Factor Income Taxation, Growth, and Investment Specific Technological Change*

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Abstract

Why do countries with different tax arrangements exhibit the same growth rate? We refer to this as a growth-tax puzzle. To explain the puzzle, we construct a tractable endogenous growth model with endogenous investment specific technological change (ISTC). Public and private capital stock externalities are assumed to augment ISTC. A specialized labor input exerts a positive externality in final good production. Our primary interest is to highlight the role of such externalities in explaining the puzzle. We show that the competitive equilibrium growth rate can be decomposed into a labor factor and a capital factor. Changes in factor income taxes, by affecting these factors, can have opposing effects leading to constancy in growth. Our model builds on the existing endogenous growth literature by providing an alternative, but compatible explanation for the offsetting growth effects of fiscal policy on growth observed in the data.

Keywords : Endogenous Investment Specific Technological Change, Factor Income Taxation, Endogenous Growth Theory, Fiscal Policy.

JEL Codes: E2; E6; H2; O4

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1 Introduction

Why do countries with different factor income tax combinations exhibit similar growth rates? In this paper, we develop an endogenous growth model with endogenous investment specific technological change to understand this question.

Figure 1 plots the average aggregate annual real GDP growth rate from 1990 to 2007 against the factor income tax ratio for several advanced economies.[1] Average growth for all countries (excluding Ireland) falls between 0.875% and 2.462%. The standard deviation of the average real GDP growth rates is low at 0.878 (excluding Ireland, the standard deviation is 0.4756). Figure 2 plots the range of individual factor income taxes for these countries where the tax on capital and labor income have been averaged over 1990–2007. What is striking is that the range in the ratios of the average capital income tax rate to the average labor income tax rate in these economies is much more pronounced: 0.3951 to 1.725.[2] Also whereas the difference between factor income taxes is large in some countries, it is quite small in others.[3] Figure 1 and Figure 2 suggest that countries with almost similar growth rates are accompanied by totally different factor income tax combinations.

[Insert Figure 1 and 2]

Figure 3 plots the levels of factor income tax rates across the G7 countries. The incidence of factor income taxation is quite disparate. In the US, UK, Canada, and Japan, the tax on capital income is greater than the tax on labor income. In contrast, for Germany, Italy, and France, the reverse is true.

[Insert Figure 3]

In other evidence, Jones (1995) also shows in a sample of 15 OECD countries from 1950 to 1987, that changes in investment rates do not have any significant long run growth effects.

[1] The growth rates are calculated from the OECD (2012) database: see Table ($VXVOB$). The countries are: Austria (AUS), Belgium (BEL), Canada (CAN), Denmark (DEN), Finland (FIN), France (FRA), Germany (GER), Greece (GRE), Ireland (IRE), Italy (ITA), Japan (JPN), Netherlands (NET), Portugal (PRT), Spain (SP), Sweden (SWE), United Kingdom (UK) and United States of America (USA). The base year is 2000

[2] Canada and Japan have data on capital and labor income tax estimates based on the approach used in Mendoza et al. (1994) and Trabandt and Uhlig (2009) from 1965 to 1996. For Germany, United Kingdom and United States of America, data is from 1965 to 2007. For France, the data is from 1970 to 2007. For Italy, the data is from 1980 to 2007. For Austria, Belgium, Denmark, Finland, Netherlands, Portugal and Sweden, the data is from from 1995 to 2007. For Spain and Greece, the data is from 2000 to 2007. Finally, for Ireland, the data is from 2002 to 2007.

[3] The data on factor income taxes are from Mendoza et al. (1994) and Trabandt and Uhlig (2009). The latter have used the approach in Mendoza et al. (1994) to estimate the tax rates for 17 OECD nations till 2007.

He shows that shocks to investments – both total and durables and in particular durable equipment – have only a short-run growth effect with no significant effect on long run growth.

Figures 1 - 3 and the evidence from Jones (1995) are suggestive of a "growth-tax" puzzle since countries with different factor income tax combinations exhibiting similar growth rates is incompatible with a standard model of endogenous growth.[4] The standard endogenous (AK) growth model predicts that fiscal policy has a large growth effect through its impact on the economy's investment rate. Taken to the data, these models would predict a high correlation between the investment rate and the growth rate. The above evidence therefore suggests that changes in fiscal policy (or factor income taxes) must have offsetting changes in investments such that growth rates do not change.

The literature has tried to find extensions to the standard endogenous growth model that can explain the apparent absence of growth effects of fiscal policy. McGrattan (1998) develops a theoretical framework where government policy can be incorporated into a standard AK growth model by incorporating two types of capital: structures and equipment capital. She shows that the equilibrium growth rate depends on the investment rate and the capital-output ratio. The reason why fiscal policy has no growth effects is because its effect on the investment rate is offset by the effect of fiscal policy on the capital-output ratio. Because of these offsetting effects, total investment does not change that much. Jaimovich and Rebelo (2012) show that changes in tax rates can have non-linear effects on long-run output growth. To capture this non-linearity, they construct a model where low tax rates have negligible effects on growth but when disincentives to invest are large, larger tax rates have a strong negative effect on output growth. The mechanism in their model is based on a skewed distribution of agents between workers and innovators, which results in a small number of highly productive workers in equilibrium. In a related literature, Glomm and Ravikumar (1998) build a growth model where public education spending, financed by distortionary taxes affect human capital accumulation. Again, they find that despite being distortionary in nature, tax rates have negligible effects on growth rates.

1.1 Description of our model and main results

We provide an alternative, but compatible, explanation for the above growth-tax puzzle, i.e., the fact that different combinations of factor income taxes can generate the same growth rate. We construct an endogenous growth model with endogenous investment specific technological change with three types of externalities: (a) an externality from the stock of private, (b) an externality from public capital in the process of innovation; and (c) an externality from

[4] Stokey and Rebelo (1995) also show in a numerical exercise that big changes in tax on capital income (up to the order 30%) do not have large growth effects on the US economy.

labor allocated to research in final good production. Investment specific technological change refers to technological change which reduces the real price of capital goods. Specifically, the public capital stock – financed by distortionary taxes – and the private capital stock augment investment specific technological change (ISTC) as a positive externality.[5] Typically in the literature, the public input is seen as directly affecting final production directly either as a stock or a flow (e.g., see Futagami, Morita, and Shibata (1993), Chen (2006), Fischer and Turnovsky (1997, 1998), and Eicher and Turnovsky (2000)). We show that embedding varying magnitudes of these externalities into a model of endogenous growth with endogenous ISTC leads to offsetting effects of factor income taxes on growth. To the best of our knowledge, we are not aware of any paper in the literature in which public capital affects ISTC.[6]

Our basic model follows Huffman (2008).[7] There are two sectors in the model: a final goods sector and a research sector. The final good sector produces a final good, using private capital and labor. Labor supply is composite in the sense that one type of labor activity is devoted to final good production, and the other to research which directly reduces the real price of capital goods in the next period. The second sector (the research sector) captures the effect of public capital and private capital stock spillovers and research activity on reducing the real price of capital goods. We assume two types of labor activities: one type is labor allocated for final goods production, or current production, and another type is labor allocated for enhancing investment specific technological change, or future capital accumulation, and therefore future production. While agents supply aggregate labor, firms optimally choose each labor activity. Crucially, in our model, however, firms might not be aware that their allocation of labor towards research also influences productivity of the current period's final goods production. Therefore, although research labor allocation is

[5] Our setup *also* allows investment specific technological change to enhance the accumulation of public capital. For instance, providing better infrastructure today reduces the cost of providing public capital in the future.

[6] In a different context, Harrison and Weder (2000) build a two sector representative agent model with increasing returns to scale driven by externalities that come from sector specific as well as aggregate economic activity. Benhabib and Farmer (1996) show that small empirically plausible external effects lead to indeterminacy. Neither of these papers has a role for public capital. Lloyd-Braga, Modesto, and Seegumuller (2008) introduce positive government spending externalities in preferences. In our model, externalities from the public stock influence ISTC directly.

[7] A growing literature has attributed the importance of investment specific technological change to long run growth (see Greenwood et al. (1997, 2000); Whelan (2003)). Greenwood et al. (1997, 2000) show that once the falling price of real capital goods is taken into account, this explains most of the observed growth in output in the US, with relatively little being left over to be explained by total factor productivity. Kamber et al. (2015) build a NK DSGE model with ISTC shocks to investigate the role and transmission mechanism of such shocks in the presence of financial frictions. Their main finding is that the introduction of financial frictions in the form of a collateral constraint materially alters which shocks are thought to be the most important drivers of the US business cycle.

done from the point of future capital accumulation and hence future output, we assume that firms might be unaware of the spillover it has on current production. This implies that the process of augmenting knowledge - which is designed to influence the price of capital in the future - may affect present output too. Effectively, this means that the process of augmenting knowledge may make routine labor (in the final goods sector) more effective.

The planner maximizes the utility of the representative agent and internalizes the externalities in the research sector and final good sector. In the planner's problem, we assume that public investment is financed by a *fixed* proportional income tax as in Barro (1990). Given a fixed tax rate, the planner's problem yields the socially efficient allocation. *Corresponding to this allocation*, we characterize the steady state balanced growth path and show that the growth rate depends on two factors: 1) a labor input devoted to research (the labor factor) and 2) the contribution to growth from public and private capital (the capital factor).

We then ask under what conditions can the planner's allocations be replicated by the competitive decentralized equilibrium with identical and different factor income taxes. We assume that public investment is financed by distortionary factor income taxes on capital and labor income. Our main result is summarized in Proposition 1 which states that under an intuitive sufficient condition, the growth rate corresponding to the efficient allocation can be replicated in the competitive equilibrium by a combination of capital income tax rates and labor income tax rates. In particular, Proposition 1 shows that raising the labor income tax and/or reductions in the capital income tax implement a higher planner's growth rate if the sufficient condition is satisfied. The expressions for the capital and labor factors - which are in closed form - allows us to see how multiple factor income tax combinations - and therefore factor income tax gaps - can implement a given planner's growth rate. In particular, an increase in the capital income tax reduces the capital factor, and *reduces* growth. However, an increase in the labor income tax exerts both offsetting income and substitution effects. We show that with ISTC, the income effect is stronger than the substitution effect, and so increases in the labor income tax *increase* labor supply. The increase in labor supply increases the labor factor (which is essentially research-labor input) which increases capital accumulation and growth. We also show that the strength of the income effect becomes stronger the larger the importance of research-labor input on ISTC. Hence, the competitive equilibrium replicates the planner's growth rate, either by an increase in the labor income tax, or a reduction in the capital income tax, or some combination of both. Proposition 1 is therefore consistent with the empirical evidence documented in Figures 1 - 3. In a numerical section we show that for a fixed set of parameters a wide range of tax rates imply the same growth rate.[8]

[8] Our definition of indeterminacy is as follows: there is no unique combination of factor income taxes on

How do the externalities affect the factor income tax gaps that implement the planner's allocations? We first consider the case of a positive spillover from the specialized research labor activity on final good production. In this case, an increase in the spillover increases the planner's allocation towards specialized labor. This is because research labor has a positive effect on final good production over and above its effect on ISTC. This increases the growth rate corresponding to the socially efficient allocation. To implement this higher growth rate, this requires an increase in the labor income tax, which raises the labor factor from the competitive growth rate, or a reduction in the capital income tax, which raises the capital factor. Implementing either leads to a widening of the equilibrium factor income tax gap.[9]

In contrast, when the weight on the positive spillover from the public and private capital stock falls, this leads to a higher weight on the existing stock of ISTC. That is, a lower weight on the stock externalities implies that the weight on the persistence of ISTC is higher since the weights sum to one. More persistent ISTC leads to a higher planner's growth rate. To raise the competitive equilibrium growth rate, as before, a reduction in the tax on capital income that raises the capital factor and/or an increase in the labor income tax that raises the labor factor is required. Such a policy increases the factor income tax gap and implements the planner's growth rate.

Our general result is that to the extent that spillovers from a specialized labor input and the public and private capital stocks exist, an increase in these spillovers from the specialized labor input, and a decrease in the spillover from public and private capital, increase the planner's growth rate, and therefore increase the factor income tax gap required to implement the growth rate corresponding to the efficient allocation. Conversely, for a given level of externalities, maintaining the constancy of growth also requires different combinations of factor income taxes as in McGrattan (1998). We also show that when there are no externalities, *equal* factor income taxes always yield the optimal growth rate from the planner's problem. Hence, the factor income tax gap is zero.

Finally, we also conduct a simple numerical exercise to show that equilibrium factor income taxes generated by our model are in accordance with Figures 1 - 3. As mentioned above, under an intuitive sufficient condition, we are able to analytically characterize replicating the growth rate corresponding to the efficient allocation. We consider two sets of policy experiments: one where the sufficient condition holds and another where the condition is violated.

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capital and labor income that replicates the planner's growth rate for a fixed set of parameters. Indeterminacy obtains because the planner's allocations yield a constant growth rate, and factor income taxes have offsetting effects on the capital factor and labor factor.

[9] Using a Pissarides type search model, Michaelis and Birk (2006) show that a revenue neutral shift from the tax on capital income to the "payroll tax" increases both employment and growth. In fact, they also show that with a larger inter-temporal elasticity of substitution, a revenue-neutral shift from a capital income tax to a wage income tax unambiguously increases employment and growth.

Our main result is to numerically show that for a fixed set of deep parameters, a wide range of tax rates implement the same growth rate when vary the externality parameters,.

1.1.1 Empirical Evidence on Externalities

Private and Public Capital With respect to the private capital stock, DeLong and Summers (1991) show that investment in machinery is associated with very strong positive externalities, and that increases in investments in equipment implies higher growth. Hamilton and Monteagudo (1998) find that capital is associated with positive external effects in an estimated Solow growth model. Greenwood et al. (1997), show that the real price of capital equipment in the US – since 1950 – has fallen alongside a rise in the investment-GNP ratio. This suggests that the private capital stock exhibits a positive externality in investment specific technological change through the aggregate capital stock. Importantly, Greenwood et al. (1997, p. 342) say: "The negative co-movement between price and quantity.....can be interpreted as evidence that there has been significant technological change in the production of new equipment. Technological advances have made equipment less expensive, triggering increases in the accumulation of equipment both in the short and long run."

With respect to the nexus between public expenditures, R&D, and growth, Griliches (1979) examines how the indirect effects of research and development affect future output through induced changes in factor inputs. In his model, the accumulation of private capital is driven by the aggregate stock of knowledge and current and past stocks of research and development (R&D). Scott (1984) and Levin and Reiss (1984) estimate that the high spillovers from federal research and development spending dominates the crowding-out effect it has on private spending on R&D. The net effect is that public spending has a positive effect on productivity. Finally, David et al. (2000), show that public R&D spending is complementary to private R&D spending.

Specialized research labor In the high-tech manufacturing sector, Davidson (2012) documents evidence on the extent to which skills required for advanced manufacturing jobs. He argues that skilled factory workers these days are typically "hybrid-workers": they are both machinists (engaging in final good production) as well as computer programmers (engaging in research). In the US metal-fabricating sector, workers not only use cutting tools to shape a raw piece of metal, but they also write the computer code that instructs the machine to increase the speed of such operations. Globerman (1975) describes a class of machinists in the manufacturing sector called "tool and die makers", or also "mold makers" (see Bryce (1997)). The machinist receives on-the-job training which enables him to work with machines and computers, which makes him multi-skilled. Even though on-the-job training is costly, Park

(1996) shows, from an empirical study on manufacturing industries in Korea that employing "multi-skilled workers" makes a firm's production more efficient in comparison to employing "single-skilled "or specialized workers to handle each individual activity.[10]

Given this, we assume that the specialized labor input which is allocated to augment future output in the research sector exerts a positive externality in the current period's production of the final good. Other examples that support this assumption are outcomes of long-term research projects undertaken by firms – in say the pharmaceutical (drug research) or the IT (software development) sectors – which may only be realized in a *future* time period. The time allocated towards future research activities however may help improve the productivity of *current* period's production, although the spillover on current period's production may not be realized by firms.[11] In other words, on the job training is undertaken for future benefits but it may also augment the efficiency of standard labor that has been assigned to produce output in the current period. We feel that this link has been ignored by the literature.

1.1.2 Related Literature

The setup of our model is technically similar to Huffman (2007, 2008) who explicitly models the mechanism by which the real price of capital falls when investment specific technological change occurs. Our model however is closer to Huffman (2008) rather than Huffman (2007). Huffman (2008) builds a neoclassical growth model with investment specific technological change. Labor is used in research activities in order to increase investment specific technological change. In particular, the changing relative price of capital is driven by research activity, undertaken by labor effort. Higher research spending in one period lowers the cost of producing the capital good in the next period.[12] Investment specific technological change is thus endogenous in the model, since employment can either be undertaken in a research sector or a production sector. His model includes capital taxes, labor taxes, and investment subsidies that are used to finance a lump-sum transfer. Huffman (2008) finds that a positive capital tax that is larger than a positive investment subsidy along with zero labor tax can replicate the first best allocation. Huffman's models however do not incorporate public

[10] Even though labor productivity in final good production is typically seen to be a function of the stock of knowledge (and therefore the externality comes from the level of ISTC), we assume that there is no difference in skills and ability in the labor force in the two productive activities, so that labor allocated to research is not an exact proxy for the stock of knowledge.

[11] Primarily a skilled artisan, a tool and die maker works in an industrial environment where producing the final good requires two different skills – creative skills and machine knowledge (such as engineering drawing). Another example is research and teaching by faculty. Presumably, better research improves teaching. Better teaching also augments future research. Hence there is a dynamic feedback.

[12] Krusell (1998) also builds a model in which the decline in the relative price of equipment capital is a result of R&D decisions at the level of private firms.

capital - a feature we show that is important in explaining the growth-tax puzzle in our paper.

Our paper is also related to the literature on fiscal policy and long run growth in the neoclassical framework. The literature started by Barro (1990) and Futagami, Morita, and Shibata (1993) – incorporate a public input – such as public infrastructure – that directly augments production. In Barro (1990), public services are a flow; while in Futagami, Morita, and Shibata (1993), public capital accumulates. However, in the large literature on public capital and its impact on growth spawned by these papers, the public input, whether it is modeled as a flow or a stock, doesn't directly influence the real price of capital goods.[13] Since public capital affects the real price of capital explicitly in our model, this means that the public input affects future output through its effect on both future investment specific technological change, as well as future private capital accumulation.

Finally, in addition to labor time deployed by the representative firm towards R&D, the public capital stock, G, plays a crucial role in lowering the price of capital accumulation. Typically the public input is seen as directly affecting final production – either as a stock or a flow (e.g., see Futagami, Morita, and Shibata (1993), Chen (2006), Fischer and Turnovsky (1997, 1998), and Eicher and Turnovsky (2000), and Agénor (2007 and 2011)). Instead, here we assume that the public input facilitates investment specific technological change. This means that the public input affects future output through future private capital accumulation directly. In the above literature, the public input affects current output directly. This is our point of departure. We therefore formalize the link between fiscal policy and growth *through the effect that* fiscal policy has on ISTC.

2 The Model

Consider an economy that is populated by identical infinitely lived agents with unit mass, who at each period t, derive utility from consumption of the final good C_t and leisure $(1-n_t)$. There is no population growth which implies that aggregate variables are also per-capita variables. The term n_t represents the fraction of time spent at time t in employment. The discounted life-time utility, U, of an infinitely lived representative agent is given by

$$U = \sum_{t=0}^{\infty} \beta^t [\log C_t + \log(1 - n_t)]. \tag{1}$$

[13] For instance, in Ott and Turnovsky (2006) - who use the flow of public services to model the public input - and Chen (2006), Fischer and Turnovsky (1998) - who use stock of public capital - the shadow price of private capital is a function of public and private capital.

where $\beta \in (0, 1)$ denotes the period-wise discount factor. The total supply of labor for the agent at any time t is given by n_t such that

$$n_t \equiv n_{1t} + n_{2t}, \tag{2}$$

where n_{1t} is labor allocated for final goods production, or current production, and n_{2t} is labor allocated for enhancing investment specific technological change, or future capital accumulation, and therefore future production.[14] Therefore, although n_{2t} is employed from the point of future capital accumulation and hence future output the agent is unaware of the spillover it has on current production.

The final good is therefore produced by a neoclassical production function with capital K_t, n_{1t}, and n_{2t}. An important point is that the planner internalizes the effect of n_{2t} on final goods production, while the agent will not. The production function is given by

$$Y_t = \underline{A} K_t^{\alpha} n_{1t}^{1-\alpha} \left(n_{2t}^{1-\alpha}\right)^{\xi} \tag{3}$$

where $\underline{A} > 0$ is a scalar that denotes the exogenous level of productivity, $\alpha \in (0, 1)$ is the share of output paid to capital and $\xi > 0$ is the externality parameter capturing the effect that n_2 has on direct production. When $\xi > 0$, the planner internalizes the effect that n_2 has on direct production. When $\xi = 0$, there is no externality from n_2 on the production of the final good. Note, in this framework, as in Huffman (2008) the two labor activities n_{1t} and n_{2t} are assumed to be equally skilled, but are optimally allocated across different activities by households.[15]

Private capital accumulation grows according to the standard law of motion augmented by investment specific technological change,

$$K_{t+1} = (1 - \delta)K_t + I_t Z_t, \tag{4}$$

where $\delta \in [0, 1]$ denotes the rate of depreciation of capital and I_t represents the amount of total output allocated towards private investment at time period t. We assume that, $\delta = 1$, to keep the model tractable. Z_t represents investment-specific technological change. The

[14] As we will discuss later, in the competitive decentralized equilibrium, households supply n_t which is then optimally allocated between n_{1t} and n_{2t} by the firm. Crucially, firms are not aware that this allocation of labor towards n_{2t} influences the current period's final goods production. We show our set-up in Figure 4. This assumption is motivated by the empirical evidence on "multi-skilled" workers discussed in the introduction.

[15] Other papers in the literature - such as Reis (2011) - also assume two types of labor affecting production. In Reis (2011), one form of labor is the standard labor input, while the other labor input is entrepreneurial labor.

higher the value of Z_t, the lower is the cost of accumulating capital in the future. Hence Z_t can also be viewed as the inverse of the price of per-unit private capital at time period t. The term, $I_t Z_t$, therefore represents the effective amount of investment driving capital accumulation in time period $t+1$.

We assume that in every period, public investment is funded by total tax revenue. Public capital therefore evolves according to

$$G_{t+1} = (1-\delta)G_t + I_t^g Z_t, \tag{5}$$

where G_{t+1} denotes the public capital stock in $t+1$, and I_t^g denotes the level of public investment made by the government in time period t:

$$I_t^g = \tau Y_t, \tag{6}$$

where $\tau \in (0,1)$ is the proportional tax rate.[16] We assume that Z_t augments I_t^g in the same way as I_t since it enables us to analyze the joint endogeneity of Z and G. To derive the balanced growth path, we further assume that the period wise depreciation rate $\delta \in [0,1]$ is same for both private capital and public capital.

2.1 Investment Specific Technological Change

To capture the effect of public capital on research and development, we assume that Z grows according to the following law of motion,

$$Z_{t+1} = B{n_{2t}}^{\theta} Z_t^{\gamma} \left\{ \left(\frac{G_t}{Y_{t-1}}\right)^{\mu} \left(\frac{K_t}{Y_{t-1}}\right)^{1-\mu} \right\}^{1-\gamma}. \tag{7}$$

Here, $B > 0$ stands for an exogenously fixed scale productivity parameter and $\mu \in (0,1)$ captures the impact of public investments on investment specific technological change. We assume that the parameters, $\theta \in (0,1)$ and $\gamma \in (0,1)$, where θ stands for the weight attached to research effort and γ is the level of persistence the current year's level of technology has on reducing the price of capital accumulation in the future.[17] The term $\frac{G_t}{Y_{t-1}}$ represents the externality from public capital in enhancing investment specific technological change in time period $t+1$. The aggregate capital-output ratio, $\frac{K_t}{Y_{t-1}}$, is also assumed to exert a

[16] Since $\delta = 1$, equation (5) implies that $G_{t+1} = I_t^g Z_t$, i.e., the ISTC adjusted public investment (flow) at period t equals the public capital stock in $t+1$.

[17] This contrasts with Huffman (2008) where $\gamma = 1$ is required for growth rates of Z and output to be along the balanced growth path. We require $\gamma \in (0,1)$ for the equilibrium growth rate to adjust to the steady state balanced growth path.

positive externality effect on investment specific technological change. In particular, a higher aggregate stock of capital in t, K_t, relative to Y_{t-1}, raises Z_{t+1}. Like the externality from n_2, the planner internalizes the effect that stock of public capital and private capital has on investment specific technological change, while agents treat the effect of $\frac{G_t}{Y_{t-1}}$ and $\frac{K_t}{Y_{t-1}}$ on $Z_{t+1}-$ the bracketed term – as given. Our assumption of $\frac{G_t}{Y_{t-1}}$ augmenting Z_{t+1} is for two reasons. First, if G_t augmented output Y_t instead, we can show that in equilibrium, the only possible balanced growth path is when the gross growth rate of all endogenous variables is 1 that is, all variables are at their steady state. This means, public capital will not affect the growth rate. Hence, allowing for ISTC to depend on the public input enables the balanced growth path to be affected by tax policy *through* ISTC. Second, if Z_{t+1} was instead parametrized as

$$Z_{t+1} = B{n_{2t}}^{\theta} Z_t^{\gamma} \left\{ G_t^{\mu} K_t^{1-\mu} \right\}^{1-\gamma},$$

i.e., G and K are not normalized by Y, the growth rate of Z will never converge to a balanced growth path. Note that when $\gamma = 1, \theta = 0$, ISTC is exogenous.

2.2 The Planner's Problem

We first solve the planner's problem who internalizes all the externalities. This yields the socially efficient allocation for a fixed tax rate. This is not a "full blown" planner's problem since the planner takes the fixed tax rate as given. This is equivalent to a constrained planning problem, an approach that is common in the literature.[18]

The aggregate resource constraint the economy faces in each time period t is given by

$$C_t + I_t \equiv Y_t(1-\tau) = \underline{A} K_t^{\alpha} n_{1t}^{1-\alpha} \left(n_{2t}^{1-\alpha} \right)^{\xi} (1-\tau) \tag{8}$$

where agents consume C_t at time period t and invest I_t at time period t. Aggregate consumption and investment add up to after-tax levels of output, $Y_t(1-\tau)$, where $\tau \in [0,1]$ is the proportional tax rate that is assumed to be fixed in every time period.

Since the planner internalizes the size of public expenditure given by

$$\frac{G_{t+1}}{Y_t} = \tau Z_t, \tag{9}$$

[18] We justify this assumption because of the main goal of our paper: to explain roughly similar growth rates with positive and varying factor income taxes in the data, as in Figures 1 - 3. While we don't show this here, the competitive equilibrium growth rate always falls short of the (unconstrained) first best growth rate. However, as we will see later, we can implement the growth rate corresponding to the constrained planner's problem by allowing the planner to tax factor incomes differentially. Differential taxes allows the planner to correct for the under-provision of private inputs in the competitive equilibrium.

which follows from (5) and (6) after imposing $\delta = 1$, he takes the following law of motion of ISTC as a restriction:

$$Z_{t+1} = B{n_{2t}}^{\theta} Z_t^{\gamma} Z_{t-1}^{(1-\gamma)\mu} \tau^{\mu(1-\gamma)} \left(\frac{K_t}{Y_{t-1}}\right)^{(1-\mu)(1-\gamma)}, \tag{10}$$

which is obtained by substituting (9) in (7).

To obtain the efficient allocation, the planner maximizes the lifetime utility of the representative agent – given by (1) – subject to the economy wide resource constraint given by (8), the law of motion (4), the equation describing investment specific technological change (10) and the identity for total supply of labor given by (2).[19]

2.2.1 First Order Conditions

The Lagrangian for the planner's problem is given by,

$$\begin{aligned} \mathcal{L} \;=\; & \sum_{t=0}^{\infty} \beta^t \left[\log C_t + \log(1 - n_{1t} - n_{2t})\right] + \sum_{t=0}^{\infty} \beta^t \lambda_{1t} \left[\underline{A} K_t^{\alpha} n_{1t}^{1-\alpha} \left(n_{2t}^{1-\alpha}\right)^{\xi} (1-\tau) - C_t - \frac{K_{t+1}}{Z_t}\right] \\ & + \sum_{t=0}^{\infty} \beta^t \lambda_{2t} \left[B{n_{2t}}^{\theta} Z_t^{\gamma} Z_{t-1}^{(1-\gamma)\mu} \tau^{\mu(1-\gamma)} \left(\frac{K_t}{Y_{t-1}}\right)^{(1-\mu)(1-\gamma)} - Z_{t+1}\right]. \end{aligned}$$

where λ_{1t} and λ_{2t} are the Lagrangian multipliers. Because our focus is on the balanced growth path corresponding to the efficient allocation, we assume that $\delta = 1$[20].

The following first order conditions obtain with respect to C_t, K_{t+1}, n_{1t}, and n_{2t}, respectively[21]:

$$\{C_t\} : \tfrac{1}{C_t} = \lambda_{1t} \tag{11}$$

$$\{K_{t+1}\} : \frac{1}{C_t Z_t} = \frac{\alpha\beta Y_{t+1}(1-\tau)}{C_{t+1}K_{t+1}} + \beta(1-\gamma)(1-\mu)\lambda_{2t+1}\frac{Z_{t+2}}{K_{t+1}} - \beta^2\lambda_{2t+2}(1-\gamma)\alpha\frac{Z_{t+3}}{K_{t+1}} \tag{12}$$

$$\{Z_{t+1}\} : \lambda_{2t} = \beta\lambda_{2t+1}\gamma\frac{Z_{t+2}}{Z_{t+1}} + \frac{\beta}{Z_{t+1}}\left(\frac{I_{t+1}}{C_{t+1}}\right) + \beta^2\lambda_{2t+2}\mu(1-\gamma)\frac{Z_{t+3}}{Z_{t+1}} \tag{13}$$

$$\{n_{1t}\} : \frac{1}{1-n_t} = \frac{(1-\alpha)Y_t(1-\tau)}{C_t n_{1t}} - \beta\lambda_{2t+1}(1-\gamma)(1-\alpha)\frac{Z_{t+2}}{n_{1t}} \tag{14}$$

[19] Clearly, $C_t + I_t + I_t^g = Y_t$.

[20] We assume $\delta = 1$ to obtain closed for solutions and for analytical tractability. In Appendix E we show that our main results are unchanged with $\delta < 1$.

[21] See Appendix A for derivations.

and,

$$\{n_{2t}\}: \frac{1}{1-n_t} = \frac{(1-\alpha)\xi Y_t\,(1-\tau)}{C_t n_{2t}} + \lambda_{2t}\theta\frac{Z_{t+1}}{n_{2t}} - \beta\lambda_{2t+1}(1-\gamma)\xi\,(1-\alpha)\,\frac{Z_{t+2}}{n_{2t}}. \tag{15}$$

Equation (11) represents the standard first order condition for consumption, equating the marginal utility of consumption to the shadow price of wealth. Equation (12) is an augmented form of the standard Euler equation governing the consumption-savings decision of the household. Equation (13) is the Euler equation with respect to Z_{t+1}. Equation (14) denotes the optimization condition with respect to labor supply (n_{1t}). Since $0 < \gamma < 1$, the second term in the RHS is positive which constitutes a reduction in the marginal utility of leisure. This reduces n_1 relative to the standard case in which there is no investment specific technological change. Finally, equation (15) is the first order condition with respect to n_{2t}.

2.2.2 Decision Rules

We now derive the closed form decision rules based on the above first order conditions using the method of undetermined coefficients, as shown in the following Lemma 1.

Lemma 1 *C_t, I_t, n_t, n_{1t}, n_{2t} are given by (16), (17), (18), where $0 < \Phi < 1$ is given by (19), and $0 < x < 1$ given by (20) is a constant.*

$$C_t = \Phi_P Y_t(1-\tau), I_t = (1-\Phi_P)Y_t(1-\tau) \tag{16}$$

$$n_t = n_P = \frac{(1-\alpha)[(1-\beta\gamma) - \beta^2\mu(1-\gamma) - \beta^2(1-\gamma)(1-\Phi_P)]}{(1-\alpha)[(1-\beta\gamma) - \beta^2\mu(1-\gamma) - \beta^2(1-\gamma)(1-\Phi_P)] + \Phi_P x_P\left[1-\beta\gamma-\beta^2\mu(1-\gamma)\right]}, \tag{17}$$

$$n_{1P} = x_P n_P, n_{2P} = (1-x_P)n_P, \tag{18}$$

where Φ_P is given by

$$\Phi_P = 1 - \frac{\alpha\beta\left[(1-\beta\gamma) - \beta^2\mu(1-\gamma)\right]}{(1-\beta\gamma) - \beta^2(1-\gamma) + \alpha\beta^3\,(1-\gamma)}, \tag{19}$$

and x_P is given by

$$x_P = \frac{(1-\alpha)\{(1-\beta\gamma) - \beta^2\mu(1-\gamma) - \beta^2(1-\gamma)(1-\Phi_P)\}}{(1+\xi)(1-\alpha)\{(1-\beta\gamma) - \beta^2\mu(1-\gamma) - \beta^2(1-\gamma)(1-\Phi_P)\} + \beta\theta(1-\Phi_P)}. \tag{20}$$

Proof. See Appendix A for derivations. ■

2.2.3 The Balanced Growth Path

We can obtain the balanced growth path (BGP) corresponding to the efficient allocation - and a fixed tax rate - by substituting (16), (17), (18), (19), and (20) into (7). Define $\widehat{M_P}$ a constant as

$$\widehat{M_P} = B((1-x_P)n_P)^{\theta}(1-\Phi_P)^{(1-\mu)(1-\gamma)}. \tag{21}$$

Given the assumptions it is easy to show that we can obtain a constant growth rate for Z, K, G and Y. This condition necessarily implies $0 < \Phi_P$, x_P, $n_P < 1$ which always holds true. We therefore have the following Lemma 2.

Lemma 2 *On the steady state balanced growth path, the gross growth rate of Z, K, G and Y are given by (22), and (23)*[22]

$$\widehat{g_{z_P}} = [\widehat{M_P}\{(\tau)^{\mu}(1-\tau)^{1-\mu}\}^{(1-\gamma)}]^{\frac{1}{2-\gamma}} \tag{22}$$

$$\widehat{g_{k_P}} = \widehat{g_{g_P}} = \widehat{g_{z_P}}^{\frac{1}{1-\alpha}}, \widehat{g_{y_P}} = \widehat{g_{k_P}}^{\alpha} = \widehat{g_{z_P}}^{\frac{\alpha}{1-\alpha}}. \tag{23}$$

There are several aspects of the equilibrium growth rate worth mentioning.[23] First, the growth rate corresponding to the socially efficient allocation is independent of the technology parameter, $\underline{A}$, but not B, as in Huffman (2008). Second, the growth rate of output, $\widehat{g_{y_P}}$, is less than $\widehat{g_{k_P}}$ along the balanced growth path because equation (7) is homogenous of degree $1+\theta$. Lemma (2) therefore clearly establishes that the effect of the stock of public capital on Z affects not just marginal productivity of factor inputs but also growth rate at the balanced growth path.

Finally, from (22), the tax rate exerts a positive effect on growth as well as a negative effect. This is similar to the equation characterizing the growth maximizing tax rate in models with public capital. The mechanism here is however different. For small values of the tax rate, a rise in τ leads to higher public capital relative to output, Y_{t-1}. This raises the future value of ISTC. An increase in ISTC reduces the real price of capital, stimulating investment and long run growth. However, for higher tax rates, further increases in the tax

[22] See Bishnu, Ghate and Gopalakrishnan (2011).

[23] With $\delta < 1$, the expression for, $\widehat{g_{z_P}}$, is given by

$$\widehat{g_{z_P}} = \left\{Bn_2^{\theta}\left[(\tau\Delta_1)^{\mu}(\chi_4(1-\tau))^{1-\mu}\right]^{1-\gamma}\right\}^{\frac{1}{2-\gamma}},$$

where Δ_1 and χ_4 are constants. The form is therefore identical to (22). The growth rankings implied by (23) also remain unchanged with $\delta < 1$. See Appendix E.

rate depresses after tax income, and investment. This reduces G relative to Y, lowering Z, and depressing investment and long run growth. Hence, there is a unique growth maximizing tax rate although the planner may not necessarily choose it since the tax rate is arbitrary.[24]

2.3 The Competitive Decentralized Equilibrium

We now solve the competitive decentralized equilibrium. Consider an economy that is populated by a set of homogenous and infinitely lived agents of unit mass with the aggregate population normalized to unity. There is no population growth and the representative firms are completely owned by agents. Firms pay taxes on capital income $\tau_k \in (0,1)$ while agents pay taxes on labor income $\tau_n \in (0,1)$. Agents derive utility from consumption of the final good and leisure given in equation (1). The wage payment w_t for both kinds of labor are the same since there is no skill difference assumed between both activities. Agents fund consumption and investment decisions from their after tax wages which they receive for supplying labor n_1 and n_2, and capital income earned from holding assets, which essentially equals the returns to capital lent out for production at each time period t.

Importantly, we assume that the planner can tax factor incomes at different rates which may or may not be equal to τ. This is because spillovers from labor and capital affect factor accumulation differentially. This gives the planner a wider set of instruments to implement the growth rate corresponding to the socially efficient allocation. Therefore, to fund public investment I_t^g, at each time period t a distortionary tax is imposed on labor, $\tau_n \in (0,1)$, and capital, $\tau_k \in (0,1)$ respectively. The following is therefore the government budget constraint:

$$I_t^g = w_t(n_{1t} + n_{2t})\tau_n + \{Y_t - w_t(n_{1t} + n_{2t})\}\tau_k.$$

2.3.1 The Firm's Dynamic Profit Maximization Problem

The representative firm produces the final good based on (3). Hence, the production function is given by

$$Y_t = \underline{A} K_t^{\alpha} n_{1t}^{1-\alpha} \underbrace{\left(\overline{n}_{2t}^{1-\alpha}\right)^{\xi}}_{\text{Externality}}$$

where the law of motion of private capital is given by (4). To determine the demand for factor inputs, competitive firms solve their dynamic profit maximization problems which, at time t, have capital stock, K_t, and the level of ISTC, Z_t. The firm chooses K_{t+1}, n_{1t}, and n_{2t} optimally, taking all externalities and factor prices as given. As noted before, the firm might not be aware that n_{2t}, employed from the point of lowering the price of future capital

[24] Equation (22) implies that that $\widehat{g_{z_P}}$ is maximized at, $\tau = \mu$. See Appendix A.

accumulation and hence future output, also has a spillover on current final good production. This is diagrammatically shown in Figure 4: the firm optimally allocates labor supplied by the agent between n_{1t}, and n_{2t} without realizing n_{2t} also has positive spillovers on final goods production.

[Insert Figure 4]

Let $v(K_t, Z_t)$ denote the value function of the firm at time t. The returns to investment in the credit markets are given by r_t and the wage is given by w_t at time period t. The firm's value function is given by:

$$v(K_t, Z_t) = \max_{K_{t+1}, n_{1t}, n_{2t}} \left\{ \left[Y_t - w_t \left(n_{1t} + n_{2t}\right)\right](1-\tau_k) - \frac{K_{t+1}}{Z_t} + \frac{1}{1+r_{t+1}} v(K_{t+1}, Z_{t+1}) \right\}, \tag{24}$$

which it maximizes subject to (7).

The firm's maximization exercise yields:[25]

$$\{K_{t+1}\} : \frac{1}{Z_t} = \left(\frac{1}{1+r_{t+1}}\right) \frac{\alpha Y_{t+1}(1-\tau_k)}{K_{t+1}}$$

$$\{n_{1t}\} : w_t = \frac{(1-\alpha)Y_t}{n_{1t}}$$

$$\{n_{2t}\} : w_t(1-\tau_k) = \left(\frac{\theta}{n_{2t}}\right) \sum_{j=0}^{\infty} \gamma^j \left[\prod_{k=0}^{j} \frac{1}{1+r_{t+k+1}}\right] I_{t+j+1}.$$

2.3.2 The Agents Problem

Since agents completely own the firms, they receive profits π_t as dividends $\forall t$. Agents are also allowed to borrow and lend at the rate r_t by participating in the credit market. The agent maximizes (1) subject to the consumer budget constraint[26],

$$a_{t+1} = \pi_t + (1+r_t)a_t + w_t n_t (1-\tau_n) - c_t, \tag{25}$$

[25] See Appendix B.

[26] Because there is an unit mass of agents, any aggregate variable is equal to its per-capita magnitude.

and takes factor prices w_t and r_t, profits π_t, and all externalities as given.[27] Agents choose how much to consume, how much labor to supply, and their assets in period $t+1$. Finally, the labor market clearing condition is given by

$$n_t = n_{1t} + n_{2t}.$$

2.3.3 First Order Conditions

The following is the Lagrangian for the agent,

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t [\log c_t + \log(1-n_t) + \lambda_t \{\pi_t + (1+r_t)a_t + w_t n_t (1-\tau_n) - c_t - a_{t+1}\}]. \tag{26}$$

The optimization conditions with respect to c_t, a_{t+1}, and n_t, are given by equations (27), (28), and (29) respectively:

$$\{c_t\} : \tfrac{1}{C_t} = \lambda_t \tag{27}$$

$$\{a_{t+1}\} : \frac{\beta(1+r_{t+1})}{c_{t+1}} = \frac{1}{c_t} \tag{28}$$

$$\{n_t\} : \frac{w_t(1-\tau_n)}{c_t} = \frac{1}{1-n_t} \tag{29}$$

Once we substitute out for factor prices into the firm's problem (equations (27), (28), and (29)), we obtain the following first order conditions for the competitive equilibrium:

$$\{K_{t+1}\} : \frac{1}{c_t Z_t} = \frac{\alpha\beta Y_{t+1}(1-\tau_k)}{c_{t+1}K_{t+1}} \tag{30}$$

$$\{n_{1t}\} : \frac{1}{1-n_t} = \frac{(1-\alpha)Y_t(1-\tau_n)}{c_t n_{1t}} \tag{31}$$

$$\{n_{2t}\} : \frac{1}{1-n_t} = \left(\frac{\beta\theta}{n_{2t}}\right)\left(\frac{1-\tau_n}{1-\tau_k}\right)\sum_{j=0}^{\infty} \beta^j \gamma^j \frac{I_{t+j+1}}{c_{t+j+1}}. \tag{32}$$

Equation (30) is the standard Euler equation for the household. Compared to equation (12) in the planner's problem, the effect of the stock-externalities because of K and G on the inter-temporal savings decision is absent. This is because agents do not internalize

[27] Note that we are not taxing the dividends, π_t, in the consumer budget constraint, but corporate capital income, $[Y_t - w_t(n_{1t}+n_{2t})]$, as in Huffman (2008). Strictly speaking, τ_k is therefore a corporate (profit) tax and not a tax on capital income. Taxing the firm's corporate income at source, i.e., $[Y_t - w_t(n_{1t}+n_{2t})]$, or at the level of the household, i.e., the dividend, π_t, does not change the qualitative results of the model. These results are available from the authors on request.

this externality. Equations (31) and (32) equate the after tax wage to the MRS between consumption and leisure. Compared to equations (14) and (15) respectively, the additional terms due to the externalities are also absent because the agents take the externality from n_2 as given.

2.3.4 Decision Rules

Based on the above first order conditions, Lemma 3 states the optimal decision rules for the agents.

Lemma 3 *C_t, I_t, n_t, n_{1t}, n_{2t} are given by (33), (34), (35), where $0 < \Phi_{CE} < 1$ is given by (36), and $0 < x_{CE} < 1$ given by (37) is a constant.*

$$C_t = \Phi_{CE} A Y_t, I_t = (1 - \Phi_{CE}) A Y_t \tag{33}$$

$$\textit{where, } A = \alpha(1-\tau_k) + (1-\alpha)(1-\tau_n) - \frac{\alpha\beta^2\theta(\tau_n - \tau_k)}{(1-\beta\gamma)}$$

$$n_t = n_{CE} = \frac{(1-\alpha)(1-\tau_n)}{(1-\alpha)(1-\tau_n) + x_{CE}\Phi_{CE}A}, \tag{34}$$

$$n_{1CE} = x_{CE} n_{CE}, n_{2CE} = (1 - x_{CE}) n_{CE}, \tag{35}$$

where Φ_{CE} is given by

$$\Phi_{CE} = 1 - \frac{\alpha\beta(1-\tau_k)}{A}, \tag{36}$$

and x_{CE} is given by

$$x_{CE} = \frac{(1-\alpha)(1-\beta\gamma)}{\alpha\beta^2\theta + (1-\alpha)(1-\beta\gamma)}. \tag{37}$$

Proof. See Appendix B for details. ■

The above decision rules imply that depending upon the parameter values, there exists a feasible range of values that τ_k and τ_n can take such that

$$0 < A, \Phi_{CE}, n_{CE} < 1,$$

are true.[28] The relationship between growth rates at the balanced growth path for private capital, public capital, output and investment specific technological change are identical to that for the planner's version, as given in Lemma 2.

[28] Restriction (50) in Appendix B is required on τ_n and τ_k for $0 < A, \Phi_{CE}, n_{CE} < 1$.

2.3.5 The Competitive Equilibrium Growth Rate

We would like to ascertain under what conditions the growth rate corresponding to the competitive equilibrium allocation replicates the growth corresponding to the efficient allocation. From equations (33), (34), (35), (36), and (37), the growth rate under the competitive equilibrium is given by:

$$g_{z_{CE}} = \left[B \underbrace{n_{2CE}^{\theta}}_{\text{Labor factor}} \underbrace{\left\{ (1-A)^{\mu} (A)^{1-\mu} (1-\Phi_{CE})^{1-\mu} \right\}^{1-\gamma}}_{\text{Capital factor}} \right]^{\frac{1}{2-\gamma}}. \tag{38}$$

The growth rate, $g_{z_{CE}}$, depends on two factors: a labor factor, n_{2CE}^{θ}, and a capital factor given by $\Upsilon = \left\{ (1-A)^{\mu} (A)^{1-\mu} (1-\Phi_{CE})^{1-\mu} \right\}^{1-\gamma}$, both of which depend on factor income taxes, τ_k and τ_n.

The capital factor In Appendix C we show that

$$\Upsilon = \left\{ \left[\frac{(1-\beta\gamma)\left[(1-\alpha)(\tau_n-\tau_k)+\tau_k\right]+\alpha\beta^2\theta(\tau_n-\tau_k)}{1-\beta\gamma} \right]^{\mu} \left[\alpha\beta(1-\tau_k)\right]^{1-\mu} \right\}^{1-\gamma}, \tag{39}$$

i.e., the capital factor, Υ, unambiguously increases in τ_n and the tax gap $(\tau_n - \tau_k)$. We also show that Υ also decreases in τ_k as long as the following sufficient condition is satisfied:

$$1 - \beta\gamma < \beta^2\theta. \tag{40}$$

Importantly, when $\tau_k = 1, \Upsilon = 0$, and there is no growth.[29]

The labor factor The research labor input n_{2CE} is given by

$$n_{2CE} = (1 - x_{CE}) n_{CE}, \tag{41}$$

where

$$\begin{aligned} (1 - x_{CE}) &= \frac{\alpha\beta^2\theta}{\alpha\beta^2\theta + (1-\alpha)(1-\beta\gamma)}, \\ n_{CE} &= \frac{(1-\alpha)(1-\tau_n)}{(1-\alpha)(1-\tau_n) + x_{CE}\Phi_{CE}A}. \end{aligned}$$

[29] Equation (40) can be re-written as, $\beta\theta + \gamma > \frac{1}{\beta}$, which implies that if the returns from allocating resources to ISTC are greater than the returns from investing in an asset (which equals $\frac{1}{\beta}$ in the steady state), an increase in the tax on capital income will depress the capital factor.

Clearly, $(1-x_{CE})$ is independent on factor income taxes. Hence, a change in taxes therefore affects n_{2CE} only through n_{CE}. In Appendix C, we show that

$$n_{CE} = \frac{(1-\alpha)\left[\alpha\beta^2\theta + (1-\alpha)(1-\beta\gamma)\right]}{(1-\alpha)\left[\alpha\beta^2\theta + (1-\alpha)(1-\beta\gamma)\right] + \Psi}, \tag{42}$$

where

$$\Psi = \frac{(1-\alpha)}{(1-\tau_n)}\left[(1-\beta\gamma)\left\{\alpha(1-\beta) + (1-\alpha) + \alpha(1-\beta)(\tau_n - \tau_k) - (1-\alpha\beta)\tau_n\right\} - \alpha\beta^2\theta(\tau_n - \tau_k)\right].$$

As shown in Appendix C, if condition (40) holds, Ψ decreases in the tax gap $(\tau_n - \tau_k)$ and τ_n, and increases in τ_k. As a result, n_{CE} increases in $(\tau_n - \tau_k)$ and τ_n, and decreases in τ_k. The effect of a change in the factor income tax gap $(\tau_n - \tau_k)$ and τ_n on labor supply, and therefore the labor factor, can be summarized by Lemma 4.

Lemma 4 *Suppose*

$$1-\beta\gamma < \beta^2\theta.$$

Then, (i) An increase in τ_k lowers the capital factor, i.e., $\frac{\partial\Upsilon}{\partial\tau_k} < 0$. (ii) A rise in the labor income tax rate, τ_n, and the factor income tax gap, $(\tau_n - \tau_k)$, increases the labor factor, i.e., $\frac{\partial n_{CE}}{\partial(\tau_n-\tau_k)} > 0$, $\frac{\partial n_{CE}}{\partial\tau_n} > 0$, and $\frac{\partial n_{CE}}{\partial\tau_k} < 0 \Longrightarrow \frac{\partial n_{2CE}^{\theta}}{\partial(\tau_n-\tau_k)} > 0$ and $\frac{\partial n_{2CE}^{\theta}}{\partial\tau_n} > 0$.

Proof. See Appendix C. ■

Lemma 4 implies that a smaller γ makes n_{CE} increase by more for an increase in τ_n. Proposition 1 summarizes the effect of tax rates on the competitive equilibrium growth rate.

Proposition 1 *Since the labor factor and capital factor are increasing in τ_n and decreasing in τ_k, the competitive equilibrium growth rate, $g_{z_{CE}}$, is increasing in the factor income tax gap, $(\tau_n - \tau_k)$. An increase in $g_{z_{CE}}$, is obtained by increasing $(\tau_n - \tau_k)$. The factor income tax gap must be increased by either raising τ_n, or lowering τ_k, or both.*

Proof. Follows from $\frac{\partial\Upsilon}{\partial\tau_n} > 0$, $\frac{\partial\Upsilon}{\partial(\tau_n-\tau_k)} > 0$, and Lemma 4. ■

The intuition behind the above proposition is as follows. Assume that the sufficient condition, (40), holds, because of a high value of θ.[30] Since the competitive equilibrium growth rate $g_{z_{CE}}$ increases in the factor income tax gap $(\tau_n - \tau_k)$, an increase in τ_k requires a higher τ_n to replicate the planner's growth rate, g_{z_P}. This suggests that fiscal policy has an offsetting effect on the agent's growth rate. A higher τ_k lowers the capital factor Υ. To

[30] We can implement the planner's allocations even if equation (40) is violated. However we assume this to be our main case because it is satisfied with reasonable parameter values. In the numerical section, we explore both possibilities.

mitigate the negative effect of τ_k on Υ, we have to raise τ_n which not only has a positive effect on the labor factor n_{2CE}^{θ}, but also on Υ.

This happens because although the substitution effect for the change in τ_n induces an increase in leisure,$1 - n_{CE}$, (the after tax wage has gone down), labor supply (and therefore the labor factor) increases because of the strong(er) income effect induced by ISTC. The strong income - in the presence of ISTC - off-sets the substitution effect. In particular, ISTC leads to an additional income effect, through consumption, compared to the case where ISTC is not endogenous. This can be seen from the below equation for, $\Phi_{CE}A$,

$$\Phi_{CE}A = \alpha(1-\tau_k) + (1-\alpha)(1-\tau_n) - \frac{\alpha\beta^2\theta(\tau_n - \tau_k)}{(1-\beta\gamma)} - \alpha\beta\left(1-\tau_k\right).$$

When $\theta > 0$, an increase in τ_n lowers after-tax labor income and lowers consumption even more. Relative to the case where there is no endogenous ISTC, the after tax fraction of income allocated for private consumption, $\Phi_{CE}A$, is lowered by the term, $\frac{\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta\gamma)}$. The drop in consumption causes leisure to fall more (relative to case when $\theta = 0$) and labor supply to increase by more (which follows from equation (29), where $c_t = w_t\left(1-\tau_n\right)\left(1-n_{CE}\right)$). An increase in n_{CE} in turn implies a higher n_{2CE}, from equation (41) and noting that $1-x_{CE}$ is also increasing in θ. Hence the labor factor rises. A rise in the labor factor increases Z_{t+1} which increases capital accumulation and therefore future output and future consumption. Without ISTC, it could be possible that labor supply falls if the substitution effect dominates the income effect. However with ISTC, the income effect dominates the substitution effect and labor supply, n_{CE}, rises.

Fiscal policy also offsets the effect of taxes because public capital crowds out private capital in our model. This is because, from (39) we know that $(1-A)$ increases in τ_k whereas, $A\left(1-\Phi_{CE}\right)$ decreases. Proposition 1 therefore suggest that we can raise $g_{z_{CE}}$ to replicate the efficient growth rate by increasing the factor income tax gap $(\tau_n - \tau_k)$ from an initial point where $g_{z_{CE}} < g_{z_P}$. Further, since ISTC in our model is endogenous, a higher θ causes a bigger increase in n_{CE} and therefore n_{2CE}. This translates into a bigger increase in $g_{z_{CE}}$ for a given increase in τ_n. In terms of the capital factor, since agents under-accumulate private capital because of taking the effect of Υ on Z as given, τ_k must be lowered. As a result, an increase in the tax gap by raising τ_n and lowering τ_k increases $g_{z_{CE}}$.

In sum, as to which effect dominates depends on the sufficient condition, (40), identified in Proposition 1. For instance, the sufficient condition, (40) is also satisfied for higher values of γ, which strengthens the income effect channel for an increase in τ_n. A higher γ also means that the weight on the capital stock externalities is weaker. As a result, the net effect is that a high γ and a high θ makes the labor factor increase for an increase in τ_n. Since condition

(40), which is satisfied for a high γ and θ, causes the capital factor to fall when τ_k increases, the planner's growth rate is replicated using a combination of a high τ_n and a low τ_k.

The Effect of γ and ξ Given the sufficient condition, (40), we graphically characterize the implementation of the socially efficient growth rate, g_{z_P} to illustrate the effect of a change in the externality parameters on the factor income tax gap required to replicate the planner's equilibrium growth rate. First, as ξ increases, the spillover from n_2 in final goods production increases. The planner therefore allocates more labor towards n_2, which increases the socially efficient growth rate, g_{z_P}. This is shown in Figure 5, where we assume $\tau_k = \overline{\tau}_k$, which yields a zero factor income tax gap. Starting with $\xi = 0$, the factor income tax gap required to replicate g_{z_P} corresponds to point 'a'. Now suppose ξ increases arbitrarily. Since the agent's allocations do not depend, on ξ, the competitive equilibrium growth rate $g_{z_{CE}}$ does not change. We know from Proposition 1 that in order to match a higher g_{z_P}, the labor income tax must be increased for a given $\overline{\tau}_k$, which causes an increase in the factor income tax gap. The new factor income tax gap corresponds to point 'b'.

[Insert Figure 5]

Now suppose γ is arbitrarily increased from a low to a high value. From equation 7, it can be seen that this makes ISTC more persistent, which increases g_{z_P}. At the same time, the competitive equilibrium growth rate also increases because the weight on the externality from the capital factor is lower for a higher γ This reduces the extent of under-accumulation of capital since the size of the spillover is low (and a lesser amount of the spillover is not internalized). As a result, the equilibrium factor income tax gap $(\tau_n - \tau_k)$ decreases. This is illustrated in Figure 6. Point 'a' corresponds to $\gamma = 0.5$ and point 'b' corresponds to $\gamma = 0.8$. The crucial difference is that both γ and ξ raise the planner's growth rate, whereas only γ raises the competitive equilibrium growth rate.

[Insert Figure 6]

3 Numerical Examples

In this section, we consider a few numerical examples to show how different factor income tax combinations may replicate the growth rate corresponding to the socially efficient allocation. We also analyze how the magnitude of externalities (γ, ξ) affect the factor income tax gap. To do this, we consider a benchmark value for the socially efficient growth rate, g_{z_P}, calculated at

$\tau = \mu$.[31] In particular, we consider two sets of numerical examples: one where the sufficient condition given by equation (40) holds and another where the condition is violated. Our main result is to numerically show that for a fixed set of deep parameters, a wide range of tax rates implement the same growth rate by varying the externality parameters,.

We first calibrate out factor income tax gaps that are broadly consistent with Figures 1 - 3. We start with two arbitrary values of $\gamma = \{0.1, 0.9\}$ corresponding to the case where the externality from the stock externalities are high and low, respectively. Then, starting with $\xi = 0$, we gradually raise ξ to make it arbitrarily large, and calibrate out the factor income tax gap, $(\tau_n - \tau_k)$, for each change in ξ. In all the numerical experiments we fix $\alpha = 0.35$ and $\beta = 0.95$ as in Huffman (2008).

Case 1: Satisfying sufficient condition (40) Suppose we set $\gamma = 0.9$.[32] Other parameters are arbitrarily chosen as: $\mu = 0.5, \theta = 0.8$, and $B = 1.46$ which yields a growth rate of 2.5% as in Figure 1. This set of parameters satisfy condition (40). Table 1 summarizes the values of τ_n for each value of τ_k such that $g_{z_{CE}} = g_{z_P}$ across different values of $\xi = \{0, 1, 2\}$ and range $\tau_k = \{0.1, 0.2, 0.3, 0.4\}$. Figure 7 plots the locus of all factor income tax combinations corresponding to the case where $\xi = 0$.

[Insert Figure 7]

Two observations emerge. First, as can be seen from the second column of Table 1, with a fixed set of parameters (and assuming $\xi = 0$) a wide range of tax rates replicate the same growth rate. For instance, when $\xi = 0$, $\{\tau_k = 0.1, \tau_n = 0.335\}$ yields the same growth rate of 2.5% as $\{\tau_k = 0.2, \tau_n = 0.41\}$. This holds for columns 3 and 4 as well where the cases of $\xi = 1$ and $\xi = 2$, are considered respectively, corresponding to different planner growth rates (because ξ has risen).

Second, as ξ increases, the equilibrium factor income tax gap needed to replicate the planners growth increases as in Figure 5. This is because, an increase in ξ increases the spillover from n_2 in final goods production. The planner therefore allocates more labor towards n_2. This increases g_{z_P}. To match a higher g_{z_P}, the labor income tax must be increased

[31] Note from equation (22), $\tau = \mu$ also maximizes the efficient growth rate, g_{z_P}. Therefore this is a useful benchmark to be implemented by the competitive decentralized equilibrium. There is a large literature on political economy and institutional motives for designing fiscal policy in which the policy setter is assumed to set fiscal policy to maximize the growth rate to maintain constituent support (see Key (1966), Tufte (1978), Fiorina (1981), Kiewiet and Rivers (1985), Lewis-Beck (1990), Harrington (1993), Ghate (2003)).

[32] We have chosen parameters such that n_2 has a large weight on Z, and the externality from public and private capital on Z has a small weight. In addition, the effect of public capital to output ratio on Z is moderate.

for a given τ_k, which causes an increase in the factor income tax gap. This requires $\tau_n > \tau_k$ to replicate g_{z_P}.

τ_k	$\tau_n - \tau_k$ $(\xi = 0)$	$\tau_n - \tau_k$ $(\xi = 1)$	$\tau_n - \tau_k$ $(\xi = 2)$
0.1	0.235	0.255	0.269
0.2	0.21	0.229	0.241
0.3	0.188	0.203	0.214
0.4	0.163	0.177	0.186

Table 1: Equilibrium factor income tax gaps under $\gamma = 0.9$

When γ is high, the spillover from the capital factor is low. This also makes ISTC more persistent. This increases the growth rate of the planner. To raise the competitive equilibrium growth rate, a reduction in the tax on capital income raises the capital factor and an increase in the labor income tax raises the labor factor. At the same time, since the effect of the externality from the capital factor is low, and the effect of public capital is low, $(\tau_n - \tau_k)$ is narrower.[33]

Case 2: Violating sufficient condition (40) Suppose now $\gamma = 0.1$. Other parameters are arbitrarily chosen to be: $\mu = 0.9, \theta = 0.01$, and $B = 1.81$ which yields a growth rate of 2.5% which is roughly equal to the average growth rate for our sample of OECD countries in Figure 1.[34] This set of parameters violates condition (40). Figure 8 plots the locus of all factor income tax combinations corresponding to the case where $\xi = 0$.

[Insert Figure 8]

Table 2 summarizes the values of τ_n for each value of τ_k.such that $g_{z_{CE}} = g_{z_P}$ across different values of $\xi = \{0, 1, 2\}$,and different values of $\tau_k = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.7, 0.9\}$. Observe that not only are the individual factor income tax combinations higher than in Table 1, for lower τ_n, the tax gaps $(\tau_n - \tau_k)$ are also higher. Crucially, this is because Table 2 corresponds to the case where there is a high weight on the externality on Z_{t+1} due to public and private capital. A high weight on the externality due to these variables implies that τ_k must either be very low (along with a high τ_n) or both must be high. A high τ_n is feasible because the direct effect of n_{2t} on Z_{t+1} (and therefore its indirect effect on Y_t) is low. The tax gaps also become negative, i.e., $\tau_k > \tau_n$, for higher values of τ_n.

[33] We show in Appendix D that when there are no externalities, *equal* factor income taxes always yield the optimal growth rate from the planner's problem. Hence, the factor income tax gap is zero.

[34] Our choice of parameters are now such that n_2 has a small weightage on Z while the externality from public and private capital on Z has a high weightage. In addition, the effect of public capital to output ratio on Z is very high while that of private capital to output ratio is very small.

τ_k	$\tau_n - \tau_k$ ($\xi = 0$)	$\tau_n - \tau_k$ ($\xi = 1$)	$\tau_n - \tau_k$ ($\xi = 2$)
0.1	0.862	–	–
0.2	0.721	0.759	0.767
0.3	0.583	0.621	0.628
0.4	0.448	0.486	0.493
0.5	0.316	0.355	0.362
0.7	0.073	0.114	0.122
0.9	−0.09	−0.038	−0.029

Table 2: Equilibrium factor income tax gaps under $\gamma = 0.1$

First, similar to Table 1, the factor income tax gap in each column corresponds to a fixed set of parameter values. As can be seen from column 2, for $\xi = 0$, both $\{\tau_k = 0.3, \tau_n = 0.883\}$ and $\{\tau_k = 0.9, \tau_n = 0.81\}$ implement a 2.5% growth rate. In other words, a reversal in the factor income tax ranking implies the same growth rate. From columns 3 and 4 we again observe that for an increase in ξ, there is a marginal increase in the tax gap $(\tau_n - \tau_k)$**,** as higher values of ξ corresponding to higher planner growth rates, as in Case 1.

Second, as τ_k increases, the value of τ_n that replicates the planner's growth rate for the given value of τ_k also increases. We also observe that as τ_k increases, the tax gap $(\tau_n - \tau_k)$ starts narrowing. For very high values of τ_k the corresponding value of τ_n could be smaller, such that the rankings get reversed and $\tau_n - \tau_k$ becomes negative. This is because the condition given by equation (40) is now violated**.** The intuition is as follows. For a low value of θ, the income effect channel because of ISTC on labor supply is weakened, for an increase in τ_n. Therefore, an increase in τ_n on the net, may not increase the labor factor. In addition, a low value of γ also means that the weight on the capital stock externalities is stronger. Since the capital stock externalities consist of public and private capital, a higher τ_k may not have offsetting effects on the labor and capital factor, as in the previous case where the sufficient condition (40) is satisfied. As a result, a high τ_k and a low τ_n replicate g_{z_P}. This is consistent with Figure 2 where we generally observe that high τ_k economies also have a lower τ_n (e.g., US, UK, Japan, and Denmark). Thus Table 1 is able to qualitatively match the factor income tax gaps in these economies even though the calibrated factor income tax gaps are smaller in magnitude in this experiment.

While differences in the tax gaps are not very high for higher values of ξ (because all factor income tax rates are less than 1, and that the effect of higher values of ξ on n, x, and therefore n_2, is dampened because the weight on n_2, in Z_{t+1}, i.e., θ, is also less than 1, The numerical results above still identify why the externalities are crucial for our results. While our model yields equilibrium factor income tax gaps that implement g_{z_P} under a fixed set of parameters we also show that a change in the magnitude of the externalities widen/narrows the equilibrium factor income tax gaps required to implement the planner's growth rate. These results are consistent with the growth-tax puzzle identified in Figures 1 - 3.

4 Conclusion

This paper constructs a simple and tractable endogenous growth model with endogenous investment specific technological change. Our theoretical model is motivated by the empirical observation that advanced economies – which are presumed to be on their balanced growth paths and therefore experience similar or identical growth rates – have widely varying factor income tax combinations. This observation is puzzling since it is incompatible with a standard model of endogenous growth: in the standard model, fiscal policy can have large growth effects through its impact on the economy's investment rate. We see our contribution as providing an alterative, but compatible, explanation based on the fact that different combinations of taxes can generate the same growth rate. Our innovation is to incorporate aggregate public and private capital stock externalities in ISTC, as well as positive spillovers driven by specialized labor in the research sector to explain this puzzle.

We characterize the balanced growth path of the economy corresponding to the socially efficient allocation for a fixed tax rate and derive conditions under which the competitive equilibrium can implement this growth rate. Our general result is that to the extent that spillovers from a specialized labor input and the public and private capital stocks exist, an increase in these spillover from specialized labor, and a decrease in the spillover from public and private capital, increases the growth rate corresponding to the socially efficient allocation, and therefore increases the factor income tax gap required to implement the higher planner's growth rate. Conversely, for a given level of externalities, maintaining the constancy of growth also requires different combinations of factor income taxes. Finally, when there are no externalities, *equal* factor income taxes always yield the socially efficient growth rate. Hence, the factor income tax gap is zero. In the numerical section, we show that we can qualitatively match the factor income tax gaps observed in the data.

In the future, we hope to extend our framework by comparing the growth and welfare effects of optimal tax policy on research and development versus funding public investment. In addition, our model characterizes the optimal tax rate along the balanced growth path. Future work can model the transitional dynamics.

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Appendix

Appendix A: Planner's problem

Using the Method of Undetermined Coefficients,

$$C_t = \Phi_P Y_t \left(1-\tau\right),\ I_t = \left(1-\Phi_P\right) Y_t \left(1-\tau\right),\ I_t^g = \tau Y_t$$

and

$$n_1 = xn, n_2 = (1-x)n.$$

From $\{Z_{t+1}\}$,

$$Z_{t+1}\lambda_{2t} = \beta\lambda_{2t+1}\gamma Z_{t+2} + \beta^2\lambda_{2t+2}\mu\left(1-\gamma\right)\frac{Z_{t+3}}{Z_{t+1}} + \beta\left(\frac{1-\Phi_P}{\Phi_P}\right).$$

From $\{n_{1t}\}$,

$$\frac{1}{1-n_t} = \frac{(1-\alpha)Y_t\left(1-\tau\right)}{C_t n_{1t}} - \beta\lambda_{2t+1}(1-\gamma)\left(1-\alpha\right)\frac{Z_{t+2}}{n_{1t}},$$

which implies

$$\frac{x_P n_P}{1-n_P} = \frac{(1-\alpha)}{\Phi_P} - \beta(1-\gamma)\left(1-\alpha\right)\lambda_{2t+1}Z_{t+2}.$$

Therefore,

$$\lambda_{2t+1}Z_{t+2} = \frac{\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P}}{\beta(1-\gamma)\left(1-\alpha\right)}.$$

This also implies for constant decision rules and a constant labor supply in every time period,

$$\lambda_{2i-1}Z_i = \frac{\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P}}{\beta(1-\gamma)\left(1-\alpha\right)}, \quad \text{for all } i = t.$$

Substituting in $\{Z_{t+1}\}$,

$$\frac{\left[\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P}\right]\left[1-\beta\gamma-\beta^2\mu\left(1-\gamma\right)\right]}{\beta(1-\gamma)\left(1-\alpha\right)} = \beta\left(\frac{1-\Phi_P}{\Phi_P}\right).$$

This on rearranging gives

$$\frac{n_P}{1-n_P} = \frac{\left(1-\alpha\right)\left[1-\beta\gamma-\beta^2\mu\left(1-\gamma\right)-\beta^2\left(1-\gamma\right)\left(1-\Phi_P\right)\right]}{x_P\Phi_P\left[1-\beta\gamma-\beta^2\mu\left(1-\gamma\right)\right]}.$$

Hence,

$$n_P = \frac{(1-\alpha)\left[1-\beta\gamma-\beta^2\mu(1-\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\right]}{(1-\alpha)\left[1-\beta\gamma-\beta^2\mu(1-\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\right]+x_P\Phi_P\left[1-\beta\gamma-\beta^2\mu(1-\gamma)\right]}.$$

Using

$$\frac{n_P}{1-n_P} = \frac{(1-\alpha)\left[1-\beta\gamma-\beta^2\mu(1-\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\right]}{x_P\Phi_P\left[1-\beta\gamma-\beta^2\mu(1-\gamma)\right]},$$

we get

$$\lambda_{2i-1}Z_i = \left(\frac{1-\Phi_P}{\Phi_P}\right)\left(\frac{\beta}{1-\beta\gamma-\beta^2\mu(1-\gamma)}\right).$$

From $\{n_{2t}\}$

$$\frac{(1-x_P)\,n_P}{1-n_P} = \frac{(1-\alpha)\xi}{\Phi_P}+\theta\lambda_{2t}Z_{t+1}-\beta(1-\gamma)\xi(1-\alpha)\lambda_{2t+1}Z_{t+2}.$$

This implies

$$\frac{(1-x_P)\,n_P}{1-n_P} = \frac{(1-\alpha)\xi}{\Phi_P}+\left[\theta-\beta(1-\gamma)\xi(1-\alpha)\right]\left(\frac{1-\Phi_P}{\Phi_P}\right)\left(\frac{\beta}{1-\beta\gamma-\beta^2\mu(1-\gamma)}\right).$$

Since

$$\frac{n_P}{1-n_P} = \frac{(1-\alpha)\left[1-\beta\gamma-\beta^2\mu(1-\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\right]}{x_P\Phi_P\left[1-\beta\gamma-\beta^2\mu(1-\gamma)\right]},$$

we get

$$\begin{aligned}
&\left(\frac{1-x_P}{x_P}\right)\frac{(1-\alpha)\left[1-\beta\gamma-\beta^2\mu(1-\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\right]}{\Phi_P\left[1-\beta\gamma-\beta^2\mu(1-\gamma)\right]} \\
&= \frac{(1-\alpha)\left[1-\beta\gamma-\beta^2\mu(1-\gamma)\right]\xi+\beta\left[\theta-\beta(1-\gamma)\xi(1-\alpha)\right](1-\Phi_P)}{\Phi_P\left[1-\beta\gamma-\beta^2\mu(1-\gamma)\right]} \\
&= \frac{(1-\alpha)\xi\left[(1-\beta\gamma)-\beta^2\mu(1-\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\right]+\beta\theta(1-\Phi_P)}{\Phi_P\left[1-\beta\gamma-\beta^2\mu(1-\gamma)\right]}.
\end{aligned}$$

Hence,

$$x_P = \frac{(1-\alpha)\left[1-\beta\gamma-\beta^2\mu(1-\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\right]}{(1-\alpha)(1+\xi)\left[(1-\beta\gamma)-\beta^2\mu(1-\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\right]+\beta\theta(1-\Phi_P)}.$$

Finally, from $\{K_{t+1}\}$,

$$\{K_{t+1}\}: \frac{1}{C_tZ_t} = \frac{\alpha\beta Y_{t+1}(1-\tau)}{C_{t+1}K_{t+1}}+\beta(1-\gamma)(1-\mu)\lambda_{2t+1}\frac{Z_{t+2}}{K_{t+1}}-\beta^2\lambda_{2t+2}(1-\gamma)\alpha\frac{Z_{t+3}}{K_{t+1}}$$

$$
\begin{aligned}
\frac{1}{\Phi_P Y_t Z_t} &= \frac{\alpha\beta}{\Phi_P\left(1-\Phi_P\right)Y_t Z_t} + \frac{\beta\left(1-\gamma\right)\left(1-\mu\right)}{\left(1-\Phi_P\right)Y_t Z_t}\left(\frac{1-\Phi_P}{\Phi_P}\right) \\
&\quad + \frac{\beta^2\left(1-\gamma\right)\left(1-\mu\right)\gamma}{\left(1-\Phi_P\right)Y_t Z_t}\lambda_{2t+2}Z_{t+3} - \frac{\beta^2\alpha\left(1-\gamma\right)}{\left(1-\Phi_P\right)Y_t Z_t}\lambda_{2t+2}Z_{t+3}.
\end{aligned}
$$

Since

$$
\lambda_{2i-1}Z_i = \left(\frac{1-\Phi_P}{\Phi_P}\right)\left(\frac{\beta}{1-\beta\gamma-\beta^2\mu\left(1-\gamma\right)}\right),
$$

we get

$$
1 = \frac{\alpha\beta}{\left(1-\Phi_P\right)} + \beta\left(1-\gamma\right)\left(1-\mu\right) - \frac{\beta^3\left(1-\gamma\right)\alpha}{\left[\left(1-\beta\gamma\right)-\beta^2\mu\left(1-\gamma\right)\right]}.
$$

On simplifying we get

$$
1-\Phi_P = \frac{\alpha\beta\left[\left(1-\beta\gamma\right)-\beta^2\mu\left(1-\gamma\right)\right]}{\left(1-\beta\gamma\right)-\beta^2(1-\gamma)+\alpha\beta^3\left(1-\gamma\right)}.
$$

Conditions

As long as $(1-\Phi_P)<1$, we will get

$$
0 < x_P < 1.
$$

We know,

$$
\left(1-\Phi_P\right) = \frac{\alpha\beta\left[\left(1-\beta\gamma\right)-\beta^2\mu\left(1-\gamma\right)\right]}{\left(1-\beta\gamma\right)-\beta^2(1-\gamma)+\alpha\beta^3\left(1-\gamma\right)}
$$

Since,

$$
0 < \left(1-\beta\gamma\right)-\beta^2(1-\gamma) = \left(1-\beta\right)\left[1+\beta(1-\gamma)\right],
$$

$$
\left(1-\Phi_P\right) > 0.
$$

To show

$$
\left(1-\Phi_P\right) = \frac{\alpha\beta\left[\left(1-\beta\gamma\right)-\beta^2\mu\left(1-\gamma\right)\right]}{\left(1-\beta\gamma\right)-\beta^2(1-\gamma)+\alpha\beta^3\left(1-\gamma\right)} < 1,
$$

we require,

$$
\begin{aligned}
& \left(1-\beta\gamma\right)-\beta^2(1-\gamma)+\alpha\beta^3\left(1-\gamma\right) \\
> \; & \alpha\beta\left[\left(1-\beta\gamma\right)-\beta^2\mu\left(1-\gamma\right)\right],
\end{aligned}
$$

or,

$$
\left(1-\beta\gamma\right)\left(1-\alpha\beta\right)-\beta^2(1-\gamma)+\alpha\beta^3\left(1-\gamma\right)+\alpha\beta^3\mu\left(1-\gamma\right) > 0.
$$

Rewriting the above LHS we get

$$(1-\beta\gamma)(1-\alpha\beta)-\beta^2(1-\gamma)\left[1-\alpha\beta(1+\mu)\right].$$

Since,

$$(1-\beta\gamma)>\beta^2(1-\gamma)$$

and

$$1-\alpha\beta>1-\alpha\beta(1+\mu),$$

therefore

$$(1-\Phi_P)\in(0,1).$$

Since,

$$x_P=\frac{(1-\alpha)\left[1-\beta\gamma-\beta^2\mu(1-\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\right]}{(1-\alpha)(1+\xi)\left[(1-\beta\gamma)-\beta^2\mu(1-\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\right]+\beta\theta(1-\Phi_P)}$$

Therefore

$$0<x_P,\Phi_P<1.$$

Finally, since

$$n_P=\frac{(1-\alpha)\left[1-\beta\gamma-\beta^2\mu(1-\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\right]}{(1-\alpha)\left[1-\beta\gamma-\beta^2\mu(1-\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\right]+x_P\Phi_P\left[1-\beta\gamma-\beta^2\mu(1-\gamma)\right]}$$

and,

$$0<x_P,\Phi_P<1,$$

therefore,

$$0<n_P<1.$$

Growth rate at the BGP

$$Y_t=\underline{A}.\left(n_{2t}^{1-\alpha}\right)^{\xi}K_t^{\alpha}n_{1t}^{1-\alpha}$$

On the balanced growth path (BGP),

$$g_{y_P}=g_{y_{Pt+1}}=\frac{Y_{t+1}}{Y_t}=\frac{K_{t+1}^{\alpha}}{K_t^{\alpha}}=g_{k_{Pt+1}}^{\alpha}=g_{k_P}^{\alpha},$$

$$\text{and } g_{k_P}=\frac{K_{t+1}}{K_t}=\frac{I_tZ_t}{I_{t-1}Z_{t-1}}=g_{y_P}.g_{z_P}.$$

Hence,

$$g_{y_P} = g_{z_P}^{\frac{\alpha}{1-\alpha}}, g_{k_P} = g_{g_P} = g_{z_P}^{\frac{1}{1-\alpha}}.$$

Comparative statics of the growth rate with respect to τ

The growth rate, $\widehat{g_{z_P}}$ is maximized at $\tau = \mu$. To see this, we first take logs, such that

$$\ln \widehat{g_{z_P}} = \frac{1}{2-\gamma}\left[\ln \widehat{M_P} + (1-\gamma)\,\mu \ln \tau + (1-\gamma)\,(1-\mu)\ln(1-\tau)\right].$$

Since $\widehat{M_P}$ is independent of τ, at the point of maximum,

$$\begin{aligned}
\frac{\partial \ln \widehat{g_{z_P}}}{\partial \tau} &= \frac{(1-\gamma)\,\mu}{(2-\gamma)}\frac{\partial \ln \tau}{\partial \tau} + \frac{(1-\gamma)\,(1-\mu)}{(2-\gamma)}\frac{\partial \ln(1-\tau)}{\partial \tau} = 0 \\
&\Longrightarrow \frac{(1-\gamma)\,\mu}{(2-\gamma)\,\tau} - \frac{(1-\gamma)\,(1-\mu)}{(2-\gamma)\,(1-\tau)} = 0 \\
&\Longrightarrow \frac{1-\tau}{\tau} = \frac{(1-\mu)}{\mu}, \\
&\Longrightarrow \tau = \mu.
\end{aligned}$$

Therefore, $\widehat{g_{z_P}}$ is maximized at $\tau = \mu$. The second order condition is also negative, as follows:

$$\begin{aligned}
&\frac{(1-\gamma)\,\mu}{(2-\gamma)}\frac{\partial\left(\frac{1}{\tau}\right)}{\partial \tau} - \frac{(1-\gamma)\,(1-\mu)}{(2-\gamma)}\frac{\partial\left(\frac{1}{1-\tau}\right)}{\partial \tau} \\
= &-\frac{(1-\gamma)\,\mu}{(2-\gamma)}\left(\frac{1}{\tau^2}\right) - \frac{(1-\gamma)\,(1-\mu)}{(2-\gamma)}\left(\frac{1}{1-\tau}\right)^2 < 0.
\end{aligned}$$

Appendix B: Competitive decentralized equilibrium

We assume $\delta = 1$. From the firm's FOC $\{K_{t+1}\}$:

$$\{K_{t+1}\} : \frac{1}{Z_t} = \left(\frac{1}{1+r_{t+1}}\right)\frac{\alpha Y_{t+1}(1-\tau_k)}{K_{t+1}}.$$

Substituting for $(1+r_{t+1})$ from $\{a_{t+1}\}$

$$\begin{aligned}
&\Rightarrow \frac{1}{Z_t} = \frac{\beta c_t}{c_{t+1}}\left[\frac{\alpha Y_{t+1}(1-\tau_k)}{K_{t+1}}\right] \\
&\Rightarrow \{K_{t+1}\} : \frac{1}{c_t Z_t} = \frac{\alpha\beta Y_{t+1}(1-\tau_k)}{c_{t+1}K_{t+1}}
\end{aligned}$$

Similarly,

$$\{n_{1t}\}:\frac{1}{1-n_t}=\frac{(1-\alpha)Y_t(1-\tau_n)}{c_t n_{1t}}$$

and,

$$\{n_{2t}\}:\frac{1}{1-n_t}=\left(\frac{\beta\theta}{n_{2t}}\right)\left(\frac{1-\tau_n}{1-\tau_k}\right)\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+1}}{c_{t+j+1}}.$$

When

$$\tau_k=\tau_k=\tau,$$

we have

$$\begin{aligned}
\{K_{t+1}\} &: \frac{1}{c_t Z_t}=\frac{\alpha\beta Y_{t+1}(1-\tau)}{c_{t+1}K_{t+1}} \\
\{n_{1t}\} &: \frac{1}{1-n_t}=\frac{(1-\alpha)Y_t(1-\tau)}{n_{1t}} \\
\{n_{2t}\} &: \frac{1}{1-n_t}=\left(\frac{\beta\theta}{n_{2t}}\right)\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+1}}{c_{t+j+1}}.
\end{aligned}$$

The Decision Rules

We use the method of undetermined coefficients to obtain the decision rules

$$\begin{aligned}
C_t &= \Phi_{CE}AY_t, \\
I_t &= (1-\Phi_{CE})AY_t \\
n_{1t} &= x_{CE}n_{CE} \\
n_{2t} &= (1-x_{CE})n_{CE} \\
n_t &= n_{CE},
\end{aligned}$$

where,

$$\{Y_t-w_t(n_{1t}+n_{2t})\}(1-\tau_k)+w_t(n_{1t}+n_{2t})(1-\tau_n)=AY_t.$$

$$\Rightarrow[\alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)]Y_t+w_t n_{2t}(\tau_k-\tau_n)=AY_t$$

$$\Rightarrow[\alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)]Y_t+\left\{\frac{\beta\theta AY_t\,(1-\Phi)}{(1-\tau_k)(1-\beta\gamma)}\right\}(\tau_k-\tau_n)=AY_t$$

$$\Rightarrow\alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)+\frac{\beta\theta A\,(1-\Phi)}{(1-\tau_k)(1-\beta\gamma)}(\tau_k-\tau_n)=A$$

$$\Rightarrow Y_t\left[\alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)+\frac{\beta\theta A\,(1-\Phi)}{(1-\tau_k)(1-\beta\gamma)}(\tau_k-\tau_n)\right]=AY_t,$$

$$\Rightarrow A=\left[\alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)+\frac{\beta\theta\,(1-\Phi)\,A}{(1-\tau_k)(1-\beta\gamma)}(\tau_k-\tau_n)\right]. \tag{43}$$

From the FOC of $\{K_{t+1}\}$

$$\{K_{t+1}\}:\frac{1}{c_tZ_t}=\frac{\alpha\beta Y_{t+1}(1-\tau_k)}{c_{t+1}K_{t+1}}$$

This implies,

$$\frac{1}{\Phi_{CE}AY_tZ_t}=\frac{\alpha\beta Y_{t+1}(1-\tau_k)}{\Phi AY_{t+1}(1-\Phi_{CE})AY_tZ_t}$$

$$\Rightarrow(1-\Phi_{CE})=\frac{\alpha\beta(1-\tau_k)}{A}. \tag{44}$$

Substituting for $(1-\Phi_{CE})A$ from 44 into 43,

$$\begin{aligned}\Rightarrow\quad A &= \left[\alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)+\frac{\beta\theta\,(1-\Phi_{CE})\,A}{(1-\tau_k)(1-\beta\gamma)}(\tau_k-\tau_n)\right] && (45)\\ &= \alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)-\frac{\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta\gamma)}.\end{aligned}$$

When $\tau_n=\tau_k=\tau$

$$\begin{aligned}A &= [\alpha(1-\tau)+(1-\alpha)(1-\tau)]\\ &= (1-\tau).\end{aligned}$$

From $\{n_{1t}\}$ we get

$$\begin{aligned}\{n_{1t}\}\quad &: \quad \frac{x_{CE}n_{CE}}{1-n_{CE}}=\frac{(1-\alpha)Y_t(1-\tau_n)}{\Phi_{CE}AY_t}\\ &\Rightarrow \quad \frac{x_{CE}n_{CE}}{1-n_{CE}}=\frac{(1-\alpha)(1-\tau_n)}{\Phi_{CE}A}\\ &\Rightarrow \quad \frac{n_{CE}}{1-n_{CE}}=\frac{(1-\alpha)(1-\tau_n)}{x_{CE}\Phi_{CE}A}\end{aligned}$$

$$\Rightarrow n_{CE}=\frac{(1-\alpha)(1-\tau_n)}{(1-\alpha)(1-\tau_n)+x_{CE}\Phi_{CE}A}. \tag{46}$$

From $\{n_{2t}\}$

$$\{n_{2t}\}:\frac{(1-x)n_{CE}}{1-n_{CE}}=\frac{\beta\theta}{(1-\beta\gamma)}\left(\frac{1-\tau_n}{1-\tau_k}\right)\frac{(1-\Phi_{CE})}{\Phi_{CE}}$$

$$\Rightarrow \frac{(1-\alpha)(1-\tau_n)}{\Phi_{CE}A}\frac{(1-x_{CE})}{x_{CE}} = \frac{\beta\theta}{(1-\beta\gamma)}\left(\frac{1-\tau_n}{1-\tau_k}\right)\frac{(1-\Phi_{CE})}{\Phi_{CE}}$$

$$\Rightarrow \frac{(1-x_{CE})}{x_{CE}} = \frac{A\beta\theta(1-\Phi_{CE})}{(1-\alpha)(1-\beta\gamma)(1-\tau_k)}.$$

$$\Rightarrow x_{CE} = \frac{(1-\alpha)(1-\beta\gamma)(1-\tau_k)}{A\beta\theta(1-\Phi_{CE})+(1-\alpha)(1-\tau_k)(1-\beta\gamma)}. \tag{47}$$

Since,

$$A(1-\Phi_{CE}) = \alpha\beta(1-\tau_k),$$

$$\Rightarrow x_{CE} = \frac{(1-\alpha)(1-\beta\gamma)}{\alpha\beta^2\theta+(1-\alpha)(1-\beta\gamma)}.$$

From (36), we need

$$0 < 1 - \frac{\alpha\beta(1-\tau_k)}{A} < 1,$$

which gives us

$$0 < \frac{\alpha\beta(1-\tau_k)}{A} < 1,$$

or

$$A > \alpha\beta(1-\tau_k). \tag{48}$$

In addition, we also need

$$0 < A < 1 \tag{49}$$

to be satisfied. If equations (48) and (49) hold, we obtain

$$0 < A, \Phi_{CE}, n_{CE} < 1.$$

Equations (48) and (49) gives us a lower limit and an upper limit on τ_n, such that

$$\frac{-\alpha\left[1-\beta\theta-\beta^2\theta\right]}{(1-\alpha)(1-\beta\gamma)+\alpha\beta^2\theta}\tau_k < \tau_n < \frac{(1-\beta\gamma)(1-\alpha\beta)}{(1-\alpha)(1-\beta\gamma)+\alpha\beta^2\theta} - \alpha\frac{\left[(1-\beta\gamma)(1-\beta)-\beta^2\theta\right]}{(1-\alpha)(1-\beta\gamma)+\alpha\beta^2\theta}\tau_k. \tag{50}$$

In other words, for each τ_k the lower and the upper bound on τ_n must satisfy Restriction (50).

Appendix C

$$\begin{aligned}
1-A &= 1-\left[\alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)-\frac{\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta\gamma)}\right] \\
&= \frac{(1-\beta\gamma)-\{\alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)\}(1-\beta\gamma)+\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta\gamma)} \\
&= \frac{(1-\beta\gamma)\left[\tau_n-\alpha\left(\tau_n-\tau_k\right)\right]+\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta\gamma)} \\
&= \frac{(1-\beta\gamma)\left[\tau_k+(1-\alpha)\left(\tau_n-\tau_k\right)\right]+\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta\gamma)}.
\end{aligned}$$

Since

$$\begin{aligned}
A\left(1-\Phi_{CE}\right) &= \alpha\beta(1-\tau_k) \\
1-A &= \frac{(1-\beta\gamma)\left[(1-\alpha)\left(\tau_n-\tau_k\right)+\tau_k\right]+\alpha\beta^2\theta\left(\tau_n-\tau_k\right)}{1-\beta\gamma}.
\end{aligned}$$

This implies,

$$\Upsilon=\left\{\left[\frac{(1-\beta\gamma)\left[(1-\alpha)\left(\tau_n-\tau_k\right)+\tau_k\right]+\alpha\beta^2\theta\left(\tau_n-\tau_k\right)}{1-\beta\gamma}\right]^{\mu}\left[\alpha\beta(1-\tau_k)\right]^{1-\mu}\right\}^{1-\gamma}.$$

In Υ, $\alpha\beta(1-\tau_k)$ decreases in τ_k. Further, suppose

$$\begin{aligned}
M_1 &= \left[\frac{(1-\beta\gamma)\left[(1-\alpha)\left(\tau_n-\tau_k\right)+\tau_k\right]+\alpha\beta^2\theta\left(\tau_n-\tau_k\right)}{1-\beta\gamma}\right] \\
M_2 &= \left[\alpha\beta(1-\tau_k)\right].
\end{aligned}$$

Therefore,

$$\frac{\partial\Upsilon}{\partial\tau_k}=(1-\gamma)\,\Upsilon^{-\frac{\gamma}{1-\gamma}}\left[M_2\mu\alpha\left\{\frac{1-\beta\gamma-\beta^2\theta}{1-\beta\gamma}\right\}-M_1\left(1-\mu\right)\alpha\beta\right]M_1^{\mu-1}M_2^{-\mu}.$$

Since, $M_1>0$ because $1-A>0$ and $M_2>0$ by assumption,

$$(1-\beta\gamma)-\beta^2\theta<0,$$

implies that Υ will fall with an increase in τ_k.

From the labor supply term

$$\begin{aligned} n_{CE} &= \frac{(1-\alpha)(1-\tau_n)}{(1-\alpha)(1-\tau_n)+x_{CE}\Phi_{CE}A} \\ &= \frac{(1-\alpha)}{(1-\alpha)+\frac{x_{CE}\Phi_{CE}A}{(1-\tau_n)}}. \end{aligned}$$

Note that

$$x_{CE}\Phi_{CE}A = \frac{(1-\alpha)(1-\beta\gamma)}{\alpha\beta^2\theta+(1-\alpha)(1-\beta\gamma)}\left[A-\alpha\beta\left(1-\tau_k\right)\right].$$

But

$$A-\alpha\beta\left(1-\tau_k\right) = \frac{(1-\beta\gamma)\left[\alpha\left(1-\beta\right)\left(1-\tau_k\right)+(1-\alpha)(1-\tau_n)\right]-\alpha\beta^2\theta(\tau_n-\tau_k)}{1-\beta\gamma}.$$

Hence,

$$x_{CE}\Phi_{CE}A = \frac{(1-\alpha)\left[(1-\beta\gamma)\left\{\alpha\left(1-\beta\right)\left(1-\tau_k\right)+(1-\alpha)(1-\tau_n)\right\}-\alpha\beta^2\theta(\tau_n-\tau_k)\right]}{\left[\alpha\beta^2\theta+(1-\alpha)(1-\beta\gamma)\right]}.$$

The term

$$(1-\beta\gamma)\left\{\alpha\left(1-\beta\right)\left(1-\tau_k\right)+(1-\alpha)(1-\tau_n)\right\}$$

can be re-written as

$$(1-\beta\gamma)\left\{\alpha\left(1-\beta\right)+(1-\alpha)-\alpha\left(1-\beta\right)\tau_k-(1-\alpha)\tau_n\right\},$$

$$\begin{aligned} &= (1-\beta\gamma)\left\{\alpha\left(1-\beta\right)+(1-\alpha)-\alpha\tau_k+\alpha\beta\tau_k-\tau_n+\alpha\tau_n\right\} \\ &= (1-\beta\gamma)\left\{\alpha\left(1-\beta\right)+(1-\alpha)+\alpha\left(\tau_n-\tau_k\right)-\alpha\beta\left(\tau_n-\tau_k\right)-\left(1-\alpha\beta\right)\tau_n\right\} \\ &= (1-\beta\gamma)\left\{\alpha\left(1-\beta\right)+(1-\alpha)+\alpha\left(1-\beta\right)\left(\tau_n-\tau_k\right)-\left(1-\alpha\beta\right)\tau_n\right\}. \end{aligned}$$

Hence,

$$n_{CE} = \frac{(1-\alpha)\left[\alpha\beta^2\theta+(1-\alpha)(1-\beta\gamma)\right]}{(1-\alpha)\left[\alpha\beta^2\theta+(1-\alpha)(1-\beta\gamma)\right]+\Psi},$$

where

$$\Psi = \frac{(1-\alpha)}{(1-\tau_n)}\left[(1-\beta\gamma)\left\{\alpha\left(1-\beta\right)+(1-\alpha)+\alpha\left(1-\beta\right)\left(\tau_n-\tau_k\right)-\left(1-\alpha\beta\right)\tau_n\right\}-\alpha\beta^2\theta\left(\tau_n-\tau_k\right)\right].$$

Proof of Lemma 4

Note that

$$\begin{aligned}\frac{x_{CE}\Phi_{CE}A}{1-\tau_n} &= x_{CE}\left[\frac{\alpha(1-\beta)(1-\tau_k)}{(1-\tau_n)}+(1-\alpha)-\frac{\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta\gamma)(1-\tau_n)}\right]\\ &= x_{CE}\left[\frac{\alpha(1-\beta)(1-\tau_k)}{(1-\tau_n)}+(1-\alpha)-\frac{\alpha\beta^2\theta\tau_n}{(1-\beta\gamma)(1-\tau_n)}+\frac{\alpha\beta^2\theta\tau_k}{(1-\beta\gamma)(1-\tau_n)}\right]\end{aligned}$$

Therefore,

$$\frac{\partial\frac{x_{CE}\Phi_{CE}A}{1-\tau_n}}{\partial\tau_n}=x_{CE}\left[\frac{\alpha(1-\beta)(1-\tau_k)}{(1-\tau_n)^2}-\frac{\alpha\beta^2\theta(1-\tau_k)}{(1-\beta\gamma)(1-\tau_n)^2}\right],$$

which will be negative if

$$(1-\beta\gamma)(1-\beta)<\beta^2\theta.$$

This condition will be satisfied if equation (40) holds. And this implies

$$\frac{\partial n_{CE}}{\partial\tau_n}>0.$$

Further, since x_{CE} is independent of taxes,

$$\frac{\partial n_{2CE}}{\partial\tau_n}>0.$$

Similarly, since

$$\Psi=\frac{(1-\alpha)}{(1-\tau_n)}\left[(1-\beta\gamma)\left\{\alpha(1-\beta)+(1-\alpha)+\alpha(1-\beta)(\tau_n-\tau_k)-(1-\alpha\beta)\tau_n\right\}-\alpha\beta^2\theta(\tau_n-\tau_k)\right],$$

$$\frac{\partial\Psi}{\partial(\tau_n-\tau_k)}=\frac{(1-\alpha)}{(1-\tau_n)}\left[\alpha(1-\beta)(1-\beta\gamma)-\alpha\beta^2\theta\right](1-\tau_k)<0,$$

if equation (40) holds, which further implies,

$$\frac{\partial n_{CE}}{\partial(\tau_n-\tau_k)}>0.$$

Finally,

$$\frac{\partial\Psi}{\partial\tau_k}=-\frac{(1-\alpha)}{(1-\tau_n)}\left[(1-\beta\gamma)\alpha(1-\beta)-\alpha\beta^2\theta\right]<0,$$

if equation (40) holds.

Appendix D

We know that,

$$
\begin{aligned}
(1-\Phi_P) &= \frac{\alpha\beta\left[(1-\beta\gamma)-\beta^2\mu(1-\gamma)\right]}{(1-\beta\gamma)-\beta^2(1-\gamma)+\alpha\beta^3\left(1-\gamma\right)} \\
x_P &= \frac{(1-\alpha)\{(1-\beta\gamma)-\beta^2\mu(1-\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\}}{(1+\xi)(1-\alpha)\{(1-\beta\gamma)-\beta^2\mu(1-\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\}+\beta\theta(1-\Phi_P)} \\
n_P &= \frac{(1-\alpha)[(1-\beta\gamma)-\beta^2\mu(1-\gamma)-\beta^2(1-\gamma)(1-\Phi)]}{(1-\alpha)[(1-\beta\gamma)-\beta^2\mu(1-\gamma)-\beta^2(1-\gamma)(1-\Phi)]+\Phi x\left[1-\beta\gamma-\beta^2\mu(1-\gamma)\right]}.
\end{aligned}
$$

When $\gamma=1$ and when $\xi=0$,

$$
\begin{aligned}
1-\Phi_P &= \alpha\beta \\
x_P &= \frac{(1-\alpha)(1-\beta)}{(1-\alpha)(1-\beta)+\alpha\beta^2\theta} \\
n_P &= \frac{(1-\alpha)}{(1-\alpha)+\Phi_P x_P}.
\end{aligned}
$$

In the competitive equilibrium under equal factor income taxes,

$$
\begin{aligned}
A &= 1-\tau. \\
&\Rightarrow\ (1-\Phi_{CE})=\alpha\beta \\
&\Rightarrow\ n_{CE}=\frac{(1-\alpha)}{(1-\alpha)+x_{CE}\Phi_{CE}} \\
&\Rightarrow\ x_{CE}=\frac{(1-\alpha)(1-\beta)}{\alpha\beta^2\theta+(1-\alpha)(1-\beta)}.
\end{aligned}
$$

Clearly, when $\gamma=1$ and $\xi=0$, and $\tau_n=\tau_k=\tau$,

As $\gamma\rightarrow 1$,

$$
\begin{aligned}
1-\Phi_P &= 1-\Phi_{CE} \\
x_P &= x_{CE} \\
n_P &= n_{CE}
\end{aligned}
$$

$$\Rightarrow g_{z_{CE}}=g_{z_P}.$$

Only equal factor income taxes under the no externality case, yields the planner's growth rate, except under a very restrictive parametric restriction,

$$\left(\frac{1-\beta}{\beta}\right)^2 = \theta.$$

Under this condition equal factor income taxes are one among infinitely many factor income tax combinations that replicate the planner's growth rate. We can show this as follows.

For growth equalization, we need

$$n_{CE} = \frac{(1-\alpha)(1-\tau_n)}{(1-\alpha)(1-\tau_n) + x_{CE}\Phi_{CE}A} = n_P.$$

$$\begin{aligned}
&\Rightarrow \quad \frac{x_{CE}\Phi_{CE}A}{(1-\tau_n)} = \Phi_P x_P \\
&\Rightarrow \quad \frac{\Phi_{CE}A}{(1-\tau_n)} = \Phi_P \\
&\Rightarrow \quad \frac{A-\alpha\beta(1-\tau_k)}{(1-\tau_n)} = 1-\alpha\beta \\
&\Rightarrow \quad A-\alpha\beta(1-\tau_k) = (1-\alpha\beta)(1-\tau_n)
\end{aligned}$$

$$\Rightarrow \alpha(1-\tau_k) + (1-\alpha)(1-\tau_n) - \frac{\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta)} - \alpha\beta(1-\tau_k) = (1-\alpha\beta)(1-\tau_n).$$

Hence,

$$(\alpha-\alpha\beta)(1-\tau_k) - (\alpha-\alpha\beta)(1-\tau_n) = \frac{\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta)}$$

which implies

$$(1-\beta)(\tau_n-\tau_k) = \frac{\beta^2\theta(\tau_n-\tau_k)}{(1-\beta)}.$$

Clearly, as long as $\frac{(1-\beta)}{\beta} \neq \sqrt{\theta}$, $\tau_n = \tau_k$ always replicates planner's growth rates. When $\frac{(1-\beta)}{\beta} = \sqrt{\theta}$, any factor income tax combination replicates planner's growth rate. As noted in the text, for $\theta = 0.2$, (or $\theta = 0.5$, as we have used in our numerical exercise) as in Huffman, the value of $\beta = 0.69098$ is very small and is not consistent with the literature. (When $\theta = 0.5$, $\beta = 0.585\,79$ which is even smaller). We therefore rule out the possibility of equality.

Appendix E: Planner's problem without full depreciation

The following first order conditions are therefore obtained with respect to C_t, K_{t+1}, Z_{t+1}, n_{1t}, and n_{2t} (with $\delta < 1$):

$$\{C_t\} : \tfrac{1}{C_t} = \lambda_{1t}$$

$$\{K_{t+1}\} : \frac{1}{C_t Z_t} = \frac{\beta\left(1-\delta\right)}{C_{t+1}Z_{t+1}} + \frac{\alpha\beta Y_{t+1}\left(1-\tau\right)}{C_{t+1}K_{t+1}} + \beta(1-\gamma)(1-\mu)\lambda_{2t+1}\frac{Z_{t+2}}{K_{t+1}} - \beta^2\lambda_{2t+2}(1-\gamma)\alpha\frac{Z_{t+3}}{K_{t+1}} \tag{51}$$

$$\{Z_{t+1}\} : \lambda_{2t} = \beta\lambda_{2t+1}\gamma\frac{Z_{t+2}}{Z_{t+1}} + \beta\lambda_{1t+1}\left(\frac{K_{t+2}-\left(1-\delta\right)K_{t+1}}{Z_{t+1}^2}\right) + \beta^2\lambda_{2t+2}\mu\left(1-\gamma\right)\tau\frac{Z_{t+3}}{\frac{G_{t+2}}{Y_{t+1}}} \tag{52}$$

$$\{n_{1t}\} : \frac{1}{1-n_t} = \frac{(1-\alpha)Y_t\left(1-\tau\right)}{C_t n_{1t}} - \beta\lambda_{2t+1}(1-\gamma)\left(1-\alpha\right)\frac{Z_{t+2}}{n_{1t}} \tag{53}$$

and,

$$\{n_{2t}\} : \frac{1}{1-n_t} = \frac{(1-\alpha)\xi Y_t\left(1-\tau\right)}{C_t n_{2t}} + \lambda_{2t}\theta\frac{Z_{t+1}}{n_{2t}} - \beta\lambda_{2t+1}(1-\gamma)\xi\left(1-\alpha\right)\frac{Z_{t+2}}{n_{2t}}. \tag{54}$$

We use the method of undetermined coefficients in order to characterize the BGP. As in the case with $\delta = 1$,

$$C_t = \Phi_P Y_t\left(1-\tau\right),\ I_t = \left(1-\Phi_P\right)Y_t\left(1-\tau\right),\ I_t^g = \tau Y_t$$

and

$$n_1 = xn, n_2 = (1-x)n.$$

We know from $\{n_{1t}\}$,

$$\{n_{1t}\} : \frac{1}{1-n_t} = \frac{(1-\alpha)Y_t\left(1-\tau\right)}{C_t n_{1t}} - \beta\lambda_{2t+1}(1-\gamma)\left(1-\alpha\right)\frac{Z_{t+2}}{n_{1t}},$$

which implies

$$\frac{x_P n_P}{1-n_P} = \frac{(1-\alpha)}{\Phi_P} - \beta(1-\gamma)\left(1-\alpha\right)\lambda_{2t+1}Z_{t+2}.$$

Therefore,

$$\lambda_{2t+1}Z_{t+2} = \frac{\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P}}{\beta(1-\gamma)\left(1-\alpha\right)}.$$

This also implies for constant decision rules and a constant labor supply in every time period,

$$\lambda_{2i-1} Z_i = \frac{\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P}}{\beta(1-\gamma)(1-\alpha)}, \quad \text{for all } i = t.$$

From $\{Z_{t+1}\}$,

$$\{Z_{t+1}\} : \lambda_{2t} = \beta \lambda_{2t+1} \gamma \frac{Z_{t+2}}{Z_{t+1}} + \beta \lambda_{1t+1} \left(\frac{K_{t+2} - (1-\delta) K_{t+1}}{Z_{t+1}^2} \right) + \beta^2 \lambda_{2t+2} \mu (1-\gamma) \tau \frac{Z_{t+3}}{G_{t+2}/Y_{t+1}}.$$

On rearranging, this gives us

$$\lambda_{2t} Z_{t+1} = \beta \lambda_{2t+1} \gamma Z_{t+2} + \beta \lambda_{1t+1} \left(\frac{K_{t+2} - (1-\delta) K_{t+1}}{Z_{t+1}} \right) + \beta^2 \lambda_{2t+2} Z_{t+3} \mu (1-\gamma) \tau \frac{Z_{t+1}}{G_{t+2}/Y_{t+1}}$$

Substituting in $\{Z_{t+1}\}$,

$$\begin{aligned} \left[\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P} \right] \frac{[1-\beta\gamma]}{\beta(1-\gamma)(1-\alpha)} &= \beta \left(\frac{I_{t+1}}{C_{t+1}} \right) \\ &\quad + \frac{\tau \beta^2 \mu (1-\gamma) \left[\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P} \right]}{\beta(1-\gamma)(1-\alpha)} \frac{Z_{t+1}}{G_{t+2}/Y_{t+1}}. \end{aligned}$$

This is of the form

$$\chi_1 = \chi_2 \left(\frac{I_{t+1}}{C_{t+1}} \right) + \chi_3 \frac{Z_{t+1}}{G_{t+2}/Y_{t+1}},$$

where

$$\begin{aligned} \chi_1 &= \left[\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P} \right] \frac{[1-\beta\gamma]}{\beta(1-\gamma)(1-\alpha)} \\ \chi_2 &= \beta \\ \chi_3 &= \frac{\tau \beta^2 \mu (1-\gamma) \left[\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P} \right]}{\beta(1-\gamma)(1-\alpha)}. \end{aligned}$$

Since

$$\left(\frac{I_{t+1}}{C_{t+1}} \right) = \left(\frac{1-\Phi_P}{\Phi_P} \right),$$

substituting, we get

$$\frac{Z_{t+1}}{G_{t+2}/Y_{t+1}} = \frac{\chi_1 - \chi_2 \left(\frac{1-\Phi_P}{\Phi_P} \right)}{\chi_3} = \text{constant}. \tag{55}$$

In equation (55) equality between the LHS and the RHS will not be restored if the LHS is not a constant. Therefore, on the BGP, equation (55) must be true.

Now, using the FOC with respect to K_{t+1},

$$\begin{aligned}
\frac{K_{t+1}}{C_t Z_t} &= \frac{K_{t+1}\beta(1-\delta)}{C_{t+1}Z_{t+1}} + \frac{\alpha\beta Y_{t+1}(1-\tau)}{C_{t+1}} + \beta(1-\gamma)(1-\mu)\lambda_{2t+1}Z_{t+2} - \beta^2\lambda_{2t+2}Z_{t+3}(1-\gamma)\alpha \\
&= \frac{K_{t+1}\beta(1-\delta)}{C_{t+1}Z_{t+1}} + \frac{\alpha\beta Y_{t+1}(1-\tau)}{C_{t+1}} + \frac{(1-\mu-\alpha\beta)}{(1-\alpha)}\left[\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P}\right].
\end{aligned}$$

On rearranging, we get

$$\frac{K_{t+1}}{C_t Z_t}\left[1-\beta(1-\delta)\left(\frac{C_t}{C_{t+1}}\right)\left(\frac{Z_t}{Z_{t+1}}\right)\right] = \frac{(1-\mu-\alpha\beta)}{(1-\alpha)}\left[\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P}\right].$$

This implies

$$\frac{K_{t+1}}{C_t Z_t} = \frac{\frac{(1-\mu-\alpha\beta)}{(1-\alpha)}\left[\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P}\right]}{\left[1-\beta(1-\delta)\left(\frac{C_t}{C_{t+1}}\right)\left(\frac{Z_t}{Z_{t+1}}\right)\right]}.$$

Again this implies Z_t is growing at the same rate at $\frac{K_{t+1}}{C_t}$, or Z_{t+1} is growing at the same rate at $\frac{K_{t+2}}{C_{t+1}}$. Since, $C_{t+1} = \Phi_P Y_{t+1}(1-\tau)$, Z_{t+1} is growing at the same rate at $\frac{K_{t+2}}{Y_{t+1}}$. This is because, on the BGP the RHS is constant. In fact,

$$\frac{K_{t+2}}{Y_{t+1}Z_{t+1}} = \chi_4(1-\tau), \tag{56}$$

where

$$\chi_4 = \frac{\frac{(1-\mu-\alpha\beta)}{(1-\alpha)}\left[\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P}\right]\Phi_P}{\left[1-\beta(1-\delta)\left(\frac{C_t}{C_{t+1}}\right)\left(\frac{Z_t}{Z_{t+1}}\right)\right]}.$$

As in equation (55), in equation (56) the equality between the LHS and the RHS will not be restored if the LHS is not a constant. Therefore, on the BGP, equation (56) must be true. Using equation (55) and (56), we conclude that on the BGP,

$$\begin{aligned}
g_{z_P} &= \frac{g_{k_P}}{g_{y_P}}, \text{ and} \\
g_{z_P} &= \frac{g_G}{g_{y_P}}.
\end{aligned}$$

We know

$$Z_{t+1} = BZ_t^{\gamma} n_2^{\theta}\left[\left(\frac{G_t}{Y_{t-1}}\right)^{\mu}\left(\frac{K_t}{Y_{t-1}}\right)^{1-\mu}\right]^{1-\gamma}.$$

This implies

$$\begin{aligned}
\frac{Z_{t+1}}{Z_t} &= \frac{Z_t^{\gamma}}{Z_{t-1}^{\gamma}}\left[\frac{\left(\frac{G_t}{Y_{t-1}}\right)^{\mu}\left(\frac{K_t}{Y_{t-1}}\right)^{1-\mu}}{\left(\frac{G_{t-1}}{Y_{t-2}}\right)^{\mu}\left(\frac{K_{t-1}}{Y_{t-2}}\right)^{1-\mu}}\right]^{1-\gamma} \\
g_{z_P} &= g_{z_P}^{\gamma} g_{z_P}^{1-\gamma} = g_{z_P}.
\end{aligned}$$

Growth rate at the BGP

Since

$$\frac{K_{t+2}}{Y_{t+1}} = \chi_4 \left(1-\tau\right) Z_{t+1},$$

$$\begin{aligned}
g_{k_P} &= g_{z_P} g_{y_P} \\
&= g_{z_P} g_{k_P}^{\alpha}
\end{aligned}$$

Therefore,

$$\begin{aligned}
g_{k_P} &= g_{z_P}^{\frac{1}{1-\alpha}}, \\
\text{and therefore, } g_{y_P} &= g_{z_P}^{\frac{\alpha}{1-\alpha}}.
\end{aligned}$$

We therefore obtain qualitatively identical results to the $\delta = 1$ case.

Growth rate of ISTC

The expression for Z_{t+1} is given by

$$\begin{aligned}
Z_{t+1} &= B Z_t^{\gamma} n_2^{\theta}\left[\left(\frac{G_t}{Y_{t-1}}\right)^{\mu}\left(\frac{K_t}{Y_{t-1}}\right)^{1-\mu}\right]^{1-\gamma} \\
&= B Z_t^{\gamma} n_2^{\theta}\left[\left(\frac{G_t}{Y_{t-1}}\right)^{\mu}\left(\frac{K_t}{Y_{t-1}}\right)^{1-\mu}\right]^{1-\gamma} \\
&= B Z_t^{\gamma} n_2^{\theta}\left[\left(\frac{\chi_3 Z_{t-1}}{\chi_1 - \chi_2\left(\frac{1-\Phi_P}{\Phi_P}\right)}\right)^{\mu}\left(\chi_4\left(1-\tau\right)Z_{t-1}\right)^{1-\mu}\right]^{1-\gamma} \\
&= B Z_t^{\gamma} n_2^{\theta} Z_{t-1}^{1-\gamma}\left[\left(\tau\frac{\frac{\beta^2\mu(1-\gamma)\left[\frac{(1-\alpha)}{\Phi_P}-\frac{x_P n_P}{1-n_P}\right]}{\beta(1-\gamma)(1-\alpha)}}{\chi_1 - \chi_2\left(\frac{1-\Phi_P}{\Phi_P}\right)}\right)^{\mu}\left(\chi_4\left(1-\tau\right)\right)^{1-\mu}\right]^{1-\gamma}.
\end{aligned}$$

We can then summarize the growth rate of Z_{t+1} on the BGP

$$g_z = \left\{ Bn_2^\theta \left[(\tau \Delta_1)^\mu \left(\chi_4 \left(1 - \tau \right) \right)^{1-\mu} \right]^{1-\gamma} \right\}^{\frac{1}{2-\gamma}} .$$

5 Figures

Figure 1: Average growth rates for select OECD economies versus the ratio of tax on capital income to tax on labor income

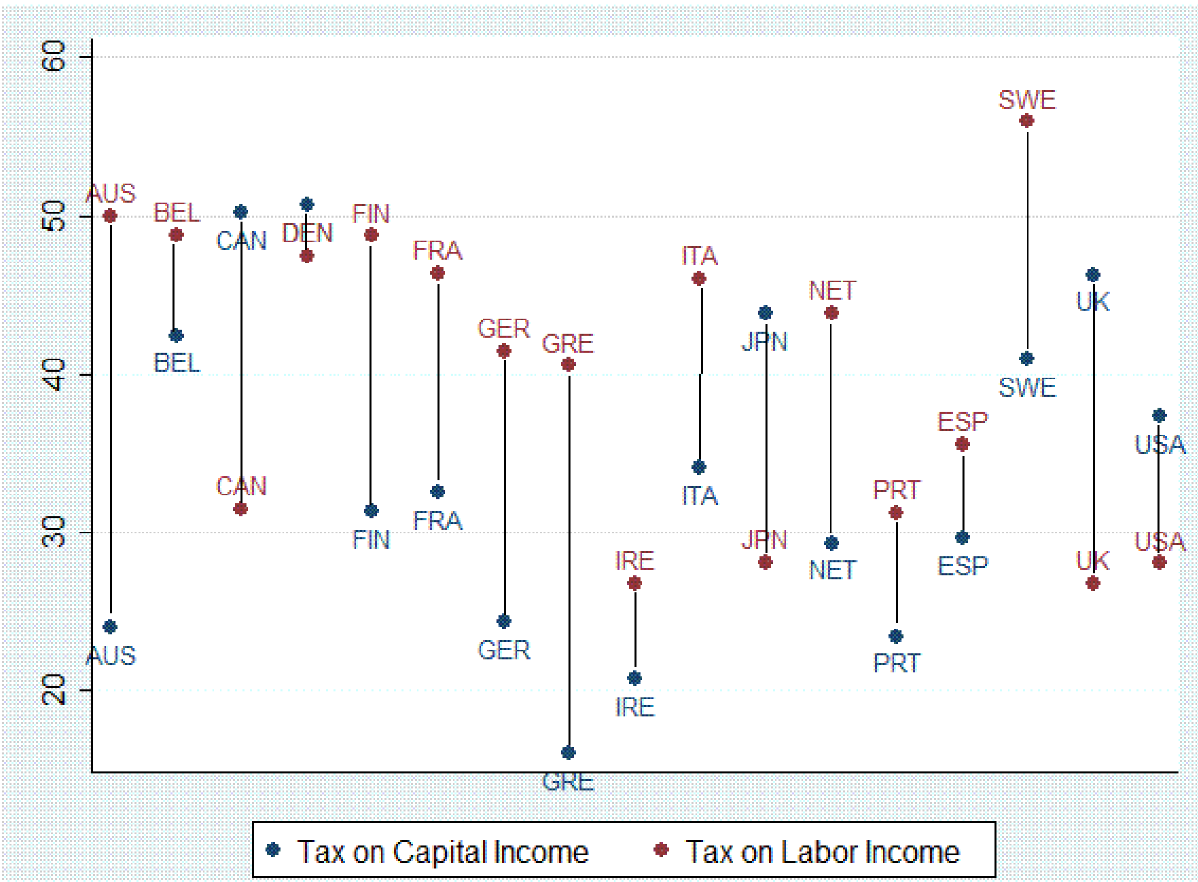

Figure 2: Average factor income tax rates for select OECD economies

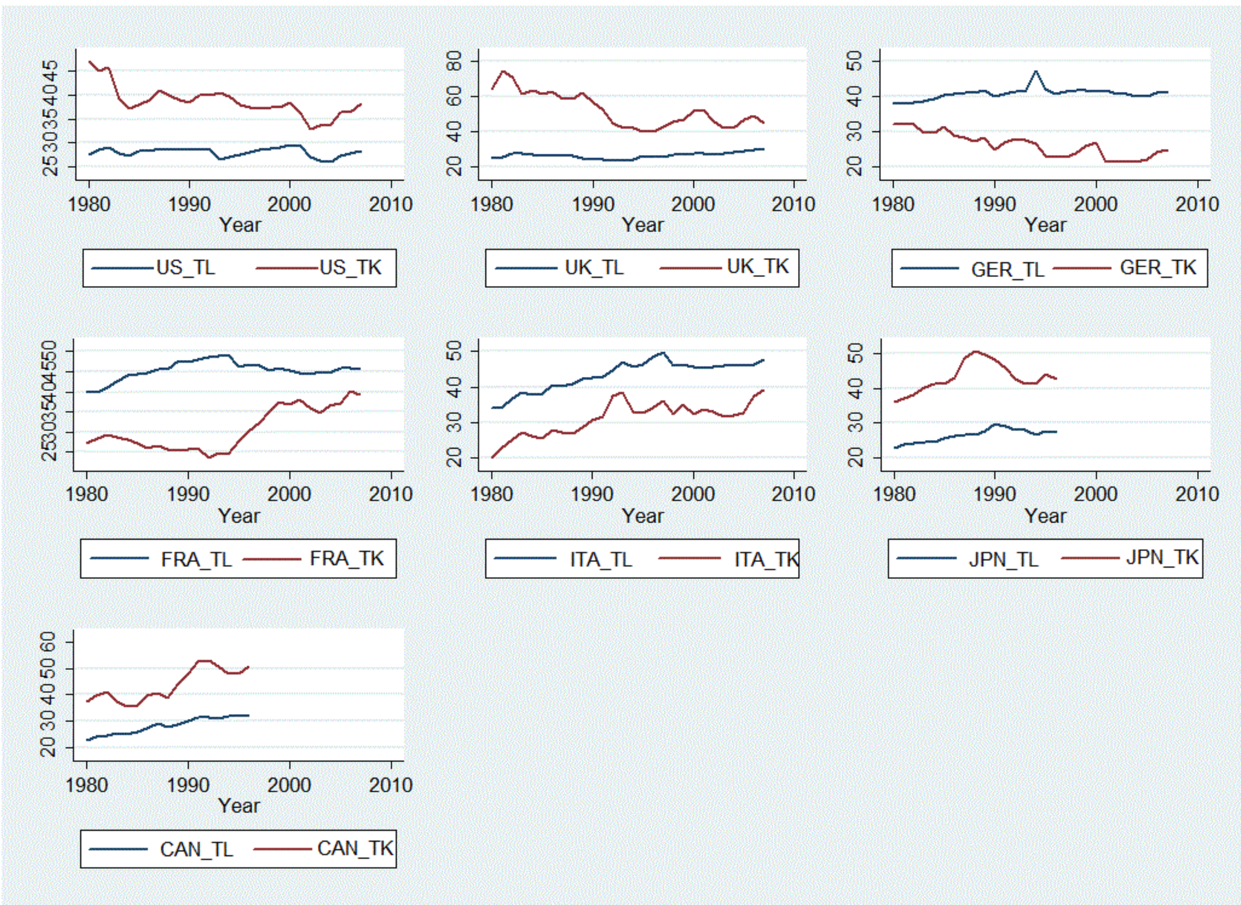

Figure 3: Time trend of factor income taxes for G7 economies

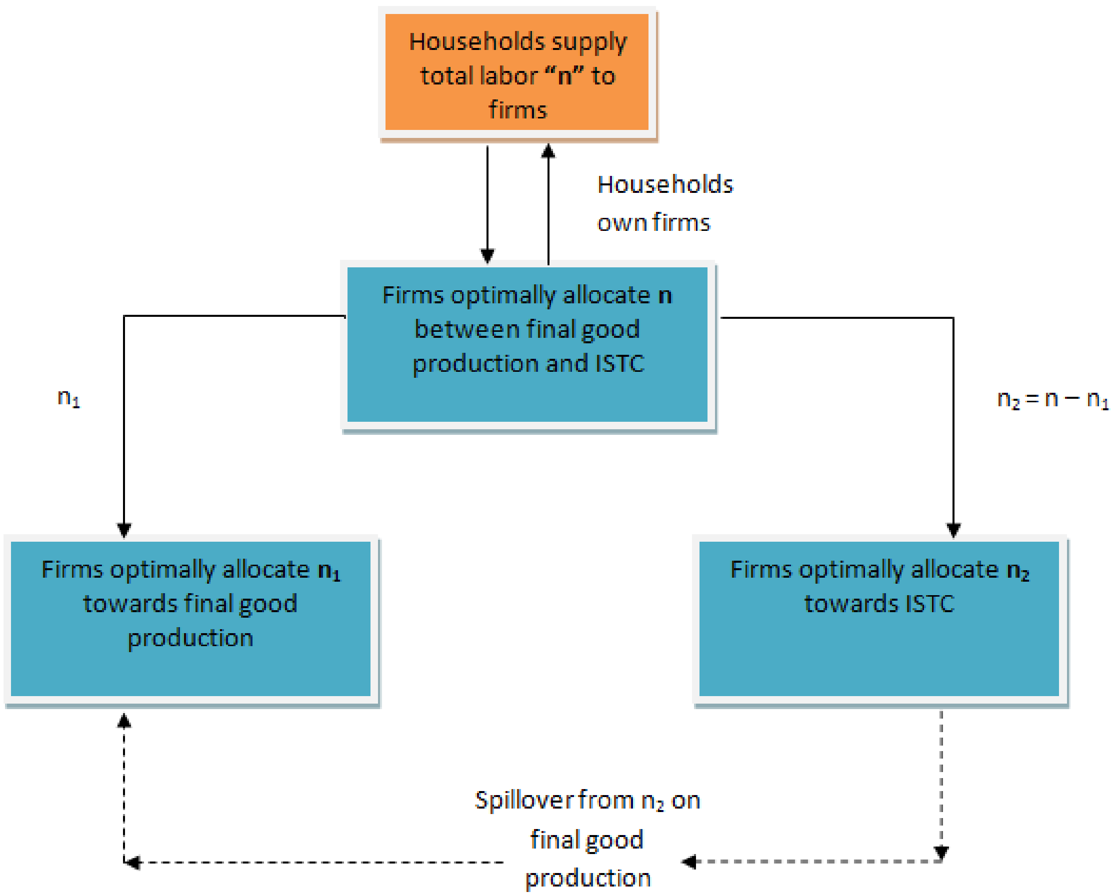

Figure 4: Allocation of n_t towards n_{1t} and n_{2t} and spillover from n_{2t} on final goods production

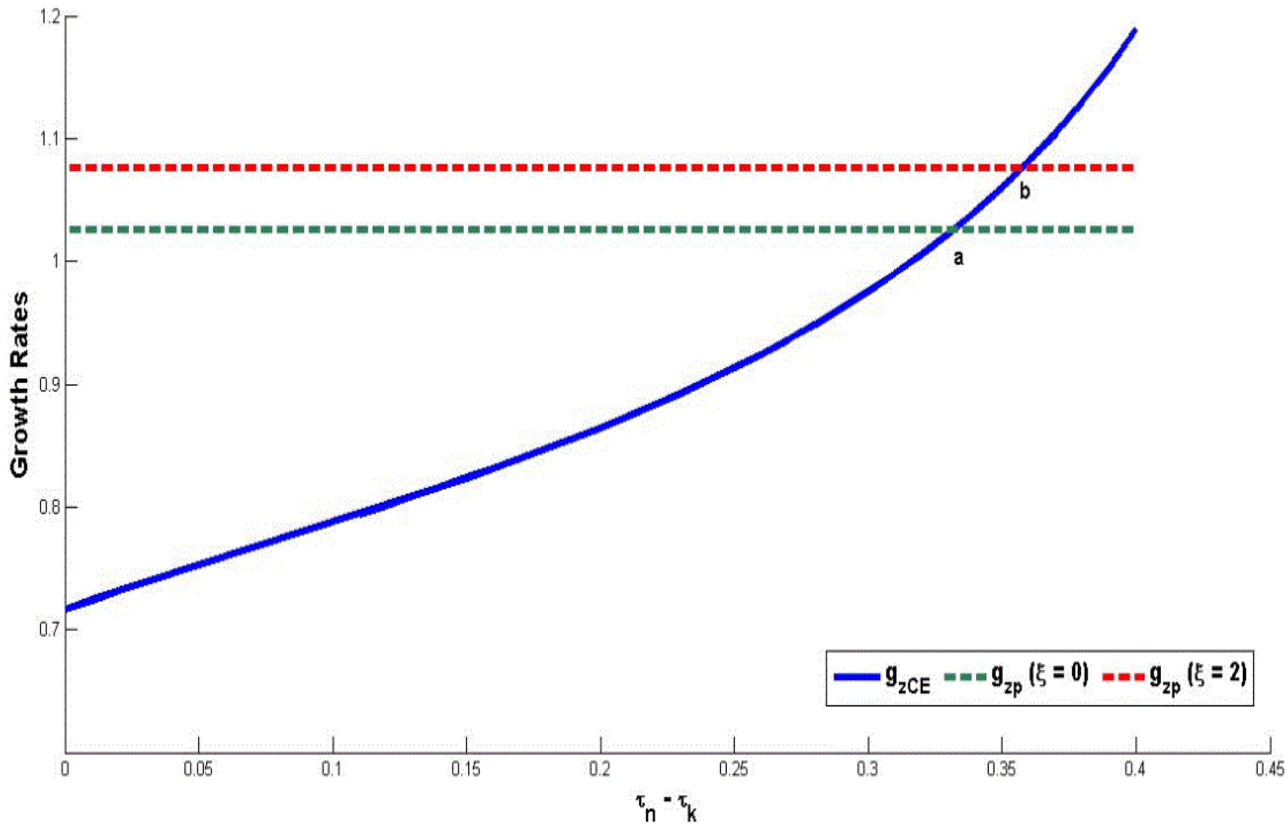

Figure 5: The effect of a change in ξ on $(\tau_n - \tau_k)$

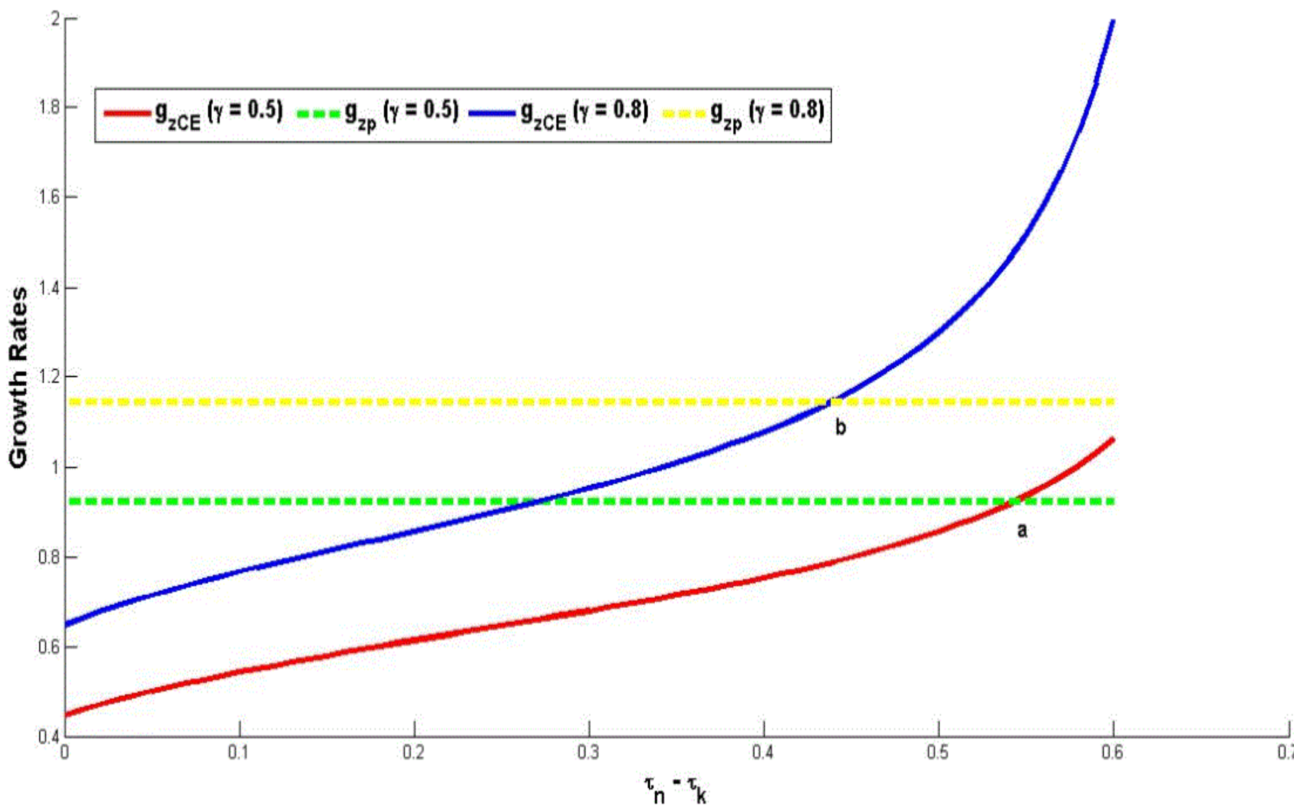

Figure 6: The effect of a change in γ on $(\tau_n - \tau_k)$

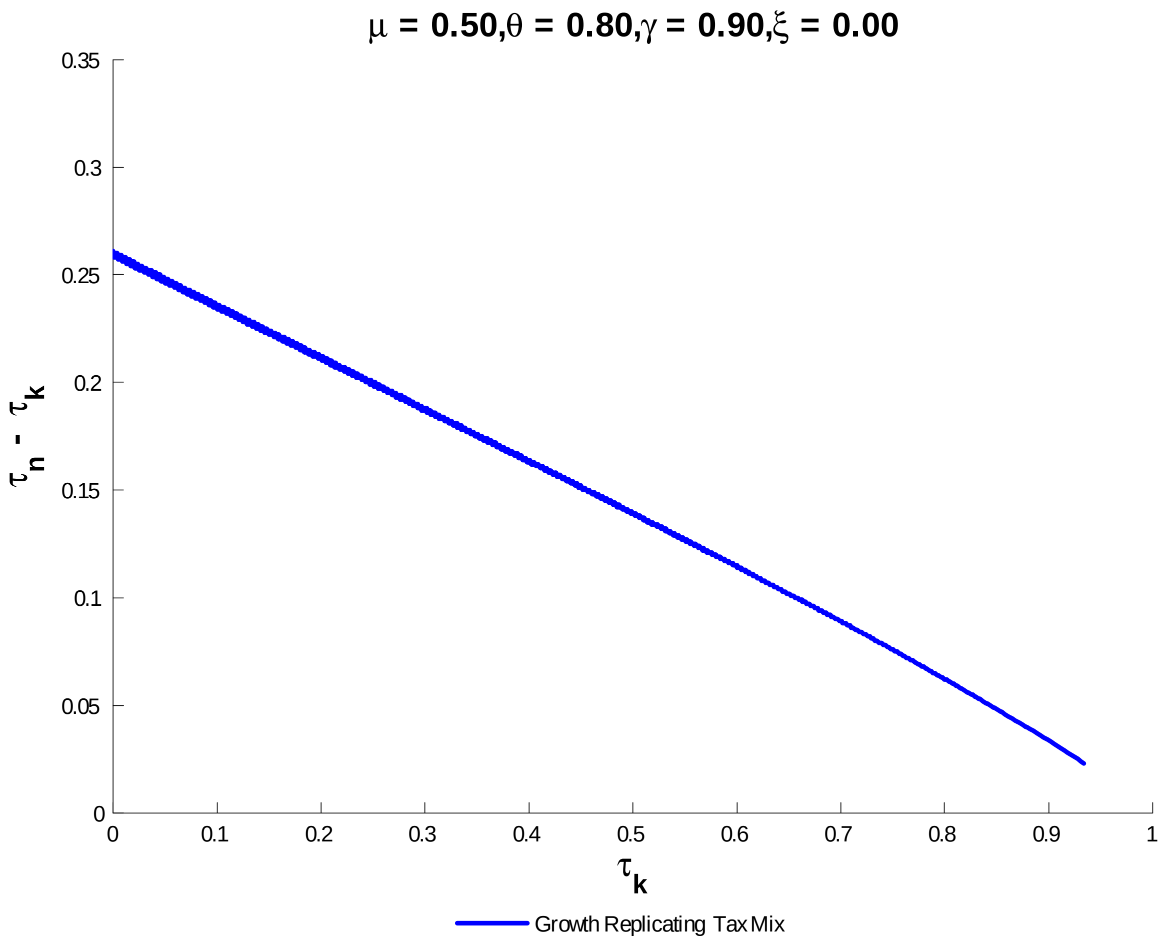

Figure 7: Growth replicating tax mix for $\xi = 0$ and satisfying condition (40)

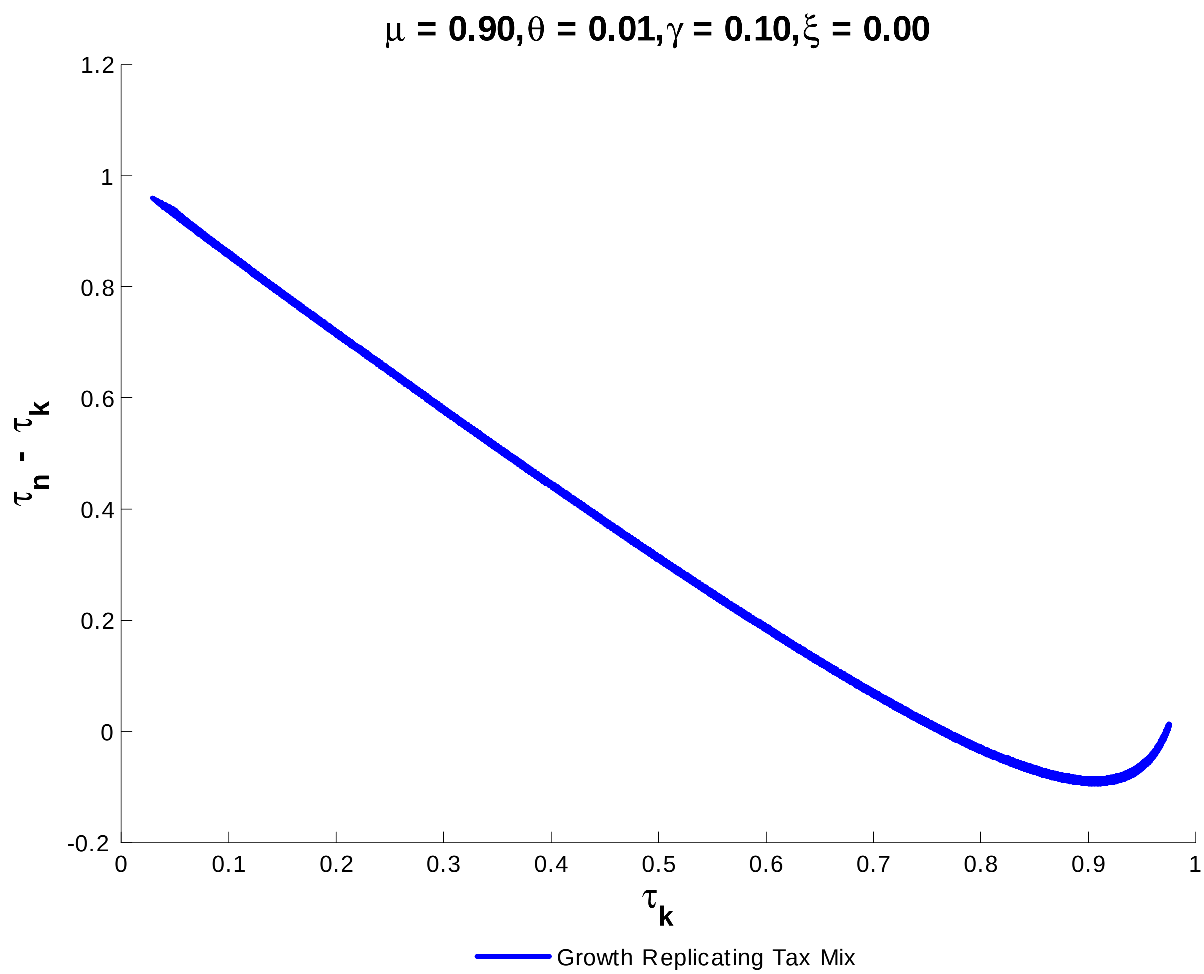

Figure 8: Growth replicating tax mix for $\xi = 0$ and violating condition (40)