

ASSIGNMENT 2

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1. Suppose G is a directed weighted complete graph. Let the weights of edges of G satisfy the following **triangle inequality**: for any three edges (i, j) , (j, k) , and (i, k) we have $w(i, j) + w(j, k) \geq w(i, k)$. Show that a potential of G exists if and only if $w(i, j) + w(j, i) \geq 0$.
2. Find a feasible solution or determine that there are no feasible solutions for the following system of difference inequalities.

$$x_1 - x_2 \leq 4$$

$$x_1 - x_5 \leq 5$$

$$x_2 - x_4 \leq -6$$

$$x_3 - x_2 \leq 1$$

$$x_4 - x_1 \leq 3$$

$$x_4 - x_3 \leq 5$$

$$x_4 - x_5 \leq 10$$

$$x_5 - x_3 \leq -4$$

$$x_5 - x_4 \leq -8.$$

3. For the directed graphs in Figure 1, verify if a potential exists. If a potential exists, then identify at least one potential.
4. Suppose $G = (N, E, w)$ is a strongly connected digraph which has no cycle of negative length. Let p be a potential of G such that $p(i) = 0$ and $p(j) = s(i, j)$ for all $j \in N \setminus \{i\}$, where $s(i, j)$ is the shortest path from i to j in G . Let q be any other potential of G such that $q(i) = 0$. Show that $p(j) \geq q(j)$ for all $j \in N$.
5. Find the maximum flow and the minimum capacity cut of the flow graph in Figure 2.

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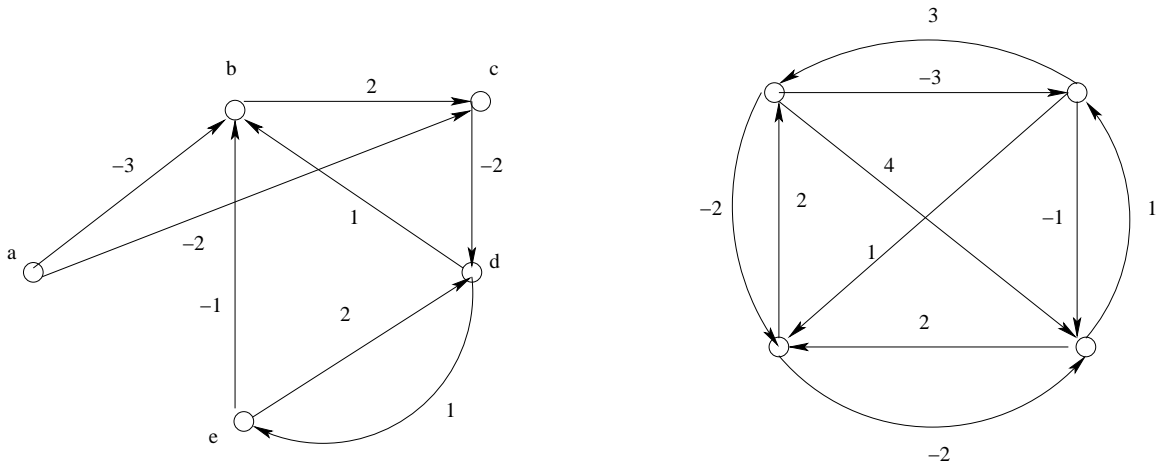


Figure 1: Two directed graphs

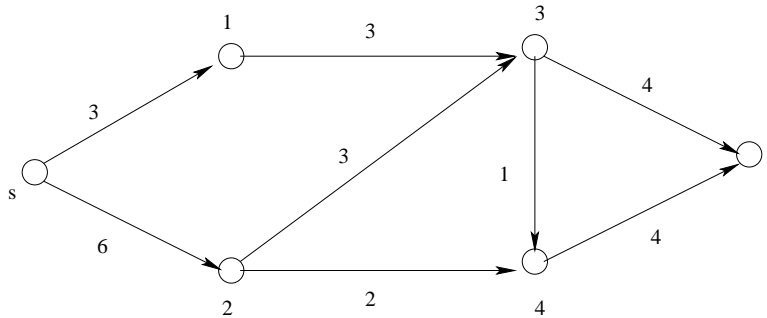


Figure 2: Maximum flow in a flow graph

6. Consider the flow graph in Figure 3. Compute the maximum flow of this flow graph using two approaches (both approaches run Ford-Fulkerson algorithm but asks you to pick a path from s to t in the residual graph in a particular way if there are more than one such path):

- In the first approach, always pick an augmenting path from s to t in the residual graph which has the maximum number of edges.
- In the second approach, always pick an augmenting path from s to t in the residual graph which has the minimum number of edges.

Compare the number of iterations of Ford-Fulkerson algorithm in both approaches.

7. Let $G = (N, E, c)$ be a flow graph and f be a feasible flow of this flow graph. Assume that $c : E \rightarrow \mathbb{Z}_+$ (integer capacities) and $f : E \rightarrow \mathbb{R}_+$ (real flow). Show that there exists another feasible flow f' of this flow graph such that $\nu(f') = \lceil \nu(f) \rceil$ and $f'(i, j) \in \{\lfloor f(i, j) \rfloor, \lceil f(i, j) \rceil\}$ for all $(i, j) \in E$.

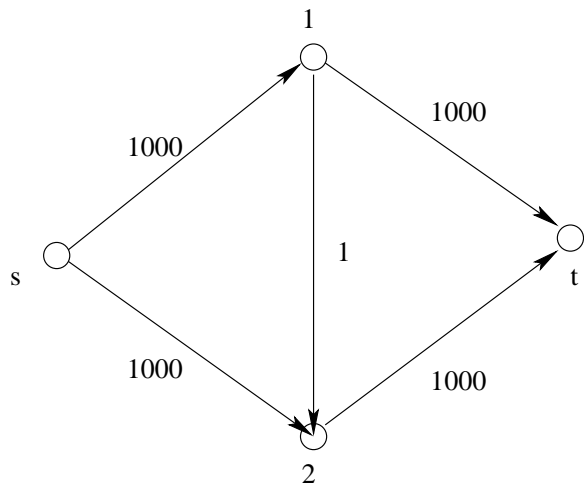


Figure 3: Maximum flow in a flow graph