

GAME THEORY 2 - ASSIGNMENT 4

The first three questions are for voting models. The final three questions are in house allocation models and house allocation models with existing tenants.

1. Show that the Gibbard-Satterthwaite theorem does not hold when the number of alternatives ($|A|$) is equal to 2.
2. Let \mathcal{P} be the set of all linear orders over the set of alternatives A (with $|A| \geq 3$). Let \mathcal{D} be a domain of orderings such that $\mathcal{P} \subseteq \mathcal{D}$. Using the Gibbard-Satterthwaite theorem, show that every onto and strategy-proof social choice function $f : \mathcal{D}^n \rightarrow A$ is a dictatorship.
3. Consider a two agent model with three alternatives $\{a, b, c\}$. Table 1 shows two preference profiles of linear orders. Suppose $f(P_1, P_2) = a$. Show that if f is strategy-proof then $f(P'_1, P'_2) = b$. You are allowed to use the result that for any preference profile (\bar{P}_1, \bar{P}_2) , $f(\bar{P}_1, \bar{P}_2) \in \{\bar{P}_1(1), \bar{P}_2(1)\}$ (but do not use any other result from the lectures).

P_1	P_2	P'_1	P'_2
a	c	b	c
b	b	a	a
c	a	c	b

Table 1: Two Preference Profiles

4. Show that in the TTC mechanism no agent can get a house which is lower in ranking than his initial endowment house.
5. In a house allocation model, consider the preferences of four agents over four houses as shown in Table 2.

\succ_1	\succ_2	\succ_3	\succ_4
a_3	a_3	a_1	a_2
a_1	a_2	a_4	a_1
a_2	a_1	a_3	a_3
a_4	a_4	a_2	a_4

Table 2: An example for housing model

- Run the TTC mechanism for some fixed endowment.

- Run the fixed priority mechanism for some priority.
6. We consider the house allocation model with existing tenants. Consider the following form of manipulation by a coalition of agents in the TTC mechanism. A coalition of agents S exchange their houses before the start of the mechanism (i.e., they end up with an endowment which is different from their actual endowment). Now, the TTC mechanism is executed. Do you think each agent in S will now get a house which is either the same house he gets if he had not done the manipulation or a house which is higher ranked than the house he gets if he does not do manipulation?