Game Theory - Final Examination

Date: November 17, 2015

Total marks: **46**

Duration: 2:00 PM to 5:00 PM

Note: Answer all questions clearly using pen. Please avoid unnecessary discussions. An answer that does not specify the strategy of a player clearly and explicitly (be it an extensive form game or Bayesian game or infinitely repeated game) may not get full credit.

1. Consider the strategic form game in Table 1.

   (a) Compute the (pure) minmax payoffs of both the players in this game. **(2 marks)**

   (b) Compute all pure Nash equilibria of this game. **(2 marks)**

| | $L_1$ | $M_1$ | $R_1$ |
|---|---|---|---|
| $L_2$ | 2,2 | 10,1 | 1,1 |
| $M_2$ | 1,10 | 7,7 | 1,1 |
| $R_2$ | 1,1 | 1,1 | 4,4 |

Table 1: Stage game

   Now, consider the infinitely repeated game where the stage game is the game in Table 1. Both players use discounting criteria to evaluate payoffs. Find the values of discounts for which $(M_1, M_2)$ is played in every period in (a) a Nash equilibrium and (b) a subgame perfect equilibrium. **(6+6 marks)**

   Note: In both the questions above, you need to clearly specify strategies of the players that sustain these equilibria.

2. A buyer with value $v$ and a seller with cost $c$ for an object are bargaining over two periods. Suppose $v > c$ and the $(v, c)$ information is common knowledge to the buyer and the seller.

   In the first period, the buyer proposes a price $p_1 \in \mathbb{R}_+$. If the seller rejects the proposal, then we move to period 2. If the seller accepts the proposal, the net utility of the buyer and the seller are $(v - p_1)$ and $(p_1 - c)$ respectively, and the game ends.

   If the seller rejects the proposal in period 1, then the seller proposes a price $p_2 \in \mathbb{R}_+$ in period 2. If the buyer rejects this proposal, then the game ends and everyone realizes a utility of zero. If the buyer accepts the proposal, then his net utility is $(\delta v - p_2)$ and the net utility of the seller is $(p_2 - \frac{c}{\delta})$, where $\delta \in (0, 1)$ but sufficiently close to 1.

(a) Describe the game as an extensive form game. Clearly define the decision vertices of each player and actions available to each player at his decision vertices. **(3 marks)**

(b) Write down any pure strategy for the buyer. **(2 marks)**

(c) Find a (pure) subgame perfect equilibrium of this game. **(5 marks)**

(d) Is this (pure) subgame perfect equilibrium unique? **(3 marks)**

3. Consider the game in extensive form in Figure 1.

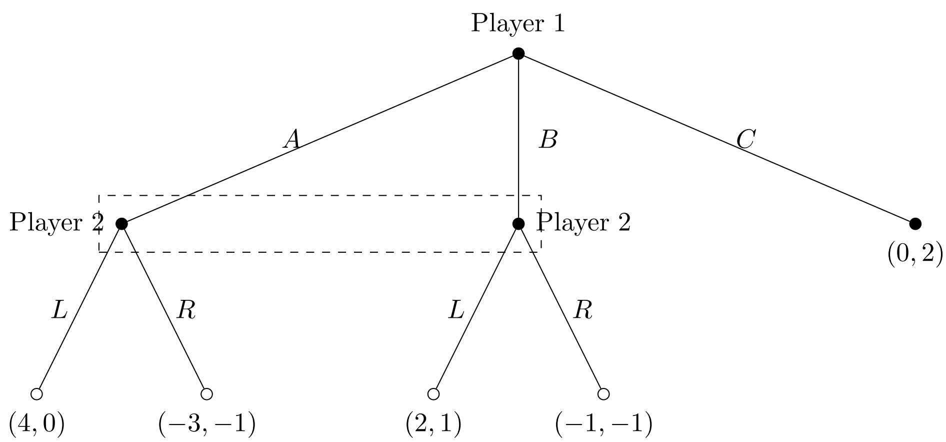


Figure 1: Sequential Equilibrium

(a) Describe a completely mixed strategy (i.e., non-pure) for each player in this game. For the completely mixed strategy profile you described above, find the Bayesian belief of Player 2. **(2 marks)**

(b) Is $(C, R)$ a Nash equilibrium of this game? **(2 marks)**

(c) Is there a sequential equilibrium of this game involving strategy profile $(C, R)$? Give clear arguments for your answer. **(4 marks)**

(d) Compute a sequential equilibrium of this game. Give clear arguments on why your claim is a sequential equilibrium. **(4 marks)**

4. Consider a standard first-price auction model with two buyers whose private values are independently drawn using uniform distribution from $[0, 10]$.

   Describe a monotone symmetric strategy of both the buyers in this Bayesian game and show that it forms a symmetric Bayes-Nash equilibrium (you do not need to show its uniqueness). **(5 marks)**