Q1:

a) Bayesian game

Players = {University (U), Student (S)}

Types:

University has no type - we can model it by assigning his

$T_U = \{\underline{t}\}$

Student has two types: research type and no research type.

$T_S = \{t_R, t_{NR}\}$.

Belief:

Prob. that student has type

$t_R = p$

and type $t_{NR} = 1-p$.

T_{t_R} and $T_{t_{NR}}$ are as specified in Tables 1 and 2.

b) Pure strategy of U = {Accept, Reject}

Pure strategy of S :

→ $s(t_R)$ = Do thesis $s(t_{NR})$ = Do thesis
→ $s(t_R)$ = Do thesis $s(t_{NR})$ = Not do thesis
→ $s(t_R)$ = Not do thesis $s(t_{NR})$ = Not do thesis
→ $s(t_R)$ = Not do thesis $s(t_{NR})$ = Not do thesis

c) First if a student is of type t_{NR}, then doing thesis is strictly dominated by not doing a thesis.

So, in any Bayesian equilibrium, t_{NR} must choose Not do thesis.

We consider possible Bayesian equilibria -

Case 1 : t_R chooses Do thesis

Then, U gets a payoff of
$8p + (-2)\,1-p = 10p - 2$
by Accept.
and
zero payoff from Reject.

If U Accepts, then S must Do thesis when t_R and Not do thesis when t_{NR} -

If U rejects, then S must Not do thesis when t_R and Not do thesis when t_{NR} -

Hence U rejecting and S Do thesis at t_R cannot be a Bayesian eqm.

So, the only pure Bayesian eqm where S Do thesis when her type is t_R is if $10p - 2 \geq 0 \iff \underline{\underline{p \geq 0.2}}$

So, if $p \geq 0.2$

Eqm 1

S choosing $\begin{cases} \text{Do thesis if } t_R \\ \text{Not do thesis if } t_{NR} \end{cases}$

U choosing {Accept

is a Bayesian equilibrium

<u>Case 2</u> t_R chooses Not do thesis.

Then U gets a payoff of $5p + (-2)(1-p) = 7p - 2$ by Accept

and zero payoff by Reject.

But t_R has a best response of Do thesis if U Accepts.

and t_R has a best response of Not do thesis if U rejects.

Note: t_{NR} always best responds by Not do thesis.

Hence the only equilibrium where t_R chooses Not do thesis is when U rejects.

This is possible if

$$7p - 2 \le 0 \iff p \le 2/7.$$

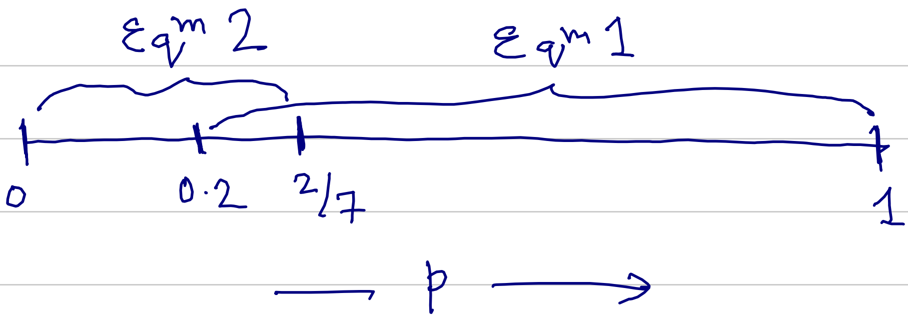
So, if $p \le 2/7$

Eq^m 2

$$\left[\begin{array}{ll} S \text{ choosing} & \left\{\begin{array}{ll} \text{Not do thesis} & \text{if } t_R \\ \text{Not do thesis} & \text{if } t_{NR} \end{array}\right. \\ & \\ U \text{ choosing} & \left\{ \text{Reject} \right. \end{array}\right.$$

is a Bayesian equilibrium.

d)



So, we have a unique Bayesian eqm

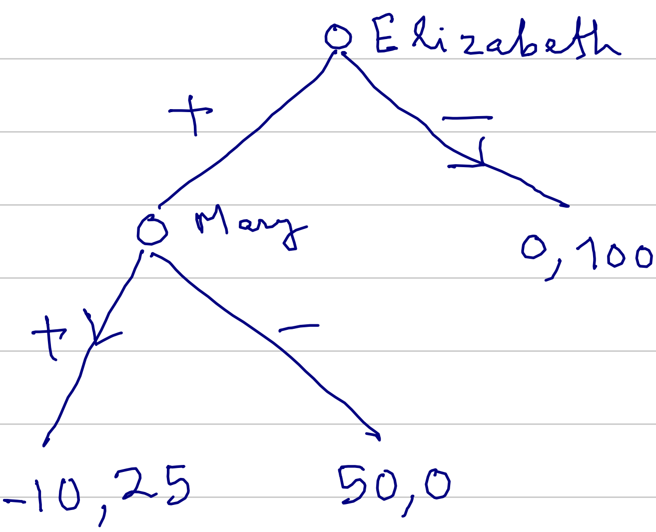
Eqm 1 if $p > 2/7$

Eqm 2 if $p < 0.2$

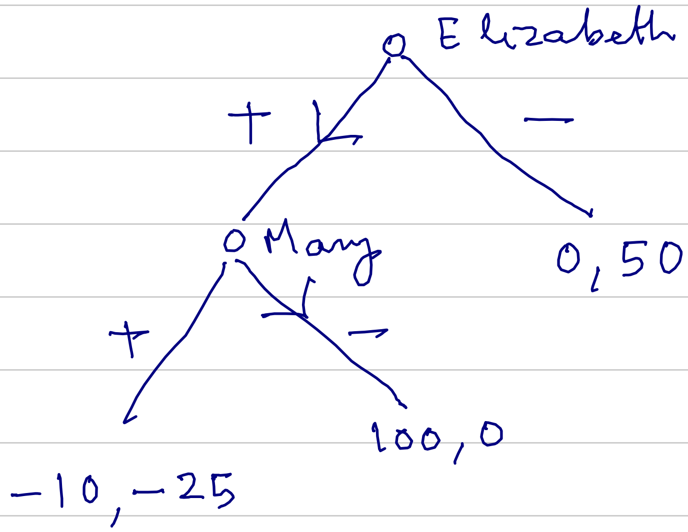
Q2.

a) + c)

Γ_M



Γ_E



First payoff is Elizabeth
Second Payoff is Mary.

SPE ⟺ Backward induction which is shown in arrows.

In T_M: E answers $-$ } SPE
M answers $+$

In T_E: E answers $+$ } SPE
M answers $-$

b) For NE, we can look at the reduced form

T_M:

Elizabeth \ Mary	+	−
+	−10, 25	50, 0
−	0, 100	0, 100

T_E:

Child \ Mary	+	−
+	−10, 25	100, 0
−	0, 50	0, 50

In T_m: If Mary plays $p(+) + (1-p)(-)$
then Elizabeth's payoff fns

$+$: $-10p + 50(1-p) = 50 - 60p$

$-$: 0

So $+$ is BR if $p \leq 5/6$.
$-$ is BR if $p \geq 5/6$

If Elizabeth plays $q(+) + (1-q)(-)$,
Mary's payoff from playing

$(+)$ $25q + 100(1-q) = 100 - 75q$

$(-)$ $100 - 100q$

So, $(+)$ is BR if $q \geq 0$

$(-)$ is BR if $q = 0$.

So, the BR maps look as follows:

BR of Mary

BR of Elizabeth

The only case where Mary mixes is when Elizabeth plays $(-)$.

But Elizabeth plays $(-)$ when Mary mixes $p(+) + (1-p)(-)$ where $p \geq 5/6$.

So, the set of all NE are: $\left(-,\ p(+) + (1-p)(-)\right)$
where $p \in [5/6, 1]$

In T_E: If Mary plays $p(+) + (1-p)(-)$ then

Elizabeth BR:

$+$ if : $-10p + 100(1-p) \geq 0$ or $p \leq \frac{10}{11}$

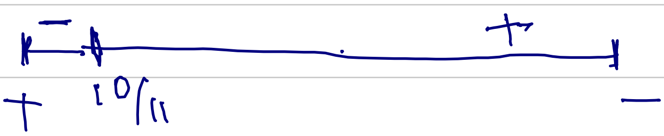
$-$ if : $p \geq 10/11$

If Elizabeth plays $q(+) + (1-q)(-)$ then Mary BR:

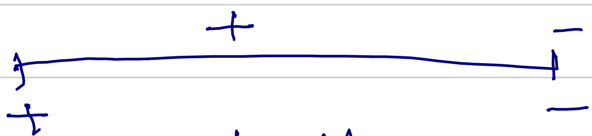
$+$ if : $25q \geq 0$ or $q \geq 0$

$-$ if : $q = 0$

So, the BR maps look as follows:



BR of Elizabeth



BR of Mary

So NE : $(-, p(+) + (1-p)(-))$, when $p \geq 10/11$.