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Labor income and risky investments: can part-time farmers compete?

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Abstract

In the developed countries, a majority of farm households receive at least as much income from nonfarm sources as from the farm. Such part-time farms have survived inspite of lower returns than full-time farms. This paper considers when lower returns to part-time farming could be compensated by risk-reduction due to diversification of income sources. The paper uses a dynamic portfolio choice model with labor income. The model and results could be applied in other contexts as well.

JEL classification: G11; Q12

Keywords: Labor income; Off-farm income; Part-time farming; Portfolio choice

1. Introduction

Farmers are in small numbers in the developed world. The economic process that reduced the relative size of this sector is well studied. However, not all aspects of the structural change are well understood. While farming is still a predominantly family business with few hired managers, the involvement of the household has diminished. In the traditional production structure, the family pooled its labor and shared its fruits. On the other hand, farm households today receive a substantial part of their income from nonfarm sources such as wage and salary jobs, nonfarm businesses and professional services.

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In US, for example, income from off-farm sources accounted for 46% of the total income for farm households in 1986. In the same year, 55% of farms in a US Department of Agriculture survey reported off-farm earnings in excess of farm income (Ahearn and Lee, 1991). Some other studies documenting the importance of off-farm income are Fuller (1991, for Canada), Huffman (1991, for Canada and US), Weiss (1999, for Austria), and Rabinowicz (1992, for Sweden). If part-time farm households are defined as those whose off-farm earnings are greater than their farm incomes, then these studies find that at least 50% of farms are part-timers. The picture does not change if part-time farm households are defined on the basis of time spent in farming. In a study of off-farm employment in upper Austria, Weiss (1997) estimates that at more than 50% of farms, the husband and wife work less than 50% of their working time on the farm.

These findings are surprising because of the general presumption, confirmed by various studies, that full-time farm operations are more efficient than part-time farms (Bollman, 1991; Cochrane, 1987; Fuller, 1991; Tweeten, 1991). Full-time operations have the advantage of being able to use scale efficient technology and have lower costs of credit. This led Cochrane to comment, "... most (part-time farms) are going to bite the dust ... cannibalized by their larger, aggressive, innovative neighbors". Yet, there is little evidence that this is happening. Instead, studies report the vanishing of the mid-sized farms as the size structure of farms settles to a bi-modal distribution where farms are either large full-time operations or small part-time activities (Weiss, 1999).

While off-farm employment has been viewed as a means of maintaining parity of incomes between farm households and nonfarm households (Gardner, 1992), this is not an explanation for part-time farming. The lower rates of return to part-time farmers should lead to their exit. So how can off-farm income make part-time farming viable? In this paper, we pursue an explanation that lower returns to part-time farming are compensated by risk-reduction due to diversification of income sources.

Imagine a farm operator household where the spouse is employed in an off-farm job. Suppose, for some reason, earnings from the off-farm job rise. What impact would this have on the household's choice of investments? In particular, would it ever lead to an increase in farm investment? To understand when this might be true, we consider a portfolio choice problem with labor income. The household receives a stochastic stream of labor income and can invest in a risk-free asset or a risky portfolio consisting of a farm asset and a nonfarm financial asset. The correlation structure between the three risky sources of revenue, i.e. the farm asset, the nonfarm financial asset and labor income is unrestricted except for the assumption that the three traded assets span labor income. We use a dynamic choice model rather than a static one because it enables us to consider wealth effects of changing levels of labor income.

In the finance literature, it is well known that labor income affects portfolio choice because of its correlation with asset returns. This insight has been developed to explain the relationship between portfolio choice and age structure (Jagannathan and Kocherlakota, 1996), the relationship between portfolio choice and labor supply flexibility (Bodie et al., 1992), the valuation of nontraded income (Svensson and Werner, 1993) and the extent of diversification between home and foreign assets (Baxter and Urban, 1997). The literature, however, does not contain any analysis about how portfolio shares and holdings vary with labor income, which is the central focus of this paper.

While our paper is motivated by the example of part-time farming, our model is applicable in other contexts where it is of interest to know whether labor income favors the holding of one kind of asset over another. We offer two examples. Consider the design of compensation schemes for managers in corporate settings. It is well known that one way to counter agency problems is to tie executive compensation to the value of the firm by means of stock options and performance contingent bonuses (Shleifer and Vishny, 1997). But would managers voluntarily want to hold the stock of their company? And how would such desired levels vary with labor income? It is important to answer these questions because if the stock ownership plans are not consistent with investor optimality, they will be costly to the company (which would have to price them low enough for manager investors to hold them). Another context in which labor income matters is in deciding the extent of portfolio diversification across domestic and foreign assets. If returns to human capital and domestic assets are highly correlated then investors should hold a higher proportion of foreign assets than would be predicted by a portfolio choice model without labor income. But how would such diversification depend on the earnings from human capital? Should high-income investors hold more or less of the foreign asset? What determines these calculations? Our paper sheds light on such questions.

2. A continuous time model of portfolio choice

A farm household has the choice of investing in a risk-free asset (yielding a rate of return R), in a farm asset (yielding a rate of return $q_{\mathrm{f}}(t)$) and in a nonfarm financial asset (yielding a rate of return $q_{\mathrm{p}}(t)$). The returns on the farm and the financial asset are risky and follow a stationary Ito process:

$$q_{\mathrm{f}}(t) = a_{\mathrm{f}}\,\mathrm{d}t + s_{\mathrm{f}}\,\mathrm{d}Z_{\mathrm{f}t} \tag{1}$$

$$q_{\mathrm{p}}(t) = a_{\mathrm{p}}\,\mathrm{d}t + s_{\mathrm{p}}\,\mathrm{d}Z_{\mathrm{p}t} \tag{2}$$

where a_{f} and a_{p} are the instantaneous expected rates of return on the farm and financial asset, s_{f} and s_{p} the instantaneous standard deviations of the return on farm and financial asset, respectively, Z_{f} and Z_{p} the standard Wiener processes and $\mathrm{d}Z_{\mathrm{f}}$ and $\mathrm{d}Z_{\mathrm{p}}$ their increments with instantaneous correlation K_{fp}. If E_t is the expectations operator at time t, then by definition, $E_t[\mathrm{d}Z_{\mathrm{f}t}] = E_t[\mathrm{d}Z_{\mathrm{p}t}] = 0$, $E_t[\mathrm{d}Z_{\mathrm{f}t}]^2 = E_t[\mathrm{d}Z_{\mathrm{p}t}]^2 = \mathrm{d}t$, and $E_t[\mathrm{d}Z_{\mathrm{f}t}\,\mathrm{d}Z_{\mathrm{p}t}] = K_{\mathrm{fp}}\,\mathrm{d}t$.

Since it will be convenient to use vector notation, write Eqs. (1) and (2) together as:

$$\boldsymbol{q} = \boldsymbol{a}\,\mathrm{d}t + \boldsymbol{s}\,\mathrm{d}\boldsymbol{Z} \tag{3}$$

where $\boldsymbol{q}$ is a 2×1 vector of asset returns, $\boldsymbol{a}$ a 2×1 vector of drift parameters, $\boldsymbol{s}$ a 2×2 diagonal matrix with standard deviations on the diagonal and $\mathrm{d}\boldsymbol{Z}$ is a 2×1 vector of increments of the Wiener processes associated with rates of return on the two assets. Let $\boldsymbol{\Sigma}$ be a 2×2 matrix representing the instantaneous covariance matrix of returns, i.e. $\boldsymbol{\Sigma}\,\mathrm{d}t = \boldsymbol{s}E_t(\mathrm{d}\boldsymbol{Z}\,\mathrm{d}\boldsymbol{Z}')\boldsymbol{s}$. Thus, the diagonal terms on the $\boldsymbol{\Sigma}$ matrix are s_{f}^2 and s_{p}^2 while the off-diagonal term is $s_{\mathrm{f}}s_{\mathrm{p}}K_{\mathrm{fp}}$. Let s_{fp} denote the covariance between the returns on the farm asset and the financial asset. Then by definition, $s_{\mathrm{fp}} = s_{\mathrm{f}}s_{\mathrm{p}}K_{\mathrm{fp}}$. $\boldsymbol{\Sigma}$ is assumed to be positive definite.

In addition to owning the financial asset and the farm asset, the farm household receives an instantaneously certain flow of off-farm income V. This income rate changes stochastically and continuously:

$$\mathrm{d}V(t) = \alpha V\,\mathrm{d}t + \sigma V\,\mathrm{d}Z_{\mathrm{v}t} \tag{4}$$

where α and σ are the mean and standard deviation of the growth rate in off-farm income over the interval dt, and dZ_{v} is the increment of a Wiener process. Therefore, $E_t(\mathrm{d}Z_{\mathrm{v}}) = 0$ and $E_t(\mathrm{d}Z_{\mathrm{v}})^2 = \mathrm{d}t$. Let the covariance structure between the processes d$\boldsymbol{Z}$ and dZ_{v} be given by $K_{\mathrm{fv}}\,\mathrm{d}t = E_t[\mathrm{d}Z_{\mathrm{f}t}\,\mathrm{d}Z_{\mathrm{v}t}]$ and $K_{\mathrm{pv}}\,\mathrm{d}t = E_t[\mathrm{d}Z_{\mathrm{p}t}\,\mathrm{d}Z_{\mathrm{v}t}]$. Accordingly, if $\boldsymbol{\Sigma}_{\mathrm{qv}}$ is a 2×1 vector containing the covariances between the risky assets and off-farm income, then, $\boldsymbol{\Sigma}_{\mathrm{qv}}\,\mathrm{d}t = \boldsymbol{s}E_t(\mathrm{d}\boldsymbol{Z}\,\mathrm{d}Z_{\mathrm{v}})\sigma V$. The elements of $\boldsymbol{\Sigma}_{\mathrm{qv}}$ vector are therefore $\sigma s_{\mathrm{f}} K_{\mathrm{fv}} V$ and $\sigma s_{\mathrm{p}} K_{\mathrm{pv}} V$.

The farm household's objective, subject to a budget constraint, is to maximize the discounted lifetime expected utility given by:

$$E_t \int_t^{\infty} \exp(-\delta x) U(C(x))\,\mathrm{d}x \tag{5}$$

where C is the rate of consumption, δ the rate of discount and U the utility function assumed to be strictly concave in C. Letting $W(t)$ be the household wealth at time t, the change in wealth over the period dt is given as follows:

$$\begin{aligned}\mathrm{d}W &= W(t+\mathrm{d}t) - W(t)\\ &= -C(t)\,\mathrm{d}t + [\omega_{\mathrm{f}}(W(t) - C(t)\,\mathrm{d}t)q_{\mathrm{f}}(t)] + [\omega_{\mathrm{p}}(W(t) - C(t)\,\mathrm{d}t)q_{\mathrm{p}}(t)]\\ &\quad + [(1-\omega_{\mathrm{f}}-\omega_{\mathrm{p}})(W(t) - C(t)\,\mathrm{d}t)R\,\mathrm{d}t] + [\mathrm{d}V(t) + V\,\mathrm{d}t]\end{aligned} \tag{6}$$

where ω_{f} and ω_{p} are the proportions of savings invested in the farm and financial asset, respectively. Netting out consumption, Eq. (6) expresses the change in wealth as the sum of: (a) the return on farm investment, (b) the return on financial investment, (c) the return on the risk-free investment, and (d) labor income. In vector notation, Eq. (6) becomes:

$$\mathrm{d}W = -C\,\mathrm{d}t + (W - C\,\mathrm{d}t)(R\,\mathrm{d}t + \boldsymbol{\Omega}'(\boldsymbol{q} - \mathbf{1}R\,\mathrm{d}t)) + \mathrm{d}V + V\,\mathrm{d}t \tag{7}$$

where $\boldsymbol{\Omega}$ is a 2×1 vector of portfolio allocations to the risky assets and $\mathbf{1}$ is a 2×1 vector of ones. Substituting for $\boldsymbol{q}$ and dV (from Eqs. (3) and (4)), the household's budget constraint over the period dt becomes:

$$\begin{aligned}\mathrm{d}W &= -C\,\mathrm{d}t + (W - C\,\mathrm{d}t)(R + \boldsymbol{\Omega}'(\boldsymbol{a} - R\mathbf{1})\,\mathrm{d}t + \boldsymbol{\Omega}'\boldsymbol{s}\,\mathrm{d}\boldsymbol{Z})\\ &\quad + (1+\alpha)V\,\mathrm{d}t + \sigma V\,\mathrm{d}Z_{\mathrm{v}}\end{aligned} \tag{8}$$

The investor's problem is to choose, given current levels of wealth and off-farm income, the consumption and portfolio paths that maximize Eq. (5) subject to Eq. (8). Let $J(W, V, t)$ be the corresponding indirect utility function, i.e.,

$$J(W, V, t) = \max\nolimits_{C,\Omega} E_t \int_t^{\infty} \exp(-\delta x) U(C(x))\,\mathrm{d}x$$

which can be written as follows:

$$J(W, V, t) = \max_{C,\Omega} E_t \times \left[\int_t^{t+\mathrm{d}t} \exp(-\delta x) U(C(x))\,\mathrm{d}x + J(W(t+\mathrm{d}t), V(t+\mathrm{d}t), t+\mathrm{d}t)\right] \quad (9)$$

By standard stochastic dynamic programming methods (Merton, 1971), we obtain the Hamilton–Jacobi–Bellman equation as follows:[1]

$$\begin{aligned} 0 = \max_{C,\Omega}[&\exp(-\delta t)U(C(t)) + J_t(W, V, t) \\ &+ J_{\mathrm{w}}(W, V, t)[-C + W(R + \boldsymbol{\Omega}'(\boldsymbol{a} - R\mathbf{1})) + (1+\alpha)V] \\ &+ J_{\mathrm{v}}(W, V, t)\alpha V + 0.5 J_{\mathrm{ww}}(W, V, t)[W^2 \boldsymbol{\Omega}'\boldsymbol{\Sigma}\boldsymbol{\Omega} + 2W\boldsymbol{\Omega}'\boldsymbol{\Sigma}_{\mathrm{qv}} + \sigma^2 V^2] \\ &+ 0.5 J_{\mathrm{vv}}(W, V, t)\sigma^2 V^2 + J_{\mathrm{wv}}(W, V, t)[W\boldsymbol{\Omega}'\boldsymbol{\Sigma}_{\mathrm{qv}} + \sigma^2 V^2]] \end{aligned} \quad (10)$$

Define the function $I(W, V) \equiv \exp(\delta t)J(W, V, t)$. The function I is independent of time t.[2] Restating Eq. (10) in terms of $I(W, V)$ and dividing the equation by $\exp(-\delta t)$, we get:

$$\begin{aligned} 0 = \max_{C,\Omega}[&U(C(t)) - \delta I(W, V) + I_{\mathrm{w}}(W, V) \\ &\times [-C + W(R + \boldsymbol{\Omega}'(\boldsymbol{a} - R\mathbf{1})) + (1+\alpha)V] + I_{\mathrm{v}}(W, V)\alpha V \\ &+ 0.5 I_{\mathrm{ww}}(W, V)[W^2 \boldsymbol{\Omega}'\boldsymbol{\Sigma}\boldsymbol{\Omega} + 2W\boldsymbol{\Omega}'\boldsymbol{\Sigma}_{\mathrm{qv}} + \sigma^2 V^2] \\ &+ 0.5 I_{\mathrm{vv}}(W, V)\sigma^2 V^2 + I_{\mathrm{wv}}(W, V)[W\boldsymbol{\Omega}'\boldsymbol{\Sigma}_{\mathrm{qv}} + \sigma^2 V^2]] \end{aligned} \quad (11)$$

The first order conditions to this problem are:

$$U_c(C^*) = I_{\mathrm{w}}(W, V) \quad (12)$$

$$\begin{aligned} &I_{\mathrm{w}} W(W, V)[W(\boldsymbol{a} - R\mathbf{1})] \\ &\quad + I_{\mathrm{ww}}(W, V)[W^2 \boldsymbol{\Sigma}\boldsymbol{\Omega}^* + W\boldsymbol{\Sigma}_{\mathrm{qv}}] + I_{\mathrm{wv}}(W, V)[W\boldsymbol{\Sigma}_{\mathrm{qv}}] = 0 \end{aligned} \quad (13)$$

where consumption and portfolio allocations have been starred to indicate that they are at optimal values. Substituting Eqs. (12) and (13) into Eq. (11), we get the following:

$$\begin{aligned} 0 = &U(C^*) - \delta I + I_{\mathrm{w}}(-C^* + WR) - 0.5\left(\frac{I_{\mathrm{w}}^2}{I_{\mathrm{ww}}}\right)(\boldsymbol{a} - R\mathbf{1})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{a} - R\mathbf{1}) \\ &+ \left[I_{\mathrm{w}} V + (I_{\mathrm{w}} + I_{\mathrm{v}})\alpha V - I_{\mathrm{w}}\left(1 + \frac{I_{\mathrm{wv}}}{I_{\mathrm{ww}}}\right)(\boldsymbol{a} - R\mathbf{1})'\boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_{\mathrm{qv}}\right] \\ &+ 0.5\left[(I_{\mathrm{ww}} + 2I_{\mathrm{wv}} + I_{\mathrm{vv}})\sigma^2 V^2 - \left(\frac{(I_{\mathrm{ww}} + I_{\mathrm{wv}})^2}{I_{\mathrm{ww}}}\right)\boldsymbol{\Sigma}_{\mathrm{qv}}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_{\mathrm{qv}}\right] \end{aligned} \quad (14)$$

which is a partial differential equation in I. For a specific utility function, Eq. (14) can be solved to obtain an explicit expression for the indirect utility function which could then be used to obtain the optimal consumption rate and investments.

[1] The appendix contains a derivation.

[2] This can be seen by substituting the first line of Eq. (9) in the definition of $I(W, V)$. For a demonstration see Ingersoll (1987, p. 274).

3. Portfolio composition

From Eq. (13), the optimal investments can be derived as follows:

$$W\boldsymbol{\Omega}^* = \left(\frac{-I_{\mathrm{w}}}{I_{\mathrm{ww}}}\right)\boldsymbol{t} + \left(1 + \left(\frac{I_{\mathrm{wv}}}{I_{\mathrm{ww}}}\right)\right)\boldsymbol{h} \tag{15}$$

where $\boldsymbol{t} = \boldsymbol{\Sigma}^{-1}(\boldsymbol{a} - R\mathbf{1})$ and $\boldsymbol{h} = (-\boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_{\mathrm{qv}})$. Eq. (15) decomposes optimal investments into a tangency portfolio $\boldsymbol{t}$ and a hedge portfolio $\boldsymbol{h}$; $\boldsymbol{t}$ is obtained as the point of tangency of the borrowing–lending line with the mean–variance frontier and $\boldsymbol{h}$ is entirely due to the riskiness of off-farm income. This portfolio has the property that its return provides the maximum negative correlation with the change in off-farm income (Ingersoll, 1987, p. 282). The hedge portfolio is scaled by $(1+(I_{\mathrm{wv}}/I_{\mathrm{ww}}))$ which reflects dual motivations for hedging. The first component here is due to the fact that the change in wealth due to off-farm income is risky. The second component of the scaling term arises because of hedging against the state variable. Since we have assumed constant drift and diffusion parameters for asset returns, the only state variable here is that which figures in the off-farm income process. By assumption, the state variable that drives the drift and the standard deviation of the income process is the level of income, V itself. Hence, the state variable hedge portfolio is the same as the portfolio which hedges against off-farm income, i.e. $\boldsymbol{h}$. However, since the state variable V enters the indirect utility function, the state variable hedge portfolio is scaled by $(I_{\mathrm{wv}}/I_{\mathrm{ww}})$ reflecting aversion to state variable risk. Note that if there is no state variable in the model such as when the income stream follows an arithmetic Brownian motion, then there is no state variable hedge portfolio either and the hedge portfolio in Eq. (15) would be scaled by 1.

If off-farm income is riskless or is uncorrelated with asset returns ($\boldsymbol{\Sigma}_{\mathrm{qv}} = 0$), the hedge portfolios vanish leaving only the tangency portfolio. The resulting solution would not, however, be equivalent to the solution in the absence of off-farm income, as off-farm income would matter to the tangency portfolio through the wealth argument in the indirect utility function. As we discuss later, a fuller characterization of the effects of off-farm income on investments requires further assumptions on the utility function.

Consider now the effect of off-farm income on the composition of the risky portfolio. A standard result in portfolio theory is that the investment in a risky asset as a proportion of investments in all risky assets is independent of risk preferences. Hence, the identification of the optimal risky portfolio with the 'market' portfolio consisting of all market assets. Here we have the following result:

Proposition 1. *If off-farm income is correlated with asset returns, the composition of risky portfolio is not independent of risk preferences.*

To see this define $\omega^* = W\boldsymbol{\Omega}^*/W\mathbf{1}'\boldsymbol{\Omega}^*$ where the denominator is the total wealth invested in risky assets. Using Eq. (15), we get the following:

$$\omega^* = \frac{(\boldsymbol{t}/\mathbf{1}'\boldsymbol{t}) + (-\boldsymbol{h}/\mathbf{1}'\boldsymbol{t})(I_{\mathrm{ww}} + I_{\mathrm{wv}})/I_{\mathrm{w}}}{1 + (-\mathbf{1}'\boldsymbol{h}/\mathbf{1}'\boldsymbol{t})(I_{\mathrm{ww}} + I_{\mathrm{wv}})/I_{\mathrm{w}}}$$

or

$$\omega^* = \frac{\omega_c^* + \boldsymbol{e}(I_{\mathrm{ww}} + I_{\mathrm{wv}})/I_{\mathrm{w}}}{1 + (\mathbf{1}'\boldsymbol{e})(I_{\mathrm{ww}} + I_{\mathrm{wv}})/I_{\mathrm{w}}} \tag{16}$$

where $\omega_c{}^* = \boldsymbol{t}/\mathbf{1}'\boldsymbol{t}$ is the composition of the risky portfolio when off-farm income is uncorrelated with asset returns and $\boldsymbol{e} = -\boldsymbol{h}/\mathbf{1}'\boldsymbol{t}$ is adjustment to the ω_c^* portfolio because of off-farm income that is correlated with asset returns. While ω_c^* is independent of risk preferences, the same is not true of ω^*.

The implication of the aforementioned result is that farm households would want to hold different proportions of the risky assets depending on the correlation of their off-farm earnings with the returns on the risky assets. Can we say anything more? Let v_{f} and v_{p} be the coefficients of variation of excess returns on the farm asset and financial asset respectively, i.e. $v_{\mathrm{f}} = s_{\mathrm{f}}/(a_{\mathrm{f}} - R)$ and $v_{\mathrm{p}} = s_{\mathrm{p}}/(a_{\mathrm{p}} - R)$. The impact of off-farm income on the composition of risky portfolio is given by the following result:

Proposition 2. *An increase in off-farm income increases* (*leaves unchanged, decreases*) *the weight of the farm asset in the risky portfolio as* $v_{\mathrm{f}}K_{\mathrm{fv}}$ *is less than* (*equal to, greater than*) $v_{\mathrm{p}}K_{\mathrm{pv}}$ *provided the ratio* $I_{\mathrm{wv}}/I_{\mathrm{ww}}$ *is invariant to V.*

A proof of this result is provided in Appendix A. According to this result, as income increases, a farm household reduces the portfolio weight of the *riskier* asset where the notion of riskiness is the asset's coefficient of variation multiplied by the correlation of that asset's returns with off-farm income. Hence, it is possible that even though the farm asset has a higher coefficient of variation than the financial asset, an increase in income increases the allocation to the farm asset if the correlation of off-farm income with farm return is much less than the correlation of off-farm income and return from financial assets. The correlation structure of off-farm income with risky asset returns is therefore important in the determination of portfolio weights. We see, therefore, that when individuals (who might otherwise be identical in terms of preferences and asset returns) differ with respect to off-farm income dynamics their optimal portfolios of risky assets necessarily differ.

4. The value of off-farm income

A sufficient condition for Proposition 2 is that the ratio $I_{\mathrm{wv}}/I_{\mathrm{ww}}$ be invariant to V. Under what conditions is this true? Suppose there exists a function $f(\cdot)$:

$$I(W, V) = f(W + bV) \tag{17}$$

where b is a parameter independent of V. It follows that $I_{\mathrm{wv}}/I_{\mathrm{ww}} = b$ is invariant to V. When is Eq. (17) true?

Proposition 3. *If off-farm income is spanned by some linear combination of the risk-free asset, the farm asset and the financial asset, then there exists a function* $f(\cdot)$ *and a parameter b which is independent of risk preferences such that* $I(W, V) = f(W + bV)$.

For a proof of this result, we refer the reader to Svensson and Werner (1993).[3] Spanning means that there exist linear combinations of the traded assets that have the same risk characteristics as off-farm income. Since it is the hedge portfolio that provides the maximum negative correlation with off-farm income, spanning means that the hedge portfolio is a perfect hedge. The sum of off-farm income and the return from the hedge portfolio would therefore have zero variance. Mathematically, this condition holds when

$$\psi \equiv \sigma^2 V^2 - \boldsymbol{\Sigma}'_{\mathrm{qv}} \boldsymbol{\Sigma}^{-1} \boldsymbol{\Sigma}_{\mathrm{qv}} = 0 \tag{18}$$

where ψ is the variance of the change in off-farm conditional on the traded assets.

The implication of spanning is that although human capital cannot be traded, one can construct a portfolio of traded assets, which generates the same return as the return on human capital. The human capital can therefore be priced as the price of the equivalent portfolio of traded assets; bV can therefore be interpreted as the capitalized value of a claim to the household's stream of off-farm income. It is independent of wealth and investor preferences because even though a claim to off-farm income cannot be traded, it can still be valued as if it were by adjusting the portfolio of traded assets (including the risk-free asset). This can be shown formally by employing the method of contingent claims analysis (see for instance, Bodie et al., 1992; Svensson and Werner, 1993).

It remains to be shown that b is independent of V. Define $\boldsymbol{\Phi}_{\mathrm{qv}} \equiv V^{-1}\boldsymbol{\Sigma}_{\mathrm{qv}}$. By the definition of $\boldsymbol{\Sigma}_{\mathrm{qv}}$, $\boldsymbol{\Phi}_{\mathrm{qv}} = \boldsymbol{s} E_t(\mathrm{d}\boldsymbol{Z}\,\mathrm{d}Z_{\mathrm{v}})\sigma$ and is invariant to V. Note also that $\boldsymbol{\Phi}_{\mathrm{qv}}$ is a vector of covariances between the growth rate of off-farm income and the returns on the risky assets.

Proposition 4. *If off-farm income is spanned by the set of traded assets, the capitalized value of off-farm income to the farm household is bV where* $b = (1+\alpha+\theta)/(R-(\alpha+\theta))$ *and* $\theta = (\boldsymbol{a} - R\mathbf{1})'(-\boldsymbol{\Sigma}^{-1}\boldsymbol{\Phi}_{\mathrm{qv}})$.

By Proposition 3, we know that, under spanning, b is independent of risk preferences. Therefore, to derive the formula for b, we pick any utility function and solve for the indirect utility function satisfying Eq. (14). Suppose preferences are described by a constant relative risk aversion utility function $U(x) = x^{\gamma}/\gamma,\ \ \gamma < 1$. It can then be shown that under spanning, Eq. (14) is satisfied by an indirect utility function of the form $I(W, V) = a(W + bV)^{\gamma}/\gamma$ where b is given by the formula in Proposition 4 and a is a function of the parameters of the utility function and the stochastic process of asset returns. Similarly, if the utility function is of the constant absolute risk aversion type, i.e. $U(x) = -\exp(-\rho x)$, then the indirect utility function is $I(W, V) = -d\exp(-\rho R(W + bV))$ where b is as mentioned before, R is the risk-free rate and d is a function of the parameters of the system. These derivations are detailed in the appendix.

To understand the expression for b, suppose for a moment that the returns on risky assets are uncorrelated with off-farm income. Then, $b = (1+\alpha)/(R-\alpha)$ which is the present value of a future stream of income growing at rate α. In the general case, when asset returns

[3] Svensson and Werner show this result for the case where income follows an arithmetic Brownian motion. However, the result remains valid for more general Brownian motions. The specification of Brownian motion matters only with respect to the expression for b and not to the result in Proposition 3.

are correlated with income, farm households adjust their portfolios. The income hedge portfolio is the adjustment and θ is the excess return on this portfolio. Hence, the present value of a risky off-farm income stream is computed by considering the risk adjusted growth rate to be $(\alpha + \theta)$.

What happens when traded assets do not span off-farm income? Could the ratio $(I_{\mathrm{wv}}/I_{\mathrm{ww}})$ still be invariant to V? In this case, the indirect utility functions are hard to characterize and analytical solutions of Eq. (14) do not exist for most utility functions.

An exception is the quadratic utility function $U(x) = -(\beta - x)^2/2$ where β is an upper bound to all feasible consumption levels. It can be shown that the corresponding indirect utility function is of the form $I(W, V) = ((aW+bV+c)^2/2)+gV^2$. Since $(I_{\mathrm{wv}}/I_{\mathrm{ww}}) = b/a$, Proposition 2 applies.

5. Wealth and portfolio rebalancing effects

Using Eq. (15) and Proposition 3, the total investments in risky assets, under the spanning assumption, is given by the following:

$$\mathbf{1}'W\boldsymbol{\Omega}^* = \left(\frac{1}{A(W+bV)}\right)\mathbf{1}'\boldsymbol{t} - (1+b)\mathbf{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\Phi}_{\mathrm{qv}}V$$

where $A(\cdot)$ is the risk aversion of the indirect utility function. Differentiating with respect to V, the following is obtained:

$$\frac{\partial(\mathbf{1}'W\boldsymbol{\Omega}^*)}{\partial V} = -\left(\frac{A'}{A^2}\right)b\mathbf{1}'\boldsymbol{t} - (1+b)\mathbf{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\Phi}_{\mathrm{qv}} \tag{19}$$

As long as risk aversion is decreasing in wealth, the first term is positive. Higher off-farm income increases wealth, reduces risk aversion and thus leads to an increase in investments in risky assets. The second term represents a portfolio rebalancing effect.[4]

As V increases, the farm household alters the hedge portfolio to reduce the risk associated with higher income. The sum of these portfolio adjustments depends on the relative magnitude of correlations and standard deviations and its sign is in general, hard to predict. It is, however, more promising to look at the demands for individual assets. Since our interest is in the demand for farm asset, the effect of off-farm income is given by the following:

$$\frac{\partial(W\omega_{\mathrm{f}}^*)}{\partial V} = -\left(\frac{A'}{A^2}\right)bt_{\mathrm{f}} - \left(\frac{1}{s_{\mathrm{f}}}\right)(1+b)(K_{\mathrm{fv}} - K_{\mathrm{fp}}K_{\mathrm{pv}})\sigma \tag{20}$$

where t_{f} is the first element of the vector $\boldsymbol{t}$. Once again, the first term is the wealth effect and leads to an increase in investment. The second term is the adjustment due to portfolio rebalancing. Its sign is opposite to the sign of $(K_{\mathrm{fv}} - K_{\mathrm{fp}}K_{\mathrm{pv}})$ and thus depends on correlations alone. It would thus seem that if K_{fv} is low enough (or high enough), the portfolio rebalancing would be positive (or negative). The intuition is that if K_{fv} is low,

[4] The terminology is borrowed from Bodie et al. (1992) who consider such portfolio adjustments in the case of a single risky asset.

the farm asset serves as a hedge to off-farm income and therefore an increase in off-farm income is countered by increasing investments in the farm asset. How low should K_{fv} be? Clearly, it is enough if it is smaller than the product of K_{fp} and K_{pv}. When would this be true?

It can be shown that the spanning restriction in Eq. (18) is equivalent to the following form:

$$K_{fv}^2 + K_{pv}^2 + K_{fp}^2 - 2K_{fv}K_{pv}K_{fp} = 1 \tag{21}$$

For given values of K_{fp} and K_{fv}, Eq. (21) is a quadratic in K_{pv} and therefore has two solutions. Denote $a = (K_{fv} - K_{fp}K_{pv})$. Consistent with the spanning restriction, there are then two values of a (of possibly opposite signs) corresponding to the two roots of Eq. (21). It is therefore possible that even if K_{fv} is very small and K_{fp} is very high, one of the values of a could be positive if the corresponding solution for K_{pv} from Eq. (21) is small. As an instance of this, consider the case when $K_{fv} = -0.8$ and $K_{fp} = 0.9$. Since the ratio (K_{fv}/K_{fp}) is very small at -0.88, it would seem that such a configuration of parameters would generate a negative a. However, the two values of K_{pv} consistent with spanning are -0.46 and -0.98. Hence a is negative for the larger root and positive for the smaller root. This example shows that a sufficient condition (on K_{fv}) that guarantees an unambiguous sign for a would not be independent of K_{fp}. The next proposition derives such a sufficient condition.

Proposition 5. *If traded assets span off-farm income, the effect on the demand for the farm asset from an increase in off-farm income decomposes into three parts*:

1. *a wealth effect which increases the demand for farm asset if risk aversion of the indirect utility function is decreasing in wealth*;
2. *a portfolio rebalancing effect which increases*(*decreases*) *the demand for farm asset if* $K_{fv} < (>) \, 0$ *and* $K_{fv}^2 > K_{fp}^2$ *for a given value of* K_{fp} *in the open interval* $(-1, 1)$;
3. *a portfolio rebalancing effect which is zero whenever* $K_{fp}^2 = 1$.

Part (a) of this proposition was noted earlier. The appendix contains a proof of parts (b) and (c). Fig. 1 illustrates the sufficient conditions when $K_{fp} = 0.1$. Fig. 1 plots $(K_{fv} - K_{fp}K_{pv})$ against K_{fv}. Since there are two solutions for K_{pv} corresponding to every value of K_{fv}, there are two curves as well for $(K_{fv} - K_{fp}K_{pv})$. The curve to the left corresponds to the smaller root of Eq. (21). It can be seen that when $K_{fv} < -0.1$, then $(K_{fv} - K_{fp}K_{pv})$ is negative for both the solutions of K_{pv}. Similarly, no matter which solution we pick $(K_{fv} - K_{fp}K_{pv})$ is positive whenever $K_{fv} > 0.1$. It can also be seen that these conditions are not necessary. In the interval $(-0.1, 0.1)$, $(K_{fv} - K_{fp}K_{pv})$ is negative for the larger root but is positive for the smaller root of Eq. (21).

The literature on capital asset pricing models in agriculture provides information about the empirical value of K_{fp}. Reviewing this literature, Bjornson (1995) concludes that farm assets contribute little to systematic risk in a portfolio comprised of farm and financial assets. For instance, from Barry's study (1980), the correlation between the return to farm asset and the return to a market portfolio (for US data) can be worked out to be 0.14. Since the market portfolio includes the farm asset, the K_{fp} implied by this figure can only be lower.

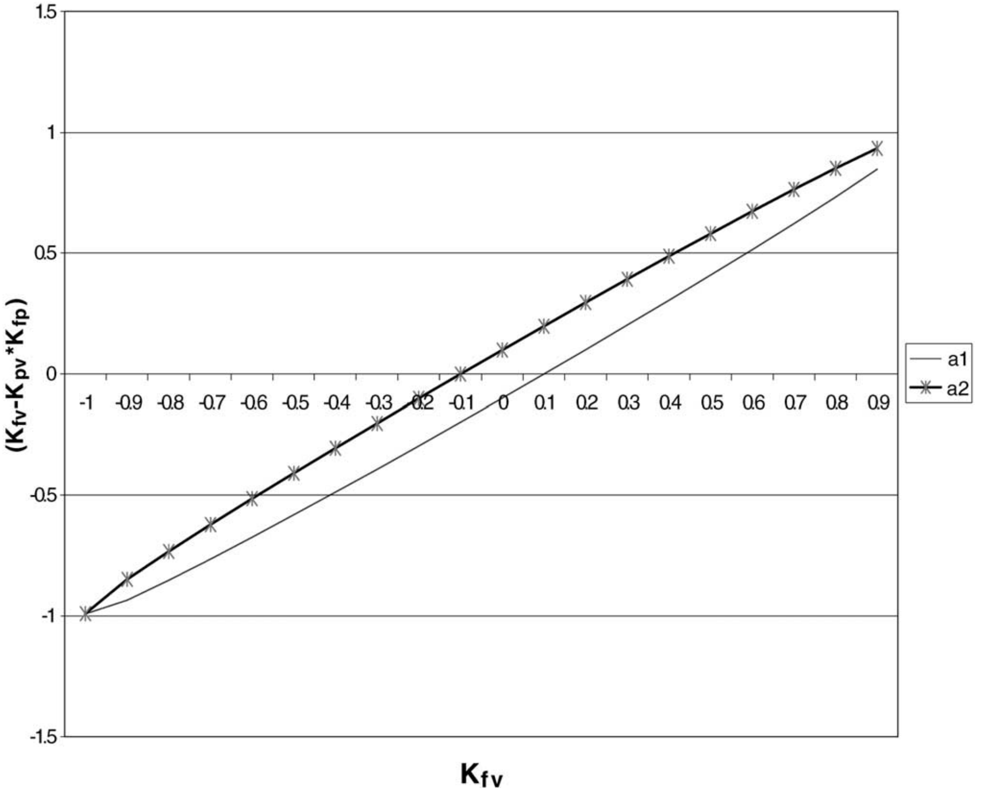

Fig. 1. Portfolio rebalancing effect, $K_{\mathrm{fp}} = 0.1$.

Empirically, it seems therefore, that an increase in off-farm income increases (or decreases) the holdings of farm assets whenever the correlation of off-farm income with farm return is negative (positive).

6. Conclusions

Using a portfolio choice model with labor income, this paper examined the investment decisions of part-time farmers. We showed that higher off-farm income would cause more wealth to be allocated to the “less riskier” asset. The less risky asset is, however, not just the one with a smaller variability of return. The riskiness of an asset also depends on its correlation with off-farm income. In terms of total investments, we found that higher off-farm income leads to two effects. First, there is a wealth effect that causes decreasing risk-averse investors to purchase more of all risky assets. Secondly, when off-farm income is correlated with asset returns, part of an investor’s portfolio is put together to counter the risk from off-farm income. An increase in off-farm income therefore leads to a portfolio rebalancing effect as the portfolio is readjusted to achieve an optimal hedge against off-farm income. The direction of portfolio rebalancing also depends on the correlation of off-farm income with asset returns. If the correlation of the return to the farm asset with off-farm income is low enough, portfolio rebalancing leads farm households to invest more in the farm asset.

Empirical evidence indicates that the importance of off-farm income varies by region as it is strongly affected by the structure of the local economy (Hearn et al., 1996). In regions, where the rural economy is diversified, the correlation of off-farm income with farm returns is likely to be low relative to the correlation of off-farm income with financial asset returns. In these circumstances, access to off-farm income might enable farm households to maintain asset portfolios that are concentrated in favor of the farm asset. Even though part-time farms might earn lower returns than full-time farms, they are compensated by lower risk. It is sometimes argued that since farm households face borrowing constraints and since they cannot sell equity to outside investors, their holdings of nonfarm financial assets tend to be small. If farm households cannot adequately diversify investments, the risk-reduction advantages of part-time farming will be even stronger.

One direction of future work would be to pursue these insights for agricultural asset pricing models. In the standard capital asset pricing model, the average excess farm return (over the risk-free rate) is determined by the extent to which the riskiness of holding farm assets can be reduced by holding a diversified portfolio. Because optimal asset holding also depends on off-farm income, the risk premium for holding farm assets, in turn, also depends on the correlations of off-farm income with asset returns. It should be possible thus to derive, estimate and test an asset pricing model where agricultural asset prices and risk premiums are sensitive to off-farm income. Such analysis would allow us to judge the quantitative significance of the risk-reduction advantages of off-farm income. Another direction for empirical work would be to directly test the theoretical implications of the model through econometric analysis of investments in farm and nonfarm assets based on cross-sectional data of farm operations. Such analysis could shed light on the extent to which wealth or portfolio rebalancing effects are significant in the agricultural sector.

In this paper, we assumed labor supply to be inelastic. Relaxing this assumption does not change any of the results as long as labor supply decisions do not affect asset returns. This is typically the case for seasonal (i.e. in the farm off-season) and spouse labor supply. During the farm season, labor supply decisions by the farm operator will affect the returns to the farm asset. Another direction of future work would be to extend our model by endogenising labor supply. This would be valuable for two reasons. First, the trade-off between farm asset returns and off-farm income will be important in explaining the bounds on the size of part-time farms. Second, it would permit richer specifications of off-farm labor supply. In the empirical literature on off-farm labor supply (Huffman, 1991; Sumner, 1982), the relation between off-farm wages and off-farm labor supply is modeled conventionally as the outcome of income effects and substitution effects between work and leisure. But if off-farm income matters to farm investment, this route affects labor supply too. The ‘asset effect’ could therefore be addressed by integrating labor supply with asset choice.

Appendix A

A.1. Derivation of Eq. (10)

The indirect utility function satisfies the following:

$$J(W, V, t) = \max_{C,\Omega} E_t \times \left[\int_t^{t+\mathrm{d}t} \exp(-\delta x) UC(x)\,\mathrm{d}x + J(W(t+\mathrm{d}t), V(t+\mathrm{d}t), t+\mathrm{d}t)\right] \tag{A.1}$$

By an exact two-term Taylor expansion the integral can be expressed as follows:

$$E_t\left[\int_t^{t+\mathrm{d}t} \exp(-\delta x) U(C(x))\,\mathrm{d}x\right] = \exp(-\delta t) U(C(t))\,\mathrm{d}t - \delta \exp(-\delta t^*) E_t(U(C(t^*))\,\mathrm{d}t^2 \tag{A.2}$$

which is satisfied by some t^* in the interval $[t, t+\mathrm{d}t]$. Let $\mathrm{o}(\mathrm{d}t)$ denote terms which are of order smaller than $\mathrm{d}t$, i.e. terms in $\mathrm{o}(\mathrm{d}t)$ will vanish as $\mathrm{d}t$ itself becomes very small. Then

$$E_t\left[\int_t^{t+\mathrm{d}t} \exp(-\delta x) U(C(x))\,\mathrm{d}x\right] = \exp(-\delta t) U(C(t))\,\mathrm{d}t + \mathrm{o}(\mathrm{d}t) \tag{A.3}$$

By Taylor's theorem, expand $J(\cdot)$ to the following:

$$\begin{aligned} &E_t J(W(t+\mathrm{d}t), V(t+\mathrm{d}t), t+\mathrm{d}t) \\ &\quad = J(W, V, t) + J_t(W, V, t)\,\mathrm{d}t + J_{\mathrm{w}}(W, V, t) E_t(\mathrm{d}W) + J_{\mathrm{v}}(W, V, t) E_t(\mathrm{d}V) \\ &\qquad + 0.5 J_{tt}(W, V, t)\,\mathrm{d}t^2 + 0.5 J_{\mathrm{ww}}(W, V, t) E_t(\mathrm{d}W)^2 + 0.5 J_{\mathrm{vv}}(W, V, t) E_t(\mathrm{d}V)^2 \\ &\qquad + 0.5 J_{\mathrm{w}t}(W, V, t) E_t(\mathrm{d}W\,\mathrm{d}t) + 0.5 J_{\mathrm{v}t}(W, V, t) E_t(\mathrm{d}V\,\mathrm{d}t) \\ &\qquad + 0.5 J_{\mathrm{wv}}(W, V, t) E_t(\mathrm{d}W\,\mathrm{d}V) + A \end{aligned} \tag{A.4}$$

where A includes terms of the form $E_t(\mathrm{d}W)^i(\mathrm{d}V)^j(\mathrm{d}t)^k$ where $i+k+j \geq 3$. To evaluate this expression, use Eqs. (3), (4) and (8) to get

$$E_t(\mathrm{d}W) = -C\,\mathrm{d}t + W(R + \boldsymbol{\Omega}'(\boldsymbol{a} - R\mathbf{1}))\,\mathrm{d}t + (1+\alpha)V\,\mathrm{d}t + \mathrm{o}(\mathrm{d}t) \tag{A.5}$$

$$E_t(\mathrm{d}W)^2 = (W^2 \boldsymbol{\Omega}'\boldsymbol{\Sigma}\boldsymbol{\Omega} + 2W\boldsymbol{\Omega}'\boldsymbol{\Phi}_{\mathrm{qv}} + \sigma^2 V^2)\,\mathrm{d}t + \mathrm{o}(\mathrm{d}t) \tag{A.6}$$

$$E_t(\mathrm{d}V) = \alpha V\,\mathrm{d}t \tag{A.7}$$

$$E_t(\mathrm{d}V)^2 = \sigma^2 V^2\,\mathrm{d}t + \mathrm{o}(\mathrm{d}t) \tag{A.8}$$

$$E_t(\mathrm{d}W\,\mathrm{d}V) = VW\boldsymbol{\Omega}'\boldsymbol{\Sigma}_{\mathrm{qv}}\,\mathrm{d}t + \sigma^2 V^2\,\mathrm{d}t + \mathrm{o}(\mathrm{d}t) \tag{A.9}$$

$$\mathrm{d}t^2 \sim \mathrm{o}(\mathrm{d}t), \quad E_t(\mathrm{d}W\,\mathrm{d}t) \sim \mathrm{o}(\mathrm{d}t), \quad E_t(\mathrm{d}V\,\mathrm{d}t) \sim \mathrm{o}(\mathrm{d}t), \quad \text{and} \quad E_t(A) \sim \mathrm{o}(\mathrm{d}t) \tag{A.10}$$

Substitute Eqs. (A.5)–(A.10) in Eq. (A.4), and then use Eqs. (A.3) and (A.4) to derive Eq. (A.1) as follows:

$$\begin{aligned} J(W,V,t) = \max\nolimits_{C,\Omega}[&\exp(-\delta t)U(C(t))\,\mathrm{d}t + J(W,V,t) + J_t(W,V,t)\,\mathrm{d}t \\ &+ J_{\mathrm{w}}(W,V,t)[-C + W(R + \boldsymbol{\Omega}'(\boldsymbol{a} - R\mathbf{1})) + (1+\alpha)V]\,\mathrm{d}t \\ &+ J_{\mathrm{v}}(W,V,t)\alpha V\,\mathrm{d}t + 0.5J_{\mathrm{ww}}(W,V,t)[W^2\boldsymbol{\Omega}'\boldsymbol{\Sigma}\boldsymbol{\Omega} + 2W\boldsymbol{\Omega}'\boldsymbol{\Sigma}_{\mathrm{qv}} \\ &+ \sigma^2V^2]\,\mathrm{d}t + 0.5J_{\mathrm{vv}}(W,V,t)\sigma^2V^2\,\mathrm{d}t \\ &+ J_{\mathrm{wv}}(W,V,t)[W\boldsymbol{\Omega}'\boldsymbol{\Sigma}_{\mathrm{qv}} + \sigma^2V^2]\,\mathrm{d}t + \mathrm{o}(\mathrm{d}t)] \end{aligned} \tag{A.11}$$

To obtain Eq. (10) of the text, subtract $J(W, V, t)$ from both sides of Eq. (A.11), divide by $\mathrm{d}t$ and take limits as $\mathrm{d}t \to 0$.

A.2. Proof of Proposition 2

From Eq. (15) we have the following:

$$\begin{aligned} W\omega_{\mathrm{f}}^*(s_{\mathrm{f}}^2s_{\mathrm{p}}^2 - s_{\mathrm{fp}}^2) &= \left(\frac{-I_{\mathrm{w}}}{I_{\mathrm{ww}}}\right)[s_{\mathrm{p}}^2(a_{\mathrm{f}} - R) - s_{\mathrm{fp}}(a_{\mathrm{p}} - R)] \\ &\quad - \left(1 + \frac{I_{\mathrm{wv}}}{I_{\mathrm{ww}}}\right)[s_{\mathrm{p}}^2\sigma_{\mathrm{fv}} - s_{\mathrm{fp}}\sigma_{\mathrm{pv}}] \\ W\omega_{\mathrm{p}}^*(s_{\mathrm{f}}^2s_{\mathrm{p}}^2 - s_{\mathrm{fp}}^2) &= \frac{-I_{\mathrm{w}}}{I_{\mathrm{ww}}}[s_{\mathrm{f}}^2(a_{\mathrm{p}} - R) - s_{\mathrm{fp}}(a_{\mathrm{f}} - R)] - \left(1 + \frac{I_{\mathrm{wv}}}{I_{\mathrm{ww}}}\right)[s_{\mathrm{f}}^2\sigma_{\mathrm{pv}} - s_{\mathrm{fp}}\sigma_{\mathrm{fv}}] \end{aligned}$$

where σ_{fv} and σ_{pv} are elements of the vector $\boldsymbol{\Sigma}_{\mathrm{qv}}$. Denote $L_{\mathrm{f}} = (a_{\mathrm{f}} - R)/s_{\mathrm{f}}$ and $L_{\mathrm{p}} = (a_{\mathrm{p}} - R)/s_{\mathrm{p}}$. Divide both sides of both equations by $s_{\mathrm{f}}^2s_{\mathrm{p}}^2$ and use the definitions, $s_{\mathrm{fp}} = s_{\mathrm{f}}s_{\mathrm{p}}K_{\mathrm{fp}}$, $\sigma_{\mathrm{fv}} = \sigma s_{\mathrm{f}}K_{\mathrm{fv}}V$ and $\sigma_{\mathrm{pv}} = \sigma s_{\mathrm{v}}K_{\mathrm{pv}}V$. Re-arranging terms the aforementioned equations become

$$Ws_{\mathrm{f}}\omega_{\mathrm{f}}^*(1-K_{\mathrm{fp}}^2) + \left(1+\frac{I_{\mathrm{wv}}}{I_{\mathrm{ww}}}\right)(K_{\mathrm{fv}} - K_{\mathrm{pv}}K_{\mathrm{fp}})\sigma V = \left(\frac{-I_{\mathrm{w}}}{I_{\mathrm{ww}}}\right)(L_{\mathrm{f}} - K_{\mathrm{fp}}L_{\mathrm{p}}) \tag{A.12}$$

$$Ws_{\mathrm{p}}\omega_{\mathrm{p}}^*(1-K_{\mathrm{fp}}^2) + \left(1+\frac{I_{\mathrm{wv}}}{I_{\mathrm{ww}}}\right)(K_{\mathrm{pv}} - K_{\mathrm{fv}}K_{\mathrm{fp}})\sigma V = \left(\frac{-I_{\mathrm{w}}}{I_{\mathrm{ww}}}\right)(L_{\mathrm{p}} - K_{\mathrm{fp}}L_{\mathrm{f}}) \tag{A.13}$$

Denote the ratio $I_{\mathrm{wv}}/I_{\mathrm{ww}}$ as b. Divide Eq. (A.12) by Eq. (A.13):

$$\frac{Ws_{\mathrm{f}}\omega_{\mathrm{f}}^*(1 - K_{\mathrm{fp}}^2) + (1+b)(K_{\mathrm{fv}} - K_{\mathrm{pv}}K_{\mathrm{fp}})\sigma V}{Ws_{\mathrm{p}}\omega_{\mathrm{p}}^*(1 - K_{\mathrm{fp}}^2) + (1+b)(K_{\mathrm{pv}} - K_{\mathrm{fv}}K_{\mathrm{fp}})\sigma V} = \frac{L_{\mathrm{f}} - L_{\mathrm{p}}K_{\mathrm{fp}}}{L_{\mathrm{p}} - L_{\mathrm{f}}K_{\mathrm{fp}}} \tag{A.14}$$

Notice that the right-hand side of Eq. (A.14) does not depend on off-farm income or on the parameters of its stochastic process. On the other hand, the left-hand side of Eq. (A.14) is a function of the riskiness of off-farm income as well as the portfolio weights. Let Γ denote the left-hand side of Eq. (A.14). Consider its derivative with respect to V when portfolio weights are held fixed at their optimal values. Under the assumption that the ratio b is invariant to V, we obtain the following expression:

$$\frac{\partial \Gamma}{\partial V} = \frac{N}{D} \tag{A.15}$$

where $N \equiv W(1+b)\sigma(1-K_{\mathrm{fp}}^2)[\omega_{\mathrm{p}}^* s_{\mathrm{p}}(K_{\mathrm{fv}}-K_{\mathrm{pv}}K_{\mathrm{fp}})-\omega_{\mathrm{f}}^* s_{\mathrm{f}}(K_{\mathrm{pv}}-K_{\mathrm{fv}}K_{\mathrm{fp}})]$ and $D = [Ws_{\mathrm{p}}\omega_{\mathrm{p}}^*(1-K_{\mathrm{fp}}^2)+(1+b)(K_{\mathrm{pv}}-K_{\mathrm{fv}}K_{\mathrm{fp}})\sigma V]^2$. Since the denominator is positive, $\partial\Gamma/\partial V$ is of the same sign as the numerator of the expression in Eq. (A.15). Suppose N is positive (negative). Then by Eq. (A.14), it is clear that $\omega_{\mathrm{f}}^*/\omega_{\mathrm{p}}^*$ must decrease (increase) in order to maintain equality in that equation. Thus

$$\frac{\partial(\omega_{\mathrm{f}}^*/\omega_{\mathrm{p}}^*)}{\partial V} > (=,<)\ 0, \quad \text{as } N < (=,>)\ 0 \tag{A.16}$$

It is, however, difficult to interpret N in its present form as it contains the endogenous variables ω_{f}^* and ω_{p}^*. Eliminate them using Eqs. (A.12) and (A.13). Then we have the following:

$$\begin{aligned} N = (1+b)\sigma \Bigg[&\left[\left(\frac{-I_{\mathrm{w}}}{I_{\mathrm{ww}}}\right)(L_{\mathrm{p}}-L_{\mathrm{f}}K_{\mathrm{fp}})-(1+b)(K_{\mathrm{pv}}-K_{\mathrm{fv}}K_{\mathrm{fp}})\sigma\right](K_{\mathrm{fv}}-K_{\mathrm{pv}}K_{\mathrm{fp}}) \\ &-\left[\left(\frac{-I_{\mathrm{w}}}{I_{\mathrm{ww}}}\right)(L_{\mathrm{f}}-L_{\mathrm{p}}K_{\mathrm{fp}})-(1+b)(K_{\mathrm{fv}}-K_{\mathrm{pv}}K_{\mathrm{fp}})\sigma\right](K_{\mathrm{pv}}-K_{\mathrm{fv}}K_{\mathrm{fp}})\Bigg] \end{aligned}$$

which simplifies to

$$N = (1+b)\sigma\left[\left(\frac{-I_{\mathrm{w}}}{I_{\mathrm{ww}}}\right)(1-K_{\mathrm{fp}}^2)(L_{\mathrm{p}}K_{\mathrm{fv}}-L_{\mathrm{f}}K_{\mathrm{pv}})\right] \tag{A.17}$$

Since $(1+b)$, σ, $(-I_{\mathrm{w}}/\mathrm{I}_{\mathrm{ww}})$ and $(1-K_{\mathrm{fp}}^2)$ are positive quantities, and since $L_{\mathrm{f}} = (1/v_{\mathrm{f}})$ and $L_{\mathrm{p}} = (1/v_{\mathrm{p}})$, we get from Eqs. (A.16) and (A.17)

$$\partial(\omega_{\mathrm{f}}^*/\omega_{\mathrm{p}}^*)/\partial V > (=,<)\ 0 \text{ as } v_{\mathrm{f}}K_{\mathrm{fv}} < (=,>)\ v_{\mathrm{p}}K_{\mathrm{pv}}$$

A.3. Proof of Proposition 4

When off-farm income is spanned, $I(W,V) = f(W+bV)$ where the constant b and the function f are to be determined. Note that

$$I_{\mathrm{w}} = f', \quad I_{\mathrm{ww}} = f'', \quad I_{\mathrm{v}} = bf', \quad I_{\mathrm{vv}} = b^2 f'', \quad \text{and} \quad I_{\mathrm{wv}} = bf''$$

Substituting in Eq. (14)

$$\begin{aligned} 0 = U(C^*) - \delta f + f'(-C^* + WR) - 0.5\left(\frac{(f')^2}{f''}\right)(\boldsymbol{a}-R\mathbf{1})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{a}-R\mathbf{1}) \\ + f'[(1+b)(\alpha V - (\boldsymbol{a}-R\mathbf{1})'\boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_{\mathrm{qv}}) + V] \\ + 0.5 f''(1+b^2)(\sigma^2 V^2 - \boldsymbol{\Sigma}_{\mathrm{qv}}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_{\mathrm{qv}}) \end{aligned}$$

By the spanning condition, the last term drops out. Denote $h = W + bV$, add and subtract bV and hence rewrite as follows:

$$\begin{aligned} 0 = &\left[U(C^*) - \delta f - f'C^* + f'hR - 0.5\left(\frac{(f')^2}{f''}\right)(\boldsymbol{a}-R\mathbf{1})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{a}-R\mathbf{1})\right] \\ &+ f'[(1+b)(\alpha - (\boldsymbol{a}-R\mathbf{1})'\boldsymbol{\Sigma}^{-1}\boldsymbol{\Phi}_{\mathrm{qv}}) - bR + 1]V \end{aligned} \tag{A.18}$$

We have a solution for $I(W, V)$ if we choose b such that the second term in brackets becomes zero and we choose a functional form f such that the first term in brackets becomes zero. Setting the second term to zero, b satisfies $(1+b)(\alpha - (\boldsymbol{a} - R\mathbf{1})'\boldsymbol{\Sigma}^{-1}\boldsymbol{\Phi}_{\mathrm{qv}}) - bR + 1 = 0$, from which we obtain $b = (1+\alpha - (\boldsymbol{a} - R\mathbf{1})'\boldsymbol{\Sigma}^{-1}\boldsymbol{\Phi}_{\mathrm{qv}})/[R - (\alpha - (\boldsymbol{a} - R\mathbf{1})'\boldsymbol{\Sigma}^{-1}\boldsymbol{\Phi}_{\mathrm{qv}})]$.

The choice of the functional form f depends on the utility function. Suppose the utility function is $U(x) = x^{\gamma}/\gamma$. Guess the solution to be $f = ah^{\gamma}/\gamma$. From Eq. (12), solve out C^* as $C^* = a^{1/(\gamma-1)}h$. Hence, $U(C^*) = a^{\gamma/(\gamma-1)}h^{\gamma}/\gamma$. Using these, the guess is verified if $a^{\gamma/(\gamma-1)}h^{\gamma}/\gamma - \delta a h^{\gamma}/\gamma - ah^{(\gamma-1)}a^{1/(\gamma-1)}h + ah^{(\gamma-1)}Rh - 0.5(a^2h^{2(\gamma-1)}/a(\gamma-1)h^{(\gamma-2)})\xi = 0$ where we have used the notation $\xi \equiv (\boldsymbol{a} - R\mathbf{1})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{a} - R\mathbf{1})$. Dividing by ah^{γ}

$$\left(\frac{a^{1/(\gamma-1)}}{\gamma}\right) - \left(\frac{\delta}{\gamma}\right) - a^{1/(\gamma-1)} + R - 0.5\left(\frac{\xi}{\gamma-1}\right) = 0$$

which is satisfied if

$$a = \frac{\gamma}{1-\gamma}\left(0.5\frac{\xi}{\gamma-1} - R + \frac{\delta}{\gamma}\right)$$

A similar procedure will work for any other choice of utility function. For instance, if $U(x) = -\exp(-\rho x)$, guess the solution to be $f = -d\exp(-\rho Rh)$ where $h = W + bV$; b is, of course, determined to be the same as before and d is determined by setting the first bracketed term in Eq. (A.18) to 0. Then it is straightforward to show

$$d = \left(\frac{1}{R}\right)\exp\left(\frac{R - \delta - 0.5\xi}{R}\right)$$

A.4. *Proof of Proposition 5*

Part (b): Consider first the case when $K_{\mathrm{fp}} \neq 0$. Let $a = (K_{\mathrm{fv}} - K_{\mathrm{fp}}K_{\mathrm{pv}})$. Then $K_{\mathrm{pv}} = (K_{\mathrm{fv}} - a)/K_{\mathrm{fp}}$. Substituting for K_{pv} in Eq. (21) yields

$$K_{\mathrm{fv}}^2 + \left[\frac{(K_{\mathrm{fv}} - a)^2}{K_{\mathrm{fp}}^2}\right] + K_{\mathrm{fp}}^2 - 2K_{\mathrm{fv}}(K_{\mathrm{fv}} - a) - 1 = 0$$

Multiplying through by K_{fp}^2

$$K_{\mathrm{fv}}^2K_{\mathrm{fp}}^2 + K_{\mathrm{fv}}^2 + a^2 - 2aK_{\mathrm{fv}} + (K_{\mathrm{fp}}^2)^2 - 2K_{\mathrm{fv}}^2K_{\mathrm{fp}}^2 + 2aK_{\mathrm{fv}}K_{\mathrm{fp}}^2 - K_{\mathrm{fp}}^2 = 0$$

Re-arranging terms yields

$$a^2 + 2aK_{\mathrm{fv}}(K_{\mathrm{fp}}^2 - 1) + (1 - K_{\mathrm{fp}}^2)(K_{\mathrm{fv}}^2 - K_{\mathrm{fp}}^2) = 0$$
$$-a^2 + 2aK_{\mathrm{fv}}(1 - K_{\mathrm{fp}}^2) = (1 - K_{\mathrm{fp}}^2)(K_{\mathrm{fv}}^2 - K_{\mathrm{fp}}^2)$$

The right-hand side of this equation is strictly positive, whenever $K_{\mathrm{fv}}^2 > K_{\mathrm{fp}}^2$ and $K_{\mathrm{fp}}^2 < 1$. Since the left-hand side of this equation must also be positive, it follows that

$$a < (>)\ 0, \quad \text{if } K_{\mathrm{fv}} < (>)\ 0$$

Suppose now $K_{\mathrm{fp}} = 0$. Multiplying a by $2K_{\mathrm{fv}}$ we get $2K_{\mathrm{fv}}a = 2K_{\mathrm{fv}}^2 - 2K_{\mathrm{fv}}K_{\mathrm{pv}}K_{\mathrm{fp}}$.

Using Eq. (21) and the assumption $K_{\text{fp}} = 0$, we have the following:

$$K_{\text{fv}}a = K_{\text{fv}}^2 - (K_{\text{pv}}^2 + K_{\text{fp}}^2 - 1) = K_{\text{fv}}^2 - (K_{\text{pv}}^2 - 1) > 0$$

It follows that if $K_{\text{fv}} < (>, =)\, 0$, $a < (>, =)\, 0$.

Part (c): Consider first the case when $K_{\text{fp}} = 1$. This leads $a = (K_{\text{fv}} - K_{\text{pv}})$. By the spanning condition (21), we have $(K_{\text{fv}} - K_{\text{pv}})^2 = 0$. Hence, $a = 0$. Now consider the case when $K_{\text{fp}} = -1$. This leads $a = (K_{\text{fv}} + K_{\text{pv}})$. By the spanning condition (21), we have $(K_{\text{fv}} + K_{\text{pv}})^2 = 0$. Hence, $a = 0$.

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