

## GAME THEORY 2 - ASSIGNMENT 5

1. Let the number of alternatives be  $m$ . Show that the number of single-peaked preference orderings with respect to  $<$  (an exogenously given ordering of alternatives) is  $2^{m-1}$ .
2. Consider the single-peaked domain model. A social choice function  $f$  is manipulable by a group of agents  $K \subseteq N$  if for some preference profile  $(P_K, P_{-K})$  there exists some preference profile  $P'_K$  of agents in  $K$  such that  $f(P'_K, P_{-K}) P_i f(P_K, P_{-K})$  for all  $i \in K$ . A social choice function  $f$  is **group strategy-proof** if cannot be manipulated by any group of agents. Is the median voter SCF group strategy-proof?
3. Consider the uniform rule  $f$  for allocating a divisible private good. Consider a profile  $(\succ_1, \dots, \succ_n)$  of single-peaked preferences of agents over the share of private good. Suppose  $\sum_{j \in N} p_j(\succ_j) \geq 1$ , where  $p_j(\succ_j)$  denotes the peak of agent  $j$  at preference ordering  $\succ_j$ . Suppose agent  $i$  reports another preference ordering  $\succ'_i$  such that  $p_i(\succ'_i) < p_i(\succ_i)$  and  $\sum_{j \neq i} p_j(\succ_j) + p_i(\succ'_i) < 1$ . Does the share of agent  $i$  decrease from  $\succ_i$  to  $\succ'_i$  (i.e.,  $f_i(\succ_i, \succ_{-i}) > f_i(\succ'_i, \succ_{-i})$ )?
4. Consider the house allocation model with the top trading cycle mechanism (with fixed endowments). A mechanism  $f$  is **non-bossy** if for every agent  $i$ , every preference profile  $\succ_{-i}$  of other agents, and every preference orderings  $\succ_i, \succ'_i$  of agent  $i$ ,  $f_i(\succ_i, \succ_{-i}) = f_i(\succ'_i, \succ_{-i})$  implies that  $f(\succ_i, \succ_{-i}) = f(\succ'_i, \succ_{-i})$ , where  $f_i(\cdot)$  denotes the allocation of agent  $i$ , i.e., if an agent  $i$  changes his preference from  $\succ_i$  to  $\succ'_i$  in a manner such that his own allocation does not change then it should not change the allocation of any other agents. Is the TTC mechanism non-bossy?