Game Theory 2
Assignment 2

We will analyze a specific model of auctions. Agents in this model have one dimensional type spaces. The objective of this exercise is to show that whatever we learnt for the single object auction case applies (more or less) to *any* single dimensional type space model.

The model is as follows. There are n agents denoted by the set $N = \{1, \ldots, n\}$. A seller has m units of a homogeneous good to sell (assume $m < n$). Each agent is interested in at most one unit of the good. So, his type can be represented by a single number, which is the value he gets from getting a unit. A deterministic allocation a is vector of 1s and 0s, where $a_i \in \{0, 1\}$ denotes whether agent i gets the object or not, and the restriction $\sum a_i \leq m$ holds. Let ΔA denote the set of all probability distributions over A. An alternative $a \in \Delta A$ will now be a vector of probabilities, where a_i will denote the probability that agent i gets a unit of the good. Assume that type of agent i lies in interval $T_i = [0, b_i]$. An allocation rule f is a mapping $f : \times_{i \in N} T_i \rightarrow \Delta A$. Assume that the valuation function is in the product form, i.e., $v_i(a, t_i) = a_i \times t_i$. A deterministic allocation rule f is a mapping $f : \times_{i \in N} T_i \rightarrow A$.

1. Describe the efficient allocation rule, i.e., given a type profile of valuations describe who gets a unit and who does not.

2. Describe the pivotal mechanism in this setting.

3. Prove that the analogue of the non-decreasing property characterizes DSIC allocation rules in this setting.

4. Given a DSIC allocation rule, compute the set of all payments (precise functional form) that make it DSIC (this is the analogue revenue equivalence result).

5. For deterministic DSIC allocation rule, show how the payments depend on threshold values where an agent starts to win a unit.