

# Distortionary Taxes and Public Investment in a Model of Endogenous Investment Specific Technological Change\*

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## Abstract

We construct a model of endogenous investment specific technological change in which the stock of public capital influences the real price of capital goods. We show that the growth and welfare maximizing tax rates coincide in the planned economy. When factor income taxes finance public investment infinitely many tax-subsidy combinations can decentralize the planner's allocations. The optimal capital income tax can be positive in this environment. We then augment the model to incorporate administrative costs. A unique combination of factor income taxes now decentralizes the planner's allocations. A simple calibration exercise suggests that changes in factor income taxes does not cause a significant change in the optimal growth rate or welfare. Our framework broadens the environment in which investment specific technological change occurs, and characterizes the role of optimal factor income taxation in raising long run growth and welfare.

**Keywords** : Investment Specific Technological Change, Endogenous Growth, Capital Income Taxation, Public Policy, Administrative Costs.

**JEL Codes**: E2; E6; H2; O4

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# 1 Introduction

A growing literature attributes the importance of investment specific technological change to long run growth (see Greenwood et al. (1997, 2000); Whelan (2003)). Investment specific technological change refers to technological change which reduces the real price of capital goods. Greenwood et al. (1997, 2000) show that once the falling price of real capital goods is taken into account, this explains most of the observed growth in output in the US, with relatively little being left over to be explained by total factor productivity. Other authors, such as Gort et al. (1999) distinguish between equipment specific technological change and structure specific technological change. These authors show that 15% of US economic growth rate can be attributed to structure specific technological change in the post war period, while equipment-specific technological progress accounts for 37% of US growth. This implies 52% of US economic growth can be attributed to technological progress in new capital goods.<sup>1</sup> However, investment specific technological change in these models is typically assumed to be an exogenous process.

In a series of recent papers, Huffman (2007, 2008) builds upon this literature by explicitly modeling the mechanism by which the real price of capital falls when investment specific technological occurs. Such models are characterized by endogenous investment specific technological change. In Huffman (2008), the changing relative price of capital is driven by research activity, undertaken by labor effort. Higher research spending in one period lowers the cost of producing the capital good in the next period.<sup>2</sup> Agents equate the utility costs of raising employment in research with the benefits of doing so. The return to increasing research employment is the discounted value of the reduction in the real cost of investment in future periods. Extra research employment also leads to higher future consumption because of the reduction in cost of investing in capital goods because of the research.<sup>3</sup> Investment specific technological change is thus endogenous in the model, since employment can either be undertaken in a research sector or a production sector. Huffman (2008) then studies a version of the model with capital and labor taxation, and a research externality in which agents or firms directly benefit from the research undertaken by others. In such an environment, Huffman shows that there exists a competitive equilibrium that delivers the optimal

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<sup>1</sup>See Greenwood and Krusell (2007) for a careful discussion of growth accounting in the presence of investment specific technological change. Cummins and Violante (2002) show that "technological gaps", i.e., the comparative productivity of a new machine relative to an average machine, can explain the dynamics of investments in new technologies as well as returns to human capital.

<sup>2</sup>Krusell (1998) also builds a model in which the decline in the relative price of equipment capital is a result of R&D decisions at the level of private firms.

<sup>3</sup>However, if the utility cost of raising research employment is high, then all employment would be devoted to production of output, and there would be no research employment. There would be no output growth, and the economy converges to a steady state with constant levels of output.

allocations.

However, there are two important aspects of investment specific technological change missing from the literature.

First, public sector involvement in enhancing productivity is well documented in developing economies (see Schmitz (2001)). We therefore have in mind an environment where the government invests in the stock of public capital – as in Barro (1990), Fischer and Turnovsky (1997) and Eicher and Turnovsky (2000) – which in turn has a direct impact on reducing the price of real capital goods. Hence, there is a role for optimal tax policy in generating endogenous growth and raising welfare, since public capital is funded by distortionary taxes.<sup>4</sup>

Second, we assume that investment specific technological change is also assisted by learning by doing (see Arrow (1962)). In particular, we assume that the aggregate capital-output ratio exerts a positive contribution on reductions in the future price of real capital goods because of learning by doing. This is consistent with Greenwood et al (1997) in which the real price of capital equipment in the US – since 1950 - has fallen even though the US investment-GNP ratio has risen. Learning by doing therefore exerts a separate - though positive - effect on the level of investment specific technological change.

To keep the model analytically tractable, we focus on the balanced growth path (BGP). We show that the balanced growth path is stable under a reasonable restriction. We characterize the growth and welfare maximizing tax rates. We show that the growth and welfare maximizing tax rate is given by the relative importance of public capital vis-a-vis learning by doing in the investment specific technological function. The implication of this is that if a planner was to choose the tax rate, he could maximize long run growth and welfare as long as the tax rate equals the relative contribution of public capital to investment specific technological change.

We then derive a restriction which enables us to decentralize the planner's allocations. Given this condition, we show that infinitely many combinations of factor income taxes can replicate the planner's allocations based on an optimal tax rule. This rule suggests that a weighted average of factor income taxes (with the weights determined by the relative elasticities that factors have in production) should equal the relative contribution of public capital in investment specific technological change.

Since raising taxes for government investments involve administrative costs, a natural extension is to incorporate a variety of specifications in which tax revenues accruing to the government change as the incidence of taxation increases. When costs increase with increases in the tax rate (see Perotti (1993) and Buiters (2011)) we show that there is a level reduction in the growth rate for all tax rates, and a lower growth maximizing tax rate. However, as in

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<sup>4</sup>Public capital however does not directly affect final good production.

the model without costs, the welfare and growth maximizing tax rates coincide – although at lower tax rates. A linear cost technology induces a level downward shift in growth for all tax rates, although it doesn’t change the growth maximizing tax rate. In the decentralized version, we assume that individual factor income taxes are associated with tax specific cost components. Our main result now – which is different from the baseline model – is that there exists a *unique* ranking between taxes on capital and labor which maximizes the steady state rate of growth. In particular, if collecting both capital and labor income taxes are associated with a rising (but common) cost component, we show that labor should be taxed at a *higher* rate than capital. However, if the cost associated with collecting both capital and labor income taxes increases at a *decreasing* rate, then the optimal tax rate on labor should be less than the tax rate on capital. In other words, a higher tax rate on capital – relative to labor – or vice versa, can be growth enhancing, depending on the curvature of the cost technology.

We quantify the general equilibrium effect of an increase in the share of government spending on infrastructure relative to aggregate output – in the investment specific technological change equation – on the equilibrium growth rate. Although a higher share of public infrastructure spending leads to a higher optimal tax rate, we show that the impact of higher taxes on growth depends on the persistence of investment specific technological change in determining the future level of the real price of capital.

Quantitatively, we show that reducing the tax on labor, while increasing the tax on capital by an equi-proportionate amount, reduces growth marginally. However, a revenue neutral change - which takes into account the elasticity adjusted factor income tax changes - a rule that we characterize analytically – increases growth in comparison to the equi-proportionate case. This result holds for large changes in the labor income tax. However, if we reduce the tax on capital, the change required in terms of a revenue neutral increase in the labor income tax is less than the equi-proportionate case. Hence, a reduction in the tax on labor increases growth if we compare the equi-proportionate case to the revenue neutral case. These results contrast with the case where we increase the tax on capital. Finally, comparing the growth effects of individual factor income taxes in partial equilibrium, we derive an analytical condition on the marginal impact of changes in factor income taxes on growth. We show that the effect of a change in capital income taxes on growth (compared to a change in the labor income tax) depends on the value of the share of output paid to capital. This is consistent with some of the results in this literature (see Stokey and Rebelo (1995)) that the growth effects of changes in the capital income tax rate are not large.

In the model with administrative costs, our policy experiments show that when we have concave administrative costs, it is easier to increase the tax on capital to re-establish optimal growth rates, compared to the model with convex administrative costs. We also show that

there is virtually no change in growth or welfare for a significant increase in the tax on capital that matches a given reduction in the tax on labor. From a policy standpoint, this suggests that it may be easier to tax capital at a higher rate without changing growth or welfare when administrative costs are concave, when the tax on labor is reduced.

Our paper is related to two strands of the literature on fiscal policy and long run growth in the neo-classical framework. The first literature - started by Barro (1990) and Futagami, Morita, and Shibata (1993) – incorporate a public input – such as public infrastructure – that directly augments production.<sup>5</sup> However, in the large literature on public capital and its impact on growth spawned by these papers, government investment doesn't directly augment investment specific technological change. Our model presents a tractable framework to think about public capital, its financing via factor income taxes, and the role of optimal tax policy on investment specific technological change. Hence, there is a role for optimal public policy.

Our results are also related to a celebrated literature started by Judd (1985) and Chamley (1986), who find that capital taxation decreases welfare and a zero capital tax is thus efficient in the long-run steady state. From a growth standpoint, models analyzing the equilibrium relationship between capital income taxes and growth also typically find that an increase of the capital income tax reduces the return to private investment, which in turn implies a decrease of capital accumulation and thus growth (see Lucas (1990) and Rebelo (1991)). In contrast, our results are consistent with some other papers in this literature which show that the optimal capital income tax is positive, i.e., taxation may *increase* growth (see Uhlig and Yanagawa (1996) and Rivas (2003)). On normative grounds, our results suggests that policy makers may want to measure precisely the relative cost associated with factor income tax collection before setting factor income taxes.

The rest of the paper proceeds as follows. Section 2 develops the baseline model. Section 3 develops the model with administrative costs. Section 4 conducts numerous policy experiments. Section 5 concludes.

## 2 The Model

Consider an economy that is populated by identical representative agents, who at each period  $t$ , derive utility from consumption of the final good  $C_t$  and leisure  $(1 - n_t)$ . The term  $n_t$  represents the fraction of time spent at time  $t$  in employment. The discounted life-time

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<sup>5</sup>See Glomm and Ravikumar (1994, 1997), Fischer and Turnovsky (1997), and Eicher and Turnovsky (2000).

utility,  $U$ , of an infinitely lived representative agent is given by

$$U = \sum_{t=0}^{\infty} \beta^t [\log C_t + \log(1 - n_t)]. \quad (1)$$

where  $\beta \in (0, 1)$  denotes the period-wise discount factor. The final good is produced by a standard Cobb-Douglas production function with a constant returns to scale technology. The production function is given by

$$Y_t = AK_t^{\alpha_1} n_{1t}^{\alpha_2} (\delta_m n_{2t})^{\alpha_3}, \quad (2)$$

Output is produced using capital,  $K_t$ , and two different sources of labor supplied by agents,  $n_{1t}$  and  $n_{2t}$ , all of which are essential to production. There is no population growth.  $A > 0$  is the productivity parameter. The first part of labor input,  $n_1$ , is devoted to direct production of final output. The second part of agent labor effort,  $n_2$ , can be thought of as a more specialized labor input required for research and development which affects the level of investment specific technological change,  $Z$ . In particular, we assume that a representative firm employs a fraction,  $\delta_m \in (0, 1)$ , of  $n_2$  in final production. The remaining fraction of  $n_2$  not used directly in production of the final good is devoted to research effort which augments the level of investment specific technological change ( $Z$ ) in the subsequent time period. Thus,  $n_2$ , is analogous to the research employment in Huffman (2008) that goes into raising  $Z$ .<sup>6</sup> In other words, the higher is the fraction of  $n_2$  allocated for research effort, the higher is the future value of  $Z$ . The total supply of labor by an agent at time  $t$  is given by the following

$$n_t \equiv n_{1t} + n_{2t}. \quad (3)$$

The shares of capital  $K_t$ ,  $n_{1t}$  and  $n_{2t}$  in final goods production are given by  $\alpha_i \in (0, 1)$ ,  $i = 1, 2, 3$  respectively. The assumption of constant returns to scale in this model ensures that they add up to unity, that is,  $\alpha_1 + \alpha_2 + \alpha_3 = 1$ .<sup>7</sup>

Private capital accumulation grows according to the standard law of motion augmented by investment specific technological change,

$$K_{t+1} = (1 - \delta)K_t + I_t Z_t, \quad (4)$$

where  $\delta \in [0, 1]$  denotes the rate of depreciation of capital and  $I_t$  represents the amount of

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<sup>6</sup>Other papers in the literature - such as Reis (2011) - also assume two types of labor affecting production. In Reis (2011), one form of labor is the standard labor input, while the other labor input is entrepreneurial labor. Our analysis considers  $n_2$  as quality enhancing labor.

<sup>7</sup>In Huffman,  $n_2$  doesn't enter into the production of final goods directly.

total output allocated towards private investment at time period  $t$ .  $Z_t$  represents investment-specific technological change. The higher the value of  $Z_t$ , the lower is the cost of accumulating capital in the future. Hence  $Z_t$  also represents the inverse of the price of per-unit private capital at time period  $t$ . Thus at every period  $t$ ,  $Z_t$  augments investment  $I_t$ .  $I_t Z_t$  thus represents the effective amount of investment driving capital accumulation in time period  $t + 1$ .<sup>8</sup>

In addition to labor time deployed by the representative firm towards R&D, the public capital stock,  $G$ , plays a crucial role in lowering the price of capital accumulation. Typically, public infrastructure, or public capital, is seen as directly affecting final production (see Barro (1990)). Rather, we assume that the presence of government support is essential to facilitate progress in research and development.<sup>9</sup> In every period, public investment is funded by a proportional tax,  $\tau \in [0, 1]$ , on income. A higher addition to  $G$  relative to the level of output,  $Y$ , ensures better support for future research and development. We assume that public infrastructure investments are subject to congestion. This implies that reductions in  $G$  relative to the current level of output,  $Y$ , lead to lower future research and development. We assume that public capital evolves according to

$$G_{t+1} = (1 - \delta)G_t + I_t^g Z_t, \quad (5)$$

where  $G_{t+1}$  denotes the  $t+1$  public capital stock, and  $I_t^g$  denotes the level of public investment made by the government in time period  $t$ . To retain tractability, we assume that  $Z_t$  augments  $I_t^g$  in the same way as  $I_t$  since it is logical to assume that  $Z_t$  is benefited by both private agents and the government in an identical manner. To derive the balanced growth path, we further assume that the period wise depreciation rate  $\delta \in [0, 1]$  is same for both private capital accumulation and public capital. The government budget constraint is given by

$$I_t^g = \tau Y_t, \quad (6)$$

where  $\tau \in (0, 1)$  is the tax rate imposed by the planner to fund addition to public input for R&D.

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<sup>8</sup>Technical progress – in the form of lowering the price of capital accumulation – can be seen as being facilitated by more learning-by-doing because of more experience that comes with using equipment and machinery (Arrow, 1962). Higher future capital accumulation relative to current levels of output therefore ensures more learning by doing and hence higher future values of  $Z$  thereby further lowering the cost of capital accumulation.

<sup>9</sup>See, for instance, Agenor (2011).

## 2.1 Investment Specific Technological Change

To capture the effect of public capital on research and development, we assume that  $Z$  grows according to the following law of motion,

$$Z_{t+1} = B((1 - \delta_m)n_{2t})^\theta Z_t^\gamma \left\{ \left( \frac{G_t}{Y_{t-1}} \right)^\mu \left( \frac{K_t}{Y_{t-1}} \right)^{1-\mu} \right\}^{1-\gamma}. \quad (7)$$

Here,  $B$  stands for an exogenously fixed scale productivity parameter,  $1 - \delta_m$  represents the fraction of labor input,  $n_{2t}$ , devoted towards R&D, and  $\mu \in (0, 1)$  captures the impact of public investments on investment specific technological change. We assume that the parameters,  $\theta \in (0, 1)$  and  $\gamma \in (0, 1)$ , where  $\theta$  stands for the weight attached to research effort and  $\gamma$  is the level of persistence the current year's level of technology has on reducing the price of capital accumulation in the future. The term  $\frac{G_t}{Y_{t-1}}$  represents public capital's influence in affecting investment specific technological change in time period  $t + 1$ . For a given  $\mu$ , increases in  $G_t$  relative to  $Y_{t-1}$  leads to increases in the future level of  $Z$ . Likewise, for a given  $\mu$ , reductions in  $G_t$  relative to  $Y_{t-1}$  leads to decreases in future values of  $Z$ . We further assume the existence of learning by doing effects captured by the aggregate capital-output ratio,  $\frac{K_t}{Y_{t-1}}$ . In particular, a higher aggregate stock of capital in  $t$ ,  $K_t$ , relative to  $Y_{t-1}$ , raises  $Z_{t+1}$ . At this stage, we make three remarks to compare our setup with that in Huffman (2008).

**Remark 1** Assuming  $\gamma = 1$ ,  $\delta_m \rightarrow 0$  and  $\alpha_3 = 0$ , in equation (7) yields Equation 2.9 in Huffman (2008) describing investment specific technological change.

**Remark 2** We require  $\gamma \in (0, 1)$  for the equilibrium growth rate to adjust to the steady state balanced growth path.<sup>10</sup>

**Remark 3** Equation (7) is homogenous of degree  $1 + \theta$ .

## 2.2 The Planner's Problem

We first solve the planner's problem. The resource constraint the economy faces in each time period  $t$  is given by

$$C_t + I_t \equiv Y_t(1 - \tau) = AK_t^{\alpha_1} n_{1t}^{\alpha_2} (\delta_m n_{2t})^{\alpha_3} (1 - \tau) \quad (8)$$

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<sup>10</sup>This contrasts with Huffman (2008) where  $\gamma = 1$  is required for growth rates of  $Z$  and output to be along the balanced growth path. In Huffman (2004),  $\gamma < 1$  implies that the effect of research spending diminishes over time. This generates technical innovation having immediate productive effects that can be maintained only with more spending in the future. Therefore  $\gamma = 1$  is not needed for balanced growth.

where agents consume  $C_t$  at time period  $t$  and invest  $I_t$  at time period  $t$ . Aggregate consumption and investment add up to after-tax levels of output,  $Y_t(1-\tau)$ , in every time period.

The planner maximizes life-time utility of a representative agent – given by (1) – subject to the economy wide resource constraint given by (8), the law of motion of private capital in equation (4), the law of motion of public capital in equation, (5), the equation describing investment specific technological change (7), the identity for total supply of labor given by (3) and finally, the government budget constraint given by (6).<sup>11</sup>

### 2.2.1 First Order Conditions

The Lagrangian for the planner's problem is given by,

$$L = \sum_{t=0}^{\infty} \beta^t [\log C_t + \log(1 - n_{1t} - n_{2t}) + \lambda_t \{AK_t^{\alpha_1} n_{1t}^{\alpha_2} (\delta_m n_{2t})^{\alpha_3} (1 - \tau) - C_t - I_t\}]. \quad (9)$$

For simplicity, we assume that  $\delta = 1$ . The following first order conditions obtain with respect to  $C_t$ ,  $K_{t+1}$ ,  $n_{1t}$ , and  $n_{2t}$ , respectively<sup>12</sup>:

$$\frac{1}{C_t} = \lambda_t \quad (10)$$

$$\frac{1}{C_t Z_t} = \frac{\alpha_1 \beta Y_{t+1} (1-\tau)}{C_{t+1} K_{t+1}} + \frac{\beta^2 I_{t+2} (1-\gamma) (1-\mu)}{C_{t+2} K_{t+1}} + \frac{\beta^3 (1-\gamma) (\gamma (1-\mu) - \alpha_1)}{K_{t+1}} \sum_{j=0}^{\infty} \frac{\beta^j \gamma^j I_{t+j+3}}{C_{t+j+3}} \quad (11)$$

$$\frac{1}{1-n_t} + \frac{\beta^2 \alpha_2 (1-\gamma)}{n_{1t}} \sum_{j=0}^{\infty} \frac{\beta^j \gamma^j I_{t+j+2}}{C_{t+j+2}} = \frac{\alpha_2 Y_t (1-\tau)}{C_t n_{1t}} \quad (12)$$

and,

$$\frac{1}{1-n_t} = \frac{\alpha_3 Y_t (1-\tau)}{C_t n_{2t}} + \frac{\beta \theta I_{t+1}}{C_{t+1} n_{2t}} + \frac{\beta^2 (\gamma \theta - \alpha_3 (1-\gamma))}{n_{2t}} \sum_{j=0}^{\infty} \frac{\beta^j \gamma^j I_{t+j+2}}{C_{t+j+2}}. \quad (13)$$

Equation (10) represents the standard first order condition for consumption, equating the marginal utility of consumption to the shadow price of wealth. Equation (11) is an augmented form of the standard Euler equation governing the consumption-savings decision of the household. The first term on the RHS of equation (11),  $\frac{\alpha_1 \beta Y_{t+1} (1-\tau)}{C_{t+1} K_{t+1}}$ , corresponds to the after tax marginal productivity of capital in  $t+1$ . The second term,  $\frac{\beta^2 I_{t+2} (1-\gamma) (1-\mu)}{C_{t+2} K_{t+1}} > 0$ , is the (further) increment to the marginal productivity of capital that agents get in period  $t+2$  by postponing consumption today. This is increasing in the investment-consumption ratio, but adjusted by the weight,  $1 - \mu$ , of the aggregate capital-output ratio, in the investment

<sup>11</sup>Clearly,  $C_t + I_t + I_t^g = Y_t$ .

<sup>12</sup>See Appendix A for details.

specific technological change equation. The third term,  $\frac{\beta^3(1-\gamma)(\gamma(1-\mu)-\alpha_1)}{K_{t+1}} \sum_{j=0}^{\infty} \frac{\beta^j \gamma^j I_{t+j+3}}{C_{t+j+3}}$ , is the discounted increase in marginal productivity of investing in capital from period  $t+3$  onwards. This expression is adjusted by the term  $(\gamma(1-\mu) - \alpha_1)$ , which can be either positive or negative – depending on the relative importance of capital in equation (7) vis-a-vis its direct contribution to increasing output, from (2). It is easy to see that when  $\gamma = 1$ , the additional terms in the Euler equation are equal to zero, yielding the standard Euler equation.

Equation (12) denotes the optimization condition with respect to labor supply ( $n_{1t}$ ). If we reorganize (12), we get the following relationship between marginal rate of substitution between consumption and leisure and the marginal product for  $n_1$ ,

$$\frac{C_t}{1-n_t} + \frac{\beta^2 \alpha_2 (1-\gamma)}{n_{1t}} \sum_{j=0}^{\infty} \frac{\beta^j \gamma^j C_t I_{t+j+2}}{C_{t+j+2}} = \frac{\alpha_2 Y_t (1-\tau)}{n_{1t}}. \quad (14)$$

The term on the right hand side of the above equation,  $\frac{\alpha_2 Y_t (1-\tau)}{n_{1t}}$ , denotes the standard after tax marginal product of  $n_1$ . The second term on the LHS,  $\frac{\beta^2 \alpha_2 (1-\gamma)}{n_{1t}} \sum_{j=0}^{\infty} \frac{\beta^j \gamma^j C_t I_{t+j+2}}{C_{t+j+2}}$ , is the increment to the marginal rate of substitution between leisure and consumption. Hence, the effective marginal rate of substitution between leisure and consumption rises. Since  $0 < \gamma < 1$ , this term is positive. In other words, leisure is more valuable at time  $t$  because of the indirect effect that  $n_1$  has on future capital accumulation and learning by doing. This reduces  $n_1$  relative to the standard case in which there is no investment specific technological change. As such, this effect acts like an additional tax on  $n_1$ .

Similarly, re-arranging the terms of (13), we obtain,

$$\frac{C_t}{1-n_t} = \frac{\alpha_3 Y_t (1-\tau)}{n_{2t}} + \frac{\beta \theta C_t I_{t+1}}{C_{t+1} n_{2t}} + \frac{\beta^2 (\gamma \theta - \alpha_3 (1-\gamma))}{n_{2t}} \sum_{j=0}^{\infty} \frac{\beta^j \gamma^j C_t I_{t+j+2}}{C_{t+j+2}}. \quad (15)$$

Unlike  $n_1$ ,  $n_2$  affects both production and investment specific technological change. The first term on the RHS of (15),  $\frac{\alpha_3 Y_t (1-\tau)}{n_{2t}}$ , is the standard after tax marginal product of  $n_2$ . The remaining terms are the  $t > 0$  increment to marginal productivity that accrue in the future because of  $n_2$ 's role in assisting *both* research effort and increasing output. However, because  $n_2$  has a direct and indirect effect (through production and investment specific technological change, respectively), the future discounted gains are adjusted by the term,  $\gamma \theta - \alpha_3 (1-\gamma)$ . This term reflects the effective contribution of  $n_2$  to future gains because of its role in investment specific technological change function (hence the term  $\gamma \theta$ ) vis-a-vis the marginal productivity gains from a higher capital output ratio adjusted by  $n_2$ 's share in production (hence the term,  $\alpha_3 (1-\gamma)$ ). In contrast to  $n_1$ , investment specific technological change acts like a subsidy on  $n_2$ , increasing it relative to the standard case. When  $\gamma = 1$ ,

the marginal productivity gains from the capital output ratio are zero for any time period from  $t + 2$  onwards.

We now derive the closed form decision rules based on the above first order conditions.

### 2.2.2 Decision Rules

**Lemma 1**  $C_t, I_t, n_t, n_{1t}, n_{2t}$  are given by (16), (17), (18), where  $0 < \Phi < 1$  is given by (19), and  $0 < x < 1$  given by (20) are constants. The supply of labor is constant. Then,

$$C_t = \Phi Y_t(1 - \tau), I_t = (1 - \Phi)Y_t(1 - \tau), \quad (16)$$

$$n_t = n = \frac{\alpha_2[(1 - \beta\gamma) - \beta^2(1 - \gamma)(1 - \Phi)]}{(\alpha_2 + \Phi x)(1 - \beta\gamma) - \alpha_2\beta^2(1 - \gamma)(1 - \Phi)}, \quad (17)$$

$$n_{1t} = n_1 = xn, n_{2t} = n_2 = (1 - x)n, \quad (18)$$

where  $\Phi$  is given by

$$\Phi = \frac{(1 - \beta\gamma)[1 - \beta^2(1 - \gamma)(1 - \mu) - \alpha_1\beta] - \beta^3(1 - \gamma)[\gamma(1 - \mu) - \alpha_1]}{(1 - \beta\gamma)[1 - \beta^2(1 - \gamma)(1 - \mu)] - \beta^3(1 - \gamma)[\gamma(1 - \mu) - \alpha_1]}, \quad (19)$$

and  $x$  is given by

$$x = \frac{\alpha_2(1 - \beta\gamma) - \beta^2\alpha_2(1 - \gamma)(1 - \Phi)}{[\alpha_2 + \alpha_3 + \beta\theta(1 - \Phi)](1 - \beta\gamma) + \beta^2(1 - \Phi)[\gamma\theta - \alpha_3(1 - \gamma) - \alpha_2(1 - \gamma)]}. \quad (20)$$

**Proof.** These expressions follow from the first order conditions as shown in (10), (11), (12) and (13). See appendix B for details. ■

How does a change in  $\mu$  affect consumption and investment? While the decision rules for consumption and investment given by (16) suggest that the levels of consumption and investment would fall if the tax rates increases (because of the  $1 - \tau$  term), the actual share of after tax income spent on consumption given by (19) rises when  $\mu$  rises, although for investment it falls.<sup>13</sup> Intuitively, the representative agent does not invest as much in private capital because of an enhanced role of public capital in augmenting investment specific technological change.

We will show later that when  $\tau \neq \mu$ , the allocations from the planner's problem are sub-optimal, even though there is balanced growth. Here, we note that labor supply,  $n$ , is

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<sup>13</sup>See Appendix F.

constant and independent of the tax rate. This is because of the assumption of log utility. However, labor supply is affected by  $\mu$ . This can be shown by the following lemma

**Lemma 2** *An increase in  $\mu$  leads to an increase in the share of  $n$  devoted to  $n_1$ , i.e.,  $\frac{\partial x}{\partial \mu} > 0$ . Since  $\frac{\partial \Phi}{\partial \mu} > 0$  from before, this implies  $\frac{\partial n}{\partial \mu} < 0$ .*

**Proof.** *Shown in Appendix F. ■*

Lemma (2) shows that while changes in  $n_1$  due to an increase in  $\mu$  is ambiguous, the actual amounts of labor supply devoted towards research effort,  $n_2$ , will always fall with an increase in  $\mu$ . However, the latter effect dominates the ambiguous effect of how  $n_1$  changes, thereby leading to an overall decrease in  $n$ . This implies that an enhanced role of public capital induces agents to supply lesser labor ( $n$ ), particularly towards research effort ( $n_2$ ).<sup>14</sup>

### 2.3 Stability of the Balanced Growth Path

To obtain the balanced growth path (BGP), we substitute the above decision rules into the law of motion for investment specific technological progress, (7), to characterize the balanced growth path (BGP). Given the decision rules (16), (17), (18), (20) and (6), we can re-write the above law of motion as

$$Z_{t+1} = \widehat{M} Z_t^\gamma Z_{t-1}^{(1-\gamma)} \{(\tau)^\mu (1-\tau)^{1-\mu}\}^{(1-\gamma)} \quad (21)$$

where  $\widehat{M}$  is a constant and is expressed as

$$\widehat{M} = B((1-\delta_m)(1-\widehat{x})\widehat{n})^\theta (1-\widehat{\Phi})^{(1-\mu)(1-\gamma)}.$$

If we define the absolute growth rate by  $\frac{Z_{t+1}}{Z_t} = g_{z_{t+1}}$ , then we can re-write (21) as

$$g_{z_{t+1}} = g_{z_t}^{\gamma-1} \widehat{M} \{(\tau)^\mu (1-\tau)^{1-\mu}\}^{(1-\gamma)}. \quad (22)$$

<sup>14</sup>To see this, we can decompose the total change in  $n$  because of changes in  $\mu$  by

$$\frac{\partial n}{\partial \mu} = \frac{\partial n_1}{\partial \mu} + \frac{\partial n_2}{\partial \mu}.$$

Given  $\frac{\partial x}{\partial \mu} > 0$  and  $\frac{\partial \Phi}{\partial \mu} > 0$  (and hence,  $\frac{\partial(1-x)}{\partial \mu} < 0$ )  $\frac{\partial n_2}{\partial \mu} < 0$  will be true. Since the change in  $n_1$  due to a change in  $\mu$  can be written as

$$\frac{\partial n_1}{\partial \mu} = x \underbrace{\frac{\partial n}{\partial \mu}}_{<0} + n \underbrace{\frac{\partial x}{\partial \mu}}_{>0},$$

$\frac{\partial n_1}{\partial \mu}$  may or may not be negative. While an increase in  $\mu$  has an ambiguous effect on  $n_1$ , it reduces  $n_2$ . However, because the latter dominates the former effect,  $n$  falls.

Using (22) and the parameter restrictions in Remark 1, we will get the same constant growth rates along the balanced growth path (BGP) as in Huffman (2008). We re-write (22) in the following way,

$$g_{z_{t+1}} = \frac{\Omega}{g_{z_t}^{(1-\gamma)}}, \quad (23)$$

where  $\Omega$  is a constant and is expressed as

$$\Omega = \widehat{M}\{(\tau)^\mu(1-\tau)^{1-\mu}\}^{(1-\gamma)}.$$

Under the condition  $0 < \gamma < 1$  it is easy to show that we can obtain a constant growth rate for  $Z$ ,  $K$ ,  $G$  and  $Y$ . This condition necessarily implies  $0 < \Phi < 1$  and  $0 < x < 1$  (as shown in Appendix B). We therefore have the following lemma. Figure [1] shows the dynamic adjustment to the steady state balanced growth graphically.

**Lemma 3** *Suppose  $0 < \gamma < 1$ . On the steady state balanced growth path, the gross growth rate of  $Z$ ,  $K$ ,  $G$  and  $Y$  are given by (24), and (25)*

$$\widehat{g}_z = \Omega^{\frac{1}{1+(1-\gamma)}} = [\widehat{M}\{(\tau)^\mu(1-\tau)^{1-\mu}\}^{(1-\gamma)}]^{\frac{1}{2-\gamma}}, \quad (24)$$

$$\widehat{g}_k = \widehat{g}_g = \widehat{g}_z^{\frac{1}{1-\alpha_1}}, \widehat{g}_y = \widehat{g}_k^{\alpha_1} = \widehat{g}_z^{\frac{\alpha_1}{1-\alpha_1}}. \quad (25)$$

**Proof.** While  $\widehat{g}_z$  can be computed directly from (23), the expressions for the remaining variables are derived in Appendix C. ■

[INSERT FIGURE 1]

There are several aspects of the equilibrium growth rate worth mentioning. First, the growth rate is independent of the technology parameter,  $A$ , as in Huffman (2008). Second, the growth rate of output,  $\widehat{g}_y$ , is less than  $\widehat{g}_k$  along the balanced growth path, which is because equation (7) is homogenous of degree  $1 + \theta$ . Third, an increase in  $\mu$ , increases the optimal growth rate, by skewing the growth rate schedule to the right. This happens because an increase in  $\mu$  lowers the amount of labor input ( $n_2$ ) by agents towards research effort. Hence, in order to finance an increased importance of public capital (a higher  $\mu$ ) in investment specific technological change, the government charges a higher tax on income.

Finally, from expression (24), the tax rate exerts a positive effect on growth as well as a negative effect. This is similar to the equation characterizing the growth maximizing tax rate in Barro (1990). The mechanism here is however different. For small values of the tax rate, a rise in  $\tau$  leads to higher public capital relative to output,  $Y_{t-1}$ . This raises the future value of investment specific technological change,  $Z$ . An increase in  $Z$  reduces the real

price of capital, stimulating investment and long run growth. However, for higher tax rates, further increases in the tax rate depresses after tax income, and investment. This reduces  $G$  relative to  $Y$ , lowering  $Z$ , and depressing investment and long run growth. Hence, there is a unique growth maximizing tax rate.

Using the expression for  $\hat{g}_z$  in (24) we can characterize the growth maximizing tax rate as follows:

**Proposition 1** *In the steady state, there exists a unique growth maximizing tax rate, given by  $\hat{\tau} = \mu$ .*

**Proof.** See appendix D. ■

Proposition [1] sets a benchmark for the planner to set the optimal tax rate. If the planner wants to maximize growth, he should set the tax rate to  $\mu$ . The higher the weight attached to  $\frac{G_t}{Y_{t-1}}$  in the investment specific technological change equation, the higher should be the optimal tax set by the planner. This result is intuitive since it suggests that the government would have to impose a higher tax rate on income if public capital were to play a greater role in driving investment specific technological change.

INSERT TABLE [1]

Figure [2] calibrates the impact of a change in  $\mu$  on the long run growth rate for two arbitrary values of  $\mu$ . Parameter values taken from Table 1.<sup>15</sup> When  $\mu = 0.5$ , the growth maximizing tax rate is given by  $t_1 = 0.5$ . When  $\mu = 0.6$ , the growth maximizing tax rate rises to  $t_2 = 0.6$ . However, because  $\mu$  has increased, there is a level downward shift in the growth tax curve associated with the higher value of  $\mu$  (the red line is lower than the blue line). This is because of the reduction in growth due to a reduction  $n_2$ . Due to this effect, the planner needs to raise taxes to maximize growth, given that now there is a higher weightage on  $\frac{G_t}{Y_{t-1}}$ . In sum, a higher value of  $\mu$  tilts the growth-tax curve in a south-westerly direction, leading to a higher growth maximizing tax rate. The net effect on growth however depends on which effect (reduction in  $n_2$  versus more weightage on  $\frac{G_t}{Y_{t-1}}$ ) dominates. This is sensitive to the value of  $\gamma$ . In particular, the effect of  $n_2$  on reducing growth is higher for higher values of  $\gamma$ . Likewise, if there is a reduction in  $\mu$ ,  $n_2$  may increase sufficiently leading to higher growth depending on the value of  $\gamma$ .

[INSERT FIGURE 2]

Finally, the growth rate is decreasing in  $\delta_m$ , the weight attached to  $n_2$  in production. In other words, as  $\delta_m$  increases, more  $n_2$  (specialized R&D labor) is devoted to production of

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<sup>15</sup>Without loss of generality we assume  $A = 1$  and  $B = 1$ . The parameters which have a (\*) against their values means that they have been borrowed from Huffman (2008). The rest of the parameters have been assigned in order to ensure feasibility.

the final good, and less to investment specific technological change. This reduces the future value of  $Z$ , and in the long run lowers optimal growth although there is no change in the growth optimizing tax rate. This is because a higher  $\delta_m$  leads to a level downward shift in the growth tax curve, for all tax rates. To illustrate the quantitative impact of higher  $\delta_m$  on growth, we first increase  $\delta_m$  arbitrarily from 0.2 to 0.3. We find that growth falls from 0.491 to 0.4622 (the difference being 0.0288). For higher values of  $\delta_m$  that is, increasing  $\delta_m$  from 0.5 to 0.6, we find that the fall in growth is 0.3968 to 0.3587 (the difference is 0.0381). This shows that the fall in the growth rate is *higher* for higher values of  $\delta_m$ . Therefore, a greater reduction in the share of  $n_2$  available for future  $Z$  has a more detrimental, and non-linear effect on growth.

## 2.4 The Decentralized Equilibrium

Consider an economy that is populated by a set of homogenous and infinitely lived agents. There is no population growth and the representative firms are completely owned by agents, who supply labor for final goods production,  $n_1$ , and R&D,  $n_2$ . Agents derive utility from consumption of the final good and leisure given in (1). Agents fund consumption and investment decisions from their after tax wages  $w_1$  and  $w_2$ , which they receive for supplying labor  $n_1$  and  $n_2$ , profits  $\Pi_t$  earned from the final goods production, which they take as given, and the returns to capital lent out for production at each time period  $t$ .

The representative firm produces the final good based on (2) where the law of motion of private capital is given by (4). Technological change is again investment specific where public capital,  $G$ , is assumed to play a crucial role in enhancing technology. The laws of motion for  $G$  and investment specific technological change are given by (5) and (7), respectively. The government funds public investment,  $I_t^g$ , at each time period  $t$  using a distortionary tax imposed on labor,  $\tau_n \in [0, 1]$ , and capital,  $\tau_k \in [0, 1]$  respectively. Like Huffman (2008), it is assumed that profits are taxed according to the same rate as capital income.

### 2.4.1 The Firm's Problem

Firms solve a dynamic optimization problem which, at time  $t$ , has capital stock,  $K_t$ , and  $Z_t$ . Let  $v(K_t, Z_t)$  denote the value function of the firm at time  $t$ . The firm's optimization problem, assuming full depreciation, is given by,

$$v(K_t, Z_t) = \max_{K_{t+1}, n_{1t}, n_{2t}} \left\{ \left( Y_t - [w_{1t}n_{1t} + w_{2t}n_{2t}] - \frac{K_{t+1}}{Z_t} \right) + \beta v(K_{t+1}, Z_{t+1}) \right\}. \quad (26)$$

which it maximizes subject to (5) and (7). The inside term in brackets denotes output minus wage income minus investment costs. The ownership of firms by households implies that firms earn profits, which accrues to them as consumers, who then pay factor income taxes.<sup>16</sup> Taking taxes and profits as given, agents make their consumption-leisure decisions. From the firm's maximization exercise, we get the following first order conditions,

$$\{K_{t+1}\} : \frac{1}{Z_t} = \beta \left( \frac{\alpha_1 Y_{t+1}}{K_{t+1}} + \frac{\beta(1-\gamma)(1-\mu)K_{t+3}}{K_{t+1}Z_{t+2}} + \frac{\beta^2(1-\gamma)(\gamma(1-\mu)-\alpha_1)}{K_{t+1}} \sum_{j=0}^{\infty} \beta^j \gamma^j \frac{K_{t+j+4}}{Z_{t+j+3}} \right)$$

$$\{n_{1t}\} : w_{1t} = \underbrace{\frac{\alpha_2 Y_t}{n_{1t}}}_{MPn_1} - \underbrace{\frac{\alpha_2 \beta^2 (1-\gamma)}{n_{1t}} \sum_{j=0}^{\infty} \frac{\beta^j \gamma^j K_{t+j+3}}{Z_{t+j+2}}}_D$$

$$\{n_{2t}\} : w_{2t} = \underbrace{\frac{\alpha_3 Y_t}{n_{2t}}}_{MPn_2} + \underbrace{\frac{\beta \theta K_{t+2}}{Z_{t+1} n_{2t}} + \frac{\beta^2 (\gamma \theta - \alpha_3 (1-\gamma))}{n_{2t}} \sum_{j=0}^{\infty} \frac{\beta^j \gamma^j K_{t+j+3}}{Z_{t+j+2}}}_E.$$

Because of investment specific technological change, factor prices are no longer equal to the standard marginal products. In particular, the wage paid to  $n_1$  is reduced by the term  $D$ , while the wage paid to  $n_2$  is increased by the term  $E$ . The general point to note is that,  $w_1 = w_2$  does not imply that  $MPn_1 = MPn_2$  unless the restriction,  $D + E = 0$ .<sup>17</sup> Figure [3] shows how investment specific technological change acts like a tax on  $n_1$  and a subsidy to  $n_2$ .

[INSERT FIGURE 3]

### 2.4.2 The Agents Problem

A representative agent maximizes (1) subject to the consumer budget constraint (CBC),

$$C_t + \frac{K_{t+1}}{Z_t} = [w_{1t}n_{1t} + w_{2t}n_{2t}](1 - \tau_n) + [Y_t - (w_{1t}n_{1t} + w_{2t}n_{2t})](1 - \tau_k), \quad (27)$$

the laws of motion given by (4), (5) and (7), total labor supply given by (3), and takes factor prices and profits as given. The first term on the right hand side denotes after tax wage income. The second term is the firm's capital income plus profits, which is taxed at the rate,

<sup>16</sup>In the expression for the value function of the firm in Huffman(2008, p. 3455), a tax on capital is imposed on capital income accruing to the firm. We will later show that this does not matter for the results.

<sup>17</sup>Intuitively, because investment specific technological change has a dynamic effect on the marginal productivity of factor inputs, we cannot restrict ourselves to the static marginal productivities to calculate factor prices.

$\tau_k$ . The following restriction required for decentralizing the planner's allocations.

**Remark 4**  $Dn_1 = En_2$  is a necessary and sufficient condition required to decentralize the planner's allocation at every time period,  $t$ .

This means a reduction in the total wage bill paid to  $n_1$  due to investment specific technological change gets offset by an equivalent increase in the wage bill paid to  $n_2$ . This ensures that the total wage bill paid by the firm to  $n$  does not depart from the marginal productivities paid to each type under investment specific technological change for any combination of factor income taxes.<sup>18</sup>

Under *Remark (4)*, the CBC – from equation (27) – can be written as

$$C_t + \frac{K_{t+1}}{Z_t} = \Theta Y_t$$

where  $\Theta = (1 - \alpha_1)(1 - \tau_n) + (\alpha_1)(1 - \tau_k)$ . This implies the government budget constraint (GBC) is given by

$$I_t^g = (1 - \Theta)Y_t. \quad (28)$$

In general, any factor income tax combination decentralizes the planner's allocations as long as *Remark (4)* is imposed, because of the offsetting effects on the total wage bill of the firm.<sup>19</sup>

### 2.4.3 The Agent's First Order Conditions

The Lagrangian given below for the agent's problem is given by

$$L = \sum_{t=0}^{\infty} \beta^t [\log C_t + \log(1 - n_{1t} - n_{2t}) + \lambda_t \{\Theta Y_t - C_t - I_t\}]. \quad (29)$$

The optimization conditions with respect to  $C_t$ ,  $K_{t+1}$ ,  $n_{1t}$ , and  $n_{2t}$ , are given by equations (30), (31), (32) and (33) respectively:

$$\frac{1}{C_t} = \lambda_t \quad (30)$$

---

<sup>18</sup>Even if Huffman's (2008) framework was to be augmented with infrastructure in the manner considered in this paper, a similar restriction would need to be placed to decentralize the planner's allocation. From Remark (1), augmenting Huffman's (2008) model with infrastructure would imply that  $\alpha_3 = 0$ ,  $n_2$  fully entering into  $Z_{t+1}$  but  $\gamma \neq 1$ . From the firm's first order conditions above, because  $\gamma \neq 1$ , a term similar to term  $D$  would exist in the FOC with respect to  $n_1$ . However, because  $n_2$  doesn't enter into direct production, the marginal product term with respect to  $n_2$  would not exist, and the payment to  $n_2$  would be a term similar to term  $E$  capturing the productivity gains to  $n_2$  from the investment specific technological equation. It can now easily be seen that  $w_1 = w_2$  requires that a similar restriction be placed, as in our model.

<sup>19</sup>If we follow Huffman (2008, p. 3455), then,  $w_1 = MP_{n_1} - \frac{D}{(1-\tau_k)}$  and  $w_2 = MP_{n_2} + \frac{E}{(1-\tau_k)}$ . *Remark (4)* is still required, and ensures that the consumer budget constraint evaluated at these factor prices yields the resource constraint of the planner.

$$\frac{1}{C_t Z_t} = \frac{\alpha_1 \beta Y_{t+1} \Theta}{C_{t+1} K_{t+1}} + \frac{\beta^2 (1-\gamma)(1-\mu) I_{t+2}}{C_{t+2} K_{t+1}} + \frac{\beta^3 (1-\gamma)(\gamma(1-\mu) - \alpha_1)}{K_{t+1}} \sum_{j=0}^{\infty} \frac{\beta^j \gamma^j I_{t+j+3}}{C_{t+j+3}} \quad (31)$$

$$\frac{1}{1-n_t} + \frac{\beta^2 \alpha_2 (1-\gamma)}{n_{1t}} \sum_{j=0}^{\infty} \frac{\beta^j \gamma^j I_{t+j+2}}{C_{t+j+2}} = \frac{\alpha_2 Y_t \Theta}{C_t n_{1t}} \quad (32)$$

$$\frac{1}{1-n_t} = \frac{\alpha_3 Y_t \Theta}{C_t n_{2t}} + \frac{\beta \theta I_{t+1}}{C_{t+1} n_{2t}} + \frac{\beta^2 (\gamma \theta - \alpha_3 (1-\gamma))}{n_{2t}} \sum_{j=0}^{\infty} \frac{\beta^j \gamma^j I_{t+j+2}}{C_{t+j+2}}. \quad (33)$$

The above first order conditions can be derived in the same way as the planner's version as shown in Appendix A. Their interpretation is also the same as before, except that  $1 - \tau$  is replaced by  $\Theta$ . The closed form decision rules are characterized by the following lemma.

**Lemma 4** *In the decentralized equilibrium, the expressions for  $C_t$ ,  $I_t$ ,  $n_t$ ,  $n_{1t}$ ,  $n_{2t}$  are given by the same decision rules derived in (34), and (17), (18) respectively, where  $0 < \Phi < 1$  is given by (19) and  $0 < x < 1$  is given by (20), with*

$$C_t = \Phi Y_t \Theta \text{ and } I_t = (1 - \Phi) Y_t \Theta. \quad (34)$$

**Proof.** The above expressions can be constructed from the first order conditions given by equations (30), (31), (32) and (33), as explained in Appendix B for the planner's version. ■

The first order conditions governing the planner's allocations can be easily be seen to be replicated by the decentralized equilibrium, once we assume  $\tau_k = \tau_n = \tau$ . In this case,  $\Theta = 1 - \tau$ , and the first order conditions characterizing the planner's allocations obtain. Hence, the comparative statics of consumption, investment, and labor input vis-a-vis changes in  $\mu$  remain unchanged. The gross growth rate of  $Z$ ,  $K$ ,  $G$  and  $Y$  at the steady state can also be derived in a similar fashion. As in the planner's version, the condition for dynamic stability is the same, i.e.,  $0 < \gamma < 1$ . We therefore have the following lemma

**Lemma 5** *In the steady state of the decentralized economy, the gross growth rate of  $Z$ , is given by (35) while the gross growth rates for  $K$ ,  $G$  and  $Y$  are given by (36)*

$$\hat{g}_z = \Omega^{\frac{1}{2-\gamma}}, \text{ where } \Omega = \widehat{M} \{ (1 - \Theta)^\mu (\Theta)^{1-\mu} \}^{(1-\gamma)}, \widehat{M} = B((1 - \delta_m)(1 - \hat{x})\hat{n})^\theta (1 - \widehat{\Phi})^{(1-\mu)(1-\gamma)} \quad (35)$$

$$\hat{g}_k = \hat{g}_g = \hat{g}_z^{\frac{1}{1-\alpha_1}}, \hat{g}_y = \hat{g}_k^{\alpha_1} = \hat{g}_z^{\frac{\alpha_1}{1-\alpha_1}}. \quad (36)$$

**Proof.** *These are derived in the same way as in the planner's version.* ■

Note that the expressions for the equilibrium long run growth rate are identical to those in the planned economy, except that the growth rates differ because  $\Theta$  need not equal  $1 - \tau$ .

We want to check under what conditions the growth maximizing allocations from the planner's problem can be replicated by the decentralized economy. From the steady state growth rate given by (35), this leads to a second proposition.

**Proposition 2** *If factor income taxes are chosen such that a linear combination of factor income taxes is equal to  $\mu$ , i.e.,*

$$(1 - \alpha_1)(\widehat{\tau}_n) + \alpha_1(\widehat{\tau}_k) = \mu = \widehat{\tau}. \quad (37)$$

*then the decentralized allocations can replicate the growth maximizing allocations from the planner's problem.*

**Proof.** *Shown in Appendix E. ■*

The proposition above suggests that there is no *unique* combination of labor and capital income taxes which maximizes growth. However, if the condition specified in the proposition holds, the planner's allocations can be achieved from the solutions of the agent's version. Since there does not exist any unique growth optimizing tax rate combination for  $\widehat{\tau}_n$  and  $\widehat{\tau}_k$  the planner could choose from a set of tax/subsidy schemes such that the convex combination will always equal the planner's growth optimizing tax rate of  $\mu$ . Under this condition, the solution to the agent's version replicates the planners optimization solution. We therefore have the following proposition

**Proposition 3**  $\widehat{\tau}_n \gtrless \widehat{\tau} = \mu \Rightarrow \widehat{\tau}_n \gtrless \widehat{\tau} = \mu \gtrless \widehat{\tau}_k$ .

**Proof.** *This is obtained from equation (37). ■*

From equation (37), it follows that if one of the factors receives a subsidy, the other factor must be sufficiently taxed such that the optimality rule given by (37) is satisfied.<sup>20</sup>

## 2.5 Welfare

We are interested in evaluating whether the optimality results for taxes vary when the rule is to maximize welfare and not growth. Since consumption is a fixed fraction of after tax income, it grows at the same gross growth rate of output at the steady state as shown in Lemma 1 and Lemma 3. The total labor supplied is also constant at each time period  $t$ . Using this information, we can re-write the life-time utility given in (1) in the planner's version as

$$\Lambda = \Gamma + \Psi \log[(\tau)^\mu (1 - \tau)^{1-\mu}] \quad (38)$$

---

<sup>20</sup>We have shown in Lemma 1 and in Lemma 4 that  $0 < \Phi < 1$  given by (19),  $0 < x < 1$  given by (20) and the supply of labor given by (17) are constants.

where  $\Gamma$  and  $\Psi$  denote all other terms independent of the flat tax rate  $\tau$ . We show this in appendix G. It is clear from the above expression for welfare that the optimal tax rate for the planner's model coincides with the growth optimizing tax rate. This gives us the following result.

**Proposition 4** *The welfare and growth maximizing taxes coincide, that is,  $\tau^w = \hat{\tau} = \mu$ .*

**Proof.** *This can be obtained by differentiating (38) with respect to  $\tau$ . It can be seen that we get the same optimal tax rate  $\tau^w = \hat{\tau} = \mu$ . ■*

In the agent's version, the life-time welfare function can be expressed as

$$\Delta = \Xi + \Psi \log[(1 - \Theta)^\mu (\Theta)^{1-\mu}] \quad (39)$$

where  $\Xi$  and  $\Psi$  all other terms independent of the tax rates  $\tau_n$  and  $\tau_k$ . We get the same result as in the case of the planner's version, as shown in the following proposition

**Proposition 5** *There is no unique welfare optimizing combination of labor and capital income tax rates. We however get the same condition as we obtained in the agent's growth optimizing tax rates; that is  $(1 - \alpha_1)(\hat{\tau}_n^w) + \alpha_1(\hat{\tau}_k^w) = \mu = \hat{\tau} = \tau^w$ .*

**Proof.** *We get the above equation from the welfare optimization conditions with respect to  $\tau_n$  and  $\tau_k$ . ■*

In sum, we have characterized both the planner's and agent's problem. We have shown that there exists a unique growth maximizing tax rate, which also maximizes long run welfare. We have also shown that the planner's allocation can be decentralized by infinitely many tax/subsidy combinations (as long as both are not subsidies) on either factor, so as to generate the planner's growth rate. In the next section, we consider an environment where it can be shown that it is possible to uniquely characterize factor income taxes to replicate the planner's allocations.

### 3 The Model with Administrative Costs

The presence of administrative costs during tax collection is one of the major reasons (the other being the dead-weight loss of imposing the tax), for the government's inability to change (particularly increase) tax rates with ease for the purpose of raising revenue (see Bovenberg and Goulder (1986), Yitzhaki (1979), Yang (1989)). We provide specific conditions under which the tax rate differs at the optimum – compared to the benchmark model – when administrative costs for implementing public investment occur. In particular, as

long as administrative costs are not linear, we show that there will always exist a unique combination of distortionary tax rates on labor income and capital income.

### 3.1 The Planner's Model with Administrative Costs

As before, the government collects taxes by imposing a proportional tax rate on income to fund  $G$  to contribute to investment specific technological change. It however incurs administrative costs with respect to tax collection. The government budget constraint takes the following form

$$I_t^g = (\tau - \omega(\tau))Y_t$$

where  $\omega(\tau)$  represents continuously varying administrative costs with respect to the tax rate  $\tau$ . Here,  $\omega'(\tau) > 0$ , which implies that the administrative cost is assumed to be increasing in the tax rate. In what follows, we will assume that these costs could be linear, convex, or concave with respect to the tax rate, and show that these assumptions have different implications for the steady state balanced growth path.

#### 3.1.1 Case 1: Convex Administrative Costs

Suppose that administrative costs are strictly convex with respect to the tax rate (e.g., Perotti (1993), Buiter and Sibert (2011)), i.e.,  $\omega''(\tau) > 0$ . By strict convexity we mean that a proportional increase in the tax rate causes a more than proportional increase in the collection costs and that governments experience greater difficulty in imposing a higher tax rate on income as compared to a lower tax rate. In such a scenario, if achievable growth rates are much lower than as compared to the case where such costs are absent, the government, at an optimum, would consider imposing a lower tax rate on income.

We parametrize the function,  $\omega(\tau) = c\tau^\chi$ , where  $0 < c \leq 1$  is like a "fixed level cost" parameter and  $\chi > 1$  is the degree of convexity. The government budget constraint takes the form

$$I_t^g = (\tau - c\tau^\chi)Y_t. \tag{40}$$

We assume that agents who are subject to these tax rates are unaffected by the administrative costs the government incurs for imposing taxes. It is like as if the government incurs an additional expenditure towards enhancing investment specific technological change. For this reason the first order conditions are the same as in the baseline model and are therefore given by (10), (11), (12) and (13). The decision rules are also the same and are given by (16), (17), (18), (19) and (20) as shown in Lemma 1.

Given the government budget constraint in (40), the law of motion for investment specific

technological change according to (7) and the decision rules for consumption, investment and the labor supplies as (16), (17), (18), (19), (20), we can rewrite (7) in terms of the steady state growth rate,

$$\hat{g}_z = [B((1 - \delta_m)(1 - \hat{x})\hat{n})^\theta (1 - \Phi)^{(1-\mu)(1-\gamma)} \{(\tau - c\tau^\chi)^\mu (1 - \tau)^{1-\mu}\}^{(1-\gamma)}]^{1/(2-\gamma)}. \quad (41)$$

As before, the growth rate, with administrative costs, is increasing in  $B$ , decreasing in  $\delta_m$ , and increasing in  $n_2$ . Further, an increase in  $c$  reduces the growth rate. We now get the following proposition.

**Proposition 6** *The growth maximizing tax rate,  $\tau_{AC}$  (tax with administrative costs) in a model with convex administrative costs is always less than the growth maximizing tax without such costs, that is,  $\hat{\tau} = \mu$ . The optimal tax is obtained from the following expression*

$$(1 - \tau_{AC})\mu[1 - c\chi(\tau_{AC})^{\chi-1}] = (1 - \mu)(\tau_{AC} - c(\tau_{AC})^\chi), \quad (42)$$

where,

$$\tau_{AC} = \hat{\tau} = \mu \text{ when } c = 0 \text{ or when } \chi = 1. \quad (43)$$

**Proof.** Shown in Appendix H ■

Given that administrative costs with respect to the tax rates are convex, the steady state optimal growth rate given by (41) will be lower than the steady state optimal growth rate in the baseline model. This is shown in Figure [3] where  $t$  is the optimal tax rate as derived in the baseline model. The tax rate  $t_1$  is the growth optimizing tax rate when the government faces convex administrative costs.

[INSERT FIGURE 4]

Some papers - such as Perotti (1993) and Buiter and Sibert (2011) - assume that convex administrative costs are quadratic in nature ( $\chi = 2$ ). Assuming  $c = 1$ , the optimal tax is now given by the following equation

$$(1 - \tau_{AC})\mu(1 - 2\tau_{AC}) = (1 - \mu)(\tau_{AC} - (\tau_{AC})^2) \quad (44)$$

which gives us

$$\tau_{AC} = \frac{\mu}{1 + \mu} < \hat{\tau} = \mu. \quad (45)$$

Therefore the general result in a model with convex administrative costs is that the planner will choose to charge a lower tax rate compared to the case when there are no administrative costs. These costs hamper the availability of resources for funding public expenditure thereby

leading to lower growth rates. But the planner would choose to charge a lower tax rate at the optimum since the costs of imposing higher tax rates are increasing in the tax rate.

### 3.1.2 Case 2: Concave Administrative Costs

The government budget constraint for the planner's version is again given by equation (40) but now instead of having strict convexity in administrative costs with respect to the tax rate, we assume strict concavity ( $\omega''(\tau) < 0$ ) in administrative costs, i.e.,  $\chi < 1$ . Anecdotally, strict concavity of administrative costs is suggestive of a more efficient administrative machinery compared to the previous case with convex costs.<sup>21</sup> Such administrative costs increase with a higher tax rate but at a decreasing rate. Therefore at optimum, the government has an incentive to impose a higher tax on income as compared to the case of the baseline model even though the steady state growth rates is lower because of the loss due to administrative costs. This again is shown in Figure [3] where now  $t_2$  is the growth optimizing tax rate when the government faces concave administrative costs.

### 3.1.3 Case 3: Linear Administrative Costs

Linear administrative costs simply cause a level downward shift in the optimal growth rate. The optimal tax however remains the same. This is shown in Figure [5].

[INSERT FIGURE 5]

## 3.2 The Decentralized Equilibrium

There are two separate factor income taxes imposed on labor and capital income, i.e.,  $\tau_n$  and capital  $\tau_k$  respectively. The administrative costs incurred by the government for imposing tax rate on labor and capital are assumed to be different, in terms of the fixed level costs although not in terms of the degree of convexity or concavity. Hence the cost of imposing  $\tau_k$  is  $c_1\tau_k^\chi$  while that for  $\tau_n$  is  $c_2\tau_n^\chi$ . The following is the government budget constraint<sup>22</sup>

$$I_t^g = [\alpha_1\tau_k + (1 - \alpha_1)\tau_n - c_1\tau_k^\chi - c_2\tau_n^\chi]Y_t. \quad (46)$$

The rest of the specification remains the same, just as in the baseline agent's model. The first order conditions are given by equations (30), (31), (32) and (33) and the decision rules

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<sup>21</sup>This may be due to employing better computerized technology that may assist in revenue collection. See Slemrod (1990).

<sup>22</sup>This equation is derived in the same way as in the baseline model, where the firm's profit maximization solutions are substituted into wages and rate of return on capital. In addition, there are also administrative costs of imposing each tax rate.

by (16), (17), (18), (19) and (20) as shown in Lemma 3. We now substitute the decision rules given by equations (17), (18), (19) and (20) as in Lemma 3 and the government budget constraint given by equation (46) into the investment specific technological change equation given by (7) to obtain the following steady state growth rate for  $Z$

$$\widehat{g}_z = [B((1 - \delta_m)(1 - \widehat{x})\widehat{n})^\theta(1 - \Phi)^{(1-\mu)(1-\gamma)}\{(1 - \Theta - c_1\tau_k^\chi - c_2\tau_n^\chi)^\mu(\Theta)^{1-\mu}\}^{(1-\gamma)}]^\frac{1}{2-\gamma}, \quad (47)$$

where,  $\Theta = 1 - \alpha_1\tau_k - (1 - \alpha_1)\tau_n$ . Unlike in the baseline framework where administrative costs were absent, we show that there exists a *unique* combination of the tax on labor income and on capital income given by the following proposition.

**Proposition 7** *There exists a unique combination of positive tax rates  $\tau_n$  and  $\tau_k$  which maximizes the steady state growth rate given by*

$$\left(\frac{\tau_n^*}{\tau_k^*}\right)^{\chi-1} = \left(\frac{1 - \alpha_1}{\alpha_1}\right) \left(\frac{c_1}{c_2}\right). \quad (48)$$

When  $c_1 = c_2$ ,

$$\chi \geq 1 \Rightarrow \tau_n^* = \left\{ \left(\frac{1 - \alpha_1}{\alpha_1}\right)^{\frac{1}{\chi-1}} \right\} \tau_k^* \geq \tau_k^*.$$

**Proof.** *Shown in Appendix I* ■

This inequality result holds for  $0 < \alpha_1 < 0.5$ . The proposition suggests that if the scale constant,  $c$ , and variable costs are identical, the government could maximize efficiency by charging a higher tax on labor income.<sup>23</sup> The tax on capital income could therefore exceed the tax on labor income only under the special case when the fixed level costs of imposing  $\tau_n^*$  relative to  $\tau_k^*$  sufficiently exceeds the relative share of total labor to capital in production. In other words, capital income could be subject to a higher tax rate rate compared to that on labor provided it is less costly to impose a higher tax on capital.

This specification also allows us to recover uniqueness, which was absent in the baseline model. This happens because  $\mu$  affects the first order conditions – relating the growth rate to the optimal factor income tax rates – symmetrically (see equations (49) and (50 in Appendix I). The ratio of factor income taxes, as seen in (48), is therefore independent of  $\mu$ . However, individual factor income taxes still depend on  $\mu$ .

Finally, in the baseline version, where the optimality results suggest that there could exist infinitely many combinations of distortionary tax rates on labor and capital, having

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<sup>23</sup>In fact for similar reasons even the recent OECD's Current Tax (2011) agenda suggests that from the point of view of efficiency that capital must be taxed at lower rates as compared to labor and this calls for a major restructuring of the tax models.

a flat tax rate is a feasible solution. As long as both tax rates were charged based on the degree of public congestion, a flat tax system cannot be sustained within this framework at the optimum. However, when  $\chi \rightarrow \infty$ , the two tax rates become perfectly identical. Hence at the optimum, the possibility of having a flat tax system on all sources of income, is only possible as a limiting case.

Concave administrative costs are suggestive of a more efficient tax collecting system. Costs do tend to increase for imposing higher tax rates. However, these might be more to do with tax evasions and not as much with administrative issues. In such a scenario, the model suggests that it is optimal to charge a higher tax on capital income than on labor income.

## 4 A Numerical Example

In this section, we calibrate the model to quantify the growth and welfare effects of factor income tax changes.<sup>24</sup> Our focus is to assess the growth and welfare effects of changing factor income taxes. We calibrate our results using parameter values from the US in Huffman (2008), and other parameters from the literature. We use arbitrary values of  $\mu$ ,  $\gamma$ , and  $\delta_m$  because of the lack of clear empirical estimates for these parameters. Table [1] contains the parameter values used in the calibration exercise. We are interested in two different policy experiments: the effects of 1) equi-proportionate changes in factor income taxes and 2) revenue neutral changes on growth and welfare. We then augment these estimates assuming different cost technologies.

We first analyze the impact of changing the tax on capital on the growth rate for a given value of the labor income tax in the decentralized model using the optimality condition (37), or

$$(1 - \alpha_1)(\widehat{\tau}_n) + \alpha_1(\widehat{\tau}_k) = \mu = \widehat{\tau}.$$

We earlier quantified the effect of an increase in the share of government spending on infrastructure relative to aggregate output – in the investment specific technological change equation – on the equilibrium growth rate. We showed that a higher share of public infrastructure spending leads to a higher optimal tax rate, and a higher growth rate. This effect on growth can be decomposed into two effects: the effect of a higher government infrastructure share on labor effort devoted to research, and the direct effect of a higher government infrastructure share on increasing the level of investment specific technological change. The quantitative results show that an increase in the government spending share *reduces* the labor input devoted to research effort which reduces growth, but increases the level

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<sup>24</sup>All calibration exercises were done in Matlab. The programs are available from the authors on request.

of technological change which increases growth. Hence, the *net* change in growth depends on the magnitude of these two effects.

The above dynamics are replicated when there are tax changes in the decentralized equilibrium, except that at the optimum, there does not exist any unique combination of tax rates on capital and labor income which maximizes growth (and welfare). For illustrative purposes, we arbitrarily pin down a particular value for the tax on labor,  $\widehat{\tau}_n = 0.4$ . According to (37) – where  $\mu = 0.5$  and  $\alpha_1 = 0.35$  – which yields  $\widehat{\tau}_k \approx 0.6857$ .<sup>25</sup> This can also be replicated if we plot the growth schedule of  $g_z$  – fixing  $\widehat{\tau}_n$  at 0.4 – and varying  $\widehat{\tau}_k$ .

[INSERT FIGURE 6]

This is shown in Figure [6] where the optimum combination of  $\widehat{\tau}_n$  and  $\widehat{\tau}_k$  is given by  $(0.4, 0.6857)$ , corresponding to  $t_1$ . Suppose we decrease the tax rate on labor income and increase the tax rate on capital by 0.1 in comparison to this combination at the optimum; that is,  $\tau_n$  and  $\tau_k$  are now given by  $(0.3, 0.7857)$ .<sup>26</sup> These tax rates no longer satisfy (37). Growth is no longer at its highest value, although in comparison to the growth maximizing factor income tax mix,  $(0.3, 0.87143)$  – given by  $t_3$  – which satisfies (37), the gross growth rate is marginally (though not significantly) less: by a magnitude of  $\approx 3.303 \times 10^{-4}$ . Hence, deviating from the optimal rule by changing factor income taxes in equal proportions has a negligible effect on long run growth and welfare. The reason why this happens is because of the assumed share of output accruing to capital in the final goods production,  $\alpha_1$  where we assume that  $0 < \alpha_1 < .5$ . Under this restriction, the absolute rate of change in growth rate due to a change in the tax rate on capital income will strictly be less than the absolute rate of change in the growth rate due to a change in the tax on labor. That is

$$\left| \frac{\partial g_z}{\partial \tau_k} \right| < \left| \frac{\partial g_z}{\partial \tau_n} \right|.$$

This means, deviating away from the tax rate on capital according to an optimal tax rule will only have a moderate effect on the growth rates. This result is consistent with the results of the policy experiments in Huffman (2008) relating the effect of capital income tax changes on growth. He finds that changes in factor income taxes have a minimal effect on the growth rate. However, as evident from Figure [6], depending upon whether changes to the capital income tax rate are equi-proportionate or revenue neutral, the effect on growth rate would be negative (but marginal), or zero, respectively. This contrasts with Huffman (2008) where a change in the tax on labor income has a negative effect on the growth rate.<sup>27</sup>

<sup>25</sup>The revenue neutral rule for changes in factor income taxes is given by,  $\frac{\Delta \widehat{\tau}_k}{\Delta \widehat{\tau}_n} = -\frac{(1-\alpha_1)}{\alpha_1}$ .

<sup>26</sup>The value of  $\tau_k = 0.7857$  is given by the point  $t_2$ .

<sup>27</sup>For a large arbitrary change in taxes, our calibrated results show that the "growth-gap" between equi-proportionate and revenue neutral changes is still not large, but larger than the case for small changes in

If now we reverse the exercise by first fixing  $\tau_k$  at the arbitrary value of 0.6 and then lower it to 0.3, and then calibrate the equi-proportionate increase and revenue neutral increase in  $\tau_n$ , we find that changes in growth are of the same order when we compare an equi-proportionate change to a revenue neutral change when we fixed tax  $\tau_n$  and varied  $\tau_k$ . However, there is an important difference. An equi-proportionate change in  $\tau_n$  means that now we increase the tax on labor from 0.44615 (when  $\tau_k$  is 0.6) to 0.74615 (when  $\tau_k$  is 0.3) which is significantly higher than the revenue neutral value (at 0.607). Since the equi-proportionate change in  $\tau_n$  exceeds the revenue neutral value of  $\tau_n$ , reducing  $\tau_n$  would increase growth.

We now conduct a similar policy experiment with administrative costs. For simplicity we have assumed that the fixed level cost parameter  $c$  and the variable cost parameter  $\chi$  are the same for both tax rates on labor income and on capital income. We first consider the case with convex administrative costs, and assume (arbitrarily) that  $c = 0.4$  and  $\chi = 2$ . Plugging the value of  $c$  and  $\chi$  into equation (48), and then substituting out the resulting value of  $\tau_n$  in terms of  $\tau_k$  into the first order condition (49) in Appendix I, the optimal tax rate on capital income  $\tau_k^*$  is found to be approximately 0.23. Using this value for  $\tau_k^*$ , and using the equation (48) we get  $\tau_n^* = 0.42$ . We now get Figure [7].

[INSERT FIGURE 7]

Here  $t_1$  represents the point  $(\tau_n^*, \tau_k^*) = (0.42, 0.24)$ .<sup>28</sup> This gives us the growth maximizing tax mix with convex administrative costs. Therefore, any change in the tax on labor such that  $\tau_n \neq \tau_n^*$ , such as at point  $t_3$ , where we arbitrarily reduce  $\tau_n$  by 0.1 to 0.32, and also re-calibrate  $\tau_k$  according to (48) reduces growth to a sub-optimal value. This is because the new tax mix at  $t_3$  does not satisfy optimality.<sup>29</sup> However, if the choice of  $\tau_k$  corresponding to  $\tau_n = 0.32$  were to satisfy optimality (hence  $\tau_k$  satisfies  $\frac{\partial g_z}{\partial \tau_k}|_{\tau_n=0.32} = 0$ ), then the optimal tax on capital would have to be significantly higher than  $\tau_k^*$ . This is denoted by point  $t_2$ , where growth has marginally fallen (by roughly  $3.6 \times 10^{-3}$ ), but by an amount greater than the growth fall in the model without administrative costs. In sum, because of convex administrative costs, deviating from the tax rule, (48), will lead to a greater fall in long run growth and welfare.<sup>30</sup>

We now look at the case with concave administrative costs. With  $c = 0.01$  and  $\chi = 0.1$  we can show that the optimal tax rate on capital income  $\tau_k^* \approx 0.73$ .<sup>31</sup> Using this value for

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taxes. For instance, a reduction in  $\tau_n$  from 0.6 to 0.3 implies that  $\tau_k$  rises from 0.315 to 0.87143. In contrast, an equiproportionate change in  $\tau_k$  is 0.61. The growth difference is still roughly  $3.303 \times 10^{-4}$ .

<sup>28</sup>Note that there are two solutions but the second solution  $\tau_k^* = 0.69$  and  $\tau_n^* = 1.28$  is not feasible.

<sup>29</sup>Here,  $\frac{\partial g_z}{\partial \tau_k} > 0$ .

<sup>30</sup>If we allow  $\tau_n$  to fall by 0.2 to 0.22 - a large arbitrary amount, we find that a decrease in  $\tau_k$  - from (48) - is 0.1209. For  $\tau_n = 0.22$ ,  $\tau_k = 0.345$ . There is larger reduction in the growth rate.

<sup>31</sup>This choice of  $c$  and  $\chi$  ensures feasibility.

$\tau_k^*$  and using the equation (48) we can show that  $\tau_n^* \approx 0.4013$ . This is shown in Figure [8].

[INSERT FIGURE 8]

Here  $t_1$  is given by  $(\tau_n^*, \tau_k^*) = (0.4013, 0.714)$ .<sup>32</sup> We conduct a similar exercise of lowering the tax on labor income by 0.1 and back out the value for the corresponding tax (shown as  $t_2$ ) on capital income from the equation (48). This will give us a tax rate on capital income equal to 0.594 which does not give us the optimal growth rate. However a tax rate on capital income, much higher than  $t_1$ , and given by  $t_3$  in Figure [8] will ensure optimal growth rates. The difference in the growth rates by moving from a tax on capital income of 0.594 as in  $t_3$  to 0.908 (moving to  $t_2$ ) is roughly around  $3.4 \times 10^{-3}$  which is also not large. Hence, having concave administrative costs suggests that there is no change in the optimal growth rate when the tax on capital income is increased.<sup>33</sup> Given concave costs, on normative grounds, a planner could charge a higher tax rate on capital without changing the optimal growth rate when there is a fall in the tax on labor income compared to  $\tau_n^*$ .

## 5 Conclusion

This paper broadens the environment in which investment specific technological change occurs. In particular, we include a role for public capital and learning by doing in assisting endogenous investment specific technological change. First we characterize the growth and welfare maximizing tax rates. We show that the growth and welfare maximizing tax rates coincide. We derive a restriction relating to the effect of investment specific technological change on the wage bill of the firm, which decentralizes the planner's allocations. Under this restriction, infinitely many combinations of factor income taxes can replicate the planner's allocations. We then add administrative costs into the model. Our main result here is that, under certain assumptions relating to the curvature of the cost function, higher capital income taxation - relative to labor income taxation - *maximizes* the steady state rate of growth. These results are consistent with some other papers in this literature which shows that capital income taxation may increase growth. A simple calibrated version of the model shows that the growth effects of sub-optimal capital income tax policy are not large.

In the current analysis we have assumed that the government can commit to a sequence of actions, making policy time consistent. Future work could formalize optimal capital income taxation under the Ramsey policy.

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<sup>32</sup>The second solutions to  $\tau_k^*$  and  $\tau_n^*$  are approximately close to 0 which means that the funding available for the public input will be close to zero. We therefore ignore this solution.

<sup>33</sup>This result is broadly consistent with the results in Huffman (2008) and Stokey and Rebelo (1995) who show that growth rates are unaffected by a deviation of tax rates on capital income away from their equilibrium values.

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# Appendix

The appendix contains all the proofs of propositions made in the paper.

## Appendix A: The planner's FOCs for the baseline model

The following is the FOC with respect to  $C_t$

$$\beta^t \frac{1}{C_t} - \beta^t \lambda_t = 0.$$

Hence, we get

$$\{C_t\} : \frac{1}{C_t} = \lambda_t.$$

This gives us equation (10). The FOC with respect to  $K_{t+1}$  is as follows:

$$\{K_{t+1}\} : \frac{-\lambda_t}{Z_t} + \beta \lambda_{t+1} \frac{\alpha_1 Y_{t+1} (1-\tau)}{K_{t+1}} - \beta \lambda_{t+1} \frac{\partial}{\partial K_{t+1}} \left( \frac{K_{t+2}}{Z_{t+1}} \right) - \beta^2 \lambda_{t+2} \frac{\partial}{\partial K_{t+1}} \left( \frac{K_{t+3}}{Z_{t+2}} \right) - \dots = 0.$$

Note that

$$\frac{\partial Z_t}{\partial K_{t+1}} = \frac{\partial Z_{t+1}}{\partial K_{t+1}} = 0, \quad \frac{\partial Z_{t+2}}{\partial K_{t+1}} = (1-\gamma)(1-\mu) \frac{Z_{t+2}}{K_{t+1}}, \quad \frac{\partial Z_{t+3}}{\partial K_{t+1}} = \frac{\gamma Z_{t+3}}{Z_{t+2}} \frac{\partial Z_{t+2}}{\partial K_{t+1}} - \alpha_1 (1-\gamma) \frac{Z_{t+3}}{K_{t+1}}.$$

We therefore have

$$\frac{\partial Z_{t+3}}{\partial K_{t+1}} = (1-\gamma) \frac{Z_{t+3}}{K_{t+1}} (\gamma(1-\mu) - \alpha_1).$$

And hence for any other future time period

$$\frac{\partial Z_{t+3+j}}{\partial K_{t+1}} = \gamma^j (1-\gamma) \frac{Z_{t+3+j}}{K_{t+1}} [\gamma(1-\mu) - \alpha_1], \quad \text{for } j \geq 0.$$

Therefore, substituting the above values in  $\{K_{t+1}\}$ , considering  $\frac{1}{C_{t+j}} = \lambda_{t+j}$  and  $I_{t+j} = \frac{K_{t+1+j}}{Z_{t+j}}$ ,  $j \geq 0$  and assuming full depreciation (that is  $\delta = 1$ ), we obtain the following FOC for  $\{K_{t+1}\}$  which is shown in equation (11)

$$\{K_{t+1}\} : \frac{1}{C_t Z_t} = \frac{\alpha_1 \beta Y_{t+1} (1-\tau)}{C_{t+1} K_{t+1}} + \frac{\beta^2 I_{t+2} (1-\gamma)(1-\mu)}{C_{t+2} K_{t+1}} + \frac{\beta^3 (1-\gamma)(\gamma(1-\mu) - \alpha_1)}{K_{t+1}} \sum_{j=0}^{\infty} \frac{\beta^j \gamma^j I_{t+j+3}}{C_{t+j+3}}.$$

Next, the FOC with respect to  $n_{1t}$  is given by

$\frac{-1}{1-n_t} + \frac{\lambda_t \alpha_2 Y_t (1-\tau)}{n_{1t}} - \lambda_t \frac{\partial}{\partial n_{1t}} \left( \frac{K_{t+1}}{Z_t} \right) - \beta \lambda_{t+1} \frac{\partial}{\partial n_{1t}} \left( \frac{K_{t+2}}{Z_{t+1}} \right) - \beta^2 \lambda_{t+2} \frac{\partial}{\partial n_{1t}} \left( \frac{K_{t+3}}{Z_{t+2}} \right) - \dots = 0$ . Given that

$$Z_{t+1} = \frac{B(1-\delta_m)^\theta n_{2t}^\theta Z_t^\gamma (G_t^\mu)^{(1-\gamma)} (K_t^{1-\mu})^{(1-\gamma)} (K_{t-1}^{-\alpha_1})^{(1-\gamma)} (n_{1t-1}^{-\alpha_2})^{(1-\gamma)} (\delta_m^{-\alpha_3} n_{2t-1}^{-\alpha_3})^{(1-\gamma)}}{A^{(1-\gamma)}},$$

this implies

$$\frac{\partial}{\partial n_{1t}} \left( \frac{K_{t+1}}{Z_t} \right) = \frac{\partial}{\partial n_{1t}} \left( \frac{K_{t+2}}{Z_{t+1}} \right) = 0.$$

We further have

$$\frac{\partial}{\partial n_{1t}} \left( \frac{K_{t+3}}{Z_{t+2}} \right) = \frac{\alpha_2 (1-\gamma) K_{t+3}}{n_{1t} Z_{t+2}},$$

and similarly,

$$\begin{aligned} \frac{\partial}{\partial n_{1t}} \left( \frac{K_{t+4}}{Z_{t+3}} \right) &= \frac{\alpha_2 (1-\gamma) \gamma K_{t+4}}{n_{1t} Z_{t+3}} \\ \frac{\partial}{\partial n_{1t}} \left( \frac{K_{t+5}}{Z_{t+4}} \right) &= \frac{\alpha_2 (1-\gamma) \gamma^2 K_{t+5}}{n_{1t} Z_{t+4}} \end{aligned}$$

for all future time periods. Therefore, substituting the above expressions into the expression for  $\{n_{1t}\}$  we get

$$\frac{1}{1-n_t} + \beta^2 \lambda_{t+2} \left( \frac{\alpha_2 (1-\gamma) K_{t+3}}{n_{1t} Z_{t+2}} \right) + \beta^3 \lambda_{t+3} \left( \frac{\alpha_2 (1-\gamma) \gamma K_{t+4}}{n_{1t} Z_{t+3}} \right) + \dots = \frac{\lambda_t \alpha_2 Y_t (1-\tau)}{n_{1t}}.$$

Recall that for every  $t$ ,  $\frac{1}{C_t} = \lambda_t$ . Since  $\delta = 1$ ,  $I_{t+j} = \frac{K_{t+1+j}}{Z_{t+j}}$ , for all  $j \geq 0$ . This gives us the

final expression of the FOC for  $n_{1t}$ , as shown in equation (12)

$$\{n_{1t}\} : \frac{1}{1-n_t} + \frac{\beta^2 \alpha_2 (1-\gamma)}{n_{1t}} \sum_{j=0}^{\infty} \beta^j \gamma^j \frac{I_{t+j+2}}{C_{t+j+2}} = \frac{\alpha_2 Y_t (1-\tau)}{C_t n_{1t}}.$$

The FOC with respect to  $n_{2t}$  is given by

$$\{n_{2t}\} : \frac{-1}{1-n_t} + \frac{\lambda_t \alpha_3 Y_t (1-\tau)}{n_{2t}} - \lambda_t \frac{\partial}{\partial n_{2t}} \left( \frac{K_{t+1}}{Z_t} \right) - \beta \lambda_{t+1} \frac{\partial}{\partial n_{2t}} \left( \frac{K_{t+2}}{Z_{t+1}} \right) - \beta^2 \lambda_{t+2} \frac{\partial}{\partial n_{2t}} \left( \frac{K_{t+3}}{Z_{t+2}} \right) \dots = 0. \text{ Given}$$

$$\begin{aligned} \frac{\partial Z_t}{\partial n_{2t}} &= 0, \quad \frac{\partial Z_{t+1}}{\partial n_{2t}} = \frac{\theta Z_{t+1}}{n_{2t}}, \\ \frac{\partial Z_{t+2}}{\partial n_{2t}} &= \frac{\gamma Z_{t+2}}{Z_{t+1}} \frac{\theta Z_{t+1}}{n_{2t}} - \alpha_3 (1-\gamma) \frac{Z_{t+2}}{n_{2t}} \end{aligned}$$

this means a change in  $n_2$  on  $Z$  has two effects - a direct effect and an indirect effect. The expression,  $\frac{\gamma Z_{t+2}}{Z_{t+1}} \frac{\theta Z_{t+1}}{n_{2t}}$ , is the direct effect, while the expression,  $-\alpha_3 (1-\gamma) \frac{Z_{t+2}}{n_{2t}}$ , is the indirect

effect. Therefore,

$$\frac{\partial Z_{t+2}}{\partial n_{2t}} = (\gamma\theta - \alpha_3(1 - \gamma)) \frac{Z_{t+2}}{n_{2t}}.$$

Similarly, the derivative of  $Z$  with respect to  $n_2$  at time period  $t + 3$  is

$$\frac{\partial Z_{t+3}}{\partial n_{2t}} = \gamma(\gamma\theta - \alpha_3(1 - \gamma)) \frac{Z_{t+3}}{n_{2t}}.$$

Hence, for any future time period  $t + j + 2$ , the derivative is as follows

$$\frac{\partial Z_{t+j+2}}{\partial n_{2t}} = \gamma^j(\gamma\theta - \alpha_3(1 - \gamma)) \frac{Z_{t+j+2}}{n_{2t}}.$$

Substituting the above expressions into the above expression for  $\{n_{2t}\}$ , we get

$$\frac{-1}{1 - n_t} + \frac{\lambda_t \alpha_3 Y_t (1 - \tau)}{n_{2t}} + \beta \lambda_{t+1} \frac{K_{t+2} \theta Z_{t+1}}{Z_{t+1}^2 n_{2t}} + \beta^2 \lambda_{t+2} \frac{K_{t+3}}{Z_{t+2}^2} (\gamma\theta - \alpha_3(1 - \gamma)) \frac{Z_{t+2}}{n_{2t}} + \dots = 0.$$

Again, for every  $t$ ,  $\frac{1}{C_t} = \lambda_t$ . Since  $\delta = 1$ ,  $I_{t+j} = \frac{K_{t+1+j}}{Z_{t+j}}$ , for all  $j \geq 0$ . This implies that

$$\frac{-1}{1 - n_t} + \frac{\lambda_t \alpha_3 Y_t (1 - \tau)}{n_{2t}} + \beta \theta \frac{\lambda_{t+1} K_{t+2}}{Z_{t+1} n_{2t}} + \frac{\beta^2 (\gamma\theta - \alpha_3(1 - \gamma))}{n_{2t}} \sum_{j=0}^{\infty} \beta^j \gamma^j \frac{I_{t+j+2}}{C_{t+j+2}} = 0.$$

Hence the final expression for  $\{n_{2t}\}$  is as follows and as shown in equation (13)

$$\{n_{2t}\} : \frac{1}{1 - n_t} = \frac{\alpha_3 Y_t (1 - \tau)}{C_t n_{2t}} + \beta \theta \frac{I_{t+1}}{C_{t+1} n_{2t}} + \frac{\beta^2 (\gamma\theta - \alpha_3(1 - \gamma))}{n_{2t}} \sum_{j=0}^{\infty} \beta^j \gamma^j \frac{I_{t+j+2}}{C_{t+j+2}}.$$

## Appendix B: Decision rules for the planner's problem for the base-line model

We first show that  $\Phi$ ,  $x$  and  $n$  are constants, and have feasible values. Feasibility requires that  $0 < \Phi, x, n < 1$ . We also derive a condition that ensures that the balanced growth rate is stable, i.e.,

$$g_{zt+1} = g_{zt} = g^z.$$

We have shown earlier

$$\begin{aligned} \{K_{t+1}\}: \frac{1}{C_t Z_t} &= \beta \frac{\alpha_1 Y_{t+1} (1 - \tau)}{C_{t+1} K_{t+1}} + \beta^2 \frac{I_{t+2}}{C_{t+2} K_{t+1}} (1 - \gamma)(1 - \mu) + \frac{\beta^3 (1 - \gamma)(\gamma(1 - \mu) - \alpha_1)}{K_{t+1}} \sum_{j=0}^{\infty} \beta^j \gamma^j \frac{I_{t+j+3}}{C_{t+j+3}} \\ \Rightarrow \frac{1}{\Phi Y_t (1 - \tau) Z_t} &= \beta \frac{\alpha_1 Y_{t+1} (1 - \tau)}{\Phi Y_{t+1} (1 - \tau) (1 - \Phi) Y_t (1 - \tau) Z_t} + \beta^2 \frac{(1 - \Phi)}{\Phi (1 - \Phi) Y_t (1 - \tau) Z_t} (1 - \gamma)(1 - \mu) + \frac{\beta^3 (1 - \gamma)(\gamma(1 - \mu) - \alpha_1)}{(1 - \Phi) Y_t (1 - \tau) Z_t} \left( \frac{1}{1 - \beta \gamma} \right) \frac{(1 - \Phi)}{\Phi} \end{aligned}$$

$$\begin{aligned} \Rightarrow (1 - \Phi) &= \frac{\alpha_1 \beta (1 - \beta \gamma)}{(1 - \beta \gamma)[1 - \beta^2(1 - \gamma)(1 - \mu)] - \beta^3(1 - \gamma)[\gamma(1 - \mu) - \alpha_1]} \\ \Rightarrow \Phi &= \frac{(1 - \beta \gamma)[1 - \beta^2(1 - \gamma)(1 - \mu) - \alpha_1 \beta] - \beta^3(1 - \gamma)[\gamma(1 - \mu) - \alpha_1]}{(1 - \beta \gamma)[1 - \beta^2(1 - \gamma)(1 - \mu)] - \beta^3(1 - \gamma)[\gamma(1 - \mu) - \alpha_1]}, \end{aligned}$$

as shown in (19).

For  $0 < \Phi < 1$ , we require that

$$(1 - \beta \gamma)[1 - \beta^2(1 - \gamma)(1 - \mu) - \alpha_1 \beta] - \beta^3(1 - \gamma)[\gamma(1 - \mu) - \alpha_1] > 0$$

and

$$1 - \beta \gamma > 0 \Rightarrow \gamma < \frac{1}{\beta}$$

Note that  $\gamma < \frac{1}{\beta}$  the condition for stability. From the FOC for  $n_{1t}$ ,

$$\begin{aligned} \{n_{1t}\} : \frac{1}{1 - n_t} + \frac{\beta^2 \alpha_2 (1 - \gamma)}{n_{1t}} \sum_{j=0}^{\infty} \beta^j \gamma^j \frac{I_{t+j+2}}{C_{t+j+2}} &= \frac{\alpha_2 Y_t (1 - \tau)}{C_t n_{1t}} \\ \Rightarrow \frac{n_t}{1 - n_t} &= \frac{\alpha_2}{\Phi x} - \frac{\beta^2 \alpha_2 (1 - \gamma)(1 - \Phi)}{x(1 - \beta \gamma)\Phi} \\ \Rightarrow n &= \frac{\alpha_2 [(1 - \beta \gamma) - \beta^2(1 - \gamma)(1 - \Phi)]}{(\alpha_2 + \Phi x)(1 - \beta \gamma) - \alpha_2 \beta^2(1 - \gamma)(1 - \Phi)} \end{aligned}$$

which is the expression in equation(17), and  $\Phi$  is derived above. Note, when

$$\alpha_2 [(1 - \beta \gamma) - \beta^2(1 - \gamma)(1 - \Phi)] > 0$$

and when

$$0 < \Phi, x < 1,$$

then

$$0 < n < 1.$$

Next, we derive the expression for  $x$ . We know the FOC with respect to  $\{n_{2t}\}$  is given by

$$\begin{aligned} \frac{1}{1 - n_t} &= \frac{\alpha_3 Y_t (1 - \tau)}{C_t n_{2t}} + \beta \theta \frac{I_{t+1}}{C_{t+1} n_{2t}} + \beta^2 (\gamma \theta - \alpha_3 (1 - \gamma)) \sum_{j=0}^{\infty} \beta^j \gamma^j \frac{I_{t+j+2}}{C_{t+j+2}} \\ \Rightarrow (1 - x) \left( \frac{n_1}{1 - n_t} \right) &= \frac{\alpha_3}{\Phi} + \beta \theta \left( \frac{1 - \Phi}{\Phi} \right) + \beta^2 (\gamma \theta - \alpha_3 (1 - \gamma)) \left( \frac{1}{1 - \beta \gamma} \right) \left( \frac{1 - \Phi}{\Phi} \right) \end{aligned}$$

$$\begin{aligned} \Rightarrow \left( \frac{1-x}{x} \right) &= \frac{\alpha_3(1-\beta\gamma) + \beta\theta(1-\Phi)(1-\beta\gamma) + \beta^2(\gamma\theta - \alpha_3(1-\gamma))(1-\Phi)}{\alpha_2(1-\beta\gamma) - \beta^2\alpha_2(1-\gamma)(1-\Phi)} \\ \Rightarrow x &= \frac{\alpha_2(1-\beta\gamma) - \beta^2\alpha_2(1-\gamma)(1-\Phi)}{[\alpha_2 + \alpha_3 + \beta\theta(1-\Phi)](1-\beta\gamma) + \beta^2(1-\Phi)[\gamma\theta - \alpha_3(1-\gamma) - \alpha_2(1-\gamma)]} \end{aligned}$$

which is the expression for  $x$  in equation (20). When  $0 < \Phi < 1$ ,  $0 < x < 1$  is automatically satisfied. This means feasible values of  $x$  require no other additional assumption other than  $0 < \gamma < 1$ . We first verify that  $0 < \Phi < 1$ . This means we need to check whether the following two inequalities are satisfied:

$$\begin{aligned} (i) (1-\beta\gamma)[1-\beta^2(1-\gamma)(1-\mu) - \alpha_1\beta] - \beta^3(1-\gamma)[\gamma(1-\mu) - \alpha_1] &> 0 \\ (ii) 1-\beta\gamma > 0 &\Rightarrow \gamma < \frac{1}{\beta} \end{aligned}$$

Since  $0 < \gamma < 1$ , we have  $0 < \gamma < 1 < \frac{1}{\beta}$ . This means (ii) is trivially satisfied when  $0 < \gamma < 1$ . To check (i), we must check whether  $(1-\beta\gamma)[1-\beta^2(1-\gamma)(1-\mu) - \alpha_1\beta] - \beta^3(1-\gamma)[\gamma(1-\mu) - \alpha_1] > 0$ , or  $(1-\beta\gamma)(1-\alpha_1\beta) - \beta^2(1-\gamma)(1-\mu - \alpha_1\beta) > 0$ ,

$$\Rightarrow (1-\beta\gamma)(1-\alpha_1\beta) > \beta^2(1-\gamma)(1-\mu - \alpha_1\beta).$$

We know  $(1-\alpha_1\beta) > (1-\mu - \alpha_1\beta)$  because  $0 < \mu < 1$ . Further,  $(1-\beta\gamma) > \beta^2(1-\gamma)$  if and only if  $(1-\beta\gamma) - \beta^2(1-\gamma) > 0$ . Clearly,  $(1-\beta)(1+\beta-\beta\gamma) > 0$ . Hence,  $(1-\beta\gamma) - \beta^2(1-\gamma) > 0$ . This implies

$$(1-\beta\gamma)(1-\alpha_1\beta) - \beta^2(1-\gamma)(1-\mu - \alpha_1\beta) > 0.$$

Hence, (i) is satisfied and  $0 < \Phi < 1$ . We now verify that  $0 < x < 1$ . We know that

$$x = \frac{\alpha_2(1-\beta\gamma) - \beta^2\alpha_2(1-\gamma)(1-\Phi)}{[\alpha_2 + \alpha_3 + \beta\theta(1-\Phi)](1-\beta\gamma) + \beta^2(1-\Phi)[\gamma\theta - \alpha_3(1-\gamma) - \alpha_2(1-\gamma)]}.$$

To show the above expression for  $0 < x < 1$ , it is sufficient to show that  $\alpha_2[(1-\beta\gamma) - \beta^2(1-\gamma)(1-\Phi)] > 0$ , since we have already shown,  $0 < \Phi < 1$ . We also have to show the denominator in the above expression is greater than the numerator so as to ensure that  $x$  is a fraction. As shown earlier,

$$(1-\beta\gamma) - \beta^2(1-\gamma) > 0$$

which implies  $(1-\beta\gamma) - \beta^2(1-\gamma)(1-\Phi) > 0$  since we have already shown  $0 < \Phi < 1$  (and

so  $0 < (1 - \Phi) < 1$ ). This implies that,  $\alpha_2[(1 - \beta\gamma) - \beta^2(1 - \gamma)(1 - \Phi)] > 0$  is true. The denominator is given by the expression

$$[\alpha_2 + \alpha_3 + \beta\theta(1 - \Phi)](1 - \beta\gamma) + \beta^2(1 - \Phi)[\gamma\theta - \alpha_3(1 - \gamma) - \alpha_2(1 - \gamma)],$$

which on re-arranging yields  $\alpha_2[(1 - \beta\gamma) - \beta^2(1 - \gamma)(1 - \Phi)] + \alpha_3[(1 - \beta\gamma) - \beta^2(1 - \gamma)(1 - \Phi)] + \beta\theta(1 - \Phi)(1 - \beta\gamma) + \beta^2(1 - \Phi)\gamma\theta$ . We earlier showed that,  $\alpha_2[(1 - \beta\gamma) - \beta^2(1 - \gamma)(1 - \Phi)] > 0$ . Similarly, we can obtain

$$\alpha_3[(1 - \beta\gamma) - \beta^2(1 - \gamma)(1 - \Phi)] > 0.$$

Also,

$$\beta\theta(1 - \Phi)(1 - \beta\gamma) + \beta^2(1 - \Phi)\gamma\theta > 0$$

follows from above and the restriction that  $(1 - \beta\gamma) > 0$ . Hence the denominator of  $x$  is the numerator plus a sum of two positive terms. This shows that  $0 < x < 1$ .

Finally, we need to check that  $0 < n < 1$ . Recall that,

$$n = \frac{\alpha_2[(1 - \beta\gamma) - \beta^2(1 - \gamma)(1 - \Phi)]}{(\alpha_2 + \Phi x)(1 - \beta\gamma) - \alpha_2\beta^2(1 - \gamma)(1 - \Phi)}.$$

This means if we just show the numerator is greater than zero,  $0 < n < 1$  is true. This is because the denominator is simply the numerator +  $\Phi x(1 - \beta\gamma)$ . From above, this term ( $\Phi x(1 - \beta\gamma)$ ) is positive. We have also seen that  $\alpha_2[(1 - \beta\gamma) - \beta^2(1 - \gamma)(1 - \Phi)] > 0$ . Hence,  $0 < n < 1$  is true.

Therefore,  $0 < \Phi, x, n < 1$ .

## Appendix C : Growth optimizing tax rate in the planner's problem

In the planner version,

$$Z_{t+1} = B((1 - \delta_m)n_{2t})^\theta Z_t^\gamma \left\{ \left( \frac{G_t}{Y_{t-1}} \right)^\mu \left( \frac{K_t}{Y_{t-1}} \right)^{1-\mu} \right\}^{(1-\gamma)}.$$

Since under full depreciation  $K_t = I_{t-1}Z_{t-1}$  and  $G_t = I_{t-1}^g Z_{t-1}$ , we can substitute for  $G$  and  $K$  in the law of motion and further substitute the decision rules for  $I$  and  $I^g$ , leading to

$$I_t = (1 - \Phi)Y_t(1 - \tau)$$

and

$$I_t^g = \tau Y_t.$$

Further,

$$n_2 = (1 - x)n$$

Hence,

$$\begin{aligned} Z_{t+1} &= B((1 - \delta_m)(1 - x)n)^\theta Z_t^\gamma \left\{ \left( \frac{\tau Y_{t-1} Z_{t-1}}{Y_{t-1}} \right)^\mu \left( \frac{(1 - \Phi) Y_{t-1} (1 - \tau) Z_{t-1}}{Y_{t-1}} \right)^{1-\mu} \right\}^{(1-\gamma)} \\ &\Rightarrow Z_{t+1} = \widehat{M} Z_t^\gamma Z_{t-1}^{(1-\gamma)} \{(\tau)^\mu (1 - \tau)^{1-\mu}\}^{(1-\gamma)} \end{aligned}$$

where

$$\widehat{M} = B((1 - \delta_m)(1 - x)n)^\theta (1 - \Phi)^{(1-\mu)(1-\gamma)}$$

is a constant. This implies

$$\begin{aligned} \frac{Z_{t+1}}{Z_t} &= g_{z_{t+1}} = \widehat{M} \frac{Z_t^{\gamma-1}}{Z_{t-1}^{\gamma-1}} Z_{t-1}^{(1-\gamma)+\gamma-1} \{(\tau)^\mu (1 - \tau)^{1-\mu}\}^{(1-\gamma)} \\ &\Rightarrow g_{z_{t+1}} = g_{z_t}^{-(1-\gamma)} \widehat{M} \{(\tau)^\mu (1 - \tau)^{1-\mu}\}^{(1-\gamma)}. \end{aligned}$$

The assumption  $0 < \gamma < 1$ , makes the system defined by

$$g_{z_{t+1}} = \frac{\Omega}{g_{z_t}^{(1-\gamma)}}$$

(where  $\Omega = \widehat{M} \{(\tau)^\mu (1 - \tau)^{1-\mu}\}^{(1-\gamma)}$  is a constant) dynamically stable. Any deviation from the point of intersection of the 45-degree line with the plot for  $g_{z_{t+1}} = \frac{\Omega}{g_{z_t}^{(1-\gamma)}}$  will eventually result in the system converging to the 45-degree line. This is shown in Figure [1]. Note that  $(1 - \gamma) = 1$  will give an oscillating system which will never ever converge. Likewise  $(1 - \gamma) > 1$  will lead to an explosive system. In our model, it is therefore sufficient to have  $0 < \gamma < 1$  to ensure a steady state BGP.

At the steady state therefore,

$$g_{z_{t+1}} = g_{z_t} = \widehat{g}_z$$

and hence,

$$\widehat{g}_z = \Omega^{\frac{1}{1+(1-\gamma)}} = [\widehat{M} \{(\tau)^\mu (1 - \tau)^{1-\mu}\}^{(1-\gamma)}]^{\frac{1}{2-\gamma}}$$

is the steady state growth rate, where  $\widehat{M} = B((1 - \delta_m)(1 - x)n)^\theta (1 - \Phi)^{(1-\mu)(1-\gamma)}$  is a constant.

To derive the growth rates of the other variables on the BGP, note that

$$\begin{aligned}
K_{t+1} &= I_t Z_t \\
\Rightarrow \frac{K_{t+1}}{K_t} &= \frac{I_t Z_t}{I_{t-1} Z_{t-1}} \\
\Rightarrow \hat{g}_k &= \frac{(1-\Phi)Y_t(1-\tau)Z_t}{(1-\Phi)Y_{t-1}(1-\tau)Z_{t-1}} = \frac{Y_t Z_t}{Y_{t-1} Z_{t-1}} = \hat{g}_y \hat{g}_z.
\end{aligned}$$

Now,

$$\begin{aligned}
Y_t &= AK_t^{\alpha_1} n_{1t}^{\alpha_2} (\delta_m n_{2t})^{\alpha_3} \\
\Rightarrow \frac{Y_t}{Y_{t-1}} &= \hat{g}_y = \frac{AK_t^{\alpha_1} n_{1t}^{\alpha_2} (\delta_m n_{2t})^{\alpha_3}}{AK_{t-1}^{\alpha_1} n_{1t-1}^{\alpha_2} (\delta_m n_{2t-1})^{\alpha_3}} = \frac{K_t^{\alpha_1}}{K_{t-1}^{\alpha_1}} = \hat{g}_k^{\alpha_1} \\
\Rightarrow \hat{g}_y &= \hat{g}_k^{\alpha_1} \\
\Rightarrow \hat{g}_y &= \hat{g}_k^{\alpha_1} = \hat{g}_z^{\frac{\alpha_1}{1-\alpha_1}}.
\end{aligned}$$

## Appendix D - Optimal tax rate in the baseline model - the planner's problem

$$\begin{aligned}
\frac{\partial \hat{g}_z}{\partial \tau} &= \frac{\partial [\widehat{M} \{(\tau)^\mu (1-\tau)^{1-\mu}\}^{(1-\gamma)}]^{1-\gamma}}{\partial \tau} = 0 \\
\Rightarrow \widehat{M}^{\frac{1}{(2-\gamma)}} \frac{(1-\gamma)}{(2-\gamma)} [(\tau)^\mu (1-\tau)^{1-\mu}]^{\frac{-1}{(2-\gamma)}} \frac{\partial [(\tau)^\mu (1-\tau)^{1-\mu}]}{\partial \tau} &= 0 \\
\Rightarrow (1-\tau)^{1-\mu} \mu (\tau)^{\mu-1} &= (\tau)^\mu (1-\mu) (1-\tau)^{-\mu} \\
\Rightarrow (1-\tau) \mu &= (\tau) (1-\mu) \\
\Rightarrow \hat{\tau} &= \mu.
\end{aligned}$$

Hence the steady state growth optimizing tax rate in the planner version is  $\hat{\tau} = \mu$ .

## Appendix E - Optimal tax rate in the baseline model - the agent's version

$$\begin{aligned}
\{\tau_n\} &: \frac{\partial \hat{g}_z}{\partial \tau_n} = \frac{\partial [\widehat{M} \{(1-\Theta)^\mu (\Theta)^{1-\mu}\}^{(1-\gamma)}]^{1-\gamma}}{\partial \tau_n} = 0 \\
\Rightarrow \widehat{M}^{\frac{1}{(2-\gamma)}} \frac{(1-\gamma)}{(2-\gamma)} [(1-\Theta)^\mu (\Theta)^{1-\mu}]^{\frac{-1}{(2-\gamma)}} \frac{\partial [(1-\Theta)^\mu (\Theta)^{1-\mu}]}{\partial \tau_n} &= 0 \\
\Rightarrow (\Theta)^{1-\mu} \mu (1-\Theta)^{\mu-1} \frac{\partial [(1-\Theta)]}{\partial \tau_n} + (1-\Theta)^\mu (1-\mu) (\Theta)^{-\mu} \frac{\partial [(\Theta)]}{\partial \tau_n} &= 0.
\end{aligned}$$

Since

$$\begin{aligned}\frac{\partial[(1-\Theta)]}{\partial\tau_n} &= \frac{\partial[(\alpha_2+\alpha_3)(\tau_n)+\alpha_1(\tau_k)]}{\partial\tau_n} = \alpha_2+\alpha_3, \\ \frac{\partial[(\Theta)]}{\partial\tau_n} &= \frac{\partial[(\alpha_2+\alpha_3)(1-\tau_n)+\alpha_1(1-\tau_k)]}{\partial\tau_n} = -(\alpha_2+\alpha_3).\end{aligned}$$

Substituting the above expressions yields,

$$\begin{aligned}\Rightarrow (\Theta)^{1-\mu}\mu(1-\Theta)^{\mu-1} - (1-\Theta)^\mu(1-\mu)(\Theta)^{-\mu} &= 0 \\ \Rightarrow \frac{\mu}{1-\mu} &= \frac{1-\Theta}{\Theta} \\ \Rightarrow \widehat{1-\Theta} &= \mu.\end{aligned}$$

Likewise

$$\{\tau_k\} : \frac{\partial\widehat{gz}}{\partial\tau_k} = \frac{\partial[\widehat{M}\{(1-\Theta)^\mu(\Theta)^{1-\mu}\}^{(1-\gamma)}]^{1/(2-\gamma)}}{\partial\tau_k} = 0.$$

Since

$$\begin{aligned}\frac{\partial[(1-\Theta)]}{\partial\tau_k} &= \frac{\partial[(\alpha_2+\alpha_3)(\tau_n)+\alpha_1(\tau_k)]}{\partial\tau_k} = \alpha_1, \\ \frac{\partial[(\Theta)]}{\partial\tau_k} &= \frac{\partial[(\alpha_2+\alpha_3)(1-\tau_n)+\alpha_1(1-\tau_k)]}{\partial\tau_k} = -(\alpha_1).\end{aligned}$$

Substituting the above yields the same FOCs just as in the case of  $\{\tau_n\}$ . This implies, both  $\{\tau_n\}$  and  $\{\tau_k\}$  give the same FOC

$$\begin{aligned}\widehat{1-\Theta} &= \mu \\ \Rightarrow (\alpha_2+\alpha_3)(\widehat{\tau}_n) + \alpha_1(\widehat{\tau}_k) &= \mu = \widehat{\tau}.\end{aligned}$$

## Appendix F - Comparative statics - the planner's problem

We know from equations (17) and (19)

$$n = \frac{\alpha_2[(1-\beta\gamma) - \beta^2(1-\gamma)(1-\Phi)]}{(\alpha_2+\Phi x)(1-\beta\gamma) - \alpha_2\beta^2(1-\gamma)(1-\Phi)}.$$

And

$$\Phi = \frac{(1-\beta\gamma)[1 - \beta^2(1-\gamma)(1-\mu) - \alpha_1\beta] - \beta^3(1-\gamma)[\gamma(1-\mu) - \alpha_1]}{(1-\beta\gamma)[1 - \beta^2(1-\gamma)(1-\mu)] - \beta^3(1-\gamma)[\gamma(1-\mu) - \alpha_1]}.$$

Further

$$\hat{\tau} = \mu.$$

We shall first see how  $\Phi$  changes with  $\mu$ . This implies, the relationship between  $1 - \Phi$  and  $\mu$  gets reversed.

$$\begin{aligned} \frac{\partial \Phi}{\partial \mu} &= \frac{\partial}{\partial \mu} \left[ \frac{(1 - \beta\gamma)[1 - \beta^2(1 - \gamma)(1 - \mu) - \alpha_1\beta] - \beta^3(1 - \gamma)[\gamma(1 - \mu) - \alpha_1]}{(1 - \beta\gamma)[1 - \beta^2(1 - \gamma)(1 - \mu)] - \beta^3(1 - \gamma)[\gamma(1 - \mu) - \alpha_1]} \right] \\ &= \frac{\partial}{\partial \mu} \left[ 1 - \frac{\alpha_1\beta(1 - \beta\gamma)}{(1 - \beta\gamma)[1 - \beta^2(1 - \gamma)(1 - \mu)] - \beta^3(1 - \gamma)[\gamma(1 - \mu) - \alpha_1]} \right] \\ &= \frac{\alpha_1\beta(1 - \beta\gamma)}{\{(1 - \beta\gamma)[1 - \beta^2(1 - \gamma)(1 - \mu)] - \beta^3(1 - \gamma)[\gamma(1 - \mu) - \alpha_1]\}^2} \times \\ &\quad \frac{\partial \{(1 - \beta\gamma)[1 - \beta^2(1 - \gamma)(1 - \mu)] - \beta^3(1 - \gamma)[\gamma(1 - \mu) - \alpha_1]\}}{\partial \mu} \\ &= \frac{\alpha_1\beta(1 - \beta\gamma)[(1 - \beta\gamma)\beta^2(1 - \gamma) + \beta^3(1 - \gamma)\gamma]}{\{(1 - \beta\gamma)[1 - \beta^2(1 - \gamma)(1 - \mu)] - \beta^3(1 - \gamma)[\gamma(1 - \mu) - \alpha_1]\}^2} > 0 \end{aligned}$$

Hence

$$\frac{\partial \Phi}{\partial \mu} > 0,$$

and therefore

$$\frac{\partial(1 - \Phi)}{\partial \mu} < 0.$$

Since

$$\begin{aligned} C_t &= \Phi Y_t(1 - \tau) \\ &\Rightarrow \frac{\partial C_t}{\partial \tau} < 0. \end{aligned}$$

Now we take the partial derivative of

$$x = \frac{\alpha_2(1 - \beta\gamma) - \beta^2\alpha_2(1 - \gamma)(1 - \Phi)}{[\alpha_2 + \alpha_3 + \beta\theta(1 - \Phi)](1 - \beta\gamma) + \beta^2(1 - \Phi)[\gamma\theta - \alpha_3(1 - \gamma) - \alpha_2(1 - \gamma)]}$$

with respect to  $\mu$ . We will use  $\frac{\partial \Phi}{\partial \mu} > 0$  in our analysis. Suppose we consider the value of  $\frac{1}{x}$ ,

$$\begin{aligned} \frac{1}{x} &= 1 + \frac{\alpha_3(1 - \beta\gamma) - \beta^2\alpha_3(1 - \gamma)(1 - \Phi)}{\alpha_2(1 - \beta\gamma) - \beta^2\alpha_2(1 - \gamma)(1 - \Phi)} + \frac{\beta\theta(1 - \Phi)}{\alpha_2(1 - \beta\gamma) - \beta^2\alpha_2(1 - \gamma)(1 - \Phi)} \\ &\Rightarrow \frac{\partial \left(\frac{1}{x}\right)}{\partial \mu} = \frac{\partial}{\partial \mu} \left( \frac{\alpha_3(1 - \beta\gamma) - \beta^2\alpha_3(1 - \gamma)(1 - \Phi)}{\alpha_2(1 - \beta\gamma) - \beta^2\alpha_2(1 - \gamma)(1 - \Phi)} \right) + \frac{\partial}{\partial \mu} \left( \frac{\beta\theta(1 - \Phi)}{\alpha_2(1 - \beta\gamma) - \beta^2\alpha_2(1 - \gamma)(1 - \Phi)} \right) \end{aligned}$$

On opening up the brackets and simplifying the above expression, we get

$$\Rightarrow \frac{\partial \left(\frac{1}{x}\right)}{\partial \mu} = \frac{\alpha_2(1 - \beta\gamma)\beta\theta \frac{\partial}{\partial \mu}(1 - \Phi)}{(\alpha_2(1 - \beta\gamma) - \beta^2\alpha_2(1 - \gamma)(1 - \Phi))^2} < 0$$

because

$$\frac{\partial \Phi}{\partial \mu} > 0 \text{ and so } \frac{\partial(1 - \Phi)}{\partial \mu} < 0.$$

Hence,

$$\begin{aligned} \frac{\partial \left(\frac{1}{x}\right)}{\partial \mu} &< 0 \\ \frac{\partial(x)}{\partial \mu} &> 0. \end{aligned}$$

We now look at the partial derivative of  $n$  with respect to  $\mu$ . We have shown earlier that

$$n = \frac{\alpha_2[(1 - \beta\gamma) - \beta^2(1 - \gamma)(1 - \Phi)]}{(\alpha_2 + \Phi x)(1 - \beta\gamma) - \alpha_2\beta^2(1 - \gamma)(1 - \Phi)}.$$

Hence, on applying the quotient rule and on re-arranging the terms, we get

$$\begin{aligned} \frac{\partial}{\partial \mu}(n) &= \frac{[\alpha_2\beta^2(1 - \gamma)\Phi x(1 - \beta\gamma)] \frac{\partial \Phi}{\partial \mu}}{[\alpha_2(1 - \beta\gamma) - \alpha_2\beta^2(1 - \gamma)(1 - \Phi) + \Phi x(1 - \beta\gamma)]^2} \\ &\quad - \frac{[\alpha_2(1 - \beta\gamma) - \alpha_2\beta^2(1 - \gamma)(1 - \Phi)](1 - \beta\gamma)\Phi \frac{\partial x}{\partial \mu}}{[\alpha_2(1 - \beta\gamma) - \alpha_2\beta^2(1 - \gamma)(1 - \Phi) + \Phi x(1 - \beta\gamma)]^2} \\ &\quad - \frac{[\alpha_2(1 - \beta\gamma) - \alpha_2\beta^2(1 - \gamma)(1 - \Phi)](1 - \beta\gamma)x \frac{\partial \Phi}{\partial \mu}}{[\alpha_2(1 - \beta\gamma) - \alpha_2\beta^2(1 - \gamma)(1 - \Phi) + \Phi x(1 - \beta\gamma)]^2} \end{aligned}$$

Given that  $\frac{\partial x}{\partial \mu} > 0$  we can easily see that the second term in the above expression

$$- \frac{[\alpha_2(1 - \beta\gamma) - \alpha_2\beta^2(1 - \gamma)(1 - \Phi)](1 - \beta\gamma)\Phi \frac{\partial x}{\partial \mu}}{[\alpha_2(1 - \beta\gamma) - \alpha_2\beta^2(1 - \gamma)(1 - \Phi) + \Phi x(1 - \beta\gamma)]^2} < 0.$$

This is because from an earlier exercise we have shown that  $\alpha_2(1 - \beta\gamma) - \alpha_2\beta^2(1 - \gamma)(1 - \Phi) >$

0. Clubbing the other two terms, we get,

$$\frac{\partial}{\partial \mu}(n) = \frac{-\alpha_2 x [(1 - \beta\gamma)^2 - \beta^2(1 - \gamma)\Phi(1 - \beta\gamma) - \beta^2(1 - \beta\gamma)(1 - \gamma)(1 - \Phi)] \frac{\partial \Phi}{\partial \mu}}{[\alpha_2(1 - \beta\gamma) - \alpha_2\beta^2(1 - \gamma)(1 - \Phi) + \Phi x(1 - \beta\gamma)]^2} - \frac{[\alpha_2(1 - \beta\gamma) - \alpha_2\beta^2(1 - \gamma)(1 - \Phi)](1 - \beta\gamma)\Phi \frac{\partial x}{\partial \mu}}{[\alpha_2(1 - \beta\gamma) - \alpha_2\beta^2(1 - \gamma)(1 - \Phi) + \Phi x(1 - \beta\gamma)]^2}.$$

If  $[(1 - \beta\gamma)^2 - \beta^2(1 - \gamma)\Phi(1 - \beta\gamma) - \beta^2(1 - \beta\gamma)(1 - \gamma)(1 - \Phi)] > 0$  then  $\frac{\partial n}{\partial \mu} < 0$ . To see this, we re-arrange

$$[(1 - \beta\gamma)^2 - \beta^2(1 - \gamma)\Phi(1 - \beta\gamma) - \beta^2(1 - \beta\gamma)(1 - \gamma)(1 - \Phi)]$$

to get

$$(1 - \beta\gamma)[(1 - \beta\gamma) - \beta^2(1 - \gamma)\Phi - \beta^2(1 - \gamma)(1 - \Phi)]$$

which is equal to

$$(1 - \beta\gamma)(1 - \beta)[1 - \beta\gamma + \beta] > 0$$

because we have shown earlier that  $[1 - \beta\gamma - \beta^2(1 - \gamma)] > 0$ . Hence

$$\frac{\partial n}{\partial \mu} < 0.$$

We now need to verify whether,  $\frac{\partial n_1}{\partial \mu} < 0$ . Note that

$$\frac{\partial}{\partial \mu}(n_1) = \frac{\partial}{\partial \mu}(xn) = x \frac{\partial}{\partial \mu}(n) + n \frac{\partial}{\partial \mu}(x),$$

the sign of which is ambiguous because  $x$  is increasing in  $\mu$  while  $n$  is decreasing in  $\mu$ . Therefore whichever term dominates will determine the way  $n_1$  behaves with  $\mu$ . However we can show

$$\frac{\partial}{\partial \mu}(n_2) < 0,$$

since

$$\frac{\partial}{\partial \mu}(n_2) = \frac{\partial}{\partial \mu}((1 - x)n) = (1 - x) \frac{\partial}{\partial \mu}(n) + n \frac{\partial}{\partial \mu}(1 - x) < 0.$$

## Appendix G: Welfare analysis

In the planner's problem, we know

$$C_t = \Phi Y_t(1 - \tau)$$

$$\begin{aligned} &\Rightarrow \frac{C_t}{C_{t-1}} = \frac{\Phi Y_t(1-\tau)}{\Phi Y_{t-1}(1-\tau)} = \hat{g}_y \\ &\Rightarrow \hat{g}_c = \hat{g}_y. \end{aligned}$$

Since  $\hat{g}_c$  is a constant,  $C_t = C_0 \hat{g}_c^t$ . On the BGP, the supply of labor is the same across time. We denote welfare by  $\Lambda$ , where,

$$\begin{aligned} \Lambda &= \sum_{j=0}^{\infty} \beta^j [\log C_t + \log(1 - n_t)] \\ \Lambda &= \sum_{j=0}^{\infty} \beta^j \log C_t + \frac{\log(1 - \hat{n})}{1 - \beta} \end{aligned}$$

$$\Rightarrow \Lambda = \log C_o + \beta \log C_1 + \beta^2 \log C_2 + \beta^3 \log C_3 + \beta^4 \log C_4 + \dots + \frac{\log(1-\hat{n})}{1-\beta}$$

$$\Rightarrow \Lambda = \frac{\log C_o}{1 - \beta} + \frac{\beta^2}{1 - \beta} \log \hat{g}_c + \frac{\log(1 - \hat{n})}{1 - \beta}$$

$$\Rightarrow \Lambda = \frac{\log C_o}{1 - \beta} + \frac{\beta^2 \alpha_1}{(1 - \beta)(1 - \alpha_1)} \log[\widehat{M}\{(\tau)^\mu(1 - \tau)^{1-\mu}\}^{(1-\gamma)}]_{2-\gamma}^{\frac{1}{2-\gamma}} + \frac{\log(1-\hat{n})}{1-\beta}$$

$$\Rightarrow \Lambda = \Gamma + \Psi \log[(\tau)^\mu(1 - \tau)^{1-\mu}],$$

where  $\Gamma$  is independent of the tax rate and

$$\Psi = \frac{\beta^2 \alpha_1 (1 - \gamma)}{(1 - \beta)(1 - \alpha_1)(2 - \gamma)}.$$

In the agent's version, we denote welfare by  $\Delta$ .

$$\begin{aligned} \Delta &= \sum_{j=0}^{\infty} \beta^j [\log C_t + \log(1 - n_t)] \\ &\Rightarrow \Delta = \sum_{j=0}^{\infty} \beta^j \log C_t + \frac{\log(1 - \hat{n})}{1 - \beta} \\ &= \frac{\log C_o}{1 - \beta} + \frac{\beta^2}{1 - \beta} \log \hat{g}_c + \frac{\log(1 - \hat{n})}{1 - \beta} \end{aligned}$$

$$\Rightarrow \Delta = \frac{\log C_o}{1 - \beta} + \frac{\beta^2 \alpha_1}{(1 - \beta)(1 - \alpha_1)} \log[\widehat{M}\{(1 - \Theta)^\mu(\Theta)^{1-\mu}\}^{(1-\gamma)}]_{2-\gamma}^{\frac{1}{2-\gamma}} + \frac{\log(1-\hat{n})}{1-\beta}$$

$\Rightarrow \Delta = \Xi + \Psi \log[(1 - \Theta)^\mu (\Theta)^{1-\mu}]$ , where  $\Xi$  is independent of the tax rates and

$$\Psi = \frac{\beta^2 \alpha_1 (1 - \gamma)}{(1 - \beta)(1 - \alpha_1)(2 - \gamma)}.$$

## Appendix H - The model with administrative costs: growth optimization in planner's version

$$\begin{aligned} \widehat{g}_z &= [B((1 - \delta_m)(1 - x)(n))^\theta (1 - \Phi)^{(1-\mu)(1-\gamma)} \{(\tau - c\tau^\chi)^\mu (1 - \tau)^{1-\mu}\}^{(1-\gamma)}]^{1-\gamma} \\ &\Rightarrow \frac{\partial \widehat{g}_z}{\partial \tau} = \frac{\partial [\{(\tau - c\tau^\chi)^\mu (1 - \tau)^{1-\mu}\}^{\frac{(1-\gamma)}{(2-\gamma)}}]}{\partial \tau} = 0 \\ &\Rightarrow (\tau - c\tau^\chi)^\mu \frac{\partial (1 - \tau)^{1-\mu}}{\partial \tau} + (1 - \tau)^{1-\mu} \frac{\partial (\tau - c\tau^\chi)^\mu}{\partial \tau} = 0 \end{aligned}$$

$$\Rightarrow -(1 - \mu)(\tau - c\tau^\chi)^\mu (1 - \tau)^{-\mu} + \mu(1 - \tau)^{1-\mu} (\tau - c\tau^\chi)^{\mu-1} (1 - c\chi\tau^{\chi-1}) = 0$$

$$\Rightarrow \mu(1 - \tau^{AC})[1 - c\chi(\tau^{AC})^{\chi-1}] = (1 - \mu)(\tau^{AC} - c(\tau^{AC})^\chi) \text{ as shown in equation (42).}$$

Substituting  $c = 0$ , we get

$$\begin{aligned} \mu(1 - \tau^{AC}) &= (1 - \mu)\tau^{AC} \\ &\Rightarrow \tau^{AC} = \mu. \end{aligned}$$

Substituting  $\chi = 1$

$$\begin{aligned} \mu(1 - \tau^{AC})[1 - c] &= (1 - \mu)(\tau^{AC} - c\tau^{AC}) \\ &\Rightarrow \mu(1 - \tau^{AC}) = (1 - \mu)\tau^{AC} \\ &\Rightarrow \tau^{AC} = \mu \end{aligned}$$

## Appendix I - The model with administrative costs: growth optimization in the agent's version

We have,

$$\widehat{g}_z = [B((1 - \delta_m)(1 - x)(n))^\theta (1 - \Phi)^{(1-\mu)(1-\gamma)} \{(1 - \Theta - c_1\tau_k^\chi - c_2\tau_n^\chi)^\mu (\Theta)^{1-\mu}\}^{(1-\gamma)}]^{1-\gamma}$$

where  $1 - \Theta = \alpha_1\tau_k + (1 - \alpha_1)\tau_n$ . Substituting for  $(1 - \Phi)$  in  $\widehat{g}_z$  we get,

$$\widehat{g}_z = [B((1 - \delta_m)(1 - x)(n))^\theta (1 - \Phi)^{(1-\mu)(1-\gamma)} \{(\alpha_1 \tau_k + (1 - \alpha_1) \tau_n - c_1 \tau_k^\chi - c_2 \tau_n^\chi)^\mu (1 - \alpha_1 \tau_k - (1 - \alpha_1) \tau_n)^{1-\mu}\}^{(1-\gamma)}]^{-\frac{1}{2-\gamma}}.$$

The FOC of  $\widehat{g}_z$  with respect to  $\tau_n$  is given by

$$\begin{aligned} \{\tau_n\} : \frac{\partial \widehat{g}_z}{\partial \tau_n} &= 0 \\ \Rightarrow \frac{\partial \{(\alpha_1 \tau_k + (1 - \alpha_1) \tau_n - c_1 \tau_k^\chi - c_2 \tau_n^\chi)^\mu (1 - \alpha_1 \tau_k - (1 - \alpha_1) \tau_n)^{1-\mu}\}}{\partial \tau_n} &= 0 \\ \Rightarrow (1 - \Theta - c_1 \tau_k^\chi - c_2 \tau_n^\chi)^\mu \frac{\partial (1 - \alpha_1 \tau_k - (1 - \alpha_1) \tau_n)^{1-\mu}}{\partial \tau_n} + (\Theta)^{1-\mu} \frac{\partial (\alpha_1 \tau_k + (1 - \alpha_1) \tau_n - c_1 \tau_k^\chi - c_2 \tau_n^\chi)^\mu}{\partial \tau_n} &= 0 \\ \Rightarrow \mu(1 - \alpha_1 - c_2 \chi \tau_n^{\chi-1})(\Theta) &= (1 - \alpha_1)(1 - \mu)(1 - \Theta - c_1 \tau_k^\chi - c_2 \tau_n^\chi). \end{aligned} \quad (49)$$

Similarly, the FOC with respect to  $\tau_k$  is given by

$$\begin{aligned} \{\tau_k\} : \frac{\partial \widehat{g}_z}{\partial \tau_k} &= 0 \\ \Rightarrow \frac{\partial \{(\alpha_1 \tau_k + (1 - \alpha_1) \tau_n - c_1 \tau_k^\chi - c_2 \tau_n^\chi)^\mu (1 - \alpha_1 \tau_k - (1 - \alpha_1) \tau_n)^{1-\mu}\}}{\partial \tau_k} &= 0 \\ \Rightarrow (1 - \Theta - c_1 \tau_k^\chi - c_2 \tau_n^\chi)^\mu \frac{\partial (\Theta)^{1-\mu}}{\partial \tau_k} + (\Theta)^{1-\mu} \frac{\partial (1 - \Theta - c_1 \tau_k^\chi - c_2 \tau_n^\chi)^\mu}{\partial \tau_k} &= 0 \\ \Rightarrow \mu(\alpha_1 - c_1 \chi \tau_k^{\chi-1})(\Theta) &= \alpha_1(1 - \mu)(1 - \Theta - c_1 \tau_k^\chi - c_2 \tau_n^\chi). \end{aligned} \quad (50)$$

Using FOC (a) and FOC (b), we have

$$\begin{aligned} \frac{\mu(1 - \alpha_1 - c_2 \chi \tau_n^{\chi-1})(\Theta)}{\mu(\alpha_1 - c_1 \chi \tau_k^{\chi-1})(\Theta)} &= \frac{(1 - \alpha_1)(1 - \mu)(1 - \Theta - c_1 \tau_k^\chi - c_2 \tau_n^\chi)}{\alpha_1(1 - \mu)(1 - \Theta - c_1 \tau_k^\chi - c_2 \tau_n^\chi)} \\ \Rightarrow \frac{(1 - \alpha_1 - c_2 \chi \tau_n^{\chi-1})}{(\alpha_1 - c_1 \chi \tau_k^{\chi-1})} &= \frac{(1 - \alpha_1)}{\alpha_1} \\ \Rightarrow \alpha_1(1 - \alpha_1 - c_2 \chi \tau_n^{\chi-1}) &= (1 - \alpha_1)(\alpha_1 - c_1 \chi \tau_k^{\chi-1}) \\ \Rightarrow \alpha_1 c_2 \chi \tau_n^{\chi-1} &= (1 - \alpha_1) c_1 \chi \tau_k^{\chi-1} \\ \Rightarrow \frac{\tau_n^{\chi-1}}{\tau_k^{\chi-1}} &= \frac{(1 - \alpha_1) c_1}{\alpha_1 c_2} \\ \Rightarrow \left(\frac{\tau_n}{\tau_k}\right)^{\chi-1} &= \left(\frac{1 - \alpha_1}{\alpha_1}\right) \left(\frac{c_1}{c_2}\right) \end{aligned}$$

as shown in equation (48). We now summarize the results for linear administrative costs. The following is the FOC with respect to  $\tau_n$

$$\{\tau_n\} : \mu(1 - \alpha_1 - c_2\chi\tau_n^{\chi-1})(\Theta) = (1 - \alpha_1)(1 - \mu)(1 - \Theta - c_1\tau_k^\chi - c_2\tau_n^\chi)$$

and the following is the FOC with respect to  $\tau_k$

$$\{\tau_k\} : \mu(\alpha_1 - c_1\chi\tau_k^{\chi-1})(\Theta) = \alpha_1(1 - \mu)(1 - \Theta - c_1\tau_k^\chi - c_2\tau_n^\chi).$$

Substituting  $\chi = 1$ , we now get the following as the FOCs with respect to  $\tau_n$  and  $\tau_k$  respectively

$$\{\tau_n\} : \mu(1 - \alpha_1 - c_2)(\Theta) = (1 - \alpha_1)(1 - \mu)(1 - \Theta - c_1\tau_k - c_2\tau_n)$$

$$\{\tau_k\} : \mu(\alpha_1 - c_1)(\Theta) = \alpha_1(1 - \mu)(1 - \Theta - c_1\tau_k - c_2\tau_n).$$

This suggests that there does not exist any unique solution to the combination of tax rates on labor and capital.

**Table 1: Parameter Values**

$\alpha_1$	0.35*
$\alpha_2$	0.4
$\alpha_3$	0.25
$\beta$	0.95*
$\gamma$	0.67
$\delta$	1*
$\delta_m$	0.2
$\mu$	0.5
$\theta$	0.2*
$\chi$	2 in the model with convex costs 0.1 in the model with concave costs
$c$	0.4 in the model with convex costs 0.01 in the model with concave costs

# Figures

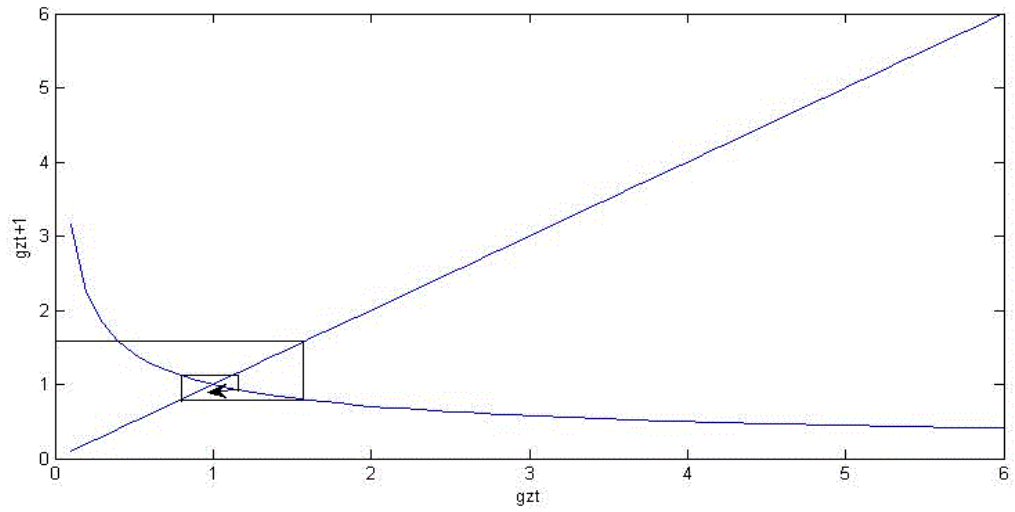


Figure 1: The dynamic adjustment process to a steady state BGP

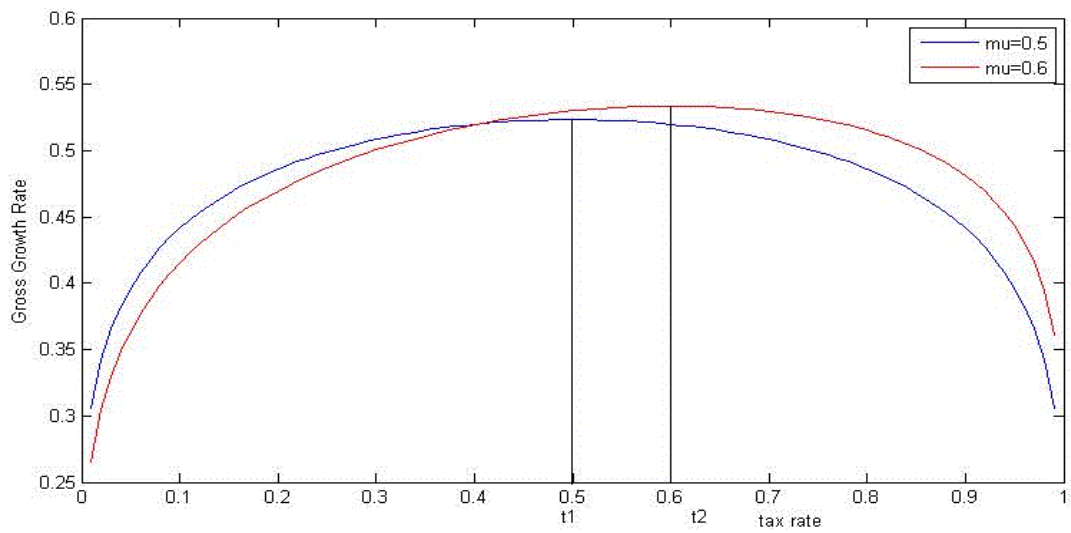


Figure 2: Comparing optimal taxes for different weights on the public input

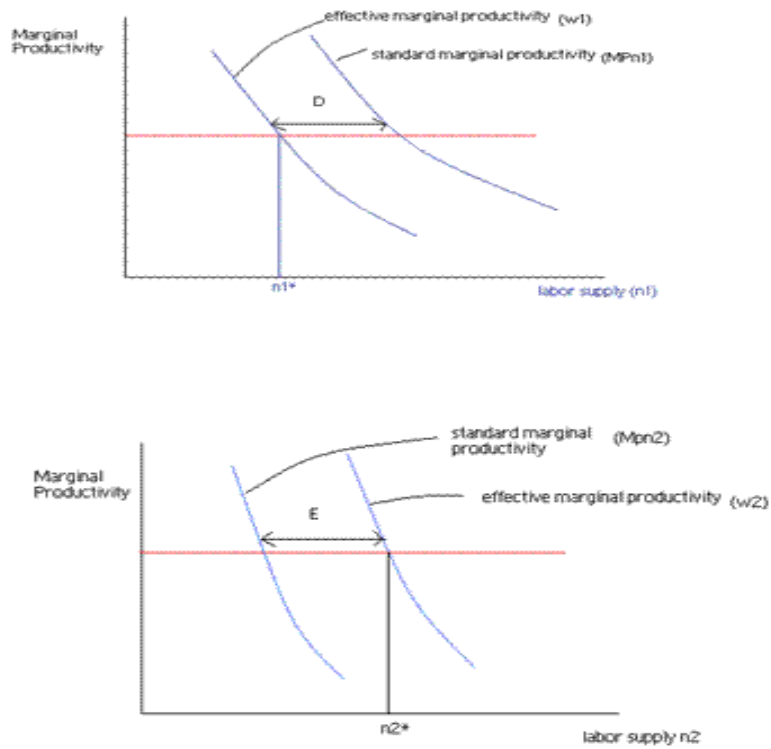


Figure 3: The Effect of ISTC on  $n_1$  and  $n_2$

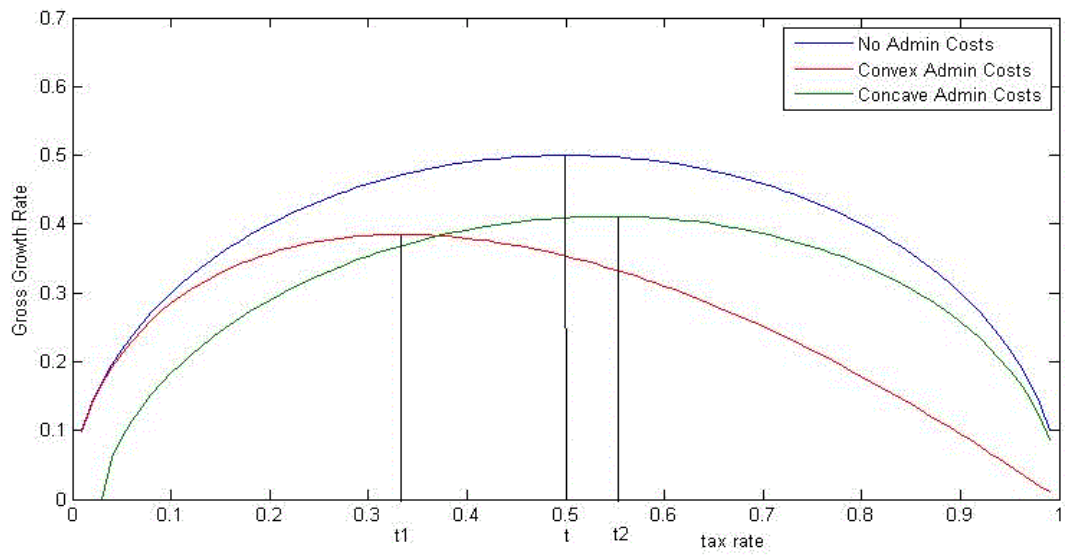


Figure 4: Optimal tax rates across different model - with and without administrative costs

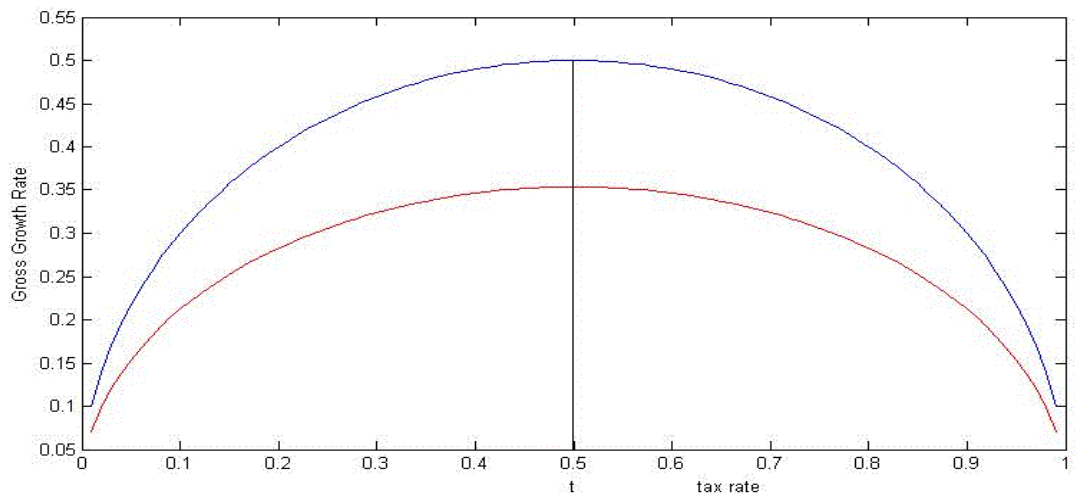


Figure 5: Baseline model versus the model with linear costs

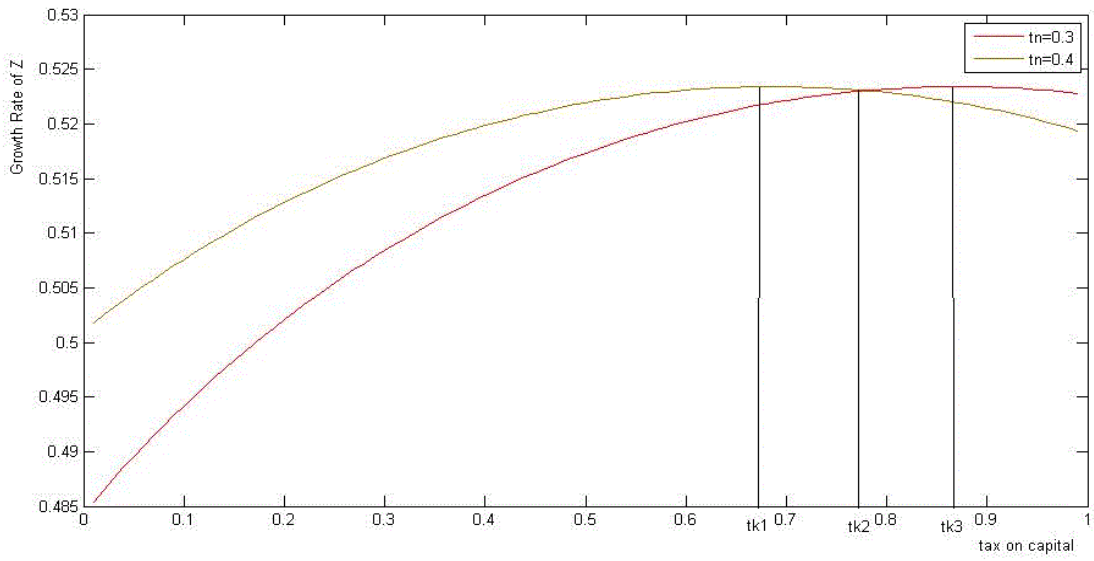


Figure 6: Impact of a change in the tax on capital - baseline model

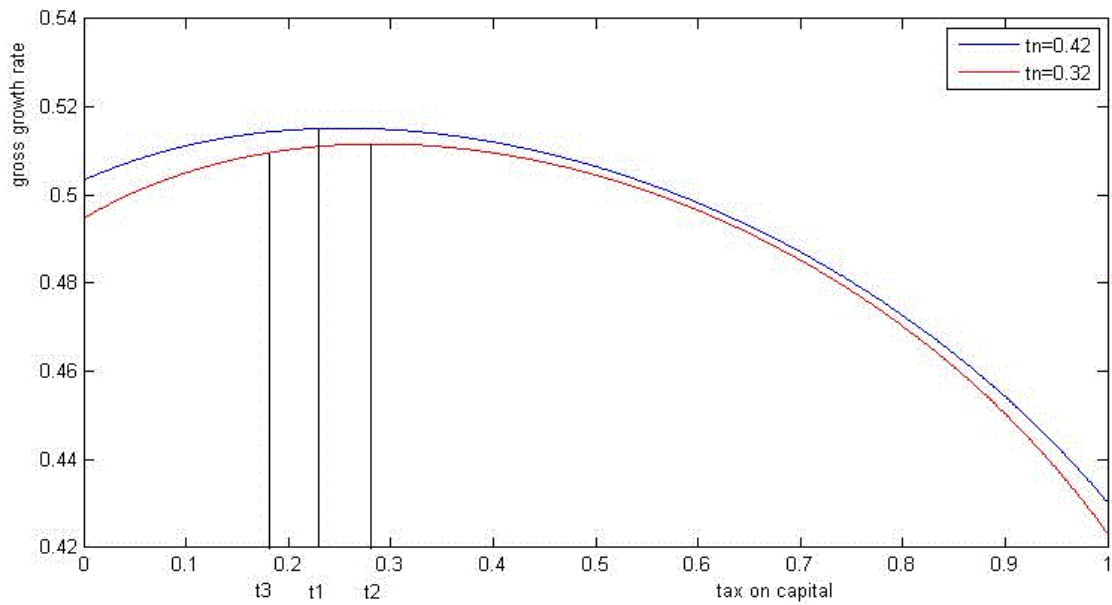


Figure 7: Policy analysis in the model with convex costs

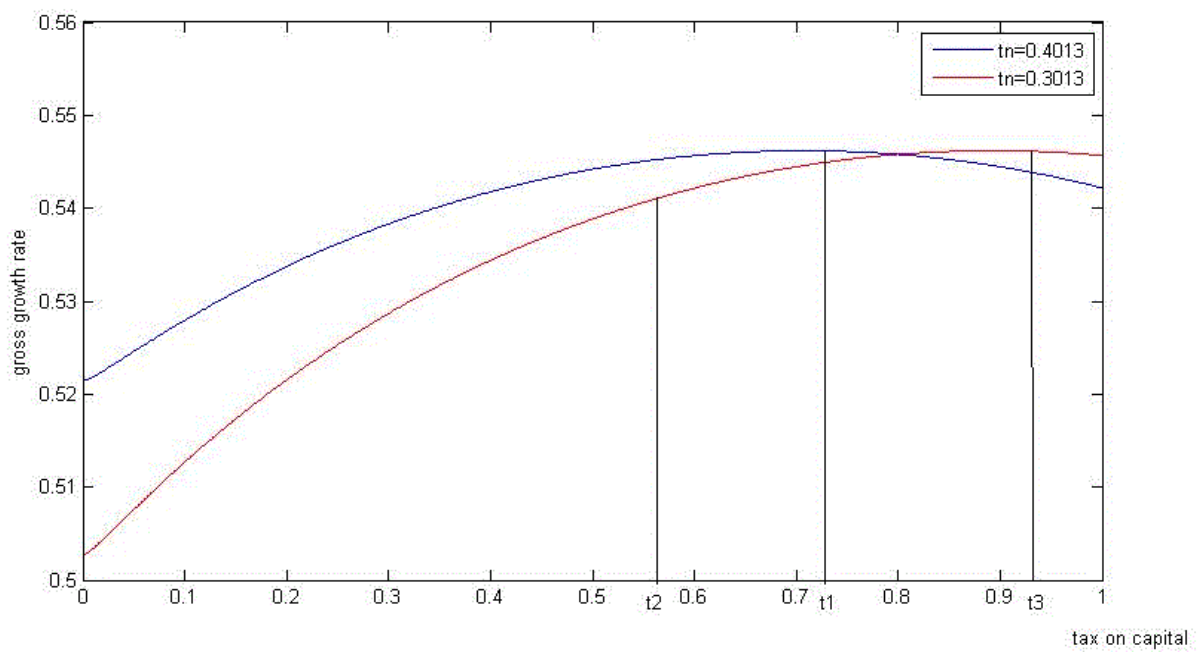


Figure 8: Policy analysis in the model with concave costs