Game Theory - Assignment 4

Due date: **September 29, 2025**.

1. Consider the standard battle-of-sexes game with two players and two strategies shown in Table 1. There is a mixed strategy Nash equilibrium $\sigma^\star$ of this game where players mix both strategies. Find this mixed strategy Nash equilibrium $\sigma^\star$.

	a	b
A	$(2, 1)$	$(0, 0)$
B	$(0, 0)$	$(1, 2)$

Table 1: Battle of sexes

Now consider a perturbation of this game shown in Table 2. Player $i \in \{1, 2\}$ privately observes ϵ_i which is drawn from $[0, \delta]$ uniformly at random.

	a	b
A	$(2 + \epsilon_1, 1)$	$(0, 0)$
B	$(0, 0)$	$(1, 2 + \epsilon_2)$

Table 2: Battle of sexes: Bayesian

(a) Consider a cutoff strategy (in the Bayesian battle-of-sexes game) for each player: there is a cutoff c such that each player i plays A or a if $\epsilon_i \geq c$ and plays B or b otherwise. Show that there is a Bayesian equilibrium where each player uses a cutoff strategy. Find the cutoff.

(b) How does this equilibrium strategy relate to the mixed strategy Nash equilibrium of the complete information game as $\delta \to 0$?

2. Consider a two-player Bayesian game where a parameter $\theta \in \{0, 3\}$ is observed by Player 1. Player 2 believes that it is equally likely that $\theta = 0$ and $\theta = 3$. For every value of θ, the strategic-form game associated with Table 3 is played.

(a) What are the strategies of the players in this Bayesian game.

(b) Compute two Bayesian equilibria of this game.

	a	b
A	$(2,2)$	$(0,\theta)$
B	$(\theta,0)$	$(1,1)$

Table 3: A Bayesian game

3. Consider a two-player Bayesian game shown in Table 4. Here, ϵ_1 is observed by Player 1 and ϵ_2 is observed by Player 2. Both ϵ_1 and ϵ_2 are distributed uniformly between $[-\frac{1}{3}, \frac{2}{3}]$ and this is common knowledge.

	a	b
A	$(2+\epsilon_1, 2)$	(ϵ_1, ϵ_2)
B	$(0,0)$	$(1, 2+\epsilon_2)$

Table 4: A Bayesian game

(a) What are the strategies of the players in this Bayesian game.

(b) Compute a Bayesian equilibrium in which each Player plays each of its actions for *some* type of hers (i.e., *do not* consider a Bayesian equilibrium where for every type a player plays the same action).