

GAME THEORY - MIDTERM EXAMINATION

Date: September 4, 2016

Total marks: **30**

Duration: 10:00 AM to 1:00 PM

Note: Answer all questions clearly using pen. Please avoid unnecessary discussions.

1. Consider the following two player game in Table 1.

	a	b	c
A	(6, 2)	(0, 6)	(4, 4)
B	(2, 12)	(4, 3)	(2, 5)
C	(0, 6)	(10, 0)	(2, 2)

Table 1: Two Player Game

- Is there a pure strategy of Player 1 that is strictly dominated by another (possibly mixed) strategy? (**2 marks**)
 - Compute and illustrate the best response maps of both the players of this game. (**4 marks**)
 - Use the best response maps to compute all Nash equilibria of this game. (**3 marks**)
2. Suppose Player i has a pure strategy s_i in a finite game that is chosen with positive probability in *each* of his maxmin strategies (maxmin strategies may be mixed). Prove that s_i is not weakly dominated by any other (pure or mixed) strategy. (**4 marks**)
- Suppose Player i has a pure strategy s_i in a finite game that is chosen with positive probability in *one* of his maxmin strategies (maxmin strategies may be mixed). Prove or provide a counter example that s_i is not weakly dominated by any other (pure or mixed) strategy. (**4 marks**)
3. Consider the two player game shown in Table 2.
- Describe all the pure maxmin strategies of Players 1 and 2. (**2 marks**)
 - Does a value exist for this game? If yes, what is the value of this game? If no, why does the value not exist in this game? (**5 marks**)
 - Without explicitly computing it, describe all mixed strategy Nash equilibria of this game. (**2 marks**)

	a	b	c
A	$(0, 0)$	$(-1, 1)$	$(1, -1)$
B	$(1, -1)$	$(0, 0)$	$(-1, 1)$
C	$(-1, 1)$	$(1, -1)$	$(0, 0)$

Table 2: Two Player Game

4. Consider a two player *partnership* game. Each player invests an amount $a_i \in [0, 1]$. The utility of player $i \in \{1, 2\}$ is given by

$$u_i(a_1, a_2) = f(a_1)f(a_2) - c(a_i),$$

where f and c are strictly increasing continuous functions with $f(0) > 0$.

Argue clearly why this partnership game has a pure strategy Nash equilibrium. (4 marks)