha_2$ as α, and $\alpha_1 e_i + \alpha_2 \theta_{1ck}$ as v_{ick}, the structural equations for the county-yield insurance scheme are

$$(18) \quad y_{ick} = \mu_{ick}(v_{ick} + \alpha_3 \theta_{2k}) \quad \text{and}$$

$$(19) \quad y_k = (\alpha + \alpha_3 \theta_{2k}) \mu_k.$$

Now, compare (16) and (18). At the county level, the systemic risk is θ_{2k} while it is θ_k at the cluster level. The nonsystemic individual-specific risk changes too. At the county level, what is measured as the nonsystemic risk is $\alpha_1 e_{ick} + \alpha_2 \theta_{1ck}$, while it is $\alpha_1 e_{ick}$ at the cluster level. Interestingly, higher aggregation reduces systemic risk and increases nonsystemic individual-specific risk. In the extreme, averages at the level of nation or group of nations may be so stable that the systemic risk component of a producer's yield might be close to zero. In such a case, all producer risk would be nonsystemic individual-specific risk.

Applying Proposition 1, the relation between individual and county yields can be represented as a LAM as in (14). Furthermore, $\beta^2_{ick} = \mu_{ick}/\mu_k$ and the disturbance term is $\varepsilon^2_{ick} = \mu_{ick}(v_{ick} - \alpha)$. It follows that for a producer with optimal county-yield insurance, the variability in profits would be $\text{Var}(\varepsilon^2_{ick}) = \mu^2_{ick}\text{Var}(v_{ick}) = (\alpha_1 \mu_{ick})^2 \text{Var}(e_{ick}) + (\alpha_2 \mu_{ick})^2 \text{Var}(\theta_{1ck})$. Consequently, the reduction in variance due to county-yield insurance schemes is $\text{Var}(\eta_i) - \text{Var}(\varepsilon^2_{ick}) = \mu_i^2 \alpha_3^2 \text{Var}(\theta_{2k})$.

Compared with the reduction achieved by cluster-yield insurance, we see that the cluster-yield insurance achieves an additional variance reduction of $\text{Var}(\mu_{ick}\alpha_2\theta_{1ck})$. This happens because, while θ_{1ck} is a systemic risk at the cluster level, it becomes a nonsystemic risk at the county level and is, therefore, not insured by the county-yield insurance scheme.[5] It is now clear that the division of producer risk into systemic and nonsystemic risks is dependent on the level of aggregation. The higher the level of aggregation, the greater are individual risks, and the smaller are systemic risks. Correspondingly, the risk reduction impacts of area-yield insurance would also be smaller.

Skees, Black, and Barnett are right in emphasizing that farms with similar soils and climate should be grouped together. In terms of the structural model, such a grouping would face risks that do not cancel out in the aggregate, and hence qualify as systemic risks. However, what our analysis has pointed out is that more risks are likely to survive aggregation (and hence be regarded as systemic) when the farmer groups are small. Hence, for area-yield insurance to have the maximum impact on risk reduction, the area boundaries for an area-yield contract should be selected so as to group together the *smallest* (and not the largest) number of farms with similar soils and climate. However, we now face the problem that large sample approximations will fail in small aggregations. The implications of this failure are investigated in a later section where we show that, fortunately, a modified linear model emerges. More importantly, none of the results on optimal insurance are affected.

A General Structural Model

The earlier sections presented a structural model that led to the LAM used in evaluations of area-yield insurance. But there might be other structural models as well, which imply a LAM. What are they? Conversely, what are instances of structural models that do not imply a LAM?

Some examples of popular specifications other than the MRAC model are the following:

(*a*) Model of additive risks with additive components (ARAC): $y_i = \mu_i + e_i + \theta$.
(*b*) The Just-Pope model with additive components (JPAC): $y_i = \mu_i + \sigma_i (\theta + e_i)$.

[5] It is easy to show that cluster yields are more correlated with producer yields than county yields.

(*c*) Model of multiplicative risks with multiplicative components (MRMC): $y_i = \mu_i e_i \theta$.

Do any or all of these models imply the LAM? To answer this, we characterize the entire class of structural models that imply the LAM. Suppose a general structural model of the form

$$y_i = f(z_i, e_i, \theta)$$

where, as before, e_i and θ are the random realizations of individual risk and aggregate shock and f is a function that maps the individual risk, the aggregate shock and a vector of parameters $\mathbf{z}$ into realized yields. In the MRAC model, $\mathbf{z}_i$ consisted of a single parameter μ_i, the ith producer's mean yield. Suppressing $\mathbf{z}_i$, we can write the model as

$$(20) \quad y_i = f_i(e_i, \theta)$$

where the function f_i is now specific to producer i.

If the relationship between individual yield and area yield is linear as in a LAM, then what restrictions must the function f_i satisfy?

PROPOSITION 2. *If the relationship between individual and area yields is described by a LAM as in* (1), *the structural model* (20) *necessarily satisfies the following. (a) For all i,* $y_i = f_i(e_i, \theta) = h_i(e_i) + g_i(\theta)$, *where h_i and g_i are functions that map nonsystemic shocks and systemic shocks, respectively, into individual yields. (b) For all i, there exists a function $k(\theta)$ and a parameter λ_i such that $g_i(\theta) = \lambda_i k(\theta) + c_i$, where c_i is a constant of integration.*

Proof: The structural model (20) satisfies

$$(\partial y_i/\partial e_i) = (\partial y_i/\partial \varepsilon_i)(\partial \varepsilon_i/\partial e_i).$$

But from (1), $\partial y_i/\partial \varepsilon_i = 1$. Hence

$$(\partial y_i/\partial e_i) = (\partial \varepsilon_i/\partial e_i).$$

Recall that the LAM splits the variation in individual yields into variation in area yield y and an individual-specific risk ε_i. By assumption, y and ε_i are orthogonal. It follows that area yield y is a function of θ alone, while ε_i is a function of e_i alone. Hence

$$\left(\partial^2 y_i / \partial e_i \partial \theta\right) = \left(\partial^2 \varepsilon_i / \partial e_i \partial \theta\right) = 0$$

that is, the cross-partial derivatives of (20) are zero. Since this can be true only if (20) is additive in the two risks, we have the result in part (*a*).

We now turn to the proof of part (*b*) of Proposition 2. Define the parameter $\delta_i = \partial y_i/\partial \theta$. δ_i measures the sensitivity of producer i's yield to aggregate shocks. Also define δ as the sensitivity of area yield to aggregate shocks, that is, $\delta = \partial y/\partial \theta$. Since $\partial y/\partial \theta = \sum w_i(\partial y_i/\partial \theta)$, we have $\delta = \sum_{i=1}^n w_i \delta_i$. Now

$$(21) \quad \delta_i = \partial y_i/\partial \theta = (\partial y_i/\partial y)(\partial y/\partial \theta) = \delta(\partial y_i/\partial y).$$

Hence, for all i

$$(22) \quad \partial y_i/\partial y = \frac{\delta_i}{\delta}.$$

Fix a producer j and define, for all i, $\lambda_i = (\partial y_i/\partial y)/(\partial y_j/\partial y)$. Clearly λ_j is 1. Using (22) we obtain $\delta_i = \lambda_i \delta_j$. Using part (a) of Proposition 4, this can be written as

$$(23) \quad \partial g_i/\partial \theta = \lambda_i(\partial g_j/\partial \theta).$$

λ_i does not vary with the aggregate shock θ. This can be seen from the LAM in equation (1), where for all i, $\partial y_i/\partial y$ is a parameter that is independent of the realization of θ. Integrating both sides of (22) with respect to θ, we therefore find that, for all i, the structural model satisfies $g_i(\theta) = \lambda_i g_j(\theta) + c_i$ where c_i is a constant of integration that varies with i. Since j is arbitrarily chosen, we define $k(\theta)$ to be $g_j(\theta)$. This proves part (*b*). ■

Proposition 2 specifies the class of structural models implied by the LAM. Notice that the LAM does not restrict the way in which risks affect production. However, the LAM does require that either the components of risk or their effects on production be additive. As a result, the model of MRMC does not satisfy the necessary conditions identified in Proposition 2. We have the important result that the LAM is inappropriate in this case. However, the ARAC and JPAC structural models meet the conditions of Proposition 1 and are, therefore, not inconsistent with a LAM. The next result considers the converse relationship: does every member of the class identified in Proposition 2 imply the LAM? The answer is yes, provided the aggregation is large enough.

PROPOSITION 3. *The structural model in (20) implies a LAM if (a) the area-weighted average of individual risks can be replaced by its large* *sample equivalent of population average and if (b) the structural model satisfies*

$$(24) \quad y_i = f_i(\theta, e_i) = a_i + b_i k(\theta) + h_i(e_i)$$

where $k(\theta)$ and $h_i(e_i)$ are monotone functions, a_i and b_i are parameters that possibly vary with i.

Proof: From (24), mean producer yield is

$$(25) \quad \mu_i = a_i + b_i E[k(\theta)] + E[h_i(e_i)].$$

Adding and subtracting μ_i to the right-hand side of (24), and using (25), we get

$$(26) \quad y_i = \mu_i + b_i[k(\theta) - Ek(\theta)] + [h_i(e_i) - E[h_i(e_i)]].$$

Now using (24), area yield is

$$(27) \quad y = a + bk(\theta) + \sum w_i h_i(e_i)$$

where $a = \sum w_i a_i$ and $b = \sum w_i b_i$. Using the weak law of large numbers, $\sum_i w_i h_i(e_i)$ can be approximated in large samples by $\sum_i w_i E[h_i(e_i)]$.[6] Hence

$$(28) \quad y(\theta) = a + bk(\theta) + \sum w_i E[h_i(e_i)].$$

Mean area yield is, therefore,

$$(29) \quad \mu = a + bEk(\theta) + \sum w_i E[h_i(e_i)].$$

From (28) and (29), $y - \mu = b[k(\theta) - Ek(\theta)]$. Substituting in (26) and defining, $(b_i/b) = \beta_i$ and $(h_i(e_i) - Eh_i(e_i)) = \varepsilon_i$, we get

$$y_i = \mu_i + \beta_i(y - \mu) + \varepsilon_i$$

where ε_i is a mean zero random variable uncorrelated with area yield. ■

The above proof also derives the relationship of the structural parameters to the parameters of the LAM model. As it is useful to identify this result separately, we have the next proposition.

PROPOSITION 4. *In the general structural model that is equivalent to the LAM, the parameters satisfy (a) $b_i/b = \beta_i$ and (b) $h_i(e_i) - Eh_i(e_i) = \varepsilon_i$.*

Two implications of Proposition 4 are worthy of special mention. First, b_i measures the sensitivity of producer *i*'s yield to aggregate shocks while b is the sensitivity of area yield to aggregate shocks. Part (*a*) of Proposition 4, therefore, states that β_i, the sensitivity of producer *i*'s yield to area yield, is that producer's sensitivity to aggregate shocks relative to the sensitivity of area yield to aggregate shocks. Second, recall that when area-yield insurance is optimal, the producer bears only the risk ε_i. From part (*b*) of Proposition 4, it can be seen, therefore, that, with optimal area-yield insurance, the variability of producer profits is $\mathrm{Var}(h_i(e_i))$.

Given Proposition 4, it is easy to compute the betas for special cases of the general structural model. We consider a few specifications that were mentioned at the beginning of this section.

(i) MRAC: $y_i = \mu_i(\gamma\theta + \alpha e_i)$.

This is the multiplicative specification considered earlier. It is additive in the interaction of systemic and nonsystemic shocks. Fix any *j* and define $k(\theta) = \mu_j\gamma\theta$. Define $b_i = (\mu_i/\mu_j)$ and $h_i(e_i) = \mu_i\alpha e_i$. Then individual yields can be written as $y_i = b_i k(\theta) + h_i(e_i)$, which is a special case of the structural model (24). Here, $b = \mu/\mu_j$. Applying Proposition 3, we compute β_i as μ_i/μ.

(ii) ARAC: $y_i = \mu_i + e_i + \theta$.

In this specification, risks are additive to mean yield. It clearly satisfies (24). Here $k(\theta) = \theta$, $b_i = 1$ and so $b = 1$. Hence $\beta_i = 1$ for all *i*. Note this result obtains even though producers are heterogeneous in mean yields. We can now see that what is important for heterogeneity in betas is heterogeneity in the way the aggregate shock affects mean yields.

(iii) JPAC: $y_i = \mu_i + \sigma_i\,(\theta + e_i)$.

This is the specification of a stochastic production function due to Just and Pope. This is also a special case of (24) where $k(\theta) = \theta$, $b_i = \sigma_i$, and, therefore, $b = \sigma$ where $\sigma = \sum_i w_i\sigma_i$. Therefore $\beta_i = \sigma_i/\sigma$.

Small Aggregations

The results following Proposition 1 point to the fact that a LAM is a consequence of additive

[6] In the MRAC model, $h_i(e_i) = \mu_i e_i$. Hence in large samples, $\sum w_i h_i$ converges in probability to $\sum w_i E(h_i) = \sum w_i \mu_i E(e_i) = \sum w_i \mu_i = \mu$, given the assumption $E(e_i) = 1$ for all *i*.

interaction of systemic and nonsystemic risks. However, while such structure of risks is necessary, it is not sufficient to ensure a LAM with conventional properties. Some structure is also required on the extent of aggregation. For this reason, Proposition 4 assumed it was valid to use large sample approximations. What if this assumption was seriously violated? What would be the relation between individual yield and area yield in small aggregations?

Suppose the structural model satisfies (24). The question is interesting only for this case because we already know that a LAM does not obtain otherwise. Given (24), equations (25)–(27) are immediate consequences and their derivation does not involve large sample approximations.

Using (27), mean area yield is

$$(30) \quad \mu = a + bEk(\theta) + \sum w_i E[h_i(e_i)].$$

From (27) and (30), we can solve for $[k(\theta) - Ek(\theta)]$ as

$$k(\theta) - Ek(\theta) = (y - \mu)/b - A/b$$

where $A = \sum w_i h_i(e_i) - \sum w_i E[h_i(e_i)]$. If the aggregation is large, the difference A could be approximated as zero by Chebychev's law of large numbers. But otherwise, it is a nonzero random variable. Substituting for $[k(\theta) - Ek(\theta)]$ in (27),

$$y_i = \mu_i - \beta_i A + \beta_i(y - \mu) + \varepsilon_i$$

where we have used the definitions $(b_i/b) = \beta_i$ and $(h_i(e_i) - Eh_i(e_i)) = \varepsilon_i$. Separating out the quantity A into its stochastic and nonstochastic components and rearranging terms, we obtain

$$y_i = \left(\mu_i + \beta_i \sum w_i E(h_i(e_i))\right. \\ \left. + \beta_i(y - \mu) + \left(\varepsilon_i - \sum w_i h_i(e_i)\right)\right).$$

Letting $\phi_i = (\mu_i + \beta_i \sum w_i E(h_i(e_i))$ and $v_i = (\varepsilon_i - \sum w_i h_i(e_i)))$, we get

$$(31) \quad y_i = \phi_i + \beta_i(y - \mu) + v_i.$$

Surprisingly, a linear relation between producer yield and area yield obtains once again. However, in other respects, the properties of (31) are different from (1). First, the intercept term is no longer the mean producer yield. Second, the error term is no longer uncorrelated across producers even when individual risks are uncorrelated. This happens because of the common random component $\sum w_i h_i(e_i)$ in each of the $v_i s$. $\sum w_i h_i(e_i)$ is nothing but the area average of individual risks. In small aggregations, this is no longer equal to the population average but is a random quantity. As the area average y is also a function of $\sum w_i h_i(e_i)$, the error term v_i is correlated with y. The important implication of this result is that, if the betas are estimated by an ordinary least squares regression, they are inconsistent. In particular, since v_i is negatively correlated with y,

$$\text{plim}\left(\hat{\beta}_i^{\text{OLS}}\right) = \beta_i - \beta_i \text{Cov}(y, v_i)/\text{Var}(y_i) \\ > \beta_i.$$

Even though (31) does not have the properties of a conventional LAM, it is easy to show that the results of earlier work will continue to hold. In particular, the slope of the optimal indemnity schedule will be $-\beta_i$ and such insurance will eliminate the systemic risk component of a producer's risk.

Multiplicative Components

As noted earlier, a structural model with multiplicative components cannot be represented as a LAM. But does that make a difference to the results of Miranda and Mahul? Suppose, for a given level of aggregation, individual yields are described by

$$(32) \quad y_i = \mu_i \eta_i \quad \text{and} \quad \eta_i = e_i \theta$$

where the variables continue to have the same meaning and properties as before. Such a specification is natural whenever the yield impacts of one risk depend on the realization of the other risk as well. For instance, even with a positive systemic shock due to timely rainfall, the impact on an individual producer's yield might be negligible because of a local risk, such as pest or fungal infestation. Conversely, very adverse aggregate shocks could nullify a good outcome in terms of local risks. Unfortunately, in an additive structure, the impact of rainfall is invariant to local risks and vice versa.[7]

To see how the multiplicative structure makes a difference, we compare it with the MRAC model. The results of Miranda and Mahul apply to the MRAC model and, therefore, we know that the slope of the optimal

[7] For an analysis of multiplicative structures arising from the interaction of price and quantity risks, see Ramaswami and Roe.

indemnity schedule is $-\beta_i = (\mu_i/\mu)$ and that it is invariant to the nonsystemic risk and its moments. Furthermore, with the optimal amount of insurance, all systemic risk is eliminated. To see whether these results extend to the MRMC model, it is necessary to directly analyze the structural form (32) as the LAM is unavailable.

The area yield associated with (32) is $y = \theta \sum_i \mu_i e_i$. By using large sample approximations, we can express area yield as

$$(33)\quad y = \mu\theta.$$

Substituting in (32),

$$(34)\quad y_i = (\mu_i/\mu)ye_i = \beta_i y e_i$$

where we have denoted (μ_i/μ) by β_i. Notice that, when the nonsystemic risk is absent and is equal to its expected value 1, (34) is identical to a LAM (without nonsystemic risk and with zero intercept). From the results that apply to a LAM, we therefore have that, the insurance schedule satisfies $I'(y) = -\beta_i$ whenever there is no nonsystemic risk. Now suppose e_i is a random variable that takes values other than one with nonzero probability. We can write producer i's revenue with insurance as

$$\pi_i = y_i + I(y) - P = \beta_i y e_i + I(y) - P.$$

An actuarially fair optimal insurance contract maximizes expected utility of producer i subject to the break-even constraint of the insurers. Hence it solves

$$(35)\quad \underset{I(y)}{\text{Max}} \int_{e_i}\int_y U(\pi_i)\, dG(y)\, dF(e_i)$$
$$\text{subject to} \qquad P = \int_y I(y)\, dG(y)$$

where U is an increasing, concave and thrice differentiable utility function, F is the cumulative density of the nonsystemic shock, and G is the cumulative density of area yield derived from the probability distribution of the systemic shock θ (from (33)). Note that since area yield is a function of θ alone, it is distributed independently of the nonsystemic risk.

Let λ be the Lagrange multiplier associated with the break-even constraint. Then the optimal function $I(y)$ satisfies for every y

$$(36)\quad \int_{e_i} U'(\pi_i) f(y)\, dG(e_i) = \lambda f(y)$$

where $f(y) = dF(y)/dy$. Clearly (36) can also be written as

$$E[U'(\pi_i)|y] = \lambda$$

that is, the optimal insurance equalizes the expected marginal utility in every state of area yield, y. Differentiating the first order condition with respect to y,

$$E[U'(\pi_i)(\beta e_i + I'(y))] = 0$$

from which we can solve for the slope of the indemnity schedule as

$$(37)\quad I'(y) = -\beta_i\left[1 + \frac{\text{Cov}(U''(\pi), e_i)}{EU''(\pi)}\right].$$

$EU'' < 0$ and so the sign of $\frac{\text{Cov}(U''(\pi), e_i)}{EU''(\pi)}$ is opposite to the sign of the covariance term. Since $\partial(U''(\pi_i)/\partial e_i) = U'''(\pi_i)\beta_i y$, the covariance term is positive, equal to zero or negative as U''' is positive, zero or negative. A risk-averse agent with a positive third derivative of utility function has been referred to as prudent (Kimball). It is easy to show that an agent with nonincreasing risk aversion must be prudent. U''' is zero for an agent with a quadratic utility function. Since constant or decreasing risk aversion is a reasonable restriction on risk-averse behavior, we concentrate below on the case when $U''' > 0$.

PROPOSITION 5. *If systemic and nonsystemic risks interact multiplicatively, the optimal insurance for a prudent producer satisfies* $-I'(y) < \beta_i$.

The proof is immediate from (37). Recall, that when nonsystemic risk is absent, $-I'(y) = \beta_i$. This can also be seen directly from (37). Thus, we obtain the important result that in the presence of an uninsured nonsystemic risk, it is optimal for a producer to choose a lower level of coverage as compared to the case where nonsystemic risk is absent. This is unlike the additive case where the demand for insurance against the systemic risk is unaffected by nonsystemic risk.

To analyze local changes in risk, consider a one-term expansion of U'' as

$$U''(\pi) = U''(E(\pi)) + (\pi - E(\pi))U'''(E(\pi)) \quad \text{or}$$

$$U''(\pi) = U''(E(\pi)) + \beta_i y(e_i - 1)U'''(E(\pi)).$$

Substituting in (37),

$$(38)\quad I'(y) = -\beta_i \left[1 + \beta_i y \mathrm{Var}(e_i) \frac{U'''(E(\pi))}{U''(E(\pi))}\right].$$

The greater is the riskiness of the nonsystemic risk, the smaller is the optimal coverage for a prudent producer. The demand for area-yield insurance depends, therefore, on the uninsured nonsystemic risks faced by an individual producer. As seen earlier, the classification of risks as either systemic or nonsystemic changes with the area size used for computing area yields. In a multiplicative model, therefore, the demand for area-yield insurance will depend on the level of aggregation at which area yields are determined. Since higher aggregations increase nonsystemic risk, they thus reduce the demand for area-yield insurance.

To see this, denote I_1 and I_2 as the optimal insurance contracts at the cluster and county levels of aggregation. Suppose also that the mean yields of all producers are equal. Then $\beta_i = 1$, irrespective of the level of aggregation. In an additive model, the optimal coverage would satisfy $-I_1'(y_{ck}) = -I_2'(y_k) = 1$, where y_{ck} and y_k are cluster and county yields.

In a multiplicative model, individual yields, cluster yields, and county yields are given by $y_{ick} = \mu_{ick} e_{ick} \theta_{1ck} \theta_{2k}$, $y_{ck} = \mu_{ck} \theta_{1ck} \theta_{2k}$, and $y_k = \mu_k\ \theta_{2k}$. Hence the nonsystemic risk for cluster insurance is $e_{ick}\theta_{1ck}$, but is only e_{ick} for a county-yield insurance. The variance of nonsystemic risk is, therefore, greater with county-yield insurance. From Proposition 5 and (38), it follows that the optimal coverage for a prudent producer satisfies $1 > -I_1'(y_{ck}) > I_2'(y_k)$.

Conclusions

The LAM decomposes individual producer yield into a systemic component due to area yield variation and to an independent additive producer-specific component. While previous work has established its convenience for analyzing area-yield insurance, its theoretical justification has been neglected. In spite of its likeness to the CAPM model of finance, the LAM cannot be validated in a similar manner.

This article has derived the LAM from aggregation of microproduction functions. The basis for LAM rests on two conditions. First, the aggregation must be large enough that all individual risk is eliminated in the area aggregate. Second, in the individual production functions, the systemic and nonsystemic individual risk components must be additive.

Knowledge of the underlying "structural" model enables analysis of the factors that determine the parameters of the LAM model. This was used to examine the relation between producer risk and the level of aggregation. Other uses are possible, such as the analysis of the relation between area-yield insurance and production decisions.

To design insurance schemes, an analysis based on the law of large numbers can be misleading. We find that dropping the large numbers restriction alone does not alter the linear relationship between individual and area yield. Neither does it affect the central results that have been obtained using the LAM. The major outcome is that the decomposition now consists of two correlated risk components. As a result, it is not valid to estimate the beta parameter by ordinary least squares.

The consequences are more serious if the assumption of additive components is dropped. Then a LAM representation does not exist. Further, previous results obtained in the literature are not likely to be valid. This was shown for the important case of multiplicative components. In such a setup, area-yield insurance does not eliminate all systemic risk. Moreover, the demand for insurance is not independent of the nonsystemic risk. The greater the nonsystemic risk, the lower the demand for insurance. As a result, the demand for area-yield insurance varies with the level of aggregation unlike the case in the additive components model.

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