Game Theory - Midterm Examination, 1

Date: September 15, 2017

Total marks: **20**

Duration: 10:00 AM to 12:00 PM

Note: Answer all questions clearly using pen. Please avoid unnecessary discussions. In all questions, unless specified, you do not need to consider the mixed extension of a game (i.e., consider only pure strategies unless specified).

1. Two candidates $\{1, 2\}$ are contesting to win a prize worth 1 to both. Candidates put in effort in $[0, 1]$ and the candidate who puts in higher effort wins the prize (the losing candidate does not win anything). In case of equal effort by both, the prize is split equally. The payoff is given by

   Prize amount won - Effort.

   (a) Describe this problem as a strategic form game. **(2 marks)**

   (b) Is the payoff of a player continuous in his own strategy? **(2 marks)**

   (c) Does a Nash equilibrium exist in this game? If yes, find it. If not, argue why. **(2 marks)**

2. Consider a two player game Γ, where player 1 has strategies $\{A, B, C\}$ and player 2 has strategies $\{a, b, c\}$. Consider the mixed extension of this game. The best response simplex of player 2 in the mixed extension of Γ is shown in Figure 1.

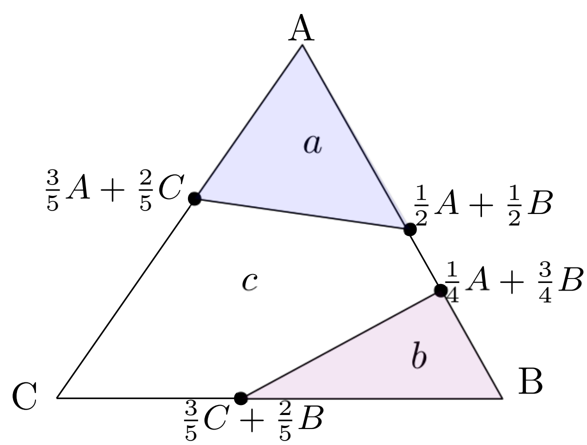

Figure 1: Best response of player 2: $B_2(\sigma_1)\ \forall\ \sigma_1$

Answer the following questions with reference to the mixed extension of Γ. Give clear and short arguments.

(a) Describe the (mixed) strategies of Player 2 that are not part of any Nash equilibrium. **(2 marks)**

(b) Describe the (mixed) strategies of Player 2 that are strictly dominated. **(2 marks)**

(c) Suppose Γ is such that the best response simplex of Player 1 looks exactly like Figure 1 with the change of labels as follows

$$a \leftrightarrow A, \quad b \leftrightarrow B, \quad c \leftrightarrow C.$$

Find all Nash equilibria of the mixed extension of Γ - i.e., find all pure and mixed Nash equilibria. **(4 marks)**

3. Suppose $N = \{1, 2\}$ and consider a finite game $\Gamma = \{N, S_1, S_2, u_1, u_2\}$. For each of the statements below, answer TRUE or FALSE with either a proof or a counter-example.

(a) Suppose (s_1^*, s_2^*) is a strategy profile in Γ such that

$$u_1(s_1^*, s_2^*) > u_1(s_1, s_2) \ \ \forall \ (s_1, s_2) \neq (s_1^*, s_2^*).$$

$$u_2(s_1^*, s_2^*) > u_2(s_1, s_2) \ \ \forall \ (s_1, s_2) \neq (s_1^*, s_2^*).$$

Then, (s_1^*, s_2^*) is a Nash equilibrium of Γ. **(2 marks)**

(b) Suppose (σ_1^*, σ_2^*) is a (mixed) strategy profile in the mixed extension of Γ that satisfies

$$u_1(\sigma_1^*, \sigma_2^*) > u_1(\sigma_1, \sigma_2^*) \ \ \forall \ \sigma_1 \neq \sigma_1^*.$$

$$u_2(\sigma_1^*, \sigma_2^*) > u_2(\sigma_1^*, \sigma_2) \ \ \forall \ \sigma_2 \neq \sigma_2^*.$$

Then, σ_i^* is a pure strategy for player i for each $i \in \{1, 2\}$. **(2 marks)**

(c) Suppose Γ is a zero-sum game. If (σ_1^*, σ_2^*) is a max-min strategy-profile in the mixed extension of Γ and $\sigma_1^*(s_1) > 0$, then s_1 is not strictly dominated. **(2 marks)**