Redistribution mechanisms

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Redistribution

Redistribution is a fundamental objective in mechanism design.

A setting with a utilitarian designer (Govt.) or a setting without any mechanism designer.

Examples:

- A bequest without a will.
- A land to acquired by Govt and reallocated.

What is different?

Incentive and participation constraints remain.

New constraints on payments/payoffs.

- Sum of payments zero or non-negative.
- *Property rights* change IR constraints.

These new constraints restrict the class of mechanisms severely.

Dissolving a partnership

Cramton, P., Gibbons, R. and Klemperer, P., 1987. Dissolving a partnership efficiently. *Econometrica*, pp.615-632.

The model

$N := \{1, \ldots, n\}$ is the set of n agents.

There is a single *divisible* object to be allocated to n agents.

Each agent i has a value v_i for the entire object and gets value $v_i q_i$ if $q_i \in [0, 1]$ part of the object is given.

Values are independently and identically distributed in $V \equiv [0, \beta]$ with F as the distribution function and f as the pdf.

Each agent i owns a share $r_i \in [0, 1]$ of the object with $\sum_{j \in N} r_j = 1$. This *property right* will play a role in defining participation constraint.

Partnership interpretation

n agents decide to form a partnership to start a firm.

Each agent is given some *share* of the firm.

Each partnership has a *dissolution clause* (a mechanism): specifies who will own the firm and how shareholders will be compensated if the firm is dissolved.

Efficiently dissolving

Efficient dissolution requires two things:

- When dissolved, firm goes to highest-valued agent.
- Entire surplus redistributed: budget-balance.

Besides this usual constraints:

- IC: Bayesian incentive compatibility.
- IR: Interim individual rationality w.r.t. property rights.

Question

What property rights structure r admits an efficient dissolution of the partnership?

For what property rights structure r, does there exist an efficient, BIC, IIR, and budget-balanced mechanism?

What kind of mechanisms can dissolve such dissolvable partnerships?

Solution

Assume agents are symmetric ex-ante.

A complete characterization of dissolvable partnerships. Simple corollaries say which partnerships are *always* dissolvable (independent of prior).

A mechanism which can dissolve every dissolvable partnership.

Simple mechanisms which can dissolve *some* dissolvable partnerships.

Main takeaways

In an *Informational symmetric* environment *property rights asymmetry* creates impossibility.

Symmetric partnerships are easy to dissolve.

Simple mechanisms dissolve many dissolvable partnerships:

- everyone bids; highest bidder wins
- highest bidder pays her bid.
- her bid amount equally distributed among all bidders.

Some partnerships

Three particular configuration of shares:

One seller many buyers model. $r_i = 1$ and $r_j = 0 \ \forall\ j \neq i$.

Equal partnership model. $r_i = \frac{1}{n}$ for all $i \in N$.

One buyer many sellers model. $r_i = 0$ and $r_j = \frac{1}{n-1} \ \forall\ j \neq i$.

Mechanism

A **mechanism** is a collection of pair of maps $(Q, T) \equiv \{Q_i, T_i\}_{i \in N}$, where for each $i \in N$,

$Q_i : V^n \to [0, 1]$ is the share allocation rule of agent i

$T_i : V^n \to \mathbb{R}$ is the transfer rule (amount paid to) of agent i.

A mechanism is **feasible** if for all $v \in V^n$

$\sum_{i \in N} Q_i(v) \leq 1$ and

transfers budget balanced, i.e., $\sum_{i \in N} T_i(v) = 0$.

Incentive and participation constraints

Fix a feasible mechanism (Q, T). Define for every $i \in N$ and every $v_i \in V$,

$$q_i(v_i) = \int_{v_{-i} \in V^{n-1}} Q_i(v_i, v_{-i}) d(F_{N-i}(v_{-i}))$$
$$t_i(v_i) = \int_{v_{-i} \in V^{n-1}} T_i(v_i, v_{-i}) d(F_{N-i}(v_{-i})),$$

where $F_{N-i} = \prod_{j \in N \setminus \{i\}} F(v_j)$.

So, every feasible mechanism (Q, T) generates interim rules $(q, t) \equiv \{q_i, t_i\}_{i \in N}$.

Incentive and participation constraints

Definition

A mechanism $\{Q_i, T_i\}_{i \in N}$ *is* **Bayesian incentive compatible (BIC)** *if for every* $i \in N$

$$v_i q_i(v_i) + t_i(v_i) \geq v_i q_i(v_i') + t_i(v_i') \qquad \forall\ v_i, v_i' \in V.$$

Participation needs to take care of property rights.

Definition

A mechanism $\{Q_i, T_i\}_{i \in N}$ *is* **individually rational (IR)** *if for every* $i \in N$

$$v_i q_i(v_i) + t_i(v_i) \geq r_i v_i \qquad \forall\ v_i \in V.$$

IR is different from optimal auction design because of property rights.

Dissolving a partnership efficiently

Definition

A parternship $\{r_i\}_{i \in N}$ *can be* **dissolved efficiently** *if there exists a*

- *feasible (allocation feasibility and budget balance)*
- *efficient*
- *Bayesian incentive compatible and*
- *individually rational mechanism*

for this partnership.

Implications of budget-balance

At every profile v, the sum of **utilities** (payoffs) of agents in mechanism (Q, T) is

$$\sum_{i \in N} \mathcal{U}_i(v) = \sum_{i \in N} \Big[v_i Q_i(v) + T_i(v) \Big] \leq \sum_{i \in N} v_i Q_i(v) \leq \max_{i \in N} v_i,$$

where the first inequality holds if $\sum_{i \in N} T_i(v) \leq 0$.

So, maximum sum of payoffs is $\max_i v_i$.

If Q is efficient and (Q, T) is budget-balanced, the above inequalities are equalities.

So, sum of payoffs is maximized by an efficient and budget-balanced mechanism.

Characterization of IC and IR

Similar to Myerson.

Lemma (IC Characterization)

A mechanism (Q, T) is Bayesian incentive compatible if and only if for every $i \in N$

- q_i *is non-decreasing*
- $t_i(v_i') - t_i(v_i) = v_i q_i(v_i) - v_i' q_i(v_i') - \int_{v_i'}^{v_i} q_i(x)dx \; \forall \; v_i, v_i' \in V.$

Interim allocation probabilities in efficiency

If type v_i, in an efficient allocation rule, agent i wins if *all others* have type less than v_i.

This happens with probability $[F(v_i)]^{n-1}$. Define for all v_i:

$$G(v_i) := [F(v_i)]^{n-1}.$$

So $q_i^e(v_i) = G(v_i) = [F(v_i)]^{n-1}$.

G is also a probability distribution and let g be its pdf: $g(v_i) := (n-1)[F(v_i)]^{n-2} f(v_i)$ for all v_i.

Payoffs

Consider interim payoffs: $U_i(v_i) := v_i q_i(v_i) + t_i(v_i)$.

Better to consider *marginal payoff*: $U_i(v_i) - r_i v_i$.

Though payoff is non-decreasing in a BIC mechanism, marginal payoff need not be.

Lemma

Suppose (Q^e, T) is an efficient and BIC mechanism. Let v_i^* be such that $G(v_i^*) = r_i$. Then, the following holds:

$$U_i(v_i) - r_i v_i \geq U_i(v_i^*) - r_i v_i^* \qquad \forall\ v_i \in V.$$

Marginal payoff is minimized at v_i^*, where $G(v_i^*) = r_i$.

Characterization of IIR

Lemma

Suppose (Q^e, T) *is an efficient and BIC mechanism. Then,* (Q^e, T) *is IIR if and only if* $t_i(v_i^*) \geq 0$ *for all* $i \in N$*, where* $G(v_i^*) = r_i$*.*

IIR is equivalent to having non-negative marginal payoff.

Since marginal payoff is minimized at v_i^*, IIR is equivalent to having non-negative marginal payoff.

But $U_i(v_i^*) - r_i v_i^* = v_i^* G(v_i^*) + t_i(v_i^*) - r_i v_i^*$ and using $G(v_i^*) = r_i$ gives the desired result.

Dissolving a partnership efficiently

Theorem (Cramton, Gibbons, Klemperer, 1987)

A partnership $\{r_i\}_{i\in N}$ can be dissolved efficiently if and only if

$$\sum_{i\in N}\left[\int_{v_i^*}^{\beta}(1-F(x))xg(x)dx-\int_0^{v_i^*}F(x)xg(x)dx\right]\geq 0, \tag{1}$$

where $G(v_i^*)=r_i$.

Complicated condition involving F, but it is a *characterization*.

Will have many interesting corollaries.

Necessary direction

Suppose (Q^e, T) is an efficient, BIC, and IR feasible mechanism for partnership r. Then, we know that

$$\sum_{i \in N} \int_0^{\beta} t_i(v_i) f(v_i) dv_i = \int_v \Big(\sum_{i \in N} T_i(v) \Big) f(v) dv = 0,$$

where the last equality follows from budget-balance.

Also, transfers are bounded due to IIR. For every $i \in N$ and every $v_i \in V$, we have

$$\begin{aligned} t_i(v_i) &= t_i(v_i^*) - v_i G(v_i) + v_i^* G(v_i^*) + \int_{v_i^*}^{v_i} G(x) dx \\ &= t_i(v_i^*) - \int_{v_i^*}^{v_i} x g(x) dx \\ &\geq \int_{v_i}^{v_i^*} x g(x) dx, \end{aligned}$$

Necessary direction

Now, we just compute the *ex-ante* value of this lower bound.

Since ex-ante expected payment is zero, a necessary condition is that ex-ante value of the lower bound is less than equal to zero.

Algebraic manipulation gives that the ex-ante value of the lower bound is exactly the necessary condition we have.

Necessary direction

$$0 = \sum_{i \in N} \int_0^{\beta} t_i(v_i) f(v_i) dv_i$$

$$\geq \sum_{i \in N} \Big[\int_0^{\beta} \Big(\int_{v_i}^{v_i^*} x g(x) dx \Big) f(v_i) dv_i \Big]$$

$$= \sum_{i \in N} \Big[\int_0^{\beta} \Big(\int_0^{v_i^*} x g(x) dx \Big) f(v_i) dv_i \Big] - \sum_{i \in N} \Big[\int_0^{\beta} \Big(\int_0^{v_i} x g(x) dx \Big) f(v_i) dv$$

$$= \sum_{i \in N} \Big[\int_0^{v_i^*} x g(x) dx - \int_0^{\beta} (1 - F(x)) x g(x) dx \Big]$$

$$= \sum_{i \in N} \Big[\int_0^{v_i^*} x g(x) dx - \int_0^{v_i^*} (1 - F(x)) x g(x) dx - \int_{v_i^*}^{\beta} (1 - F(x)) x g(x) d$$

$$= \sum_{i \in N} \Big[\int_0^{v_i^*} F(x) x g(x) dx - \int_{v_i^*}^{\beta} (1 - F(x)) x g(x) dx \Big]$$

Sufficiency direction

Suppose the Inequality holds for a partnership r. Define for every $i \in N$ and every $v_i \in V$,

$$W(v_i) := \int_{v_i}^{\beta} (1 - F(x))xg(x)dx - \int_0^{v_i} F(x)xg(x)dx \qquad (2)$$

We are given $\sum_{i \in N} W(v_i^*) \geq 0$.

Define for each agent $i \in N$:

$$c_i = \frac{1}{n} \sum_{j \in N} W(v_j^*) - W(v_i^*).$$

Note that $\sum_{i \in N} c_i = 0$.

Sufficiency direction

Now, we define the transfer functions for our efficient mechanism.

For every $i \in N$ and for every type profile $v \in V^n$, let

$$T_i(v) := \left[c_i - \int_0^{v_i} xg(x)dx + \frac{1}{n-1} \sum_{j \in N \setminus \{i\}} \int_0^{v_j} xg(x)dx\right]$$

Since $\sum_{i \in N} c_i = 0$, we get $\sum_{i \in N} T_i(v) = 0$ for all $v \in V^n$.

Sufficiency direction

Expected payment of agent i with type v_i is

$$
\begin{aligned}
t_i(v_i) &= \Big[c_i - \int_0^{v_i} xg(x)dx + \frac{1}{n-1} \sum_{j \in N \setminus \{i\}} \int_0^{\beta} \big(\int_0^{v_j} xg(x)dx \big) f(v_j) dv_j \Big] \\
&= \Big[c_i - \int_0^{v_i} xg(x)dx + \frac{1}{n-1} \sum_{j \in N \setminus \{i\}} \int_0^{\beta} (1 - F(x)) xg(x)dx \Big] \\
&= \Big[c_i - \int_0^{v_i} xg(x)dx + \int_0^{\beta} (1 - F(x)) xg(x)dx \Big] \\
&= \Big[c_i + W(v_i)\Big].
\end{aligned}
$$

IIR: $t_i(v_i^*) = c_i + W(v_i^*) = \frac{1}{n} \sum_{j \in N} W(v_j^*) \geq 0$.

Sufficiency direction

For BIC, note that allocation rule is efficient (satisfies NDE) and

$$\begin{aligned} t_i(v_i) - t_i(v_i^*) &= W(v_i) - W(v_i^*) \\ &= \int_{v_i^*}^{v_i} xg(x)dx \\ &= v_i G(v_i) - v_i^* G(v_i^*) - \int_{v_i^*}^{v_i} G(x)dx \\ &= v_i G(v_i) - v_i^* G(v_i^*) - \int_{v_i^*}^{v_i} q_i^e(x)dx \end{aligned}$$

Comments on the proof and result

Proof works out even if we replace budget-balance by **feasibility** (sum of payments is less than or equal to zero).

The mechanism proposed in the proof (call it the **CGK mechanism**) is quite prior-heavy and complicated to describe.

The paper discusses *simpler* mechanisms which can dissolve a large subset of dissolvable partnerships:

- Every agent bids and highest bidder wins.
- Highest bidder pays her bid.
- Her bid is equally divided between all.

One-seller many-buyer partnerships

Partnerships with $r_1 = 1, r_2 = \ldots = r_n = 0$ (agent 1 is seller and others are buyers).

Theorem

One-seller many-buyer partnerships cannot be dissolved efficiently.

One-seller one-buyer version of the theorem is known as the Myerson-Satterthwaite impossibility result (Myerson and Satterthwaite, 1983).

Proof of theorem

Let $r_s = 1$ for some $s \in N$ and $r_i = 0$ for all $i \neq s$.

Then $v_s^* = \beta$ and $v_i^* = 0$ for all $i \neq s$. Now, that

$$
\begin{aligned}
\sum_{i \in N} W(v_i^*) &= W(\beta) + (n-1)W(0) \\
&= (n-1)\int_0^\beta (1 - F(x))xg(x)dx - \int_0^\beta F(x)xg(x)dx \\
&= (n-1)\int_0^\beta xg(x)dx - n\int_0^\beta F(x)xg(x)dx.
\end{aligned}
$$

Proof of theorem

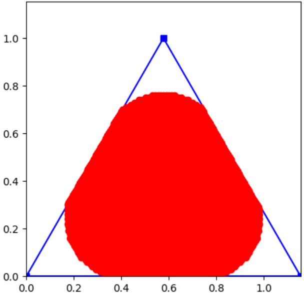
Using the fact that $g(x) = (n-1)[F(x)]^{n-2}f(x)$, we simplify as:

$$\begin{aligned}
\frac{1}{n-1}\sum_{i\in N} W(v_i^*) &= (n-1)\int_0^{\beta} x[F(x)]^{n-2}f(x)dx \\
&- n\int_0^{\beta} x[F(x)]^{n-1}f(x)dx \\
&= \left[x[F(x)]^{n-1}\right]_0^{\beta} - \int_0^{\beta}[F(x)]^{n-1}dx \\
&- \left[x[F(x)]^{n}\right]_0^{\beta} + \int_0^{\beta}[F(x)]^{n}dx \\
&= \int_0^{\beta}[F(x)]^{n-1}\big(F(x)-1\big)dx] \\
&< 0.
\end{aligned}$$

Equal partnerships

Theorem

Equal partnerships can always be dissolved efficiently. The set of partnerships that can be dissolved efficiently is a convex subset around the equal partnership.



Proof of theorem

Convexity follows straightforwardly.

Equal partnership $r_i = \frac{1}{n}$ for all $i \in N$. Let $G(v^*) = [F(v^*)]^{n-1} = \frac{1}{n}$. Need to show $W(v^*) \geq 0$,

$$W(v^*) = \int_{v^*}^{\beta} xg(x)dx - \int_{0}^{\beta} xF(x)g(x)dx$$
$$= (n-1)\int_{v^*}^{\beta} x[F(x)]^{n-2}f(x)dx - (n-1)\int_{0}^{\beta} x[F(x)]^{n-1}f(x)dx$$

Proof of theorem

Hence, we get

$$\begin{aligned}
\frac{1}{n-1}W(v^*) &= \int_{v^*}^{\beta} x[F(x)]^{n-2}f(x)dx - \int_0^{\beta} x[F(x)]^{n-1}f(x)dx \\
&= \frac{1}{n-1}\left[\beta - (v^*)[F(v^*)]^{n-1} - \int_{v^*}^{\beta}[F(x)]^{n-1}dx\right] \\
&- \frac{1}{n}\left[\beta - \int_0^{\beta}[F(x)]^n dx\right] \\
&= \frac{1}{n}\int_0^{\beta}[F(x)]^n dx + \frac{1}{n(n-1)}\left[\beta - v^*\right] \\
&- \frac{1}{(n-1)}\int_{v^*}^{\beta}[F(x)]^{n-1}dx \\
&= \frac{1}{n}\int_0^{v^*}[F(x)]^n dx + \frac{1}{n(n-1)}\left[\beta - v^*\right] \\
&- \frac{1}{n(n-1)}\int_{v^*}^{\beta}\left[n[F(x)]^{n-1} - (n-1)[F(x)]^n\right]dx
\end{aligned}$$

Proof of theorem

Consider the function $\phi(x) := nF(x)^{n-1} - (n-1)F(x)^n$ for all $x \in [v^*, \beta]$.

Note that $\phi(v^*) = 1 - \frac{n-1}{n}F(v^*) < 1$ and $\phi(\beta) = 1$.

Further,

$$\begin{aligned}\phi'(x) &= n(n-1)[F(x)]^{n-2}f(x) - n(n-1)[F(x)]^{n-1}f(x) \\ &= n(n-1)f(x)[F(x)]^{n-2}(1 - F(x)) \\ &> 0.\end{aligned}$$

Hence, ϕ is a strictly increasing function. So, $\phi(x) \leq 1$ for all $x \in [v^*, \beta]$. This means,

$$\frac{1}{n-1}W(v^*) \geq 0,$$

Comments

The CGK result is remarkable because it clearly spells out what is possible and what is not when dissolving a partnership efficiently.

Symmetry of agents is crucial – see Figueroa and Skreta (2012).

If agents values are interdependent, difficult to dissolve – see Moldovanu (2002).

Given an arbitrary share structure, what is the *optimal* mechanism? What is an optimal share structure (with asymmetric agents)? – see Loertscher and Wasser (2019).

Dominant strategy redistribution

A BIC redistribution mechanism may require too much information on priors.

Is it possible to strengthen the solution concept to DSIC?

Answer. *No.* DSIC is too demanding (as is the case often).

Bilateral trading model

Two agents, a buyer and a seller: $\{b, s\}$.

Values of both distributed in $[0, \beta]$.

Consider any DSIC, efficient, and budget-balanced mechanism with net utility functions

$$\mathcal{U}_b : [0, \beta] \to \mathbb{R} \text{ and } \mathcal{U}_s : [0, \beta] \to \mathbb{R}.$$

Efficiency means $Q_b^e(v_b, v_s) = 1$ if $v_b > v_s$ and $Q_s^e(v_b, v_s) = 1$ if $v_s > v_b$.

Budget balance (and efficiency) means at any profile (v_b, v_s) we have

$$\mathcal{U}_b(v_b, v_s) + \mathcal{U}_s(v_b, v_s) = \begin{cases} v_b & \text{if } v_b > v_s \\ v_s & \text{otherwise.} \end{cases}$$

Impossibility

DSIC implies *payoff equivalence* formula.

For any $v_b > v_s > 0$, we know trade happens at (v_b, v_s). So,

$$\begin{aligned}
\mathcal{U}_b(v_b, v_s) &= \mathcal{U}_b(0, v_s) + \int_0^{v_b} Q_b^e(x, v_s)dx \\
&= \mathcal{U}_b(0, v_s) + \int_{v_s}^{v_b} dx \\
&= \mathcal{U}_b(0, v_s) + (v_b - v_s) \\
&= v_s - \mathcal{U}_s(0, v_s) + (v_b - v_s) \\
&= (v_b - v_s) - \mathcal{U}_s(0, 0)
\end{aligned}$$

Impossibility

Identical argument gives

$$\begin{aligned}\mathcal{U}_s(v_b, v_s) &= \mathcal{U}_s(v_b, 0) + \int_0^{v_s} Q_s^e(v_b, x)dx \\ &= \mathcal{U}_s(v_b, 0) \\ &= v_b - \mathcal{U}_b(v_b, 0) \\ &= -\mathcal{U}_b(0, 0)\end{aligned}$$

Since $v_b > v_s$, we have

$$\begin{aligned}v_b &= \mathcal{U}_b(v_b, v_s) + \mathcal{U}_s(v_b, v_s) \\ &= (v_b - v_s) - \mathcal{U}_s(0, 0) - \mathcal{U}_b(0, 0) \\ &= (v_b - v_s),\end{aligned}$$

which is a contradiction if $v_s > 0$.

Green-Laffont impossibility theorem

Theorem

In the bilateral trading problem, there is no DSIC, efficient, and budget-balanced mechanism.

Notice no participation constraint is required.

Possible resolutions

Suppose participation constraint was not an issue. How can we achieve IC, efficiency, and budget-balance?

Three possible approaches:

- Relax efficiency.
- Relax budget-balance.
- Relax solution concept.

Relax efficiency

In other words, what is the (ex-ante) *total payoff maximizing* DSIC and budget-balanced mechanism.

Extremely difficult to solve.

Green-Laffont mechanisms: allocate to highest-valued agent with prob $1 - \frac{1}{n}$ and second highest-valued agent with prob $\frac{1}{n}$.

Asymptotically *efficient*.

Relax budget-balance

Burn money: we can raise money.

what is the (ex-ante) *total payoff maximizing* DSIC and efficient mechanism (Groves mechanism).

Difficult to solve but asymptotic budget-balance possible.

Cavallo mechanism

A Vickrey auction is conducted and its revenue is redistributed smartly to maintain DSIC.

Take a valuation profile v with $v_1 \geq v_2 \geq \ldots \geq v_n$.

Winner 1 makes payment v_2; Agents 1 and 2 are given back $\frac{v_3}{n}$; Others are given $\frac{v_2}{n}$.

Total money collected is:

$$v_2 - \frac{n-2}{n} v_2 - \frac{2}{n} v_3 = \frac{2}{n}(v_2 - v_3),$$

which approaches zero for large n.

Asymptotically redistributes all the surplus (v_1 here).

Relax solution concept

Is there a BIC, efficient and budget-balanced mechanism?

We know from Cramton, Gibbons, Klemperer (1987) that in the presence of property rights, imposing IIR brings impossibility for many property rights structures.

Without any IR constraints, the answer to the above question is *YES*.

The dAGV mechanism

Let A be a finite set of alternatives and $v_i \in \mathbb{R}^{|A|}$ be the valuation vector of agent i.

Let $\mathcal{V}_i$ be the type space of agent i.

Let $\mathcal{V} \equiv \mathcal{V}_1 \times \ldots \times \mathcal{V}_n$. We will assume that types are drawn **independently**.

Efficient mechanism

An efficient mechanism is $(Q^e, T_i)_{i \in N}$ such that

$$Q^e(v) = \arg\max_{a \in A} \sum_{i \in N} v_i(a) \ \forall \ v \in \mathcal{V}.$$

Key construction: the map $r_i : \mathcal{V}_i \to \mathbb{R}$ for every $i \in N$.

$$r_i(v_i) = \mathbb{E}_{v_{-i}}\Big[\sum_{j \in N \setminus \{i\}} v_j(Q^e(v_i, v_{-i}))\Big] \qquad \forall \ v_i \in \mathcal{V}_i,$$

where $\mathbb{E}_{v_{-i}}$ is the expectation over valuations of other agents besides agent i.

Critical that the expectation in $r_i(v_i)$ can be computed without conditioning on the true type v_i.

Example

Suppose single object is allocated. Values are drawn from $[0, \beta]$ for all n agents.

Efficient allocation: highest agent gets the object.

What $r_i(v_i)$? Others get value when agent i is *not* the winner. Then, the value is $\max_{j \neq i} v_j$.

So, it is the conditional expectation of maximum of $(n-1)$ random draws conditioned on the fact that the maximum is higher than v_i.

Efficient mechanism

Residual utility. $r_i(v_i)$ captures the **expected welfare of others** when her type is v_i.

Arrow (1979) and d'Aspremont and Gerard-Varet (1979) define the transfer rules $\{T_i^{dagv}\}_{i \in N}$ as follows: for every $i \in N$,

$$T_i^{dagv}(v) = r_i(v_i) - \frac{1}{n-1} \sum_{j \in N \setminus \{i\}} r_j(v_j) \qquad \forall\ v \in \mathcal{V}.$$

The payment of agent i is the difference between the average residual utility of other agents and her own residual utility.

An interim analogue of the VCG idea - agents pay their **expected externality**.

The dAGV mechanism

Theorem

The dAGV mechanism is efficient, budget-balanced, and Bayesian incentive compatible.

Proof of theorem

Efficiency and budget-balancedness follows from the definition. To see BIC, fix agent i and two types v_i, v_i'. Note the following.

$$
\begin{aligned}
&\mathbb{E}_{v_{-i}}\Big[v_i(Q^e(v_i, v_{-i})) + T_i^{dagv}(v_i, v_{-i})\Big] \\
&= \mathbb{E}_{v_{-i}}\Big[v_i(Q^e(v_i, v_{-i})) + r_i(v_i) - \frac{1}{n-1}\sum_{j \in N \setminus \{i\}} r_j(v_j)\Big] \\
&= \mathbb{E}_{v_{-i}}\Big[v_i(Q^e(v_i, v_{-i})) - \frac{1}{n-1}\sum_{j \in N \setminus \{i\}} r_j(v_j) + r_i(v_i') - r_i(v_i') + r_i(v_i)\Big] \\
&= \mathbb{E}_{v_{-i}}\Big[v_i(Q^e(v_i, v_{-i})) + r_i(v_i') - \frac{1}{n-1}\sum_{j \in N \setminus \{i\}} r_j(v_j)\Big] \\
&+ \mathbb{E}_{v_{-i}}\Big[\sum_{j \in N \setminus \{i\}} v_j(Q^e(v_i, v_{-i}))\Big] - \mathbb{E}_{v_{-i}}\Big[\sum_{j \in N \setminus \{i\}} v_j(Q^e(v_i', v_{-i}))\Big]
\end{aligned}
$$

Proof of theorem

$$
\begin{aligned}
&= \mathbb{E}_{v_{-i}}\Big[\sum_{j\in N} v_j(Q^e(v_i, v_{-i})) + r_i(v_i') - \frac{1}{n-1}\sum_{j\in N\setminus\{i\}} r_j(v_j)\Big] \\
&- \mathbb{E}_{v_{-i}}\Big[\sum_{j\in N\setminus\{i\}} v_j(Q^e(v_i', v_{-i}))\Big] \\
&\geq \mathbb{E}_{v_{-i}}\Big[\sum_{j\in N} v_j(Q^e(v_i', v_{-i})) + r_i(v_i') - \frac{1}{n-1}\sum_{j\in N\setminus\{i\}} r_j(v_j)\Big] \\
&- \mathbb{E}_{v_{-i}}\Big[\sum_{j\in N\setminus\{i\}} v_j(Q^e(v_i', v_{-i}))\Big] \\
&= \mathbb{E}_{v_{-i}}\Big[v_i(Q^e(v_i', v_{-i})) + r_i(v_i') - \frac{1}{n-1}\sum_{j\in N\setminus\{i\}} r_j(v_j)\Big] \\
&= \mathbb{E}_{v_{-i}}\Big[v_i(Q^e(v_i', v_{-i})) + T_i^{dagv}(v_i', v_{-i})\Big],
\end{aligned}
$$

So, BIC holds.

Comments on the dAGV mechanism

General possibility result with independent types.

A way to escape the Green-Laffont impossibility.

Main issue: participation constraint. The dAGV may give negative interim payoffs to agents (even in a simple bilateral trading problem).