Assignment 4

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1. Consider the linear program (**LP**).

$$
\begin{aligned}
& Z = \max 2x_1 + x_2 \\
\text{s.t.} & \\
& 2x_1 + 3x_2 \leq 3 \\
& x_1 + 5x_2 \leq 1 \\
& 2x_1 + x_2 \leq 4 \\
& 4x_1 + x_2 \leq 5 \\
& x_1, x_2 \geq 0.
\end{aligned} \tag{LP}
$$

   (a) Draw the feasible region of (**LP**).

   (b) Solve (**LP**) using the simplex method.

   (c) At every step of the simplex method, read the feasible solution from the dictionary, and locate it in your drawing of the feasible region.

   (d) Which inequalities and non-negativity constraints are **tight** at the optimal solution of (**LP**)?

   (e) Write down the dual of (**LP**).

   (f) Find the optimal solution of dual of (**LP**) from the final dictionary of the simplex method of (**LP**).

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2. Consider the linear program (**LP-2**).

$$
\begin{aligned}
Z = \max\, & 3x_1 + 2x_2 + 4x_3 \\
\text{s.t.} & \\
x_1 + x_2 + 2x_3 &\leq 4 \\
2x_1 + 3x_3 &\leq 5 \\
2x_1 + x_2 + 3x_3 &\leq 7 \\
x_1, x_2, x_3 &\geq 0.
\end{aligned}
\tag{LP-2}
$$

Solve (**LP-2**) using the simplex method, and identify the tight constraints at the optimal solution.

3. Consider the linear program (**SP**)

$$
\begin{aligned}
Z = \max & \sum_{j=1}^{n} c_j x_j \\
\text{s.t.} & \\
\sum_{j=1}^{n} a_j x_j &\leq b \\
x_j &\geq 0 \qquad \forall\, j \in \{1, \ldots, n\}.
\end{aligned}
\tag{SP}
$$

Assume that $c_j > 0$ and $a_j > 0$ for all $j \in \{1, \ldots, n\}$, and $b > 0$. Prove that $Z = b \max_{j \in \{1,\ldots,n\}} \frac{c_j}{a_j}$.

4. Solve the following linear program using the (two-phase) simplex method.

$$
\begin{aligned}
\max\, & -x_1 - 2x_2 \\
\text{s.t.} & \\
-3x_1 + x_2 &\leq -1 \\
x_1 - x_2 &\leq 1 \\
-2x_1 + 7x_2 &\leq 6 \\
9x_1 - 4x_2 &\leq 6 \\
-5x_1 + 2x_2 &\leq -3 \\
7x_1 - 3x_2 &\leq 6 \\
x_1, x_2 &\geq 0.
\end{aligned}
$$

5. Consider the linear program (**P**).

$$\begin{aligned} & \max \sum_{j=1}^{n} c_j x_j \\ & \text{s.t.} \\ & \sum_{j=1}^{n} a_{ij} x_j \leq b_i \qquad \forall\, i \in \{1, \dots, m\} \\ & x_j \geq 0 \qquad \forall\, j \in \{1, \dots, n\}. \end{aligned} \tag{P}$$

(1)

Show that if (**P**) is unbounded then there exists x_k ($k \in \{1, \dots, n\}$) such that the following LP (**P**-k) is unbounded.

$$\begin{aligned} & \max x_k \\ & \text{s.t.} \\ & \sum_{j=1}^{n} a_{ij} x_j \leq b_i \qquad \forall\, i \in \{1, \dots, m\} \\ & x_j \geq 0 \qquad \forall\, j \in \{1, \dots, n\}. \end{aligned} \tag{P-k}$$

(2)