Theory of Mechanism Design

Assignment 2

Due: 7, September, 2017.

The assignment asks to you derive "optimal" mechanisms in the three models below. The objective of the exercise is to convince you that whatever we did for the single object auction model can also be extended to other models as long as type is "one-dimensional" and value function remains linear in type. In each of the questions below, you need to do the following. Assume transfers are permitted and quasilinear utility with respect to transfers.

- Formulate the problem mathematically writing down all the assumptions you make.
- Characterize the set of Bayesian incentive compatible mechanisms in your model.
- Characterize the set of Bayesian incentive compatible (BIC) and interim individually rational (IIR) mechanisms in your model.
- Derive an "optimal" mechanism (that maximizes the expected payment to the designer under BIC and IIR) under regularity.

1. Suppose a seller (broadcasting company) is selling advertisement slot of duration T. Each agent (advertiser) i has an advertisement of length L_i (this information is common knowledge). Agent i gets a value of t_i (his type) if his advertisement is chosen. Else, he gets zero value. Assume that $T \geq \max_i L_i$ (i.e., each agent's advertisement is eligible to be displayed). A seller can choose multiple advertisers but their total duration must be less than or equal to T. Design a mechanism that maximizes the payment to the designer under BIC and IIR.

2. A set of n agents are located along a circular park. Denote the agents as $\{1, \ldots, n\}$. The **neighbors** of agent i are $(i-1)$ and $(i+1)$, where $i-1 \equiv n$ if $i = 1$ and $i+1 \equiv 1$ if $i = n$. A designer wants to locate a street light at one of the n locations along the park - it may also decide not to locate the street light. Each agent has a value for the street light, which is his type. This value is realized if the street light is located either at his location or at the location of one of his neighbors. Design a mechanism that maximizes the payment to the designer under BIC and IIR.

3. Consider a set of n agents who each own a job that needs service at a server. The server can process only one job at a time. Each job takes one unit of time. The server decides on a "sequence" (ordering) of agents and processes jobs one at a time according to the sequence. Agents incur **waiting cost** due to the sequence. If agent i is at the k-th position in the sequence, then he incurs a waiting cost of $(k-1)\theta_i$, where θ_i is the unit time waiting cost of agent i - the type of agent i is θ_i.

The server needs to serve all the agents - so, the set of alternatives is the $n!$ sequences (you can restrict attention to deterministic allocation rules here). Transfers are permitted to compensate the agents with quasilinearity assumption. Design a BIC and IIR mechanism that minimizes the payment of the designer over all BIC and IIR mechanisms.