*Some results on*
*Transition Probabilities*

*S. M. SRIVASTAVA*

## ABSTRACT

Motivated by some applications in modal logic, using measurable selection theorems, we prove the following results on the existence of transition probabilities. In what follows, $X$ and $Y$ are uncountable Polish spaces with $\lambda$ a Borel probability on $X$ and $B \subset X \times Y$ a Borel set.

**Result 1.** *Let $\mathcal{A}$ be a countably generated sub $\sigma$-algebra of the Borel $\sigma$-algebra $\mathcal{B}_Y$. Suppose for each $x \in X$, $P(x, \cdot)$ is a probability measure on $\mathcal{A}$ such that for every $A \in \mathcal{A}$, $x \to P(x, A)$ is Borel measurable. Then, for each $x \in X$, there is a Borel probability $Q(x, \cdot)$ on $Y$ extending $P(x, \cdot)$ with $x \to Q(x, C)$ $\lambda$-measurable for every Borel $C \subset Y$. We also give a necessary and sufficient condition under which we can choose the extensions $Q(x, \cdot)$ so that for every Borel set $C \subset Y$, $x \to Q(x, C)$ is Borel measurable.*

**Result 2.** *Assume that for every $x \in X$, the section $B_x = \{y \in Y : (x, y) \in B\}$ is uncountable. Then for every $x \in X$, there is a continuous Borel probability $P(x, \cdot)$ on $Y$ with topological support contained in $B_x$ such that for every Borel $C \subset Y$, $x \to P(x, C)$ is $\lambda$-measurable.*