GAME THEORY - FINAL EXAMINATION

Date: October 21, 2016

Total marks: **24**

Duration: 9:30 AM to 11:30 AM

Note: Answer all questions clearly using pen. Please avoid unnecessary discussions. An answer that does not specify the strategy of a player clearly and explicitly (be it an extensive form game or Bayesian game) may not get full credit.

1. An MSQE student either likes research or does not like research. At the end of her first year, she decides whether to do a thesis or not. Irrespective of her decision to do thesis or not, she simultaneously applies for higher studies to a University. The University can either accept the student or reject her. The University cannot observe whether the student likes research or not.

   The payoffs in both the situations are given below in Tables 1 and 2. Row belongs to University and Column belongs to Student in both the tables. Further, first entry of the payoff is for the University in both the tables.

	Do thesis	Not do thesis
Accept	8,8	5,6
Reject	0,-3	0,0

   Table 1: When student likes research

	Do thesis	Not do thesis
Accept	2,-3	-2,0
Reject	1,-3	0,0

   Table 2: When student *does not* like research

   Suppose there is a probability p with which the student likes research and this is common knowledge.

   (a) Model this problem as a Bayesian game. **(2 marks)**

   (b) Enumerate all the pure strategies of the players in this game. **(2 marks)**

   (c) Describe all pure strategy Bayesian equilibria of this game (as a function of p). **(6 marks)**

   (d) For what values of p, do we have a unique pure strategy Bayesian equilibrium in this game? **(2 marks)**

2. King Solomon is faced with two women, Elizabeth and Mary, who both claim to be the mother of an infant. King Solomon does not know the true mother. If the true mother gets the child, then she gets a utility of 100. On the other hand, the woman who is not the true mother only gets a utility of 50 if given the child. By not getting the child, both women get zero utility.

King Solomon sets up the following game.

Step 1. He will ask Elizabeth whether the child is hers. If she answers negatively, the child will be given to Mary. If she answers affirmatively, the king will continue to the next step.

Step 2. He will ask Mary whether the child is hers. If she answers negatively, the child will be given to Elizabeth. If she answers affirmatively, the king will ask Mary to pay 75 and Elizabeth to pay 10, and give the child to Mary. Utility from money is linear, i.e., paying p gives a utility of $-p$.

Since King Solomon does not know the true mother, there are two extensive form games possible - denote them as Γ_M (where Mary is the true mother) and Γ_E (where Elizabeth is the true mother).

(a) Describe Γ_M and Γ_E. **(2 marks)**

(b) Describe all Nash equilibria (pure and mixed) of Γ_M and Γ_E. **(4 marks)**

(c) Argue that there is a unique subgame perfect equilibrium of each of the games where the true mother gets the infant. **(6 marks)**