Theory of Mechanism Design - Assignment 2

1. We consider the house allocation model with existing tenants. Consider the following form of manipulation by a coalition of agents in the TTC mechanism. A coalition of agents S exchange their houses before the start of the mechanism (i.e., they end up with an endowment which is different from their actual endowment). Now, the TTC mechanism is executed. Do you think each agent in S will now get a house which is either the same house he gets if he had not done the manipulation or a house which is higher ranked than the house he gets if he does not do manipulation?

2. Consider the house allocation model with three agents $N = \{1, 2, 3\}$ and three objects $M = \{a, b, c\}$. Let f be a social choice function defined as follows. At any preference profile $\succ \equiv (\succ_1, \ldots, \succ_n)$, if $\succ_2(1) = a$, then agent 1 gets the best element in $\{b, c\}$ according to his preference ordering $\succ_1$, agent 2 gets a, and agent 3 gets the remaining object (i.e., a serial dictatorship with the highest priority to agent 2, followed by agent 1, and finally to agent 3). In all other cases, agent 1 gets the best object in M, agent 2 gets the best remaining object according to $\succ_2$, and agent 3 gets the remaining object (i.e., a serial dictatorship with the highest priority to agent 1, followed by agent 2, and finally to agent 3).

   Is f strategy-proof? Is f non-bossy, i.e., can an agent change the outcome at a profile without changing the object assigned to him?

3. Consider a two-sided matching model with men and women. Let $\succ$ be a profile of preference orderings as shown in Table 1.

$\succ_{m_1}$	$\succ_{m_2}$	$\succ_{m_3}$	$\succ_{w_1}$	$\succ_{w_2}$	$\succ_{w_3}$
w_1	w_2	w_2	m_2	m_1	m_1
w_3	w_1	w_1	m_1	m_2	m_2
w_2	w_3	w_3	m_3	m_3	m_3

Table 1: Preference orderings of men and women

   Suppose μ is the outcome of the women-proposing deferred acceptance algorithm for the preference profile $\succ$. Let μ' be the outcome of the fixed-priority TTC mechanism where the priorities of men are fixed according to their preference orderings in $\succ$ in Table 1, and then each woman points to the woman with her favorite man in every stage of the TTC.

   (a) Verify that $\mu \neq \mu'$.

(b) Verify that μ' is not stable by identifying a blocking pair.

(c) Verify that μ' women-dominates μ.

4. Prove that if m and w are matched to each other in the men-proposing and women-proposing DAA, then they are matched to each other in *every* stable matching.

5. A stage in a men-proposing DAA involves proposals from rejected men in the previous stage and tentative acceptances and rejections by women based on these proposals. What is the maximum possible number of stages in a DAA with n men and n women.

6. Let μ and μ' be two stable matchings. Define $\mu \wedge^m \mu'$ as follows: for every $m \in M$,

$$(\mu \wedge^m \mu')(m) := \min_{\succ_m}(\mu(m), \mu'(m)).$$

   Either show that $(\mu \wedge^m \mu') = (\mu \vee^w \mu')$ or provide a counterexample.