# Further Results on Strategy-proof Social Choice under Categorization *

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**Abstract**

We consider a social choice problem where the set of alternatives can be partitioned into categories based on some exogenous criteria. We extend the results in Sato (2012) by proving that a social choice problem under categorization satisfying richness* property, which is weaker than richness property in Sato (2012), is sufficient for every strategy-proof social choice function to be decomposable.

KEYWORDS: Strategy-proofness, richness* property, domain restriction, categorization.
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# 1 Introduction

The coincidence of strategy-proofness and non-dictatorship has always been an intriguing question since Alan Gibbard and Mark Satterthwaite proposed their impossibility result (Gibbard (1973), Satterthwaite (1975)) - famously known as the Gibbard-Satterthwaite (GS) Theorem - which states that any strategy-proof and unanimous social choice function defined over unrestricted domain of preferences over at least three alternatives would be dictatorial. On the other hand, it is well known that possibility results can emerge if we restrict the domain of preferences. For instance, if preferences are restricted to be single-peaked, then the generalized median rules are unanimous and non-manipulable (Moulin (1980), Weymark (2011), Achuthankutty and Roy (2016)).

Motivated from practical settings (Saari (2001), Mbih et al. (2008)), we consider social choice problems where the set of alternatives can be partitioned into categories based on some exogenous criteria. We propose a condition which we call richness* property on a social choice problem under categorization and prove that every strategy-proof social choice function on a social choice problem under categorization satisfying richness* property is decomposable. The richness* property requires that for each admissible preference $P$ and each category, there exists an admissible preference $P'$ such that: (i) both $P'$ and $P$ restricted to the category is the same; and (ii) when considering the admissible preferences induced on the set of categories, the upper contour set of the category at the preference $P'$ is the *smallest* containing the upper contours sets of the category at all the preferences $\tilde{P}$ such that both $\tilde{P}$ and $P$ restricted to the category is the same. Richness* property is weaker than the richness property in Sato (2012). To see this, observe that in a social choice problem under categorization, richness property requires the upper contour set of the preference $P'$ induced on the set of categories as required in (i) and (ii) is empty. Hence, our result offers a generalization of the decomposability result in Sato (2012).

We discuss some related literature to put our results in perspective. Some of previous results in this direction include Inada (1964) who consider two categories in an Arrovian framework and Sakai and Shimoji (2006) who also consider two categories but with weak preferences. Our framework bears a close resemblance with two strands of literature in social choice theory: (i) our assumption that the set of alternatives can be partitioned into categories based on some exogenous criteria closely resembles the social choice models with exogenous indifference classes (Barberà and Ehlers (2011), Sato (2009), Pramanik and Sen (2016)) where the indifference classes of the agent's preferences are exogenously given, and (ii) our decomposability theorem adds to the existing decomposability results in social choice theory such as the result in a multi-dimensional social choice framework (Breton and Sen (1999)) and the results in a multiple public goods setup (Reffgen and Svensson (2012)).

The rest of the paper is organized as follows. We introduce the basic framework of a social choice problem under categorization in the Section 2 and state our main result in the Section 3. The last section concludes the paper.

## 2 Preliminaries

Let $N = \{1, 2, \ldots, n\}$ be a finite set of *agents*. For a finite set $A$, a *preference* over $A$ is a linear order over $A$. By $\mathbb{L}(A)$, we denote all preferences over $A$. An element $a \in A$ is called the $k^{th}$ ranked alternative in a preference $P$ over $A$, denoted by $P(k)$, if $|\{x \in A \mid xPa\}| = k - 1$. For ease of presentation, sometimes we write a preference $P$ as $abc\ldots$ meaning that $a$ is the top-ranked alternative, $b$ is the second-ranked alternative and so on. For $P \in \mathbb{L}(A)$ and $B \subsetneq A$, the restriction of $P$ to $B$, denote by $P|B$, is defined as follows: $xP|By$ if and only if $xPy$ for all $x, y \in B$. A subset $\mathbb{D} \subseteq \mathbb{L}(A)^n$ is called a *domain* over $A$ and a subset $\mathcal{D}$ of $\mathbb{L}(A)$ is called an *agent domain* over $A$, and the restriction of an agent domain $\mathcal{D}$ over $A$ to a subset $B$ of $A$, denoted by $\mathcal{D}|B$, is defined as $\mathcal{D}|B = \{P|B \mid P \in \mathcal{D}\}$. In the rest of the paper, the domain $\mathbb{D} = \mathcal{D} \times \mathcal{D} \times \ldots \times \mathcal{D}$.

We consider a finite set of alternatives $X$. We call the tuple $\langle X, \mathbb{D}\rangle$ a *social choice problem*. A *social choice function* on the social choice problem $\langle X, \mathbb{D}\rangle$ is a mapping $f : \mathbb{D} \to X$. Throughout this paper, we are interested in the SCFs that are *strategy-proof* which has the usual meaning.

**Definition 2.1.** Let the triple $\langle X, \mathbb{D}, \mathcal{C}\rangle$ be a *social choice problem under categorization* if $\mathcal{C} = \{C_1, C_2, \ldots, C_t\}$ is a partition of $X$ with $t > 2$ such that for each $P_i \in \mathcal{D}$ and each pair $s, s' \in \{1, \ldots, t\}$:

$$[xP_iy \text{ for some } x \in C_s \text{ and } y \in C_{s'}] \implies [x'P_iy' \text{ for each } x' \in C_s \text{ and each } y' \in C_{s'}].$$

For $P \in \mathcal{D}$ and $C_s \in \mathcal{C}$, we define $B(C_s, P) = \{x \in X \mid xPy \text{ for all } y \in C_s\}$.

**Example 2.1.** Let $X = \{a, b, c, d, e\}$. Let $\mathcal{D} = \{P^1, P^2, P^3, P^4\}$ as represented in Table 2.1. Let $C_1 = \{a, b, c\}, C_2 = \{d, e\}, C'_1 = \{a, b\}, C'_2 = \{c, d, e\}$ and let $\mathcal{C} = \{C_1, C_2\}$ and $\mathcal{C}' = \{C'_1, C'_2\}$.

Both $\mathcal{C}$ and $\mathcal{C}'$ are partitions of $X$, but $\langle X, \mathbb{D}, \mathcal{C} \rangle$ is a social choice problem under categorization and $\langle X, \mathbb{D}, \mathcal{C}' \rangle$ is not.

| $P_1$ | $P_2$ | $P_3$ | $P_4$ |
|---|---|---|---|
| $a$ | $a$ | $d$ | $c$ |
| $c$ | $b$ | $e$ | $b$ |
| $b$ | $c$ | $c$ | $a$ |
| $d$ | $e$ | $b$ | $e$ |
| $e$ | $d$ | $a$ | $d$ |

Table 1: Social Choice Problem under Categorization

One interesting feature of strategy-proof SCFs in a social choice problem under categorization is *decomposability*. Next, we formally define *decomposability* of an SCF.

**Definition 2.2.** A SCF $f : \mathbb{D} \to X$ on the social choice problem under categorization $\langle X, \mathbb{D}, \mathcal{C} \rangle$ is *decomposable* if there exist a strategy-proof SCF $h : \mathbb{D}|\mathcal{C} \to \mathcal{C}$ and for each $s \in \{1, \ldots, t\}$ a strategy-proof SCF $g^s : \mathbb{D}|C_s \to C_s$ provided that $\mathbb{D}|C_s \neq \emptyset$ such that for each $P_N \in \mathbb{D}$,

$$f(P_N) = \begin{cases} g^1(P_N|C_1) \text{ if } h(P_N|\mathcal{C}) = C_1, \\ \cdots \\ g^k(P_N|C_k) \text{ if } h(P_N|\mathcal{C}) = C_k, \\ \cdots \\ g^t(P_N|C_t) \text{ if } h(P_N|\mathcal{C}) = C_t. \end{cases}$$

Sato (2012) provides a sufficient condition, called *richness* property, for every strategy-proof SCF to be decomposable in a social choice problem under categorization. For $Q_{C_s} \in \mathbb{L}(C_s)$, let $\mathcal{D}(Q_{C_s}) = \{P \in \mathcal{D} \mid P|C_s = Q_{C_s}\}$ be the set of preferences in $\mathcal{D}$, restriction of which to the category $C_s$, coincides with the preference $Q_{C_s}$. Formally,

**Definition 2.3.** A social choice problem under categorization $\langle X, \mathbb{D}, \mathcal{C} \rangle$ satisfies *richness* property if for $C_s \in \mathcal{C}$ and all $Q_{C_s} \in \mathcal{D}$, $\mathcal{D}(Q_{C_s}) \neq \emptyset$ means there is $P \in \mathcal{D}(Q_{C_s})$ such that $P|\mathcal{C}(1) = C_s$.

We provide a much weaker condition, called *richness** property, and prove that it is sufficient for decomposability of strategy-proof SCFs.

**Definition 2.4.** A social choice problem under categorization $\langle X, \mathbb{D}, \mathcal{C}\rangle$ satisfies *richness** property if for all $C_s \in \mathcal{C}$ and all $Q_{C_s} \in \mathbb{L}(C_s)$, $\mathcal{D}(Q_{C_s}) \neq \emptyset$ means there is $P \in \mathcal{D}(Q_{C_s})$ such that

$$B(C_s, P) \subseteq B(C_s, \tilde{P}) \text{ for all } \tilde{P} \in \mathcal{D}(Q_{C_s}).$$

**Example 2.2.** We present an example of a social choice problem with categorization that satisfies richness* property but violates richness property. We see that every strategy-proof SCF is decomposable. Let $X = \{a, b, c, d, e, f, g, h\}$ with $\mathcal{C} = \{C_1, C_2, C_3, C_4\}$ where $C_1 = \{a, b\}, C_2 = \{c, d\}, C_3 = \{e, f\}, C_4 = \{g, h\}$. Let $\mathbb{D} = \mathcal{D} \times \mathcal{D} \times \ldots \times \mathcal{D}$. The domain $\mathcal{D}$ is as presented below:

| $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ |
|---|---|---|---|---|---|---|
| $b$ | $b$ | $d$ | $c$ | $f$ | $f$ | $h$ |
| $a$ | $a$ | $c$ | $d$ | $e$ | $e$ | $g$ |
| $c$ | $e$ | $h$ | $a$ | $c$ | $g$ | $f$ |
| $d$ | $f$ | $g$ | $b$ | $d$ | $h$ | $e$ |
| $e$ | $c$ | $f$ | $h$ | $g$ | $b$ | $c$ |
| $f$ | $d$ | $e$ | $g$ | $h$ | $a$ | $d$ |
| $g$ | $g$ | $a$ | $e$ | $a$ | $c$ | $a$ |
| $h$ | $h$ | $b$ | $f$ | $b$ | $d$ | $b$ |

Table 2: Richness* Property

Take $C_1 = \{a, b\}$ and consider the ordering $Q_{C_1} = ab$. Then, $\mathcal{D}(Q_{C_1}) = \{P_3, P_4, P_5, P_7\}$. Note that, $B(C_1, P_4) = \{c, d\}$. Further note that, $\{c, d\} \subseteq B(C_1, P_i)$ for all $i = 3, 5, 7$. Thus, richness* property is satisfied for $C_1 = \{a, b\}$ and $Q_{C_1} = ab$. Further, observe that richness property is *violated* for $C_1$ and $Q_{C_1}$ as there doesn't exist $P \in \mathcal{D}(Q_{C_1})$ such that $P|\mathcal{C}(1) = C_1$. Similarly, richness* property can be verified that for all $C_s \in \mathcal{C}$ and $Q_{C_s} \in \mathbb{L}(C_s)$.

# 3 Results

In this section, we state and prove our main result.

**Theorem 3.1.** *Let the social choice problem under categorization $\langle X, \mathbb{D}, \mathcal{C}\rangle$ satisfy richness* property. Then an SCF $f : \mathbb{D} \to X$ is strategy-proof if and only if $f$ is decomposable.*

*Proof.* (If part) Assume for contradtiction that SCF $f : \mathbb{D} \to X$ is not strategy-proof and is decomposable into a strategy-proof SCF $h : \mathbb{D}|\mathcal{C} \to \mathcal{C}$ and if for each $s \in \{1, \ldots, t\}$ with $\mathbb{D}|C_s \neq \emptyset$, a strategy-proof SCF $g^s : \mathbb{D}|C_s \to C_s$ as in Definition 2.2. Since $f$ is not strategy-proof, then there exists $i \in N$, $P_N \in \mathbb{D}$ and $\tilde{P}_i \in \mathcal{D}$ such that $f(\tilde{P}_i, P_{N\setminus i})\ P_i\ f(P_N)$. We consider two cases:
CASE 1: In this case, we assume $f(P_N) \in C_s$ and $f(\tilde{P}_i, P_{N\setminus i}) \in C_s$ for some $C_s \in \mathcal{C}$. Since $f$ is decomposable, $f(P_N) \in C_s$ and $f(\tilde{P}_i, P_{N\setminus i}) \in C_s$ implies $h(P_N|\mathcal{C}) \in C_s$ and $h(\tilde{P}_1|\mathcal{C}, P_{N\setminus 1}|\mathcal{C}) \in C_s$. Therefore, $f(P_N) = g^s(P_N|C_s)$ and $f(\tilde{P}_i, P_{N\setminus i}) = g^s(\tilde{P}_i|C_s, P_{N\setminus i}|C_s)$. Then $f(\tilde{P}_i, P_{N\setminus i})\ P_i\ f(P_N)$ implies $g^s(\tilde{P}_i|C_s, P_{N\setminus i}|C_s)\ P_i|C_s\ g^s(P_N|C_s)$ which contradicts our earlier assumption that $g^s$ is strategy-proof.
CASE 2: In this case, we assume $f(P_N) \in C_s$ and $f(\tilde{P}_i, P_{N\setminus i}) \in C_{s'}$ for some $C_s, C_{s'} \in \mathcal{C}$ where $s \neq s'$. Since $f$ is decomposable, this means that $h(P_N|\mathcal{C}) = C_s$ and $h(\tilde{P}_i|\mathcal{C}, P_{N\setminus i}|\mathcal{C}) = C_{s'}$. Then $f(\tilde{P}_i, P_{N\setminus i})\ P_i\ f(P_N)$ implies $h(\tilde{P}_i|\mathcal{C}, P_{N\setminus i}|\mathcal{C})\ P_i|\mathcal{C}\ h(P_N|\mathcal{C})$ which contradicts the fact that $h$ is strategy-proof.

(Only-if part) Let $f : \mathbb{D} \to X$ be a strategy-proof SCF in a social choice problem under categorization $\langle X, \mathbb{D}, \mathcal{C} \rangle$ satisfying richness* property. For each $P_N \in \mathbb{D}$, we define an SCF $h : \mathbb{D}|\mathcal{C} \to \mathcal{C}$ as follows:

$$h(P_N|\mathcal{C}) = \begin{cases} C_1 \text{ if } f(P_N) \in C_1, \\ \ldots \\ C_k \text{ if } f(P_N) \in C_k, \\ \ldots \\ C_t \text{ if } f(P_N) \in C_t. \end{cases}$$

We establish the result using the following sequence of lemmas in proving our result.

**Lemma 3.1.** *The SCF h is well-defined.*

*Proof.* In this lemma, we prove that $h$ is well-defined. In particular, we prove that for every pair $P_N, P'_N \in \mathbb{D}$ with $P_N|\mathcal{C} = P'_N|\mathcal{C}$, we have $h(P_N|\mathcal{C}) = h(P'_N|\mathcal{C})$. Assume for contradiction that $h(P_N|\mathcal{C}) = C_s$ and $h(P'_N|\mathcal{C}) = C_{s'}$ where $s \neq s'$. Without loss of generality, let $C_{s'}\ P_i|\mathcal{C}\ C_s$ for some $i \in N$. By construction, $h(P_N|\mathcal{C}) = C_s$ implies $f(P_N) \in C_s$. This means that agent $i$ manipulates at $P_N$ via $P'_i$. This is a contradiction to our assumption that $f$ is strategy-proof. Therefore, $h$ is well-defined and this completes the proof of the lemma. ■

**Lemma 3.2.** *The SCF h is strategy-proof.*

*Proof.* Let $Q_N \in \mathbb{D}|\mathcal{C}$ and $Q'_i \in \mathbb{L}(\mathcal{C})$ be such that $(Q'_i, Q_{N\setminus i}) \in \mathbb{D}|\mathcal{C}$. We prove that $g^s(Q_N)\ Q_i\ g^s(Q'_i, Q_{N\setminus i})$. Let $P_N, \tilde{P}_N \in \mathbb{D}$ such that $P_N|\mathcal{C} = Q_N$ and $\tilde{P}_N|\mathcal{C} = (Q'_i, Q_{N\setminus i})$. Because $f$ is strategy-proof, either $f(P_N) = f(\tilde{P}_N)$ or $f(P_N)\ P_i\ f(\tilde{P}_N)$. If $f(P_N) = f(\tilde{P}_N)$ then there is nothing to prove. Therefore, $f(P_N)\ P_i\ f(\tilde{P}_N)$. Let $f(P_N) \in C_s$ and $f(\tilde{P}_N) \in C_{s'}$ for some $C_s, C_{s'} \in \mathcal{C}$ where $C_s \neq C_{s'}$. Now $f(P_N) \in C_s$ implies $h(Q_N) = C_s$ and $f(\tilde{P}_N) \in C_{s'}$ implies $h(Q'_i, Q_{N\setminus i}) = C_{s'}$. Also since $P_N|\mathcal{C} = Q_N$ and $\tilde{P}_N|\mathcal{C} = (Q'_i, Q_{N\setminus i})$, we have $C_s\ Q_i\ C_{s'}$. Therefore, $h(Q_N)\ Q_i\ h(Q'_i, QN \setminus i)$ which proves that $h$ is strategy-proof. This completes the proof of this lemma. ∎

**Lemma 3.3.** *For each $s \in \{1, \dots, t\}$ and each pair $P_N, \tilde{P}_N \in \mathbb{D}$ such that $P_N|C_s = \tilde{P}_N|C_s$, $f(P_N) \in C_s$ and $f(\tilde{P}_N) \in C_s$ implies $f(P_N) = f(\tilde{P}_N)$.*

*Proof.* Let $f(P_N) = x \in C_s$. Let $P'_N = (P'_i)_{i \in N} \in \mathbb{D}$ such that:

$$B(C_s, P'_i) \subseteq B(C_s, \tilde{P}_i).$$

for all $\tilde{P}_i \in \mathcal{D}(P_i|C_s)$. For each $i \in N$, the existence of such $P'_i$ is guaranteed by the *richness** property.

We claim that $f(P'_N) = x$. To see this, successively change agents' preferences one-by-one from the profile $P_N$ to $P'_N$. It must be that $f(P'_1, P_{N\setminus 1}) \in C_s$. Assume for contradiction that $f(P'_1, P_{N\setminus 1}) \in C_{s'}$ where $C_{s'} \neq C_s$. Observe that $B(C_s, P'_1) \subseteq B(C_s, P_1)$ which implies $(X \setminus B(C_s, P_1)) \subseteq (X \setminus B(C_s, P'_1))$. Hence by strategy-proofness, $C_{s'} \notin B(C_s, P'_1) \cup (X \setminus B(C_s, P_1))$. If $C_{s'} \in (B(C_s, P_1) \setminus B(C_s, P'_1))$ then agent 1 manipulates at $(P'_1, P_{N\setminus 1})$ via $P_1$. Therefore, $f(P'_1, P_{N\setminus 1}) \in C_s$ and since $P'_1|C_s = P_1|C_s$, by strategy-proofness, $f(P'_1, P_{N\setminus 1}) = x$. Continuing in this manner, we conclude that $f(P'_N) = x$.

Next, we claim that $f(\tilde{P}_N) = x$. To see this, successively change agents' preferences one-by-one from the profile $P'_N$ to $\tilde{P}_N$. Take agent 1. Assume for contradiction that $f(\tilde{P}_1, P_{N\setminus 1}) \notin C_s$. By strategy-proofness, $f(\tilde{P}_1, P_{N\setminus 1}) \notin B(C_s, P'_1)$. Therefore, $f(\tilde{P}_1, P_{N\setminus 1}) \in (X \setminus B(C_s, P'_1))$. Now take agent 2. By strategy-proofness, $f(\tilde{P}_1, \tilde{P}_2, P'_{N\setminus\{1,2\}}) \notin B(f(\tilde{P}_1, P_{N\setminus 1}), P'_2)$ and therefore, $f(\tilde{P}_1, \tilde{P}_2, P'_{N\setminus\{1,2\}}) \in (X \setminus B(f(\tilde{P}_1, P_{N\setminus 1}), P'_2)) \cup \{f(\tilde{P}_1, P_{N\setminus 1})\}$. In general,

$$f(\tilde{P}_{S\cup\{i\}}, P'_{N\setminus(S\cup\{i\})}) \in (X \setminus B(f(\tilde{P}_S, P'_{N\setminus S}), P'_i)) \cup \{f(\tilde{P}_S, P'_{N\setminus S})\}.$$

for all $S \subseteq N$ and $i \notin S$. However, this contradicts the fact that $f(\tilde{P}_N) \in C_s$. Therefore, $f(\tilde{P}_1, P_{N\setminus 1}) \in C_s$. Since $\tilde{P}_1|C_s = P'_1|C_s$, by strategy-proofness, $f(\tilde{P}_1, P'_{N\setminus 1}) = x$. Continuing in this manner, we conclude that $f(\tilde{P}_N) = x$. This completes the proof of the lemma. ■

Let $s \in \{1, \ldots, t\}$ and $\mathbb{D}|C_s \neq \emptyset$. For each $P_N \in \mathbb{D}$ such that $f(P_N) \in C_s$, define $g^s(P_N|C_s) = f(P_N)$.

**Lemma 3.4.** *For each $s \in \{1, \ldots, t\}$ with $\mathbb{D}|C_s \neq \emptyset$, the SCF $g^s$ is well-defined.*

*Proof.* In particular, we prove that $g^s(P_N|C_s) = g^s(P'_N|C_{s'})$ for every pair $P_N, P'_N \in \mathbb{D}$ with $f(P_N) \in C_s$ and $f(P'_N) \in C_s$ which immediately follows from Lemma 3.3. ■

**Lemma 3.5.** *For each $s \in \{1, \ldots, t\}$ with $\mathbb{D}|C_s \neq \emptyset$, the SCF $g^s$ is strategy-proof.*

*Proof.* Let $Q_N \in \mathbb{D}|C_s$ and $Q'_i \in \mathbb{L}(C_s)$ be such that $(Q'_i, Q_{N\setminus i}) \in \mathbb{D}|C_s$. We claim that $g^s(Q_N)\ Q_i\ g^s(Q'_i, Q_{N\setminus i})$. Consider $P_N \in \mathbb{D}$ and $\tilde{P}_i \in \mathcal{D}$ such that $P_N|C_s = Q_N$, $f(P_N) \in C_s$, $(\tilde{P}_i, P_{N\setminus i})|C_s = (Q'_i, Q_{N\setminus i})$ and $f(\tilde{P}_i, P_{N\setminus i}) \in C_s$. Since $f$ is strategy-proof, $f(P_N)\ P_i\ f(\tilde{P}_i, P_{N\setminus i})$. Also $f(P_N) \in C_s$ and $f(\tilde{P}_i, P_{N\setminus i}) \in C_s$ implies that $f(P_N)\ Q_i\ f(\tilde{P}_i, P_{N\setminus i})$ and therefore, $g^s(Q_N)\ Q_i\ g^s(Q'_i, Q_{N\setminus i})$ as desired. This completes the proof of the lemma. ■

Lemmas 3.1-3.5 shows that every strategy-proof $f$ is decomposable. This completes the proof of the only-if part and hence completes the proof of the theorem. ■

**Corollary 3.1** (Sato (2012))**.** *Let the social choice problem under categorization $\langle X, \mathbb{D}, \mathcal{C} \rangle$ satisfy richness property. Then every strategy-proof SCF $f : \mathbb{D} \to X$ is decomposable.*

# 4 Conclusion

This paper is concerned with providing weaker sufficient conditions so that every strategy-proof SCF is decomposable in a social choice problem under categorization where the set of alternatives is partitioned into categories based on some exogenous criteria. The main result of this paper proves that in a social choice problem under categorization satisfying the richness* property, every strategy-proof SCF is decomposable. Our result generalizes Sato (2012) as our richness* property is a weaker condition than the richness condition provided in Sato (2012). We intend to pursue the quest for a necessary and sufficient condition for decomposability of strategy-proof SCFs in a social choice problem under categorization.

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