Assignment 3

Due date: **September 6, 2018**.

1. Let $C \subseteq \mathbb{R}^n$. Then show that C is a closed convex set if and only if $C = \cap\, \mathbb{F}$, where $\mathbb{F}$ is a collection (possibly infinite of them) of half-spaces. (Hint: Consider the intersection of half-spaces that contain C and use separating hyperplane theorem)

2. Let A be a $m \times n$ matrix. Define $F = \{x \in \mathbb{R}^n_+ : Ax = 0, \sum_{j=1}^n x_j = 1\}$ and $G = \{y \in \mathbb{R}^m : yA > 0\}$. Show that either $F \neq \emptyset$ or $G \neq \emptyset$ but not both.

3. Let A be a $m \times n$ matrix. Prove that the system $Ax = 0$ has a **non-zero, non-negative** solution (i.e, $x \geq 0$ and $x \neq 0$) or there is a $y \in \mathbb{R}^m$ such that $yA > 0$, but not both. (Hint: Use the result from the previous question.)

4. Use Farkas Lemma to show that a weighted directed graph has a potential if and only if it has no cycles of negative length.