An Introduction to Mechanism Design Theory *

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Abstract

The Nobel prize in economics was awarded to Leonid Hurwicz, Roger B. Myerson, and Eric S. Maskin in 2007 for "having laid the foundations of mechanism design theory". This article aims to explore these very foundations of mechanism design theory. In the process, it highlights one important contribution each of Myerson and Maskin, and describes some fundamental concepts of mechanism design, which are due to Hurwicz. While it briefly describes Maskin's Nash implementation work, it goes into the details of Myerson's optimal auction design work, its extensions, and ongoing work.

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1 Introduction

Consider a seller who owns an indivisible object, say a house, and wants to sell to a set of buyers. Each buyer has a value for the object, which is the utility of the house to the buyer. The seller wants to design a selling procedure, an auction for example, such that he gets the maximum possible price (revenue) by selling the house. If the seller knew the values of the buyers, then he would simply offer the house to the buyer with the highest value and give him a "take-it-or-leave-it" offer at a price equal to that value. Clearly, the (highest value) buyer has no incentive to reject such an offer. Now, consider a situation where the seller is unaware of the values of the buyers. What selling procedure will give the seller the maximum possible revenue? A clear answer is impossible if the seller knows nothing about the values of the buyer. However, the seller may have some information about the values of the buyers. For example, the possible range of values, the probability of having these values etc. Given these information, is it possible to design a selling procedure that guarantees maximum (expected) revenue to the seller?

In this example, the seller had a particular objective in mind - maximizing revenue. Given his objective he wanted to *design* a selling procedure such that when buyers participate in the selling procedure and try to maximize their own payoffs within the rules of the selling procedure, the seller will maximize his expected revenue over all such selling procedures.

The study of mechanism design looks at such issues. A planner needs to design a *mechanism* (a selling procedure in the above example) where strategic agents can interact. The interactions of agents result in an outcome. While there are several possible ways to do design the rules, the planner has a particular objective in mind. For example, the objective can be efficiency (maximization of the total welfare of agents) or maximization of his own surplus (as was the case in the last example). Depending on the objective, the mechanism needs to be designed in a manner such that when strategic agents interact, the resulting outcome gives the desired objective. One can think of mechanism design as the *reverse engineering* of game theory. In game theory terminology, a mechanism induces a game whose equilibrium outcome is the objective that the mechanism designer has set.

The Nobel prize in economics was awarded to Leonid Hurwicz, Roger B. Myerson, and Eric S. Maskin in 2007 for "having laid the foundations of mechanism design theory". This article aims to explore these very foundations of mechanism design theory. Since the literature is too large to cover in a brief survey, I focus on important contributions of the three laureates. While I will be brief on the contributions of Huwicz and Maskin, I will be more elaborate on the work of Myerson, mainly because it is an active research area currently.

Before I begin a formal treatment of mechanism design, let me motivate it by giving some recent success stories of this theory in practice.

1.1 Some Applications

If a theory is judged by the applications it has in practice, then the mechanism design theory passes this test. A recent book (Cramton et al., 2006) on combinatorial auctions, where a set of objects are sold to a set of buyers who have value on bundles of goods, goes into the details of many such applications. Below, I will describe two recent success stories of mechanism design in practice.

The Spectrum Auction Design

The spectrum licences in US is auctioned by Federal Communications Commission (FCC) almost every year. It raises enormous revenues - from 1994 to 2001, the revenue from these auctions is about 40 billion US dollars. The sale of spectrum is a classic case of (multi-object or combinatorial) auction design. Usually spectrum licences are auctioned separately for various regions. However, spectrum licences for adjoining regions show synergy, i.e., if one obtains licences for two neighboring states his value of the two licences will be higher than sum of values of these licences. Auctioning licences separately cannot capture such synergies of bidders. Such issues led to rethinking of the auction design by FCC. Prominent economic theorists suggested their proposals to the FCC on how best to design auctions for selling spectrum licences (Cramton, 2002). This led to major redesign of spectrum auctions by the FCC, and subsequent increase in revenues.

Other countries in the world, including India and many countries in Europe, have used auctions to sell spectrum licences. Because of the amount of revenue it generates, the application of auctions in these settings remain one of the biggest success stories of mechanism design.

The Sponsored Search Auction Design

Prominent Internet search companies use auctions as a means of generating revenue. When we enter a keyword, say "Running Shoes", for searching in Google, we see a list of advertisements on the right hand side of the screen once the search is visible. Google auctions the slots on these search pages dynamically to companies, e.g., in case of "Running Shoes" companies like Nike and Reebok may be bidders for slots. Such auctions are popularly called *sponsored search auctions* (SSA). While the technology behind conducting SSA in fractions of a second is elegant, the design of such SSA have generated enormous interests (Edelman et al., 2007). Google claims to use a "generalized" second-price auction, which we will describe later (Edelman et al. (2007) show that this is not the appropriate generalization). To understand the stakes involved in SSA, I note that Google's revenues in 2005 was about 6 billion US dollars, 90 percent of which came from revenues of such auctions. A recent book (Nisan et al.,

2007) goes into the theory of sponsored search auction design.

The rest of the essay is organized as follows. In Section 2, I give a general model of mechanism design and describe some fundamental results. Since Hurwicz laid the foundations of mechanism design, several concepts I describe in Section 2 reflect the contributions of Hurwicz. I go into some details of Hurwicz's contributions in Section 3. In Section 4, I describe the optimal auction design work in Myerson (1981). In Section 5, I describe the Nash implementation work in Maskin (1999). I conclude in Section 6.

2 A General Framework of Mechanism Design

2.1 A General Model

The set of agents is denoted by $N = \{1, \dots, n\}$. The set of potential social decisions (or outcomes) is denoted by the set D, which can be finite or infinite. Every agent has a private information, called his **type**. The type of agent $i \in N$ is denoted by θ_i which lies in some set Θ_i. I emphasise that θ_i can be multi-dimensional. I denote a profile of types as $\theta = (\theta_1, \dots, \theta_n)$ and the cross product of type spaces of all agents as $\Theta = \times_{i \in N} \Theta_i$.

Agents have preferences over decisions which depends on their respective types. This is captured using a utility function. The utility function of agent $i \in N$ is $v_i : \Theta_i \times D \to \mathbb{R}$. Thus, $v_i(d, \theta_i)$ denotes the utility of agent $i \in N$ for decision $d \in D$ when his type is $\theta_i \in \Theta_i$. I will restrict attention to this setting, called the **private values** setting, where the utility function of an agent is independent of the types of other agents. Below are two examples to illustrate the ideas.

A Public Project

Suppose a bridge needs to be built across a river in a city. The residents need to take a decision whether to build the bridge or not. Hence, $D = \{0, 1\}$, where 0 indicates that the bridge is not built and 1 indicates that it is built. There is a total cost c from building the bridge which the residents share. The value for the bridge for resident $i \in N$ is $\theta_i \in \mathbb{R}$ (his type). Hence, utility of agent i with type θ_i when the decision is $d \in \{0, 1\}$ can be written as $v_i(d, \theta_i) = d\big(\theta_i - \frac{c}{n}\big)$. This utility function has a particular form. The utility is linear in the payment $(\frac{c}{n})$ of the agent. Such utility functions are called **quasi-linear** utility functions.

Allocating Multiple Objects

A set of indivisible goods $M = \{1, \dots, m\}$ need to be allocated to a set of agents $N = \{1, \dots, n\}$. Let $\Omega = \{S : S \subseteq M\}$ be the set of bundles of goods. The type of an agent $i \in N$ is a multi-dimensional vector $\theta_i \in \mathbb{R}_+^{|\Omega|}$, where $\theta_i(S)$ indicates the value of agent $i \in N$ for a bundle $S \in \Omega$. Here, a decision is an allocation vector $x \in \{0,1\}^{n \times |\Omega|}$, where $x_i(S) \in \{0,1\}$ indicates whether bundle $S \in \Omega$ is allocated to agent $i \in N$. Of course, an allocation x must satisfy the feasibility constraints:

$$\sum_{i \in N} \sum_{S \in \Omega : j \in S} x_i(S) \leq 1 \qquad \forall\, j \in M$$

$$\sum_{S \in \Omega} x_i(S) \leq 1 \qquad \forall\, i \in N.$$

The first constraint says that no good can be allocated to more than one agent. The second constraint says that if an agent is allocated multiple goods, then it should be treated as a bundle of goods - hence, every agent can be allocated at most one bundle. Let $\mathbb{X}$ be the set of all allocations (satisfying the feasibility constraints). Then $D = \mathbb{X}$. The utility of agent i when his type is θ_i and allocation is $x \in \mathbb{X}$ is given by $v_i(\theta_i, x) = \sum_{S \in \Omega} \theta_i(S) x_i(S)$.

2.2 Efficient Decision

A **decision rule** d is a mapping $d : \Theta \to D$. Hence, a decision rule gives a decision as a function of the types of the agents. A decision rule d is **efficient** if for every $\theta \in \Theta$

$$\sum_{i \in N} v_i(\theta_i, d(\theta)) \geq \sum_{i \in N} v_i(\theta_i, d') \qquad \forall\, d' \in D.$$

Hence, efficiency implies that the total value of agents is maximized in all states of the world (i.e., for all possible type profiles of agents).

Consider an example where a seller needs to sell an object to a set of buyers. In any allocation, one buyer gets the object and the others get nothing. The buyer who gets the object realizes his value for the object, while others realize no utility. Clearly, to maximize the total value of the buyers, we need to maximize this realized value, which is done by allocating the object to the buyer with the highest value.

2.3 Transfer Functions

The fact that the decision maker is uncertain about the types of the agents makes room for agents to manipulate the decisions by misreporting their types. To give agents incentives against such manipulation, transfers are often used. Formally, a transfer function is a mapping $t : \Theta \to \mathbb{R}^n$, where $t_i(\theta)$ represents the transfer of agent i when type profile is $\theta \in \Theta$. Note that $t_i(\theta)$ can be negative or positive or zero. A positive $t_i(\theta)$ indicates that the agent is receiving money.

In many situations, we want the total transfer of agents to be either non-positive (i.e., decision maker does not incur a loss) or to be zero. A transfer rule t is **feasible** if $\sum_{i \in N} t_i(\theta) \leq 0$ for all $\theta \in \Theta$. Similarly, a transfer rule t is **balanced** if $\sum_{i \in N} t_i(\theta) = 0$ for all $\theta \in \theta$.

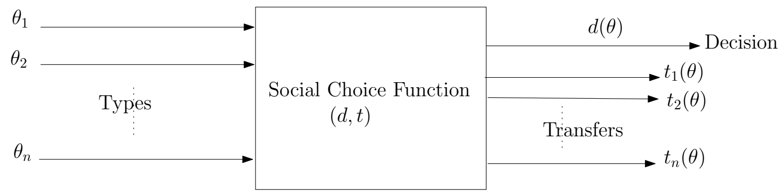

Figure 1: Social Choice Function

2.4 Social Choice Functions and Mechanisms

A **social choice function** is a pair $f = (d, t)$, where d is a decision rule and t is a transfer function. Hence, the input to a social choice function is the types of the agents. The output is a decision and transfers given the reported types. Figure 1 gives a pictorial description of a social choice function.

Under a social choice function $f = (d, t)$ the utility of agent $i \in N$ with type θ_i when all agents "report" $\hat{\theta}$ as their types is given by

$$u_i(\hat{\theta}, \theta_i, f = (d, t)) = v_i(d(\hat{\theta}), \theta_i) + t_i(\hat{\theta}).$$

This is the quasi-linear utility function.

A **mechanism** is a pair (M, g), where $M = M_1 \times \ldots \times M_n$ is the cross product of message spaces of agents and g is a mapping $g : M \to D \times \mathbb{R}^n$. The mapping g is called an outcome function. For every profile of messages $m = (m_1, \ldots, m_n) \in M$, the outcome function $g(m) = (g_d(m), g_1(m), \ldots, g_n(m))$ gives a decision $g_d(m) \in D$ and a transfer $g_i(m)$ to every agent $i \in N$. Hence, a mechanism is more general than a social choice function. In a social choice function, the input is types of agents (for example, values of bidders in an auction) but in a mechanism it is messages from agents. A message can be anything arbitrary. For example, in an auction setting it can be a sequence of bounds on the value of a bidder. The output of a mechanism and a social choice function is the same - a decision and a vector of transfers. Clearly, a social choice function is also a mechanism where the messages are restricted to be types only. This will be discussed later. Figure 2 gives a pictorial description of a mechanism.

Notice the difference between a mechanism, often referred to as a **game form**, and a game. An outcome in a mechanism is a decision and a vector of transfers but not a vector of payoffs (as in a game). Of course, once the types and utility functions of all agents are specified, it induces a game.

The goal of mechanism design is to design the message space and outcome function in a way such that when agents participate in the mechanism they have (best) strategies (messages) that they can choose as a function of their private types such that the desired outcome is achieved. The most fundamental, though somewhat demanding, notion in mechanism design is the notion of dominant strategies. A strategy $m_i \in M_i$ is a **dominant strategy** at

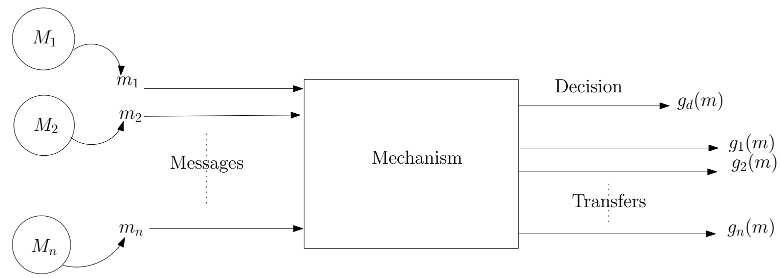

Figure 2: Mechanism

$\theta_i \in \Theta_i$ in a mechanism (M, g) if for every $m_{-i} \in M_{-i}$ [1] we have

$$v_i(g_d(m_i, m_{-i}), \theta_i) + g_i(m_i, m_{-i}) \geq v_i(g_d(\hat{m}_i, m_{-i}), \theta_i) + g_i(\hat{m}_i, m_{-i}) \qquad \forall\ \hat{m}_i \in M_i.$$

Notice the strong requirement that m_i has to be the best strategy for *every* strategy profile of other agents. Such a strong requirement limits the settings where dominant strategies exist.

A social choice function $f = (d, t)$ is **implemented** in dominant strategies by a mechanism (M, g) if there exists mappings for every agent $i \in N$, $m_i : \Theta_i \to M_i$ such that $m_i(\theta_i)$ is a dominant strategy at θ_i for every $\theta_i \in \Theta_i$ and $g(m(\theta)) = f(\theta)$ for all $\theta \in \Theta$.

Note that a social choice function $f = (d, t)$ can be viewed as a mechanism where $M = \Theta$ and $g = f$. This is known as the **direct mechanism**. A direct mechanism (or associated social choice function) is **strategy-proof** if for every agent $i \in N$ and every $\theta_i \in \Theta_i$, θ_i is a dominant strategy at θ_i. A fundamental result in mechanism design says that one can restrict attention to the direct mechanisms in a variety of contexts.

PROPOSITION **1 (Revelation Principle)** *If a mechanism (M, g) implements a social choice function $f = (d, t)$ in dominant strategies then the direct mechanism f is strategy-proof.*

The proof of the revelation principle is trivial. Because $g(m(\theta)) = f(\theta)$ for all θ, dominant strategy in (M, g) immediately implies that the direct mechanism is also strategy-proof. Revelation principle is due to Myerson (Myerson, 1979).

2.5 Absence of Transfers and Impossibility

There are a variety of contexts where transfers are undesirable. For example, consider the elections where voters vote. The private information of voters is their preferences over alternatives in the election. The mechanism may not involve any transfers to voters.

[1] Here, m_{-i} is the profile of messages of agents except agent i and M_{-i} is the cross product of message spaces of agents except agent i.

In such cases, one can restrict t to take value zero always and denote it as t^0. Hence, a decision rule d is strategy-proof if the social choice function (or direct mechanism) (d, t^0) is strategy-proof. Suppose D is finite. We say a type space is **rich** if for any ordering $\rho : D \to \{1, \ldots, |D|\}$ and any $i \in N$, there exists a type $\theta_i \in \Theta_i$ such that $v_i(d, \theta_i) < v_i(d, \theta_i')$ when $\rho(d) < \rho(d')$ for all $d, d' \in D$. A decision rule d is **dictatorial** if there exists an agent $i \in N$ such that $d(\theta) \in \arg\max_{d \in R_d} v_i(d, \theta_i)$, where $R_d = \{d \in D : \text{there exists } \theta \in \Theta \text{ with } d(\theta) = d\}$ is the range of the decision rule d. Dictatorial decision rules play a pivotal role in mechanism design. It says that there exists a **dictator** agent whose utility is maximized for every type profile of agents. For example, in the election set up, consider the presence of a powerful voter who always decides who will be elected. Hence, types of agents other than the dictator is disregarded by the mechanism. This is clearly a strategy-proof social choice function, although not efficient. Unfortunately, this is the only strategy-proof rule in many contexts.

PROPOSITION **2 (Gibbard-Satterthwaite Impossibility)** *Suppose D is finite and the type space is rich. A decision rule with at least three elements in its range is strategy-proof if and only if it is dictatorial.*

Proposition 2 was proved independently by Gibbard (1973) and Satterthwaite (1975) [2]. Though Proposition 2 is a very negative result, one need not lose hope. Proposition 2 critically relies on the assumptions made. There are social choice functions other than the dictatorial one if one relaxes the richness of type space assumption or allow for transfers. For example, under a class of type space called the **single-peaked preferences**, Moulin (1980) shows the existence of a social choice function that is not dictatorial and has nice properties (Pareto efficient, anonymous). As we will see next, in the context of auctions, we can get many social choice functions that are strategy-proof by allowing for transfers.

2.6 Efficiency with Transfers

Here, I show a class of social choice functions which are strategy-proof and whose decision rule is efficient. This is possible by allowing for transfers. These social choice functions are famously known as the Groves mechanisms (Groves, 1973).

PROPOSITION **3 (Groves Mechanisms)** *Suppose d is an efficient decision rule. For every $i \in N$, let there be a function $h_i : \Theta_{-i} \to \mathbb{R}$ such that*

$$t_i(\theta) = h_i(\theta_{-i}) + \sum_{j \in N \setminus \{i\}} v_j(d(\theta), \theta_j) \qquad \forall\ \theta \in \Theta.$$

Then (d, t) is strategy-proof.

[2] For an elegant and direct proof, see Sen (2001).

Before, I give a detailed proof, let us examine the transfer function of the Groves mechanism. The first term in the transfer is a function, specific for every agent i, which depends on the types of other agents but not on the type of i. The second term is the total value of agents other than i in an efficient decision. We need to add the utility of agent i under efficient decision to get the net utility i in a Groves mechanism. This will consist of a term which is independent of his own type and the total welfare of agents in the efficient decision. Intuitively, since efficient decision maximizes this net utility over all decisions, an agent cannot get more net utility by deviating. This is the idea of the proof.

Proof: Suppose (d, t) is not strategy-proof. Then there exists an agent $i \in N$, $\hat{\theta}_i \in \Theta_i$ and $\theta \in \Theta$ such that

$$v_i(d(\hat{\theta}_i, \theta_{-i}), \theta_i) + t_i(\hat{\theta}_i, \theta_{-i}) > v_i(d(\theta), \theta_i) + t_i(\theta).$$

Expanding the $t_i(\cdot)$ terms,

$$v_i(d(\hat{\theta}_i, \theta_{-i}), \theta_i) + \sum_{j \neq i} v_j(d(\hat{\theta}_i, \theta_{-i}), \theta_j) + h_i(\theta_{-i}) > v_i(d(\theta), \theta_i) + \sum_{j \neq i} v_j(d(\theta), \theta_j) + h_i(\theta_{-i})$$

$$\Leftrightarrow \sum_{j \in N} v_j(d(\hat{\theta}_i, \theta_{-i}), \theta_j) > \sum_{j \in N} v_j(d(\theta), \theta_j).$$

This contradicts efficiency. Hence, the Groves mechanism is strategy-proof. ■

Under some richness condition on type spaces, Groves mechanisms are the only mechanisms that are efficient. [3] Hence, Groves mechanisms occupy a central role in mechanism design.

A major drawback of Groves mechanism is that it is not balanced, a necessary requirement in many public goods settings. To overcome this difficulty, researchers have resorted to weaker notions of incentive compatibility, namely Bayesian incentive compatibility, which we will examine later.

The Pivotal Mechanism

A particular mechanism in the class of Groves mechanism is intuitive and has many nice properties (Moulin, 1986). It is commonly known as the **pivotal mechanism** or the Vickrey-Clarke-Groves (VCG) mechanism (Vickrey, 1961; Clarke, 1971; Groves, 1973). The VCG mechanism is characterized by a unique $h_i(\cdot)$ function. In particular, for every agent $i \in N$ and every $\theta_{-i} \in \Theta_{-i}$, $h_i(\theta_{-i}) = -\max_{d \in D} \sum_{j \neq i} v_j(d, \theta_j)$. This gives the following transfer

[3] Holmstrom (1979) shows that if the type space is *smoothly connected* then Groves mechanisms are the only strategy-proof social choice functions that are efficient.

	$\emptyset$	$\{1\}$	$\{2\}$	$\{1,2\}$
$v_1(\cdot)$	0	8	6	12
$v_2(\cdot)$	0	9	4	14

Table 1: An Example of VCG Mechanism with Multiple Objects

function. For every $i \in N$ and for every $\theta \in \Theta$, the transfer in the VCG mechanism is

$$t_i(\theta) = \sum_{j \neq i} v_j(d(\theta), \theta_j) - \max_{d \in D} \sum_{j \neq i} v_j(d, \theta_j). \tag{1}$$

A careful look at Equation 1 shows that the first term on the right hand side is the sum of values of agents other than i in the efficient decision. The second term on the right hand side is the maximum sum of values of agents other than i (note that this corresponds to an efficient decision when agent i is excluded from the economy). Hence, negative of the transfer in Equation 1 is the *externality* agent i inflicts on other agents because of his presence, and this is the amount he *pays*. Thus, every agent pays his externality to other agents in the VCG mechanism.

Consider the sale of a single object using the VCG mechanism. Fix an agent $i \in N$. Efficiency says that the object must go to the bidder with the highest value. Consider the two possible cases. In one case, bidder i has the highest value. So, when bidder i is present, the sum of values of other bidders is zero (since no other bidder wins the object). But when bidder i is absent, the maximum sum of value of other bidders is the second highest value (this is achieved when the second highest value bidder is awarded the object). Hence, the externality of bidder i is the second-higest value. In the case where bidder $i \in N$ does not have the highest value, his externality is zero. Hence, for the single object case, the VCG mechanism is simple: award the object to the bidder with the highest (bid) value and the winner pays the amount equal to the second highest (bid) value but other bidders pay nothing. This is the well-known second-price auction or the Vickrey auction. By Proposition 3, it is strategy-proof.

We illustrate the VCG mechanism for the sale of multiple objects by an example. Consider the sale of two objects, with values of two agents on bundles of goods given in Table 1. The efficient allocation in this example is to give bidder 1 object 2 and bidder 2 object 1 (this generates a total value of $6 + 9 = 15$, which is higher than any other allocation). Let us calculate the externality of bidder 1. The total value of bidders other than bidder 1, i.e. bidder 2, in the efficient allocation is 9. When bidder 1 is removed, bidder 2 can get a maximum value of 14 (when he gets both the objects). Hence, externality of bidder 1 is $14 - 9 = 5$. Similarly, we can compute the externality of bidder 2 as $12 - 6 = 6$. Hence, the payments of bidders 1 and 2 are 5 and 6 respectively.

2.7 Bayesian Incentive Compatibility

Bayesian incentive compatibility was introduced in Harsanyi (1967-68). It is a weaker requirement than the dominant strategy incentive compatibility. While dominant strategy incentive compatibility required the equilibrium strategy to be the best strategy under all possible strategies of opponents, Bayesian incentive compatibility requires this to hold in *expectation*. This means that in Bayesian incentive compatibility, an equilibrium strategy must give the highest expected utility to the agent, where we take expectation over types of other agents. To be able to take expectation, agents must have information about the probability distributions from which types of other agents are drawn.

To understand Bayesian incentive compatibility, fix a mechanism (M, g). A Bayesian strategy for such a mechanism is a vector of mappings $m_i : \Theta_i \to M_i$ for every $i \in N$. Notice the difference from dominant strategy settings, where a message was a mapping from type spaces of *all* agents to his own message space. A profile of such mapping $m : \Theta \to M$ is a **Bayesian equilibrium** if for all $i \in N$, for all $\theta_i \in \Theta_i$, and for all $\hat{m}_i \in M_i$ we have

$$\begin{aligned} E_{-i}\big[v_i(g_d(m_{-i}(\theta_{-i}), m_i(\theta_i)), \theta_i) + g_i(m_{-i}(\theta_{-i}), m_i(\theta_i))|\theta_i\big] \geq \\ E_{-i}\big[v_i(m_{-i}(\theta_{-i}), \hat{m}_i) + g_i(m_{-i}(\theta_{-i}), \hat{m}_i)|\theta_i\big], \end{aligned}$$

where $E_{-i}[\cdot]$ denotes the expectation over type profile θ_{-i}.

A dominant strategy incentive compatible mechanism is Bayesian incentive compatible. A direct mechanism (social choice function) $f = (d, t)$ is **Bayesian incentive compatible** if $m_i(\theta_i) = \theta_i$ for all $i \in N$ and for all $\theta_i \in \Theta_i$ is a Bayesian equilibrium, i.e., for all $i \in N$ and for all $\theta_i, \hat{\theta}_i \in \Theta_i$ we have

$$E_{-i}\big[v_i(d(\theta_{-i}, \theta_i), \theta_i) + g_i(\theta_{-i}, \theta_i)|\theta_i\big] \geq E_{-i}\big[v_i(d(\theta_{-i}, \hat{\theta}_i), \theta_i) + g_i(\theta_{-i}, \hat{\theta}_i)|\theta_i\big]$$

A mechanism (M, g) **realizes** a social choice function f in Bayesian equilibrium if there exists a Bayesian equilibrium $m(\cdot)$ of (M, g) such that $g(m(\theta)) = f(\theta)$ for all $\theta \in \Theta$ (that occurs with positive probability). Analogous to the revelation principle for dominant strategy incentive compatibility, we also have a revelation principle for Bayesian incentive compatibility.

Proposition 4 **(Revelation Principle)** *If a mechanism (M, g) realizes a social choice function $f = (d, t)$ in Bayesian equilibrium, then the direct mechanism $f = (d, t)$ is Bayesian incentive compatible.*

Again, the proof is trivial. It appears in Myerson (1979). It is one of the most widely used concepts in mechanism design, and Myerson has used it in some of his other seminal work (Myerson, 1981; Myerson and Satterthwaite, 1983). I will go into the details of Myerson (1981), where he uses the revelation principle to design optimal auctions.

3 A Brief Summary of Contributions of Hurwicz

As noted earlier, Hurwicz is regarded as the founder of mechanism design. Hurwicz's idea of a mechanism is similar to what was discussed in the last section. According to Hurwicz, a mechanism is like a machine which takes messages from agents, which may contain private information of agents, and produces an outcome. Each agent is strategic, and tries to maximize his utility given a mechanism. This may force an agent to manipulate his messages, and this creates the necessity that a mechanism be incentive compatible. He goes on to say that different mechanisms wil produce different outcomes as equilibrium of these "message games", and the comparision of different mechanisms should be based on these equilibrium outcomes.

Hurwicz elaborates these ideas in Hurwicz (1972). He also introduces the notion of individual rationality - no agent should be made worse off by participating in a mechanism. One of his seminal contributions, discussed in detail in Hurwicz (1972), is a negative result: in a standard exchange economy, no strategy-proof mechanism satisfying individual rationality can produce Pareto optimal outcomes.

The work of Hurwicz (1972) initiated the study of mechanism design. Researchers began to ask several important questions that Huriwcz's work had inspired. Here are some of those questions, most of which have been answered. Can Pareto optimal outcomes be implemented by considering a wider class of mechanisms? Can Pareto optimal outcomes be implemented by weakening the notion of incentive compatibility? If answers to these questions are negative, then what is the worst loss of efficiency? What other objectives can be achieved if efficiency cannot be achieved?

With the fundamental framework set by Hurwicz (1972), most of which was discussed in a formal manner in Section 2, researchers began answering some of these questions. The work of Myerson and Maskin are the most important answers to these questions. The revelation principle theorem in Myerson (1979), the optimal auction design work in Myerson (1981), and Nash implementation work of Maskin (Maskin, 1999) are examples of this. We elaborate on these next.

4 Optimal Auction Design

This section will describe the design of optimal auction for selling a single indivisible object to a set of bidders (buyers) who are risk neutral. The seminal paper in this area is (Myerson, 1981). We present a detailed analysis of this work [4], and point out some open questions and ongoing research in this area. Before I describe the formal model, let me describe some popular auction forms used in practice.

[4] An excellent treatment of the optimal auction design and related topics is (Krishna, 2002).

4.1 Auctions for a Single Indivisible Object

A single indivisible object is for sale. Let us consider four bidders (agents or buyers) who are interested in buying the object. Let the valuations of the bidders be $10, 8, 6$, and 4 respectively. I describe four commonly discussed auction formats using this example. As before I assume risk neutral bidders with quasi-linear utility functions and private values.

- **First-price auction:** In the first-price auction, every bidder is asked to report a bid, which indicates his value. The highest bidder wins the auction and pays the price he bid. Of course, the bid amount need not equal the value. But if the bidders bid their value, then the first bidder will win the object and pay an amount of 10.

- **Second-price auction:** In the second-price auction, like the first-price auction, each bidder is asked to report a bid. The highest bidder wins the auction and pays the price of the second highest bid. This is the Vickrey auction we have already discussed. As we saw, a dominant strategy in this auction is that bidders will bid their values. Hence, the first bidder will win the object but pay a price equal to 8, the second highest value.

- **Dutch auction:** The Dutch auction, popular for selling flowers in the Netherlands, falls into a class of auctions called the *open-cry* auctions. The Dutch auction starts at a high price and the price of the object is lowered by a small amount (called the *bid decrement*) in iterations. In every iteration, bidders can express their interest to buy the object. The price of the object is lowered only if no bidder shows interest. The auction stops as soon as any bidder shows interest. The first bidder to show interest wins the object at the current price.

  In the example above, suppose the Dutch auction is started at price 12 and let the bid decrement be 1. At price 12, no bidder should express interest since valuation of all bidders are less than 12. After price 10, the first bidder may choose to express interest since he starts getting non-negative utility from the object for any price less than or equal to 10. If he chooses to express interest, then the auction would stop and he will win the object. Clearly, it is not an equilibrium for the bidder to express interest at 10 since he can potentially get more payoff by waiting for the price to fall. Indeed, in equilibrium (under some conditions), the bidder will show interest at a price just below his valuation (see Krishna (2002) for details).

- **English auction:** The English auction is also an open-cry auction. The seller starts the auction at a low price and raises it by a small amount (called the *bid increment*) in iterations. In every iteration, like in the Dutch auction, the bidders are asked if they are interested in buying the object. The price is raised only if more than one bidder shows interest. The auction stops as soon as less than one bidder shows interest. The last bidder to show interest wins the auction at the price he last showed interest.

In the example above, suppose the English auction is started at price 0 and let the bid increment be 1. Then, at price 4 the bidder with value 4 will stop showing interest (since he starts getting non-positive payoff from that price onwards). Similarly, at prices 6, bidder with value 6 will drop out. Finally, bidder with value 8 will drop out at price 8. At this price, only bidder with value 10 will show interest. Hence, the auction will stop at price 8, and the bidder with value 10 will win the object at price 8. Notice that the outcome of the auction is the same as the second-price auction. This is no coincidence. It can be argued easily that it is an equilibrium (under private values model) for bidders to show interest (bid) till the price reaches their value in the English auction. Hence, the outcome of the English auction is the same as the second-price auction.

One can think of many more auction formats - though they may not be used in practice. Having learnt and thought about these auction formats, some natural questions arise. Is there an equilibrium strategy for the bidder in each of these auctions? What kind of auctions are incentive compatible? What is the ranking of these auctions in terms of expected revenue? Which auction gives the maximum expected revenue to the seller over all possible auctions?

Myerson (1981) answers many of these questions. First, using the revelation principle (for Bayesian incentive compatibility), he concludes that for every auction (sealed-bid or open-cry or others) there exists a direct mechanism with the same social choice function, and thus giving the same expected revenue to the seller. So, he focuses on direct mechanisms without loss of generality. Second, he characterizes direct mechanisms which are Bayesian incentive compatible. Third, he shows that all Bayesian incentive compatible mechanisms which have the same allocation rule (e.g., the first-price auction and the second-price auction have the same allocation rule since they both allocate the object to the bidder with the highest bid), differ in revenue by a constant amount. Using these results, he is able to give a precise description of an auction which gives the maximum expected revenue. He calls such an auction an *optimal auction*. Under some conditions on the valuation distribution of bidders, the optimal auction is a modified second-price auction. Next, we describe these results formally.

4.2 The Model

There is a single indivisible object for sale, whose value for the seller is zero. The set of bidders is denoted by $N = \{1, \ldots, n\}$. Every bidder has a value (this is his *type*) for the object. The value of bidder $i \in N$ is drawn from $[0, h_i]$ using a distribution with density function f_i and cumulative density F_i. We assume that each bidder draws his value independently and this value is completely determined by this draw (i.e., knowledge of other information such as value of other bidders does not influence his value). This model of valuation is referred to as the **private independent value model**. We let the joint density function of values of all the bidders as f and the joint density function of values of all the bidders except bidders i as f_{-i}. Due to the independence assumption,

$$f(x_1, \ldots, x_n) = f_1(x_1) \times \ldots \times f_n(x_n)$$
$$f(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n) = f_1(x_1) \times \ldots \times f_{i-1}(x_{i-1}) \times f_{i+1}(x_{i+1}) \times \ldots f_n(x_n).$$

Let $X_i = [0, h_i]$ and $X = [0, h_1] \times \ldots \times [0, h_n]$. Similarly, let $X_{-i} = \times_{j \in N \setminus \{i\}} X_j$. A typical valuation of bidder i will be denoted as $x_i \in X_i$, a valuation profile of bidders will be denoted as $x \in X$, and a valuation profile of bidders in $N \setminus \{i\}$ will be denoted as $x_{-i} \in X_{-i}$. The valuation profile $x = (x_1, \ldots, x_i, \ldots, x_n)$ will sometimes be denoted as (x_i, x_{-i}). We assume that $f_i(x_i) > 0$ for all $i \in N$ and for all $x_i \in X_i$.

4.3 The Direct Mechanism

Though a mechanism can be very complicated, a direct mechanism is simpler to describe. By virtue of the revelation principle (Proposition 4), we can restrict attention to direct mechanisms only. Henceforth, I will refer to a direct mechanism as simply a mechanism.

Let Δ be the set of probability distributions over bidders in N. A mechanism M in this context is a pair of mappings $M = (a, p)$, where $a : X \to \Delta$ is the **allocation rule** and $p : X \to \mathbb{R}^n$ is the **payment rule**. Given a mechanism $M = (a, p)$, a bidder $i \in N$ with (true) value $x_i \in X_i$ gets the following utility when all the buyers report values $z = (z_1, \ldots, z_i, \ldots, z_n)$

$$u_i(z; x_i) = a_i(z)x_i - p_i(z).$$

Every mechanism (a, p) induces an expected allocation rule and an expected payment rule (α, π), defined as follows. The expected allocation of bidder i when he reports $z_i \in X_i$ in allocation rule a is

$$\alpha_i(z_i) = \int_{X_{-i}} a_i(z_i, z_{-i}) f_{-i}(z_{-i}) dz_{-i}.$$

Similarly, the expected payment of bidder i when he reports $z_i \in X_i$ in payment rule p is

$$\pi_i(z_i) = \int_{X_{-i}} p_i(z_i, z_{-i}) f_{-i}(z_{-i}) dz_{-i}.$$

So, the expected utility from a mechanism (a, p) to a bidder i with true value x_i by reporting a value z_i is $\alpha_i(z_i)x_i - \pi_i(z_i)$.

Definition 1 *A mechanism (a, p) is* **Bayesian incentive compatible** *if for every bidder $i \in N$ and for every possible true value $x_i \in X_i$ we have*

$$\alpha_i(x_i)x_i - \pi_i(x_i) \geq \alpha_i(z_i)x_i - \pi_i(z_i) \qquad \forall\ z_i \in X_i. \tag{\textbf{BIC}}$$

Equation **BIC** says that a bidder maximizes his expected utility by reporting true value. So, when bidder i has value x_i, he gets more expected utility by reporting x_i than by reporting any other value $z_i \in X_i$.

4.4 Characterization of Bayesian Incentive Compatibility

Myerson (1981) shows that Bayesian incentive compatibility is characterized by a special class of allocation rules.

Definition **2** *An allocation rule a is* **weakly monotone (w-mon)** *if for every bidder $i \in N$ and for every $x_i, z_i \in X_i$ with $x_i > z_i$, we have $\alpha_i(x_i) \geq \alpha_i(z_i)$.*

Theorem **1 (Incentive Comaptibility)** *For an allocation rule a, there exists a payment rule p such that (a, p) is Bayesian incentive compatible if and only if $\alpha_i(\cdot)$ is w-mon for every $i \in N$.*

Proof: Suppose (a, p) is a Bayesian incentive compatible mechanism. Consider a buyer $i \in N$ and $x_i, z_i \in X_i$ with $x_i > z_i$. Applying Equation **BIC** twice, we get

$$\begin{aligned} \alpha_i(x_i)x_i - \pi_i(x_i) &\geq \alpha_i(z_i)x_i - \pi_i(z_i) \\ \alpha_i(z_i)z_i - \pi_i(z_i) &\geq \alpha_i(x_i)z_i - \pi_i(x_i). \end{aligned}$$

Adding these two equations, we get

$$\begin{aligned} &\alpha_i(x_i)x_i + \alpha_i(z_i)z_i \geq \alpha_i(z_i)x_i + \alpha_i(x_i)z_i \\ \Leftrightarrow &\big[\alpha_i(x_i) - \alpha_i(z_i)\big](x_i - z_i) \geq 0. \end{aligned}$$

Since $x_i > z_i$, this implies that $\alpha_i(x_i) \geq \alpha_i(z_i)$.

Now, suppose that a is w-mon. We will show that there exists a p such that (a, p) is Bayesian incentive compatible. Consider a p whose expected payment is defined as follows. For every $i \in N$ and every $x_i \in X_i$

$$\pi_i(x_i) = \pi_i(0) + \alpha_i(x_i)x_i - \int_0^{x_i} \alpha_i(t_i)dt_i.$$

Since $\alpha_i(\cdot)$ is non-increasing, it is Riemann integrable, and hence, the above payment rule is well defined. Now for any $i \in N$ and any $x_i, z_i \in X_i$, we have

$$\begin{aligned} \pi_i(x_i) - \pi_i(z_i) &= \alpha_i(x_i)x_i - \int_0^{x_i} \alpha_i(t_i)dt_i - \alpha_i(z_i)z_i + \int_0^{z_i} \alpha_i(t_i)dt_i \\ &= \alpha_i(x_i)x_i - \alpha_i(z_i)x_i + \alpha_i(z_i)x_i - \alpha_i(z_i)z_i + \int_{x_i}^{z_i} \alpha_i(t_i)dt_i \\ &= \alpha_i(x_i)x_i - \alpha_i(z_i)x_i + \alpha_i(z_i)(x_i - z_i) - \int_{z_i}^{x_i} \alpha_i(t_i)dt_i. \end{aligned}$$

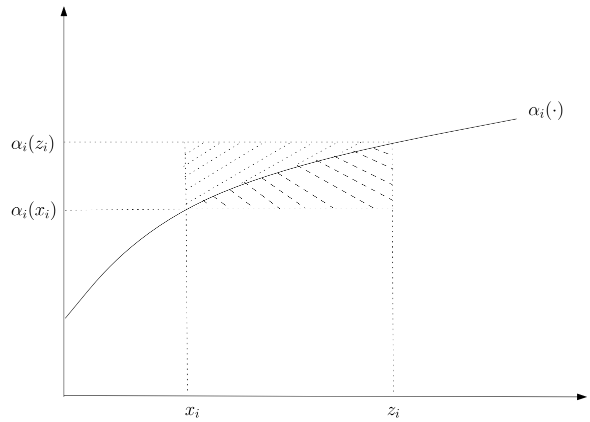

Figure 3: An Illustration

Now consider two cases. First, let $x_i > z_i$. In that case, since $\alpha_i(\cdot)$ is non-decreasing, $\int_{z_i}^{x_i} \alpha_i(t_i)dt_i \geq \alpha_i(z_i)(x_i - z_i)$. Second, let $x_i < z_i$. Again using the fact that $\alpha_i(\cdot)$ is non-decreasing, we get $\alpha_i(z_i)(z_i - x_i) \geq \int_{x_i}^{z_i} \alpha_i(t_i)dt_i$. Thus, in both cases $\alpha_i(z_i)(x_i - z_i) - \int_{z_i}^{x_i} \alpha_i(t_i)dt_i$ is non-positive. The argument is illustrated in Figure 3. If $z_i > x_i$ (as is the case in Figure 3), the term $\alpha_i(z_i)(z_i - x_i) - \int_{x_i}^{z_i} \alpha_i(t_i)dt_i$ is the dotted area above the curve $\alpha_i(\cdot)$ in Figure 3. However, if we switch x_i and z_i in Figure 3, then we get the case $z_i < x_i$. If $z_i < x_i$, then the term $\alpha_i(z_i)(z_i - x_i) - \int_{x_i}^{z_i} \alpha_i(t_i)dt_i$ is the dashed area just below the curve $\alpha_i(\cdot)$ in Figure 3. Note that both areas exist since $\alpha_i(\cdot)$ is non-decreasing.

Hence, $\alpha_i(x_i)x_i - \alpha_i(z_i)x_i \geq \pi_i(x_i) - \pi_i(z_i)$, which implies that $\alpha_i(x_i)x_i - \pi_i(x_i) \geq \alpha_i(z_i)x_i - \pi_i(z_i)$. So, there exists a p such that (a, p) is Bayesian incentive compatible. ■

Theorem 1 says that w-mon allocation rule is equivalent to a Bayesian incentive compatible mechanism. A dictatorial allocation rule, which allocates the object to a dictator bidder all the time, satisfies w-mon trivially. Let us consider the allocation rules in the first-price and the second-price auction. The allocation rules in the first-price and the second-price auction are the same - the object goes to bidder with the highest bid. In this allocation rule, if a bidder wins the object with a given bid, he will continue to win it if he increases his bid. Hence, this is an allocation rule which satisfies w-mon. However, Theorem 1 is silent about the exact payment needed to make an allocation rule incentive compatible. Since the first-price and the second price auctions use particular payment rules, we cannot conclude from Theorem 1 whether they are Bayesian incentive compatible mechanisms. Of course, we know that the second-price auction is strategy-proof, and hence, Bayesian incentive compatible. One can also derive a Bayesian equilibrium in the first-price auction (see Krishna (2002)).

The next theorem states a powerful fact about the payments in a Bayesian incentive compatible mechanism. It says that once we fix a w-mon allocation rule, the payment rule is uniquely determined upto an additive constant. This is known as the **revenue equivalence** result, and proved in Myerson (1981).

THEOREM **2 (Revenue Equivalence)** *Suppose (a, p) is Bayesian incentive compatible. Then, for every bidder $i \in N$ and every $x_i \in X_i$ we have*

$$\pi_i(x_i) = \pi_i(0) + \alpha_i(x_i)x_i - \int_0^{x_i} \alpha_i(t_i)dt_i.$$

Proof: For every $i \in N$ and every $x_i \in X_i$ denote $U_i(x_i) = \alpha_i(x_i)x_i - \pi_i(x_i)$. Consider a bidder $i \in N$ and $x_i, z_i \in X_i$. Since (a, p) is Bayesian incentive compatible, we can write

$$\begin{aligned} U_i(x_i) &\geq \alpha_i(z_i)x_i - \pi_i(z_i) \\ &= \alpha_i(z_i)(x_i - z_i) + \alpha_i(z_i)z_i - \pi_i(z_i) \\ &= U_i(z_i) + \alpha_i(z_i)(x_i - z_i). \end{aligned}$$

Switching the role of x_i and z_i we get

$$U_i(z_i) \geq U_i(x_i) + \alpha_i(x_i)(z_i - x_i).$$

Hence, we can write

$$\alpha_i(x_i)(z_i - x_i) \leq U_i(z_i) - U_i(x_i) \leq \alpha_i(z_i)(z_i - x_i).$$

Let $z_i = x_i + \delta$ for $\delta > 0$. Then, we get

$$\alpha_i(x_i)\delta \leq U_i(x_i + \delta) - U_i(x_i) \leq \alpha_i(x_i + \delta)\delta.$$

Hence, $\alpha_i(\cdot)$ is the derivative of $U_i(\cdot)$. Using the fundamental theorem of calculus, and the fact that $\alpha_i(\cdot)$ is Riemann integrable since it is non-decreasing, we can write

$$\int_0^{x_i} \alpha_i(t_i)dt_i = U_i(x_i) - U_i(0).$$

Substituting $U_i(0) = -\pi_i(0)$ and $U_i(x_i) = \alpha_i(x_i)x_i - \pi_i(x_i)$, we get

$$\int_0^{x_i} \alpha_i(t_i)dt_i = \alpha_i(x_i)x_i - \pi_i(x_i) + \pi_i(0).$$

This gives us the desired inequality

$$\pi_i(x_i) = \pi_i(0) + \alpha_i(x_i)x_i - \int_0^{x_i} \alpha_i(t_i)dt_i.$$

■

Theorem 2 says that the (expected) payment of a bidder in a mechanism is uniquely determined by the allocation rule once we fix the expected payment of a bidder with the lowest type. Hence, a mechanism is uniquely determined by its allocation rule and the payment of a bidder with the lowest type. Suppose the payment to the lowest type is always zero. Then, by Theorem 2, the expected revenue in the first-price and the second-price auction (ad hence the English auction) is the same. This remarkable result hinges on the assumptions we made earlier. For example, if we drop the assumption of private values and allow a particular kind of interdependent valuation (value of every bidder depends on the information of other bidders), then the English auction generates more expected revenue than the second-price auction, which in turn generates more expected revenue than the first-price auction. Similarly, if bidders are risk-averse then the expected revenue ranking of popular auction formats change (Maskin and Riley, 1984).

We next impose a condition on the mechanism which determines the payment of a bidder when he has the lowest type.

Definition 3 *A mechanism* (a, p) *is* **individually rational** *if for every bidder* $i \in N$ *we have* $\alpha_i(x_i)x_i - \pi_i(x_i) \geq 0$ *for all* $x_i \in X_i$.

Notice that if (a, p) is Bayesian incentive compatible and individually rational, then $\pi_i(0) \leq 0$ for all $i \in N$. Note that if $\alpha_i(\cdot)$ is non-decreasing, then $\alpha_i(x_i) - \int_0^{x_i} \alpha_i(t_i)dt_i \geq 0$. Hence, if $\pi_i(0) = 0$, then $\pi_i(x_i) \geq 0$ for all $x_i \in X_i$. As we will see, the optimal auction has $\pi_i(0) = 0$, and thus $\pi_i(x_i) \geq 0$ for all $x_i \in X_i$, i.e., biddders pay the auctioneer. This is a standard feature of auctions in practice where bidders are never paid.

4.5 Optimal Mechanism

Denote the expected revenue from a mechanism (a, p) as

$$\Pi(a, p) = \sum_{i \in M} \int_0^{h_i} \pi_i(x_i) f(x_i) dx_i.$$

We say a mechanism (a, p) is an **optimal mechanism** if

- (a, p) is Bayesian incentive compatible and individually rational,
- and $\Pi(a, p) \geq \Pi(a', p')$ for any other Bayesian incentive compatible and individually rational mechanism (a', p').

Fix a mechanism (a, p) which is Bayesian incentive compatible and individually rational. For any bidder $i \in N$, the expected payment of bidder $i \in N$ is given by

$$\int_0^{h_i} \pi_i(x_i) f(x_i) dx_i = \pi_i(0) + \int_0^{h_i} \alpha_i(x_i) x_i f(x_i) dx_i - \int_0^{h_i} \int_0^{x_i} \big(\alpha_i(t_i) dt_i\big) f(x_i) dx_i,$$

where the last equality comes by using revenue equivalence (Theorem 2). By interchanging the order of integration in the last term, we get

$$\begin{aligned} \int_0^{h_i} \int_0^{x_i} \big(\alpha_i(t_i) dt_i\big) f(x_i) dx_i &= \int_0^{h_i} \big(\int_{t_i}^{h_i} f(x_i) dx_i\big) \alpha_i(t_i) dt_i \\ &= \int_0^{h_i} (1 - F_i(t_i)) \alpha_i(t_i) dt_i. \end{aligned}$$

Hence, we can write

$$\Pi(a, p) = \sum_{i \in N} \pi_i(0) + \sum_{i \in N} \int_0^{h_i} \big(x_i - \frac{1 - F_i(x_i)}{f_i(x_i)}\big) \alpha_i(x_i) f_i(x_i) dx_i.$$

We now define the **virtual valuation** of bidder $i \in N$ with valuation $x_i \in X_i$ as

$$v_i(x_i) = x_i - \frac{1 - F_i(x_i)}{f_i(x_i)}.$$

Note that since $f_i(x_i) > 0$ for all $i \in N$ and for all $x_i \in X_i$, the virtual valuation $v_i(x_i)$ is well defined. Also, not that virtual valuations can be negative. Using this and the definition of $\alpha_i(\cdot)$, we can write

$$\begin{aligned} \Pi(a, p) &= \sum_{i \in N} \pi_i(0) + \sum_{i \in N} \int_0^{h_i} v_i(x_i) \alpha_i(x_i) f_i(x_i) dx_i \\ &= \sum_{i \in N} \pi_i(0) + \sum_{i \in N} \int_0^{h_i} \big(\int_{X_{-i}} a_i(x_i, x_{-i}) f_{-i}(x_{-i}) dx_{-i}\big) v_i(x_i) f_i(x_i) dx_i \\ &= \sum_{i \in N} \pi_i(0) + \sum_{i \in N} \int_X v_i(x_i) a_i(x) f(x) dx \\ &= \sum_{i \in N} \pi_i(0) + \int_X \big(\sum_{i \in N} v_i(x_i) a_i(x)\big) f(x) dx. \end{aligned}$$

We need to maximize $\Pi(a, p)$ subject to Bayesian incentive compatibility and individual rationality constraints. Let us sidestep Bayesian incentive compatibility constraint for the moment. So, we are only concerned about maximizing

$$\Pi(a, p) = \sum_{i \in N} \pi_i(0) + \int_X \big(\sum_{i \in N} v_i(x_i) a_i(x)\big) f(x) dx, \tag{2}$$

subject to individual rationality constraint. But individual rationality says $\pi_i(0) \leq 0$ for all $i \in N$. Hence, if we want to maximize $\Pi(a, p)$, then $\pi_i(0) = 0$ for all $i \in N$. A careful look at the second term on the right hand side of Equation 2 is necessary. Consider a profile of valuations $x \in X$. Consider $\sum_{i \in N} v_i(x_i) a_i(x)$ for a valuation profile $x \in X$. This is maximized by setting $a_i(x) = 1$ if $v_i(x_i) = \max_{j \in N} v_j(x_j) \geq 0$, else setting $a_i(x) = 0$ for all $i \in N$. That is, we allocate the object to the buyer with the highest non-negative virtual valuation, and we do not allocate the object if the highest virtual valuation is negative. This way, we will maximize $\sum_{i \in N} v_i(x_i) a_i(x)$, and hence will maximize $\sum_{i \in N} v_i(x_i) a_i(x) f(x)$. This in turn will maximize the second term on the right hand side of Equation 2.

Now, we come back to Bayesian incentive compatibility requirement. By virtue of Theorem 1, we need to ensure that the suggested allocation rule is w-mon. In general, it is not w-mon. However, it is w-mon under the following condition. We say the **regularity** condition holds if for every bidder $i \in N$, $v_i(x_i) \geq v_i(z_i)$ for all $x_i, z_i \in X_i$ with $x_i > z_i$. In other words, for all $i \in N$, for all $x_i, z_i \in X_i$ with $x_i > z_i$, regularity is satisfied if $\frac{1-F_i(x_i)}{f_i(x_i)} \leq \frac{1-F_i(z_i)}{f_i(z_i)}$. The term $\frac{f_i(x_i)}{1-F_i(x_i)}$ is called the **hazard rate**. So, regularity is satisfied if the hazard rate is non-decreasing. The uniform distribution satisfies the regularity condition.

If the regularity condition holds, then a is w-mon. To see this, consider a bidder $i \in N$ and $x_i, z_i \in X_i$ with $x_i > z_i$. Regularity gives us $v_i(x_i) \geq v_i(z_i)$. By the definition of the allocation rule, for all $x_{-i} \in X_{-i}$, we have $a_i(x_i, x_{-i}) \geq a_i(z_i, x_{-i})$. Hence, a is w-mon. The associated payment for bidder $i \in N$ for a profile of valuation x can be computed from Theorem 2 subject to the fact that $\pi_i(0) = 0$.

$$p_i(x) = a_i(x) x_i - \int_0^{x_i} a_i(t_i, x_{-i}) dt_i$$

This describes an optimal mechanism. From Equation 2 and the description of the optimal mechanism, the expected highest revenue is the expected value of the highest virtual valuation provided it is non-negative.

This mechanism can be simplified further. Define for all $i \in N$ and all $x_{-i} \in X_{-i}$

$$q_i(x_{-i}) = \inf\{z_i : v_i(z_i) \geq 0 \text{ and } v_j(x_j) \leq v_i(z_i) \ \forall\ j \neq i\}.$$

Hence, $q_i(x_{-i})$ is the valuation whose corresponding virtual valuation is non-negative and "beats" the virtual valuations of other bidders. Thus the optimal allocation rule under regularity condition can be rewritten as, for all $i \in N$, for all $z_i \in X_i$, and for all $x_{-i} \in X_{-i}$,

$$\begin{aligned} a_i(z_i, x_{-i}) &= 1 \qquad \text{if } z_i > q_i(x_{-i}) \\ a_i(z_i, x_{-i}) &= 0 \qquad \text{otherwise.} \end{aligned}$$

Hence, for all $i \in N$, for all $x_i \in X_i$, and for all $x_{-i} \in X_{-i}$

$$\int_0^{x_i} a_i(z_i, x_{-i}) dz_i = x_i - q_i(x_{-i}) \qquad \text{if } x_i > q_i(x_{-i})$$
$$\int_0^{x_i} a_i(z_i, x_{-i}) dz_i = 0 \qquad \text{otherwise.}$$

This simplifies the payment rule. For all $i \in N$, for all $x_i \in X_i$, and for all $x_{-i} \in X_{-i}$

$$p_i(x) = q_i(x_{-i}) \qquad \text{if } a_i(x) = 1$$
$$p_i(x) = 0 \qquad \text{if } a_i(x) = 0.$$

Thus, the optimal mechanism is the following auction. We order the virtual valuations of bidders. Award the object to the highest non-negative virtual valuation bidder (breaking ties arbitrarily - ties will happen with probability zero), and the winner, if any, pays the valuation corresponding to the second highest virtual valuation, while the losers pay nothing. In other words, the seller sets a reserve price [5] equal to $\max_{i \in N} v_i^{-1}(0)$ and for the valuations that exceed this reserve price, he conducts the above auction. This leads to the seminal result in (Myerson, 1981).

Theorem 3 (Optimal Auction) *Suppose the regularity condition holds. Then, the following mechanism is optimal. For all $i \in N$,*

$$a_i(x) = 1 \qquad \textit{if } v_i(x_i) > \max_{j \neq i} v_j(x_j) \textit{ and } v_i(x_i) \geq 0$$
$$a_i(x) = 0 \qquad \textit{otherwise}$$

$$p_i(x) = q_i(x_{-i}) \qquad \textit{if } a_i(x) = 1$$
$$p_i(x) = 0 \qquad \textit{if } a_i(x) = 0.$$

Finally, we look at the special case where the buyers are **symmetric**, i.e., they draw the valuations using the same distribution - $f_i = f$ for all $i \in N$. So, virtual valuations are the same: $v_i = v$ for all $i \in N$. Hence, maximum virtual valuation corresponds to the maximum valuation. Thus, $q_i(x_{-i}) = \max\{v^{-1}(0), \max_{j \neq i} x_j\}$. This is exactly, the second-price auction with the reserve price of $v^{-1}(0)$. Hence, when the buyers are symmetric, then the second-price auction with a reserve price is optimal. Typically, the optimal mechanism is inefficient.

Though the second-price auction is rarely used in practice, but is weakly equivalent to the popular English auction. Under regularity and symmetric bidders, the optimal mechanism can be implemented using an English auction. The auction starts at price $v^{-1}(0)$ and the price is raised till exactly one bidder is interested in the object.

[5] A reserve price in an auction indicates that if bids are less than the reserve price than the object will not be sold.

4.6 Extensions of Myerson's Optimal Auction Work

The contribution in Myerson (1981) has been extended in many directions. In the single object case, several authors have tried to extend the results in Myerson (1981) by relaxing some of the assumptions and putting more qualifications in the auction design. For example, by relaxing the independent private value assumption, Cremer and McLean (1988) show that a slight degree of correlation allows the seller to extract the entire surplus from the bidders, i.e., expected payoff of every bidder can be made zero [6]. For a full account of results on the extensions of Myerson (1981) and other topics in single object auction, the readers are directed to Krishna (2002).

We discuss some recent extensions to the multi-object auction case. To do so, let us first examine the complexity of the multi-object model. In a multi-object model, there is a set of objects $M = \{1, \ldots, m\}$. The objects can be homogeneous or heterogeneous. A bidder $i \in N$ has a valuation function $v_i : 2^M \to \mathbb{R}_+$, i.e., for every bundle of objects $S \subseteq M$, bidder $i \in N$ associates a value $v_i(S)$. Thus, the value of a buyer is multi-dimensional. The multi-dimensional nature of the problem creates challenges. It is not clear whether Theorems 1 and 2 extend to the multi-dimensional case.

To describe the multi-dimensional problem, for every agent $i \in N$, his type space is k dimensional, i.e., $X_i \subseteq \mathbb{R}^k$. Let A be a finite set of outcomes. Let $v_i : A \times X_i \to \mathbb{R}$ be the valuation function of agent $i \in N$. As before $X = \times_{i \in N} X_i$ and $X_{-i} = \times_{j \neq i} X_j$. An allocation rule a is a mapping $a : X \to A$. A payment rule p is a mapping $p : X \to \mathbb{R}$. A mechanism (a, p) is **dominant strategy incentive compatible** if for every agent $i \in N$, every $x_{-i} \in X_{-i}$, and every $x_i, z_i \in X_i$ we have

$$v_i(a(x), x_i) - p_i(x) \geq v_i(a(z_i, x_{-i}), x_i) - p_i(z_i, x_{-i})$$

Writing this constraint by switching the roles of x_i and z_i, and adding these two equations we get the following necessary condition for dominant strategy incentive compatibility. For every agent $i \in N$ and for every $x_{-i} \in X_{-i}$ we need

$$\big[v_i(a(x_i, x_{-i}), x_i) - v_i(a(z_i, x_{-i}), x_i)\big] + \big[v_i(a(z_i, x_{-i}), z_i) - v_i(a(x_i, x_{-i}), z_i)\big] \geq 0. \tag{3}$$

Condition in Equation 3 is referred to as the **weak monotonicity (wmon)** condition for multi-dimensional types [7]. As was shown, this is clearly a necessary condition for incentive compatibility. For a variety of domains (i.e., restriction on X), this is also sufficient.

Some ongoing research has focused on extending the w-mon characterization to multi-object auctions. The extensions of w-mon characterizations are mainly for dominant strat-

[6] Cremer and McLean (1988) make the assumption that valuations are drawn from a discrete distribution, a condition called the *single crossing* is satisfied, and the *matrix of beliefs* has full rank.

[7] An obvious modification in notation gives an equivalent condition for Bayesian incentive compatibility.

egy incentive compatibility, except Muller et al. (2007). As long as the closure of the domain of valuations is convex, w-mon is necessary and sufficient for dominant strategy incentive compatibility in the multi-object case, as is shown in Monderer (2007) [8]. Other notable extensions of Theorem 1 to multi-dimensional case are Bikhchandani et al. (2006), Saks and Wu (2005), Muller et al. (2007) (they characterize Bayesian incentive compatibility), and Gui et al. (2004).

The revenue equivalence result of Theorem 2 have also been extended to the multi-dimensional case. Hydenreich et al. (2007) give a directed graph interpretation of the incentive compatibility constraints, and use it to characterize multi-dimensional domains where revenue equivalence holds. Other notable extensions of Theorem 2 are Milgrom and Segal (2002), Krishna and Maenner (2001), and Chung and Olszewski (2007).

The optimal auction design problem for the multi-object case is still an open problem. Armstrong (2000) analyzes this problem for two objects case when buyers valuations are additive (i.e., value for a bundle of object is sum of values of objects in the bundle). When values are drawn from a binary distribution, they construct optimal auctions under some technical conditions on the distributions of the bidders. His work shows the difficulty involved in solving the problem. Recent work by Malakhov and Vohra (2008), again assuming discrete distribution of valuations of bidders, using directed graph interpretation of incentive compatibility constraints make some advance in the multi-dimensional optimal auction design. However, a general solution still alludes the literature.

5 Nash Implementation

In this section, we briefly describe an important contribution of Eric Maskin. In order to do so, we first give a general setting for implementation. Then, we describe the seminal results on Nash implementation of Maskin (1999). For a detailed discussion on implementation, see Jackson (2001). Jackson (2001) is also an excellent place to know about the contributions of Leonid Hurwicz.

Having discussed the mechanism design setting, the implementation setting differs from it in one major way. In the implementation setting, the agents know each others preferences (complete information setting). However, the designer does not know the preferences of the agents. This is still a useful setting for many situations. Consider a situation where a firm and one of his suppliers are in a dispute about the quality of the good supplied by the supplier. They decide to take the dispute to a regulatory authority. The firm and his supplier are aware of the actual quality of the goods supplied, but the regulator is not aware of the quality. What mechanism can the regulator design to elicit true information from the

[8] Monderer (2007) also shows that any domain of valuations where w-mon allocation rule characterizes incentive compatibility must be a domain whose closure is convex.

supplier and the firm? The implementation literature answers such questions.

5.1 The Model

As before $N = \{1, \dots, n\}$ denotes a finite set of agents. The set of **outcomes** is denoted by A, which can be finite or infinite. The set of outcomes is an arbitrary set. Let $\Omega = \{S \subseteq A : S \neq \emptyset\}$. The preference of agent i is represented by a binary relation R_i over A which is complete and transitive. We let aR_ib for any $a, b \in A$ to denote that a is at least as good as b for agent i. The strict preference associated with a binary relation R_i is denoted as P_i. We use the standard notation R to denote a profile of binary relations and R_{-i} to denote a profile of binary relations of agents other than agent i. The set of all admissible preference profiles is $\mathbb{P}$.

A **social choice correspondence** F is a mapping $F : \mathbb{P} \to \Omega$. So, for a profile of preferences $R \in \mathbb{P}$, $F(R)$ represents the desirable set of alternatives. We say $F(\cdot)$ is a **social choice function** if it is single-valued.

As before, a mechanism is a pair (M, g), where M is the product of message spaces of agents and g is the outcome function $g : M \to A$. A message profile m is a **Nash equilibrium** of a mechanism (M, g) at preference profile R if for every $i \in N$, $g_i(m_i, m_{-i})R_ig_i(m'_i, m_{-i})$ for all $m'_i \in M_i$. Let $NE^{\Gamma}(R) \subseteq M$ denote the set of Nash equilibria of mechanism $\Gamma = (M, g)$ at preference profile R. Let $g(NE^{\Gamma}(R)) = \{a \in A : a = g(m) \text{ for some } m \in NE^{\Gamma}(R)\}$ be the set of outcomes obtained in Nash equilibrium of mechanism Γ at preference profile R. A social choice correspondence F is **implemented** by the mechanism $\Gamma = (M, g)$ in Nash equilibrium if $g(NE^{\Gamma}(R)) = F(R)$ for all $R \in \mathbb{P}$. A social choice correspondence F is said to be implementable in Nash equilibrium if there exists a mechanism (M, g) which implements it.

5.2 Nash Implementation and Monotonicity

The seminal work of Maskin (1999) not only gives us necessary and sufficient conditions for Nash implementable social choice correspondences but also provides a technique, which is extensively used in subsequent literature.

The condition identified by Maskin is a monotonicity condition which is necessary for Nash implementation. Suppose a social choice correspondence F is Nash implementable by a mechanism (M, g). Consider a preference profile R and let $F(R) = a$. So, there exists $m \in M$ such that $g(m) = a$ and m is a Nash equilibrium at R. Now, suppose there exists another preference profile R' such that $a \notin F(R')$. Since F is Nash implementable by (M, g), this implies that m is not a Nash equilibrium, and there exists an agent i and a message

$m_i' \in M_i$ such that

$$g(m_i', m_{-i}) P_i' g(m_i, m_{-i}).$$

Since m was a Nash equilibrium at R we can write

$$g(m_i, m_{-i}) R_i g(m_i', m_{-i})$$

If we let $b = g(m_i', m_{-i})$, we get $bP_i'a$ and aR_ib. Thus, we have arrived at a necessary condition for Nash implementation.

Definition 4 *A social choice correspondence F is* **Maskin monotonic** *if for every $R, R' \in \mathbb{P}$ and every $a \in F(R) \setminus F(R')$, there exists $i \in N$ and $b \in A$ such that $bP_i'a$ and aR_ib.*

Maskin monotonicity says that if some alternative is in the social choice correspondence at a profile but not in another profile, then it must have fallen in some agent's preference ranking (to break the Nash equilibrium). By our arguments earlier, the following theorem is immediate, which was proved in Maskin (1999).

Theorem 4 **(Necessity of Monotonicity)** *If a social choice correspondence is Nash implementable, then it is Maskin monotonic.*

There is an equivalent way to state Maskin monotonicity. Consider a profile R and $a \in F(R)$. Consider R' such that for each $i \in N$ we have aR_ib implies $aR_i'b$, i.e., ranking of a in i's preference has not fallen from R to R'. Since F is Nash implementable by (M, g), there exists $m \in M$ such that $g(m_i, m_{-i})R_ig(m_i', m_{-i})$ for all $m_i' \in M_i$, where $g(m_i, m_{-i}) = a$. But for each b such that aR_ib we have $aR_i'b$. Hence, $g(m_i, m_{-i})R_i'g(m_i', m_{-i})$ for all $i \in N$ and for all $m_i' \in M_i$. Hence, m is also a Nash equilibrium at R', i.e., $a \in F(R')$. So, an equivalent statement of Maskin monotonicity is the following. A social choice correspondence F is Maskin monotonic if for any R and $a \in F(R)$ and for R' such that aR_ib implies that $aR_i'b$ for all $i \in N$, we have $a \in F(R')$.

While Maskin monotonicity is necessary for Nash implementation, we need more sufficient conditions for Nash implementation. When $n \geq 3$, Maskin (1999) shows that monotonicity along with a condition called no veto power is sufficient for Nash implementation.

Definition 5 *A social choice correspondence satisfies* **no veto power** *if whenever i, R, and a are such that aR_jb for all $j \neq i$ and all $b \in A$, we have $a \in F(R)$.*

Clearly, no veto power is very restrictive when there are two agents. Maskin (1999) proved the following.

Theorem 5 **(Sufficiency)** *Suppose $n \geq 3$. If a social choice correspondence satisfies Maskin monotonicity and no veto power, then it is Nash implementable.*

We omit the proof, but note that the proof is constructive. For every social choice correspondence F satisfying Maskin monotonicity and no veto power, Maskin (1999) constructs a mechanism which implements it.

5.3 Extensions

Maskin's original work was circulated as a discussion paper in 1977, and it started a flurry of extensions. For $n = 2$, the sufficiency result in Maskin (1999) does not work. Dutta and Sen (1991) provide a sufficient condition for the $n = 2$ case. Researchers have also studied the consequence of other kinds of solution concepts for implementation. Virtual (approximate) implementation is studied in Matsushima (1988); Abreu and Sen (1991), implementation by sequential mechanisms is studied in Moore and Repullo (1988); Abreu and Sen (1990), and implementation in (incomplete information settings) Bayesian equilibrium is studied in Dutta and Sen (1994). Several new solution concepts of equilibrium have since been proposed, and researchers continue to find mechanisms that can be implemented in these new solution concepts. For details, the reader is referred to Jackson (2001).

6 Conclusion

Two surveys by Jackson (2001, 2003) have more details on the topic of implementation and mechanism design. The present survey is more succinct than these surveys. I focused on the foundations of mechanism design and went into the details of Myerson's optimal auction design work. I specially highlighted some of the extensions and ongoing research of Myerson's work. I also touched on, though briefly, the implementation setting and Maskin's seminal Nash implementation work.

Another place to learn about the contributions of 2007 economics Nobel prize winners is the scientific background document compiled by the Prize committee of Royal Swedish Academy of Sciences titled "Mechanism Design Theory" (it can be accessed on the Internet web site of Nobel Prize: `http://nobelprize.org`). This scientific background document gives an overview of all the major contributions of the three laureates.

The applications of mechanism design theory is growing. With the advent of Internet, new types of markets are being created. This is generating new fundamental questions in mechanism design theory, and mechanism design theory is being applied in many new areas (some of which we discussed in Section 1.1). These applications owe their success to the sound theory of mechanism design, foundations of which were laid by Hurwicz, Maskin, and Myerson.

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