Assignment 2

Due date: **August 30, 2018**.

1. Suppose G is a directed weighted complete graph. Let the weights of edges of G satisfy the following **triangle inequality**: for any three edges (i,j), (j,k), and (i,k) we have $w(i,j)+w(j,k)\geq w(i,k)$. Show that a potential of G exists if and only if $w(i,j)+w(j,i)\geq 0$.

2. Find a feasible solution or determine that there are no feasible solutions for the following system of difference inequalities.

$$
\begin{aligned}
x_1 - x_2 &\leq 4 \\
x_1 - x_5 &\leq 5 \\
x_2 - x_4 &\leq -6 \\
x_3 - x_2 &\leq 1 \\
x_4 - x_1 &\leq 3 \\
x_4 - x_3 &\leq 5 \\
x_4 - x_5 &\leq 10 \\
x_5 - x_3 &\leq -4 \\
x_5 - x_4 &\leq -8.
\end{aligned}
$$

3. Suppose $G=(N,E,w)$ is a strongly connected digraph which has no cycle of negative length. Let p be a potential of G such that $p(i)=0$ and $p(j)=s(i,j)$ for all $j\in N\setminus\{i\}$, where $s(i,j)$ is the shortest path from i to j in G. Let q be any other potential of G such that $q(i)=0$. Show that $p(j)\geq q(j)$ for all $j\in N$.

4. Suppose $G=(N,E,w)$ is strongly connected digraph which has no cycle of negative length. Suppose G satisfies the following property: for every cut $(S,N\setminus S)$ of G, there exists $i\in S$ and $j\in N\setminus S$ such that $w(i,j)+w(j,i)=0$. Show that G satisfies potential equivalence.

5. Consider the flow graph in Figure 1. Compute the maximum flow of this flow graph using two approaches (both approaches run Ford-Fulkerson algorithm but asks you to pick a path from s to t in the residual graph in a particular way if there are more than one such path):

   - In the first approach, always pick an augmenting path from s to t in the residual graph which has the maximum number of edges.

- In the second approach, always pick an augmenting path from s to t in the residual graph which has the minimum number of edges.

Compare the number of iterations of Ford-Fulkerson algorithm in both approaches.

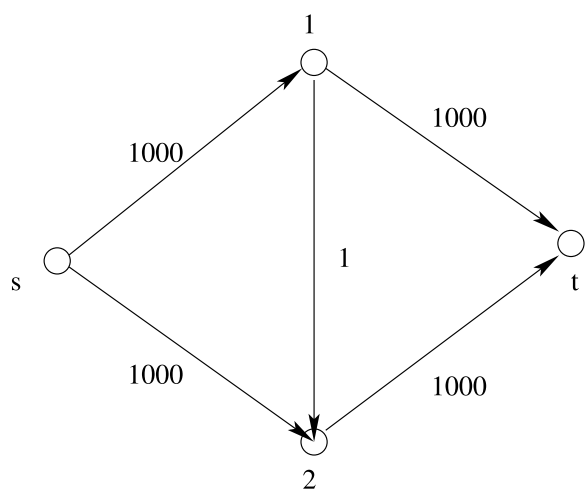

Figure 1: Maximum flow in a flow graph

6. Let $G = (N, E, c)$ be a flow graph and f be a feasible flow of this flow graph. Assume that $c : E \to \mathbb{Z}_+$ (integer capacities) and $f : E \to \mathbb{R}_+$ (real flow). Show that there exists another feasible flow f' of this flow graph such that $\nu(f') = \lceil \nu(f) \rceil$ and $f'(i,j) \in \{\lfloor f(i,j) \rfloor, \lceil f(i,j) \rceil\}$ for all $(i,j) \in E$.