Game Theory - Final Examination

Date: November 18, 2016

Total marks: **26**

Duration: 2 PM

Note: Answer all questions clearly using pen. Please avoid unnecessary discussions. An answer that does not specify the strategy of a player clearly and explicitly (be it an extensive form game or a Bayesian game or a repeated game) may not get full credit. Payoffs in repeated games are evaluated using the discounting criterion.

1. Consider the following two player game Γ in Table 1.

	a	b	c
A	$(4,4)$	$(-1,5)$	$(2,2)$
B	$(5,-1)$	$(1,1)$	$(2,2)$
C	$(2,2)$	$(2,2)$	$(3,3)$

Table 1: Two Player Game

   - What are the pure strategy Nash equilibria of the game Γ? **(1 mark)**
   - Suppose Γ is repeated for two periods. Describe a subgame perfect equilibrium strategy of Γ^2 and a corresponding discount factor such that (A, a) is played in equilibrium in the first period. **(3 marks)**
   - Suppose Γ is repeated for infinitely many periods.
     - Describe a subgame perfect equilibrium strategy of Γ^∞ and a corresponding discount factor such that (A, a) is played in equilibrium in every period. **(5 marks)**
     - Describe a Nash equilibrium strategy of Γ^∞, which is different from the subgame perfect equilibrium strategy described by you for the previous question, and a corresponding discount factor such that (A, a) is played in equilibrium in every period. **(5 marks)**

2. A firm goes to a bank for loan. The firm is one of two types: (a) a honest (H) type or (b) a cheat (C) type. The probability that the firm is of type C is $p = \frac{2}{3}$ and of type H is $1 - p$. Bank does not know the type of the firm. The bank can either *approve* or *reject* the loan request of the firm. If the loan request is approved, then the firm can either *default* the loan or *repay* the loan.

If the bank rejects the loan request, then both the bank and the firm receive a payoff of 10 each. If the bank approves the loan request and the firm repays the loan, then the bank receives a payoff of 40 and the firm receives a payoff of 60. If the bank approves the loan request and the firm defaults, then the bank has a **loss** of 100 (i.e., payoff is -100). On the other hand, if the firm defaults, his payoff is zero if he is of type H and 150 if he is of type C.

- Describe this as an extensive form game of incomplete information (a graphical representation describing all information sets is good enough). **(2 marks)**
- Describe a perfect Bayesian equilibrium of this game. **(4 marks)**
- Verify if the perfect Bayesian equilibrium is a sequential equilibrium. **(4 marks)**

3. Consider an extensive form game of incomplete information (with perfect recall) and let $\mathcal{N}$, $\mathcal{S}$, $\mathcal{B}$, and $\mathcal{Q}$ denote the set of all Nash equilibria, subgame perfect equilibria, perfect Bayesian equilibria, and sequential equilibria of this game.

   Describe the inclusion relations between $\mathcal{N}, \mathcal{S}, \mathcal{B}$, and $\mathcal{Q}$. **(2 marks)**