Theory of Mechanism Design - Final Examination

November, 2015; Duration: 3 hours; Total marks: 55.

Explain your answers clearly, but avoid unnecessary elaboration.

You can use all the results proved in class. All questions are on models with transfers. The standard assumptions (on private values etc.) discussed in the class apply.

1. A seller plans to allocate an object to two buyers whose values are drawn uniformly from the following range: (a) $[0, 10]$ for buyer 1 and (b) $[0, 20]$ for buyer 2. Answer the following questions.

   (a) The seller considers using the following allocation rule. At any valuation profile (v_1, v_2), if $\max(v_1^2, v_2) < 3$, then the object is not allocated. Else, the object is allocated to an agent $i \in \arg\max(v_1^2, v_2)$ with ties broken in favor of agent 1.

      i. Without constructing an explicit payment rule, argue that this allocation rule is implementable in dominant strategies. **(3 marks)**

      ii. Construct a payment rule which implements this allocation rule in dominant strategies such that (a) payments are non-negative and (b) the resulting mechanism satisfies individual rationality. **(5 marks)**

   (b) Write the precise form of the (expected revenue maximizing) optimal auction for this problem. **(3 marks)**

   (c) Pick $\theta \in [0, 10]$. What is the expected net utility of buyer 1 in the optimal auction if he has type θ? What is the expected net utility of buyer 2 in the optimal auction if he has type θ? **(6 marks)**

   (d) Give a type profile where the object is allocated in the optimal auction but inefficiently. **(3 marks)**

2. A set of agents are drawing their values for an object from $[0, 10]$. The seller can either sell it using the Vickrey auction or some other Bayesian incentive compatible, efficient, and interim individually rational mechanism. The criteria the seller uses to compare across mechanisms is the sum of interim net utility of the agents from the mechanisms - it will choose a mechanism that minimizes the sum of interim net utility of agents. Should he use the Vickrey auction? Explain your answer with a crisp argument. **(5 marks)**

3. A seller has two units of an object to sell to a single agent. He can either decide not to allocate any object or allocate one unit or allocate both the units to the agent. The

type of the agent is captured by its marginal value for the units - $t = (t_1, t_2)$, where t_1 is the value for unit 1 and t_2 is the marginal value for the second unit, i.e., if he gets both the units then his value is $t_1 + t_2$. Assume that the value for zero units is zero.

The agent can have three possible types $T = \{\theta^0, \theta^1, \theta^2\}$, where $\theta^0 = (55, 15), \theta^1 = (60, 25), \theta^2 = (40, 35)$. Consider the allocation rule $f : T \to \{0, 1, 2\}$ (it decides how many units to allocate):

$$f(\theta^0) = 0, f(\theta^1) = 1, f(\theta^2) = 2.$$

Is f implementable? Explain your answer. **(10 marks)**

4. In a bilateral trading problem with a buyer and a seller, the value of the buyer and the cost of the seller are distributed uniformly in $[0, 10]$.

   (a) Use an efficient, Bayesian incentive compatible, and individually rational mechanism to compute the allocation and payments when value of the buyer is 7 and cost of the seller is 5. **(5 marks)**

   (b) Use an efficient, Bayesian incentive compatible, and budget balanced mechanism to compute the allocation and payments when value of the buyer is 7 and cost of the seller is 5. **(5 marks)**

5. Consider a "combinatorial auction" setting with three bidders and two object $\{a, b\}$. The values of the bidders for objects is shown in Table 1.

	$\emptyset$	$\{a\}$	$\{b\}$	$\{a, b\}$
$v_1(\cdot)$	0	8	5	12
$v_2(\cdot)$	0	9	4	10
$v_3(\cdot)$	0	0	6	6

Table 1: Combinatorial auction

   (a) Suppose the seller uses a Groves mechanism, where $h_i(v_{-i}) = 0$ for all i and for all v_{-i}. What are the allocations and payments of the bidders using this mechanism at the valuation profile of Table 1. **(5 marks)**

   (b) What are the allocations and payments of the bidders using the VCG mechanism at the valuation profile of Table 1. **(5 marks)**