Midterm-1 solution; Sept 2017

Q1

a// Set of candidates $\equiv$ Set of Players $= \{1,2\} = N$

Set of strategies of each player
$=$ Set of effort levels $= [0,1] = S_1 = S_2$

Denote the prize winning prob. as
$\forall s_1, s_2 [0,1]$

$$\pi_i(s_1,s_2) = \begin{cases} 1 & \text{if } s_i > s_j \\ 1/2 & \text{if } s_i = s_j \\ 0 & \text{if } s_i < s_j \end{cases}$$

Then, $u_i(s_1,s_2) = \pi_i(s_1,s_2) - s_i \quad \forall i$
$\forall (s_1,s_2)$

b// Payoff is <u>not</u> continuous.

Pick $s_2 = 1/2$

$u_1(s_1, 1/2) = 1 - s_1 \ \forall \ s_1 > 1/2$
but $u_1(1/2, 1/2) = 0 \Rightarrow$ discontinuity at $s_1 = s_2$

c// <u>No</u> Nash eqm exists. Assume for contradiction (s_1, s_2) is a NE.

If $s_1 = s_2$, then payoff of each player

$\frac{1}{2} - s_i$.

Each player i can ensure zero payoff for himself by choosing $s_i = 0$

Hence, $\frac{1}{2} - s_i \geq 0$ or $s_i \leq \frac{1}{2}$.

But then, choosing $s_i + \varepsilon$ ensures a payoff of

$1 - (s_i + \varepsilon) > \frac{1}{2} - s_i$ if $\varepsilon > 0$ is very small.

So, (s_1, s_2) is not a NE.

If $s_1 \neq s_2$; then assume $s_1 > s_2$ (without loss of generality)

Then, payoff of Player 1 is
$1 - s_1$
but he can improve this payoff by lowering his effort to some $s_1 - \varepsilon$, where $\varepsilon > 0$ but very close to zero.

2// a// Describing all mixed strategies not part of NE will be impossible at this point (but it will be clear in (c) part).

We can consider never best response strategies of Player 2.

From figure 1,

- No mixing of (a+b) only is part of best response.
- No mixing of (a+b+c) is part of best response.

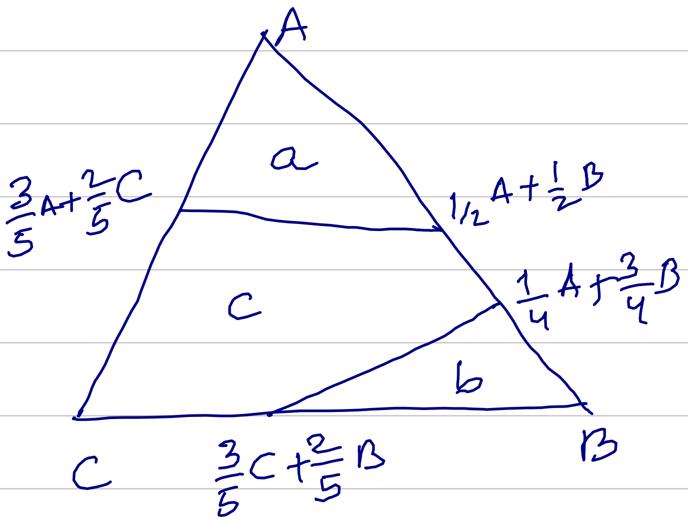
Hence, such mixed strategies cannot be part of NE.

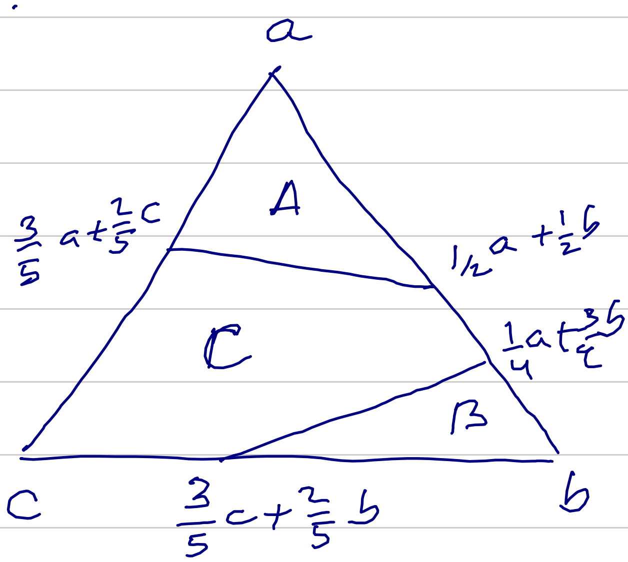
b// In 2-player games a strategy is strictly dominated if and only if it is never a best response.

Hence, the strategies in (a) part are also the strategies that are strictly dominated.

If you do not use this result, you can directly prove that those strategies are strictly dominated.

c/ Best response maps are:





Pure Nash: (A,a), (B,b), (C,c)

Mixed Nash: We know never best response strategies.
So, we check $a+c$ $(A+C)$ mix
$b+c$ $(B+C)$ mix

If Player 1 mixes $A+C$,
Player 2 mixes $(a+c)$ if it is $\frac{3}{5}A+\frac{2}{5}C$.
If Player 2 mixes $\frac{3}{5}a+\frac{2}{5}c$ Player 1 mixes $A+C$

So, $\left(\frac{3}{5}A+\frac{2}{5}C, \frac{3}{5}a+\frac{2}{5}c\right)$ is NE.

Similar logic gives $\left(\frac{3}{5}C+\frac{2}{5}B, \frac{3}{5}c+\frac{2}{5}b\right)$ is NE.

3// a// Yes.

$$u_1(s_1^*, s_2^*) > u_1(s_1, s_2^*) \quad \forall s_1 \neq s_1^*.$$
$$u_2(s_1^*, s_2^*) > u_2(s_1^*, s_2) \quad \forall s_2 \neq s_2^*$$

b// Yes. Consider Player 1.

σ_1^* is a best responese to σ_2^*.
By indifference lemma all pure strategies in support of σ_1^* are <u>also</u> best response to σ_2^*.

So, if σ_1^* has s_1 and s_1' in support.

$$u_1(\sigma_1^*, \sigma_2^*) = u_1(s_1, \sigma_2^*) = u_1(s_1', \sigma_2^*).$$

But this contradicts
$$u_1(\sigma_1^*, \sigma_2^*) > u_1(\sigma_1, \sigma_2^*) \quad \forall \sigma_1 \neq \sigma_1^*.$$

Similar argument works for Player 2.

c// Yes. If (σ_1^*, σ_2^*) is max-min strategy profile in a 2-player zero-sum game, it is a NE. Hence σ_1^* is a best response to σ_2^*. If $\sigma_1^*(s_1) > 0$, then by indifference lemma, s_1 is a BR to σ_2^*. Hence, s_1 is NOT strictly dominated.