Fertility, Child Labor and Physical Activity among the Nutritionally Poor: A Bio-economic Exploration

Abstract

This paper examines the role of nutritional poverty in household decision making. Preliminary micro-evidence from India show that households in hard labor-intensive occupations exhibit different nutritional status, fertility and child work-intensity relative to other households. Integrating insights from the physiological literature, the paper develops a household optimization model to explain occupational variation in nutritional status, and trade-offs between fertility and child health in an intertemporal equilibrium. Results suggest that fertility-child-health trade-offs do not exist among households engaged in physically demanding occupations; while other households may exhibit such trade-offs. These findings provide new insights into occupational variations in nutritional status and quantity-quality trade-offs.

Keywords: Physical activity level, Energy deficiency, Child labor, Fertility, Quantity-quality trade-off

JEL Classification: O1, I1, J1

1 Introduction

According to the latest Food and Agricultural Organization report, about 795 million individuals - more than one in nine - are undernourished. A vast majority of them, about 780 million, live in developing countries, where about 13% of the population suffer from nutritional deficiency (FAO, 2015). Inadequate access to food and nutrition is a manifestation of absolute poverty. However, a decline in poverty has not always improved nutritional status. The Indian experience elucidates the disconnect between poverty and nutritional status. India achieved significant reduction in poverty - from 37% to 28% over 1993-2004. During the same decade, the proportion of nutritionally deficient population increased from 68% to 76% (Gaiha et al., 2010). According to the National Nutrition Monitoring Bureau (1999) of India, the concentration of malnutrition is highest among those engaged in physically demanding activities (for example, agricultural laborers), whereas the workers in less strenuous activities (for example, artisans) experience much lower incidence of malnutrition despite consuming less calories than most others.[1] Similarly, the prevalence rates of chronic energy deficiency among women are found to be much higher among agriculture (41%) and manual labor activities (36%) compared to those in professional, sales and services (21%), controlling for standard of living, education, age, etc. (Bharati et al., 2007). A theoretical explanation of these observations is absent in the literature.

This research is further motivated by preliminary observations based on Indian household survey data: households in 'hard' activities engage their children more intensively at work and tend to have less children than their 'softer' counterparts after controlling for household income. An early evidence of a similar pattern is provided by Babu et al. (1993). In so far as child labor is a disinvestment in child 'quality', these observations run contrary to Becker's (1960) theory of quantity-quality trade-off that spawned a large literature on fertility and child outcome. However, empirical evidence has hardly been unanimous. While Rosenzweig and Wolpin (1980) showed a negative relation between family size and child's education attainment, recent studies contradict this finding (Black et al., 2005), Angrist et al., 2005). Millimet et al. (2011) find that this trade-off

[1] See, Tables 6.12, 7.12, Average Nutrient Report of Second Repeat Survey,1999.

holds only for some. Another piece of puzzle in the world of malnutrition, also supported by Indian data, is that despite their limited physical capabilities, children often work intensively in physically demanding activities. According to the FAO, about 60% of child labor is found in agriculture.[2] The complementarity between parent and child work is empirically observed by Goldin (1979) in nineteenth century Philadelphia. But, why children are employed by parents in strenuous activities in poor societies remains largely unexplained.

The objective of this paper is to complement the insights from the poverty-based literature by offering a novel bio-economic mechanism that explains why (i) quantity-quality trade-offs may vary across occupations and (ii) there exists a positive relationship between adult and child labor in hard occupations. To this end, I integrate insights from physiological sciences to develop a simple theoretical model of household decisions on child and adult work intensities and fertility. This paper provides the first formalization of the physiological mechanism in the fertility and child labor literature.

The interaction between the physiological and economic mechanisms is quite intuitive. Individuals employed in activities that require high physical activity level (PAL), spending more energy per unit of time than those employed in relatively sedentary occupations (Rai, 2012). Production in labor-intensive activities requires a high degree of PAL, which raises the subsistence calorie-needs of workers of similar stature (Fogel, 1994). Thus, subsistence consumption rises with PAL, given body-size. If increase in calorie requirement outweighs the increase in calorie intake from the extra output, future health is likely to deteriorate, resulting in loss of future productivity and consumption. Thus, adults living at subsistence levels, face a trade-off between current work-effort and productivity and consumption at an older (mature) age. Mature-age consumption also depends on income transfer from now-adult children.[3] However, high PAL in childhood reduces adulthood income and the magnitude of this transfer. Since child-bearing is costly, parents face a second trade-off between quantity of children on the one hand, and child work intensity on the other.

Workers engaged in non-labor-intensive sectors (e.g. those in sales and services) face a lower

[2] See: http://www.fao.org/childlabouragriculture/en/

[3] Vlassoff and Vlassoff (1980) and many subsequent studies show the importance of old-age support as a determinant of fertility in rural India.

likelihood of running an energy deficiency, because PAL is lower. This obviates any trade-off involving current PAL and own future productivity, and unlike the high-PAL workers, they face only one trade-off: quantity versus quality of children. Therefore, occupational variations in PAL is critical in understanding why nutritional status, fertility and child labor vary with occupational categories.[4]

This study is connected to an influential body of research on undernutrition, health and productivity (for example, Bliss and Stern, 1978a; Strauss, 1986; Deolalikar, 1988; Haddad and Bouis, 1991). Most studies examine the effect of undernutrition on health and productivity. This paper highlights the importance of the reverse channel: how work-effort affects nutritional status and health. So far only a few studies, most notably, Dalgaard and Strulik (2007, 2011), investigated this channel. They use similar physiological mechanism to highlight the role of nutritional requirements in the trade-off between fertility and nutritional status of children, and in re-interpreting the adult efficiency-wage argument. Similar to theirs, nutritional imbalance in this model affects stature and consequently, future productivity. However, these studies do not explore the implications on child labor and its interactions with fertility.

In a related work Glomm and Palumbo (1993) suggest that adverse nutritional shocks due to, say, crop failures, induce parents to raise child labor to boost current consumption and survival chances. Unlike their model, here health is endogenous and the foremost means of saving and consumption smoothing occurs through choice of fertility and work-intensities of family members. As an explanation to the adult-child labor complementarity, Genicot (2005) suggests employers tend to employ children along with their parents to internalize the productivity enhancing effects of efficiency-wages paid to the adults. However, Wahba (2006) empirically observes a negative correlation between incidence of wage-work and unskilled wages for boys in Egypt. Different from Genicot (2005), in the present model 'wages' are endogenously determined by worker's occupation and effort-levels in informal markets where efficiency wages are absent.

The paper is structured as follows. The next section discusses some empirical preliminaries using household survey data from India. The biological foundation of minimum consumption is discussed

[4]Differences in basal metabolic rates (BMR) is the other source of individual variation in energy expenditure, and is largely determined by attributes such as age, sex, and body-size (body mass index) (James et al., 1998).

in section 3. Section 4 develops the baseline model. Section 5 discusses the household optimal choices of fertility, own and child's supply of effort in intertemporal equilibrium and compare them with that of a standard model. Section 6 concludes with some policy implications.

2 Empirical Preliminaries

This section presents some preliminary insights on the role of adult-PAL in determining fertility and work-intensities of children, using the India Human Development Survey (IHDS, 2005) - a nationally representative survey dataset on India.[5] This dataset contains observations on most of the relevant covariates such as calorie intake, adult health status, work-hours of adults and children, as well as occupational classification. Like other survey data, observations on individual PAL is absent in this dataset as well, but occupational classification based on physical labor intensity of production provides a reasonable approximation to individual worker's PAL.

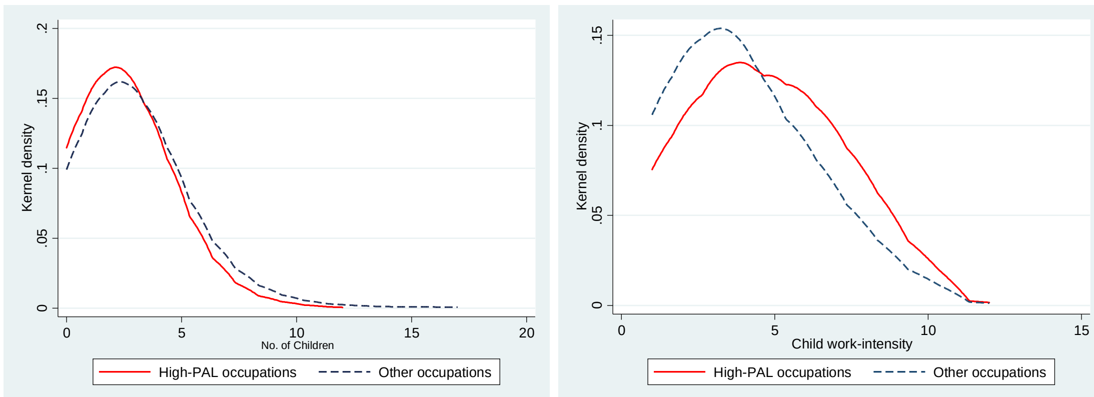
The occupational categories are categorized into two broad occupation groups: high-PAL labor, which includes agricultural labor and other physically demanding activities, and 'other' (non high-PAL) labor.[6] In the IHDS (2005) sample, there are $24,423$ workers in high-PAL and $191,329$ in the other occupations. Of the high-PAL workers, $17,845$ (73.5%) are agricultural laborers. Fig. 1 presents kernel-density plots of number of children (weighted by the number of married women) and child work-hours per day (weighted by the number of children), respectively, at the household level. The average age of women in the sample is 32.77 years, hence number of children do not necessarily reflect completed fertility. However, because there is no reason to expect future fertility of women in hard occupations will be greater than those in the other occupations, these densities provide a reasonable approximation of the difference in distributions of completed fertility. Fig. 1(a) shows that fewer high-PAL workers have greater than two children, while Fig. 1(b) depicts they are more

[5] IHDS is jointly organized by researchers from the University of Maryland and the National Council of Applied Economic Research (NCAER), New Delhi. The data used here come from the first round survey completed in 2004-05, covering 41,554 households in 1503 villages and 971 urban neighborhoods across India.

[6] In addition to agricultural labor, the other physically demanding work comprises of Plantation Labour and Related Work, Mining, Quarrying, Well Drilling and Related Work, Plumbing, Welding, Sheet Metal and Structural Metal Preparation, Bricklaying and Other Constructions, and Loading.

likely to have children work for longer hours than workers in other occupations. Epps-Singleton (1986) test results confirm that the difference in the distributions are statistically significant.[7]

Figure 1: Kernel density plots of fertility and child work-intensity



(a) (b)

Table 1 tests for the mean differences of the key variables at the household level. As in Fig. 1(b), work-intensity is measured by hours worked per day in farm work. Household income includes all sources of income. Adult health-status is measured in three ways - body weight, number of days in illness, and number of days unable to work. The last column in Table 1 shows that by all measures, adult health status is significantly worse among high-PAL workers - that is, they have lower body weight, higher days of illness and days unable to work. Additionally, the high-PAL adults on average work less per year, possibly due to seasonal nature of these occupations. In contrast, daily work-hours for children are higher among the high-PAL households, and more curiously, these are spread across fewer number of children in such households. Finally, the high-PAL workers have significantly lower household income and consumption (calorie intake) than their counterparts in other occupations.[8]

[7] Epps-Singleton test, instead of more common Kolmogorov-Smirnov two-sample (KS) test is used because the former is more powerful than the latter, and the latter is not suitable when data are drawn from discrete distributions such as fertility (measured in discrete integers) and work-hours (usually measured in 15 or 30-minute intervals).

[8] Assuming prices of basic food items do not vary significantly across regions in the rural areas, per capita consumption expenditure can be used as a proxy for calorie intake.

Table 1: Summary statistics for key variables by occupational category

	Occupational Category				
	High-PAL		Other		Mean Diff.
	Obs.	(1)	Obs.	(2)	(1) − (2)
Adult health status:					
Adult weight (kg.)	5174	45.57(19.62)	27252	50.62(16.73)	-5.05^{***}
Days of illness/yr.	1930	8.56(7.44)	16280	7.15(6.05)	1.41^{***}
Days unable to work/yr.	1930	7.38(6.87)	16280	5.85(5.44)	1.53^{***}
Adult work hrs./day	24032	7.94(1.12)	23751	8.00(1.92)	-0.06^{***}
Adult work hrs./yr.	24032	1346.14(753.07)	23751	2112.15(851.49)	-766.00^{***}
No. of children in hh$^{\flat}$	24032	1.92(1.66)	184423	2.23(1.89)	-0.31^{***}
Child work-hrs./day$^{\sharp}$	507	5.61(2.05)	7379	3.93(2.18)	1.68^{***}
Hh. income (Rs.)	23781	29919.64(29973.42)	182475	63871.46(98077.84)	-33951.81^{***}
Cons. per head (Rs.)	23781	543.04(419.92)	182475	895.11(895.28)	-352.08^{***}

Note: Standard deviations in the parentheses; $\flat$ denotes ages 1-14; $\sharp$ denotes ages 5-17; *,**, and *** denote statistical significance at $10\%, 5\%$, and 1% levels, respectively.

While these occupational comparisons are informative, it is hard to rule out the role of income or consumption poverty among the high-PAL households as drivers of these differences. In order to control for these potential correlations, we report ordinary least squares estimates in Table 2. Using two specifications that differentiate between consumption and income per head, regression results in Table 2 suggest adult work-hours not only predict more child hours but to a greater extent for high-PAL households. Fertility on the other hand, is likely to be associated with lower child work-hours and to a greater extent in high-PAL households. Although potential endogeneity of fertility does not allow for causal interpretation of these results, this preliminary exercise highlights the need for an alternative explanation for fertility-work-intensity-malnutrition relationships.

Table 2: Estimation results for fertility and child work-hours in farms

Dependent Variable: Child work hrs./day	Model 1	Model 2
High-PAL dummy	-0.25	-0.40
Adult work hrs./day	0.09^{***}	0.07^{***}
Adult work hrs./day x High-PAL dummy	0.19^{***}	0.22^{***}
Adult weight	-0.003^{**}	-0.01^{***}
Adult weight x High-PAL dummy	-0.006	-0.01
No. of children	-0.12^{***}	-0.08^{***}
No. of children x High-PAL dummy	-0.09^{***}	-0.08^{***}
Urban dummy	0.38^{***}	0.16
Consumption per head (in log)	-0.50^{***}	
Income per head (in log)		0.10^{***}
Constant	6.76^{***}	2.77^{***}
N	9746	9631
$Wald\ \chi^2(p-value)$	0.00	0.00

Note: *,** ,and *** denote statistical significance at $10\%, 5\%$, and 1% levels, respectively.

In what follows I provide a plausible underlying explanation based on an occupation-specific measure of minimum consumption requirement integrated in a model of household optimization.

3 A Biological Foundation of Minimum Consumption

Humans perform a variety of routine physical activities that are determined by technology of production, socioeconomic conditions, and lifestyle. They range from occupational labor to daily essential chores. Table 3 shows the total energy costs, expressed as multiples of basal metabolic rate for some activities in a typical agricultural household. The first step in formulating a measure of minimum consumption requirement is to focus on the relationship between energy requirement under physical activity as well as at rest. Let the energy consumed at rest, or the basal metabolic rate of an organism be E_0 calories per day. According to Kleiber's Law, $E_0 \propto m^b$, where m is the mass of the organism and $b = 0.75$ (Kleiber, 1932). At an intuitive level, the Law states that higher the body mass, the less is the energy requirement per unit of body mass in order to sustain life. This formulation has been given rather deep micro-foundations by West et al. (1997) and Banavar et al. (1999).[9] Since basal metabolic rate captures the energy needs of a body at rest, that is, it is the nutritional needs of a worker not participating in any physical activity. This energy need is proportional to body mass index (BMI). Let $E_0 = am^b$, where a is the proportionality constant.

Table 3: Energy requirements in adults

Main daily activities	Energy cost/hr.	Main daily activities (contd.)	Energy cost/hr.
Sleeping	1.0	Bundling rice	3.7
Eating and drinking	1.4	Chopping wood (for fuel)	4.2
Cooking	2.1	Spraying crops	4.3
Personal care (dressing/bathing)	2.3	Mopping/washing floor	4.4
Child care	2.5	Collecting water (from well)	4.5
Washing clothes (sitting/squatting)	2.8	Fertilizing (spreading manure)	5.2
Carrying straw	3.1	Threshing	5.4
Carrying 20-30 kg load on head	3.5	Digging	5.6

Source: Annex 5, FAO/WHO/UNU expert consultation report, World Health Organization (1985).

On the other hand, exerting 'effort' uses up additional energy in order to support the muscular contractions involved in body postures and movements. Both the proportionality constant a and the scaling exponent b have been empirically observed to rise with effort levels for animals and humans.[10] Therefore, energy needs of an active body rises with effort level. Let e represent a measure of physical activity per day, and be normalized such that $e \in [0, 1]$. At one extreme, $e = 0$ implies complete physical rest, whereas $e = 1$ denotes maximum level of physical activity during the day. In contrast to Bliss and Stern (1978a, 1978b) this notion of 'activity' is independent of the

[9] The theory has been used to explain a wide variety of biological problems from "genomes to ecosystems" (West and Brown, 2005). Dalgaard and Strulik (2007) provide a brief introduction to the energy network theory and an economic application on the development of human body size and population size over the long run.

[10] See Dalgaard and Strulik (2011) and the references therein.

number of 'tasks' being performed. Instead, following the nutritional and physiological literature, e is conceptualized as having a close positive correspondence with PAL for a given work intensity.

According to the famous Harris-Benedict equation (Harris and Benedict, 1919), the energy intake of an active body is proportional to the product of basal metabolism and extra energy needs for activity, $E \propto m^b m^c$. Energy expenditure of a person with body mass m, exerting effort e per day (and resting for the remainder of the day), is computed by the weighted geometric mean: $E(e) = \left(am^b\right)^{1-e}\left(a_e m^{b+c}\right)^e$, where a_e denotes a proportionality constant when $e = 1$ (Dalgaard and Strulik, 2011). At the maximum daily activity level an organism reaches maximum metabolism, $a_e m^{b+c}$, whereas complete inactivity implies basal metabolism in keeping with Kleiber's Law, am^b. This formulation is validated by experimental data on human subjects. In particular, total energy expenditure is found to rise with e in a manner consistent with the m^{b+ec} specification.[11] Therefore, energy expenditure function of a person with body mass m is given by:

$$E(e, m) = \left[am^b\right]^{1-e}\left[a_e m^{b+c}\right]^e \tag{1}$$

Letting $\rho \equiv a_e/a$ denote a proportionality constant capturing the ratio of energy need when $e = 1$ to the energy need when $e = 0$, (1) can be simplified to:

$$E(e, m) = a\rho^e m^{b+ce} \tag{2}$$

Using the definition of BMI, the energy requirement of a person with BMI B^i, height h^i,exerting an effort-level e^i ($i = adult, child$), is given by $E(e^i, B^i, h^i) = a\rho^e[B^i(h^i)^2]^{b+ce^i}$. Let one unit of consumption good (c) yield η units of energy (in KJ or $kcal$), that is, η is the energy exchange rate. Thus, the level of consumption sufficient to cover this energy needs defines the *minimum consumption* requirement, which for adults and children at time t are given by, respectively:

$$\bar{c}_t^a = (a/\eta)\rho^{e_t^a}[B_t^a(h_t^a)^2]^{b+ce_t^a} \tag{3}$$

$$\bar{c}_t^c = (a/\eta)\rho^{e_t^c}[B_t^c(h_t^c)^2]^{b+ce_t^c} \tag{4}$$

[11] See Westerterp (2001) and Dalgaard and Strulik (2011) for a discussion.

Note that, minimum consumption is increasing and convex in effort-level, ($\overline{c}^{i\prime} > 0, \overline{c}^{i\prime\prime} > 0$).

4 The Model

4.1 Production

The production side represents an informal subsistence economy with no formal labor-leisure trade-offs. The consumption good can be produced by adults and children alike, with one difference: while the level of output produced by an adult depends only on her physical effort, $E(B_t^a, h_t^a, e_t^a)$, that of the children depends on a combination of their number (n) and their individual physical effort $E(e^c, B^c, h^c)$. Thus, the production technologies are given by, respectively:

$$E(B_t^a, h_t^a, e_t^a) \equiv y(B_t^a, e_t^a) \text{ given } h_t^a \tag{5}$$

$$x(E(B_t^c, h_t^c, e_t^c), n_t) \equiv g(B_t^c, e_t^c, n_t) \text{ given } h_t^c \tag{6}$$

The production functions are assumed to satisfy the INADA conditions with respect to each input. We also assume $y(B_t^a, 0) = g(B_t^c, 0, n_t) = 0$ - that is, all forms of production processes require non-zero physical effort. In the reduced form, health status is a key input to production, as in Dasgupta (1997).

4.2 Households

The economy is populated by individuals who live for three periods - childhood, young adulthood and mature adulthood. In childhood an individual consumes and works, in young adulthood she takes all decisions such as current consumption, number of children, own and children's work-effort (PAL), given the resource constraints. The utility function is given by $U(c_t, c_{t+1}) = u(c_t) + \beta u(c_{t+1})$, where $u' > 0, u'' < 0$, and $\beta \in (0, 1)$ is the subjective discount factor. A household's earning in period t is comprised of output production from adult work-effort $e_t^a \in [0, 1]$ and child work-effort

$e_t^c \in [0,1]$. To focus on children's energy expenditure, child labor is defined in terms of child's work-intensity instead of a time allocation problem between work and school.[12] In less developed societies, children typically do not allocate their time independently of their parents. A child is often entrusted to perform certain 'chores' requiring a certain physical intensity, and often indivisible in time - for example, carrying straw from the field to the cattle shed requires a fixed amount of time. Once engaged in this chore, a child has no flexibility to perform a fraction of the job. Parental decision to engage a child in carrying straw or mopping floor, more or less determines the child's energy requirement from this activity. Hence, in an environment of obligatory chores that are by nature time-indivisible, child labor can be viewed as a choice of her PAL rather than her time-allocation at work.

Health status (BMI) of an individual lies at the heart of the mechanism of the fertility-child labor-health relationship through its effect on human productivity.[13] Consumption smoothing adults care about mature-age health. In addition to aging-related depreciation, mature-age health depends on energy imbalance in adulthood. Specifically, an energy shortfall (surplus) in period t has a deleterious (positive) effect on BMI or health in period $t+1$. For simplicity, individuals are assumed to fully internalize the effect of this energy imbalance in their decisions.

A young adult (parent) solves the following problem:

$$Max\ \{u(c_t) + \beta u(c_{t+1})\} \tag{7}$$

$$\begin{aligned}
s.t.\ c_t &= (1-\gamma)[y(B_t^a, e_t^a) + g(B_t^c, e_t^c, n_t) - q(e_t^c)n_t] && (8)\\
c_{t+1} &= y(B_{t+1}^{ma}) + \gamma n_t \widetilde{y}(B_{t+1}^a) && (9)\\
B_{t+1}^{ma} &= B_t^a(1-\delta) + \theta(c_t - \overline{c}_t^a),\ \ \overline{c}_t^{a\prime}, \overline{c}_t^{a\prime\prime} > 0 && (10)\\
B_{t+1}^{a} &= B_t^c(1+\mu) + \theta[q(e_t^c) - \overline{c}_t^c];\ \ q', \overline{c}_t^{c\prime} > 0, q'' < 0, \overline{c}_t^{c\prime\prime} > 0 && (11)
\end{aligned}$$

[12] This does not imply that the model is unable to incorporate schooling. As long as schooling involves only fixed cost, which is likely to be the case for public school attendees, it can be considered to be part of child rearing cost, q. Although schooling may raise productivity in adulthood, such dividends are likely to be low in labor intensive activities.

[13] Body mass is known to be a good predictor of risk to morbidity and mortality as shown in Floud (1992) and Fogel (1997).

To minimize notational clutter, only the relevant arguments of a function are carried in the text. For example, $y(B^a_{t+1}, e^a_{t+1})$ and $g(B^c_t, e^c_t, n_t)$ are written as $y(B^a_{t+1})$ and $g(e^c_t, n_t)$, respectively. In terms of notations, the superscripts denote the generation, while the subscripts denote time period. Thus, B^{ma}_{t+1} denotes health of individuals who belong to mature-adult generation at time $t+1$. An adult transfers a fraction γ of her earnings to her parent. The value of γ is assumed to be exogenous, determined by the existing social norm. At time t, an adult gives birth to n_t identical children. Rearing each child costs $q(e^c_t)$ units of consumption good that rises with e^c_t. Therefore, a child's nutritional status is endogenous to her work-intensity. Children provide an additional source of income both in childhood and adulthood. Each child possesses health level B^c_t and exerts effort e^c_t, and together they produce a net output of $g(e^c_t, n_t) - q(e^c_t)n_t$. Note that children's physical effort is not an input to adult production function. This is in line with the available evidence that child work is substitute to that of an adult. For example, using detailed household data from Rwanda, Bhargava (1997) confirms that children enable women to increase their resting time. Given these, the budget constraint of an adult is given by (8).

Effort-level in the mature-age is assumed to be given, and normalized to unity without loss of generality. Therefore, the level of production at the mature-age is solely determined by the mature-age BMI, B^{ma}_{t+1}. A person with lower B^{ma}_{t+1} therefore needs larger amount of transfer from now-adult children to attain her optimal level of old-age consumption. The aggregate transfer from children is given by $\gamma n_t \widetilde{y}(B^a_{t+1})$, where $\widetilde{y}(B^a_{t+1}) \equiv [y(B^a_{t+1}) + g(e^c_{t+1}, n_{t+1}) - q(e^c_{t+1})n_{t+1}]$. Note that this transfer is increasing in 'quantity' (n_t), as well as 'quality' (B^a_{t+1}) of children. Since B^a_{t+1} decreases in e^c_t, parents face a trade-off between current income from child labor, and future transfer. This trade-off is critical in determining the number and effort-level of children.

Eq. (10) denotes a simplified form of an adult's health accumulation function. Health in young adulthood (B^a_t) is assumed to depreciate through a natural ageing process at the rate $\delta \in (0, 1)$. More importantly, mature-adulthood health (B^{ma}_{t+1}) depends on the extent of energy-deficit in young adulthood. Energy-deficit in adults could be positive, zero or negative depending on whether calorie intake (ηc_t) has been less, equal or more than calorie spent through physical activity ($\eta \overline{c}^a_t$) , i.e. whether $c_t \lesseqqgtr \overline{c}^a_t$. In case of a prolonged energy deficiency ($c_t < \overline{c}^a_t$), B^{ma}_{t+1} deteriorates, eventually leading to chronic energy deficiency (or BMI < 18.5), leading to mature-age morbidity and loss in productivity. Likewise, a surplus energy *i.e.*, $c_t > \overline{c}_t^a$, improves future health and productivity. The parameter $\theta \in (0,1)$ measures the responsiveness to an energy disequilibrium on mature-age health. Children experience natural growth in stature at a rate $\mu > 0$. A child's future health deteriorates (improves) if $q(e_t^c) < (>)\ \overline{c}_t^c$. The literature does not provide a clear indication if higher work-intensity improves or harms future stature.[14] Child's work-effort is assumed to negatively affect long-term health in the high-PAL occupations, but not in other occupations.

5 Household Equilibrium

Assuming all the constraints hold with quality, maximizing the utility given in 7 with respect to e_t^a, e_t^c and n_t yields the following first order conditions:

$$(1-\gamma)u_t' y_e'(e_t^a) + \beta\theta u_{t+1}' y_{B^{ma}}'(B_{t+1}^{ma})\frac{\partial B_{t+1}^{ma}}{\partial e_t^a} = 0 \tag{12}$$

$$(1-\gamma)u_t'[g_e'(e_t^c, n_t) - q'(e_t^c)n_t] + \beta u_{t+1}' \left[y_{B^{ma}}'(B_{t+1}^{ma})\frac{\partial B_{t+1}^{ma}}{\partial e_t^c} + \gamma n_t y_{B^a}'(B_{t+1}^a)\frac{\partial B_{t+1}^a}{\partial e_t^c} \right] = 0 \tag{13}$$

$$(1-\gamma)u_t'[g_n'(e_t^c, n_t) - q(e_t^c)] + \beta u_{t+1}' \left[y_{B^{ma}}'(B_{t+1}^{ma})\frac{\partial B_{t+1}^{ma}}{\partial n_t} + \gamma \widetilde{y}(B_{t+1}^a) \right] = 0 \tag{14}$$

Condition (12) implies that e_t^a is chosen such that the marginal utility of increased effort in terms of consumption good, $(1-\gamma)u_t' y_e'(e_t^a)$, is equal to the discounted value of marginal disutility arising from energy imbalance due to the increased effort, $\beta\theta u_{t+1}' y_B'(B_{t+1}^{ma})\frac{\partial B_{t+1}^{ma}}{\partial e_t^a}$. The optimal level of e_t^c equates the two-fold gain - marginal gain from child's income, $(1-\gamma)u_t'[g_e'(e_t^c, n_t) - q'(e_t^c)n_t]$, and gain in discounted future productivity $\beta u_{t+1}' y_{B^{ma}}'(B_{t+1}^{ma})\frac{\partial B_{t+1}^{ma}}{\partial e_t^c}$, with the discounted marginal loss

[14] A higher work-intensity may imply higher child income, which in turn may supplement calorie intake and improve future health status. On the other hand, higher work-intensity in hard occupations (e.g., agriculature, construction or mining) may expose working children to health hazards from injury or illness that harm their growth potential. Evidence on this topic using micro-data is ambiguous. For example, while O'Donnel et al. (2005) find no effect of child labor on growth of children, Wolff and Maliki (2008) find childhood work worsen future health outcome.

due to reduced transfer, $\beta\gamma u'_{t+1} n_t y'_{B^a}(B^a_{t+1})\frac{\partial B^a_{t+1}}{\partial e^c_t}$, as a result of adverse productivity consequences of child labor. Finally, the optimal value of n_t satisfies the condition that 'net' marginal cost of having a child, $(1-\gamma)u'_t\left\{q(e^c_t) - g'_n(e^c_t, n_t)\right\}$, is equal to the discounted marginal benefit of transfer the child will make, $\beta\gamma u'_{t+1}\widetilde{y}(B^a_{t+1})$.

5.1 Assumptions:

For interior solutions to the above optimization exercise to exist we need to make the following assumptions for high-PAL workers:

The first order condition (12) requires that a change in e^a_t leads to a change in the opposite direction in energy imbalance in adulthood - that is, $\frac{\partial B^{ma}_{t+1}}{\partial e^a_t} < 0$, or

$$\overline{c}^{a\prime}_t > (1-\gamma) y'_e(B^a_t, e^a_t) \tag{15}$$

That is, given everything else, higher current work-effort always reduces mature-age health of high-PAL workers.

The net marginal contribution of child work effort to current consumption (and hence mature-age health) is positive, and that of fertility is negative - that is: $\frac{\partial B^{ma}_{t+1}}{\partial e^c_t} = \theta(1-\gamma)[g'_e(e^c_t, n_t) - q'(e^c_t)n_t] > 0$, or

$$g'_e(e^c_t, n_t) - q'(e^c_t)n_t > 0 \tag{16}$$

and $\frac{\partial B^{ma}_{t+1}}{\partial n_t} = \theta(1-\gamma)[g'_n(e^c_t, n_t) - q(e^c_t)] < 0$ - that is,

$$g'_n(e^c_t, n_t) - q(e^c_t) < 0 \tag{17}$$

The assumption in (16) implies, $y'_{B^{ma}}(B^{ma}_{t+1})\frac{\partial B^{ma}_{t+1}}{\partial e^c_t} + \gamma n_t y'_{B^a}(B^a_{t+1})\frac{\partial B^a_{t+1}}{\partial e^c_t} < 0$ in (13), which asserts that on the margin the effect of intensity of child labor on transfers (*via* her own future health) is stronger than the indirect effect on her parent's mature-age health (*via* current consumption). Equation (17), on the other hand is required to ensure the net cost of child-rearing is positive and parents do not choose extremely large number of children.

For both high-PAL and non-high-PAL households, child labor entails future health-cost - that is, $\frac{\partial B^a_{t+1}}{\partial e^c_t} = \theta[q'(e^c_t) - \overline{c}^{c\prime}] < 0$, or

$$[q'(e^c_t) - \overline{c}^{c\prime}] < 0 \tag{18}$$

The Non-high PAL households do not face any threats of mature-age malnutrition from marginal increase in adult effort level and decrease in current consumption due to marginal changes in e^c_t and n_t. Hence, these assumptions do not hold these households. Indeed, $\frac{\partial B^{ma}_{t+1}}{\partial e^a_t} = \frac{\partial B^{ma}_{t+1}}{\partial e^c_t} = \frac{\partial B^{ma}_{t+1}}{\partial n_t} = 0$ for non high-PAL workers. However, for children (18) is still valid. In what follows I summarize the results of an intertemporal equilibrium.

5.2 Intertemporal Equilibrium

In an intertemporal equilibrium, an adult takes n_{t+1} and e^c_{t+1} as given. Using (12) - (18), the relationships between the endogenous variables in the intertemporal equilibrium can be summarized as follows.

Proposition 1 *: In high-PAL occupations (i) work-effort of adults (e^a_t) and fertility (n_t) are negatively related if mature-age consumption is sufficiently responsive to changes in e^a_t and n_t, (ii) work-effort of adults (e^a_t) and that of children (e^c_t) are positively related, and (iii) when (i) holds, households exhibit no quantity-quality trade-off - that is, $\frac{dn_t}{de^c_t} < 0$.*

Proof. See Appendix. ■

The intuition behind $\frac{dn_t}{de^a_t} < 0$ is as follows. An adult's mature-age health (B^{ma}_{t+1}) and children are perfectly substitutable assets, whose returns must be equalized in an intertemporal equilibrium. An increase in e^a_t raises c_t and reduces u'_t. It, however increases u'_{t+1}through reduction in B^{ma}_{t+1}, $y(B^{ma}_{t+1})$, and c_{t+1}. The intertemporal movement reallocates resources from future to current consumption. This reallocation can be matched either by lowering n_t, given e^c_t, or by increasing e^c_t, given n_t. In the first option, a lower n_t raises c_t (reduces u'_t), raises B^{ma}_{t+1}, but reduces transfers from children, which on the net reduces c_{t+1} (raises u'_{t+1}) because the transfer-effect dominates. A sufficient condition for this result to hold is $\zeta_{y'_{B^{ma}}}\zeta_{B^{ma}} \geqslant 1$, where $\zeta_{y'_{B^{ma}}} < 0$ and $\zeta_{B^{ma}_{t+1}} < 0$ are elasticity of $y'_{B^{ma}}$ with respect to B^{ma}_{t+1}, and that of B^{ma}_{t+1} with respect to n_t, respectively - that is when c_{t+1} is responsive enough to changes in e^a_t and n_t via B^{ma}_{t+1}. Similarly, given n_t, an increase in e^c_t achieves all of these. Hence, $\frac{de^c_t}{de^a_t} > 0$ and $\frac{dn_t}{de^a_t} < 0$. Thus, $\frac{dn_t}{de^c_t} = \frac{dn_t}{de^a_t}/\frac{de^c_t}{de^a_t} < 0$, which implies there is no quantity-quality trade-off among these subsistence families in the sense of Becker (1960) and Becker and Lewis (1973). This is also in contrast to the positive association between fertility and child labor - defined as labor-time (as opposed to work-intensity here) - found in Hazan and Berdugo (2002) among others.

Proposition 2 *: In non high-PAL occupations, where mature-age health (B^{ma}_{t+1}) does not depend on c_t, there is an ambiguous relationship between work-effort of children (e^c_t) and fertility (n_t) - that is, quantity-quality trade-off may or may not hold. A sufficient condition for no quantity-quality trade-off is $n_t \leq \frac{g''_e(e^c_t)}{q''(e^c_t)}$.*

Proof. See Appendix. ■

For non high-PAL households, higher e^c_t raises c_t (reduces u'_t), and reduces c_{t+1} (raises u'_{t+1}) by reducing future health of children (see 18). Since higher e^c_t does not increase B^{ma}_{t+1}, c_{t+1} declines more compared to the high-PAL case. These effects, however, depend on the magnitude of n_t. If n_t is small, the effects are weak. Under this scenario, an increase in n_t would reduce c_t and increase c_{t+1}, but the latter may or may not be sufficient to restore the equilibrium level of c_{t+1}. However, if n_t is large, the size-effects of an increasing n_t could be similar in magnitude (but opposite in sign) to that of higher e^c_t, which would re-establish the intertemporal equilibrium. Hence, quantity-quality trade-off may hold among the non-high-PAL households if n_t is larger than a threshold value. The empirical regularities in section 2 suggests that fertility is higher among these families compared to the high-PAL ones, suggesting that the former is more likely to exhibit quantity-quality trade-offs.

Propositions (1) and (2) highlight the variation in the direction of the trade-offs between fertility and child-quality across occupational categories defined in terms of physical work-intensity. If households were categorized solely in terms of income, these trade-offs would have averaged out and produced a confounding picture.

5.2.1 A Model with Standard Assumptions

To appreciate the importance of the bio-economic channel it is instructive to compare the trade-offs in the above intertemporal equilibria with those of a benchmark model with standard assumptions of fixed minimum consumption requirements. Thus, in the standard model $\overline{c}^a$ and $\overline{c}^c$ are constants. Also, in absence of the biological mechanism, energy deficiency (or surplus) would not be consequential for future health. Consequently, B^{ma}_{t+1} and B^a_{t+1} and therefore, $y(B^{ma}_{t+1})$ and $\widetilde{y}(B^a_{t+1})$ are exogenous. Thus, $\frac{\partial B^{ma}_{t+1}}{\partial e^a_t}, \frac{\partial B^{ma}_{t+1}}{\partial e^c_t}, \frac{\partial B^a_{t+1}}{\partial e^c_t} = 0$. In such a model higher adult effort would always leads to higher current consumption, implying $e^a_t = 1$ would be the optimal solutions for e^a_t. Also, child work-effort would affect only current consumption. The household optimization is expressed as:

$$Max\ \{u(c_t) + \beta u(c_{t+1})\} \tag{19}$$

$$s.t.\ c_t = (1-\gamma)[y(B^a_t, e^a_t) + g(e^c_t, n_t) - q(e^c_t)n_t];\ q' > 0, q'' < 0 \tag{20a}$$

$$c_{t+1} = y(B^{ma}_{t+1}) + \gamma n_t \widetilde{y}(B^a_{t+1}) \tag{20b}$$

$$B^{ma}_{t+1} = B^a_t(1-\delta),\ B^a_{t+1} = B^c_t(1+\mu); \tag{20c}$$

The first order conditions for e^c_t and n_t are given by, respectively:

$$(1-\gamma)u'_t[g'_e(e^c_t, n_t) - q'(e^c_t)n_t] = 0 \tag{21}$$

and

$$(1-\gamma)u'_t[g'_n(e^c_t, n_t) - q(e^c_t)] + \beta\gamma u'_{t+1}\widetilde{y}(B^a_{t+1}) = 0 \tag{22}$$

From (21) equilibrium n_t solves $n_t = \frac{g'_e(e^c_t, n_t)}{q'(e^c_t)}$. Assuming $g''_{e,n}(e^c_t, n_t) \approx 0$, $n^* = \frac{g'_e(e^c_t)}{q'(e^c_t)}$, while the equilibrium value of e^{c^*} is obtained by substituting $n^* = \frac{g'_e(e^c_t)}{q'(e^c_t)}$ in (22). The direction of quantity-quality trade-off is given by the sign of $\frac{dn^*}{de^{c^*}} = \frac{q'(e^c_t)g''_e(e^c_t) - q''(e^c_t)g'_e(e^c_t)}{q'(e^c_t)^2}$. Such trade-offs exist in equilibrium if $\frac{dn^*}{de^{c^*}} \geqslant 0$, or if $\frac{g''_e(e^c_t)}{g'_e(e^c_t)} \geqslant \frac{q''(e^c_t)}{q'(e^c_t)}$ – that is, if the marginal benefit schedule from child work-effort is at least as flat as the marginal cost schedule of child-rearing.

In contrast to the bio-economic model, the solutions to the standard model is much simpler.

Of course, there is no occupational difference under these set of assumptions, as every adult would exert the same level of effort. The quantity-quality trade-off in the standard model, although not unambiguous, is intuitively more straightforward as everything boils down to elasticities of marginal benefit and marginal cost schedules for children. But the main drawback of the standard model is its inability to capture the inter-occupational difference in fertility, work-effort and nutritional status.

In summary, the theoretical results yield two key hypotheses in Propositions (1) - (2) regarding effort-level (or work-intensity) and fertility that are in line with the empirical observations in section 2. First, the intertemporal results show a negative relationship between adult effort-level and fertility on one hand, and a positive association between adult and child efforts on the other, in 'hard' occupations. The two together imply absence of quantity-quality trade-off. Second, this trade-off is likely to be present among parents in 'soft' occupations in intertemporal equilibrium.

6 Discussion and Policy Implications

Occupational variation in nutritional poverty and its implications for child work-intensity and fertility are so far poorly understood. The paper integrates biological and economic insights, and develops a conceptual framework to fill this gap in the literature. Specifically, the study uses a standard life-cycle model augmented by the physiological pathways between current physical activity level, subsistence calorie requirements, and future health status. The analysis provides a deep microfoundation to the links between occupational variation in nutritional status, fertility, and child labor, and explains the broad preliminary findings from a representative household survey in India.

The distinctive feature of this theoretical framework is the link between occupational category and malnutrition. In particular, the subsistence consumption levels are endogenously determined by the physical activity levels that vary across occupations. Malnutrition is a natural outcome of actual consumption persistently falling short of minimum consumption requirements. Analytical results show that workers in higher labor-intensity occupations, such as agriculture, tend to have lower health status, lower fertility, and engage children at work with higher intensity, compared to others. These workers are unable trade-off child quantity for quality. Workers employed in less physically demanding jobs, are less likely to experience nutritional deficiency. These households tend to exhibit a trade-off between fertility and child-quality.

The results have significant relevance for policy to reduce undernourishment and child labor. Treating malnutrition solely as a problem of inadequate food availability and neglecting the energy expenditure through work can render food policies ineffective. For example, food-for-work programs may fail to improve nutritional status of participants if increased work-effort required offsets the effects of additional food intake. Moreover, since energy expenditure determines nutritional status, omission of energy expenditure may produce an omitted variable bias in the estimation of the effectiveness of 'nutritional intake' on 'nutritional status'.

How to break the cycle of intergenerational nutritional poverty and child labor? This analysis provides a potential solution - nutritional intervention that improves BMI of adults engaged in physical labor, which will allow these families escape from the vicious cycle. A more effective long term solution involves modernization of production processes by increased use of labor-saving technology that will reduce the calorie expenditure (and calorie-deficiency) among the nutritionally poor. Indeed, the FAO report (FAO, 2001) observes that during 1990-99, countries that experienced a sharp decline (increase) in capital-labor ratio in agriculture, also registered significant increase (decrease) in the incidence of undernourishment (see Table 2, p. 6). The policy implications of this analysis do not undermine the importance of increasing aggregate production and access to food availability as important ways to tackle the problem of undernutrition. Instead, it shows it is equally important to reduce the 'returns to brawn' in these poor economies to break the cycle of undernourishment, fertility and child labor.

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Appendix

Proof of Proposition 1:

The first order conditions (12), (13) and (14) constitute a system of 3 equations. First, we reduce the dimensionality and convert it into a 2-equation system. Given the set of assumptions in section 5.1, and noting that second-order cross-partials such as $\partial/dn[g_e'(B_t^c, e_t^c, n_t)]$, $\partial/de_t^c[g_n'(B_t^c, e_t^c, n_t)]$, etc. are negligible, the first order conditions (12) and (13) yield:

$$y'_{B^{ma}}(B^{ma}_{t+1})\overline{c}^{a\prime}[g_e'(B_t^c, e_t^c) - q'(e_t^c)n_t] + \gamma n_t y_e'(B_t^a, e_t^a) y'_{B^a}(B^a_{t+1})[q'(e_t^c) - \overline{c}^{c\prime}] = 0 \qquad (23)$$

Note that the assumption of small second-order cross-partials is purely for analytical convenience. Without this restriction, some partial derivatives tend to have ambiguous signs.

Next, from (13) and (14) we get:

$$\theta n_t y'_{B^a}(B^a_{t+1})[q'(e_t^c) - \overline{c}^{c\prime}]\{g_n'(B_t^c, n_t) - q(e_t^c)\} - \gamma\{g_e'(B_t^c, e_t^c) - q'(e_t^c)n_t\}\widetilde{y}(B^a_{t+1}) = 0 \qquad (24)$$

where $\widetilde{y}(B^a_{t+1}) \equiv y(B^a_{t+1}) + g(e^c_{t+1}, n_{t+1}) - q(e^c_{t+1})n_{t+1}$. Eqs. (23) and (24) constitute a system of 2 equations in 3 endogenous variables (e_t^a, e_t^c, n_t) and 1 exogenous variable of interest (B_t^a).

The objective is to find the signs of the partials $\frac{\partial e_t^c}{\partial e_t^a}, \frac{\partial n_t}{\partial e_t^a}, \frac{dn_t}{de_t^c}$,etc.

Using the implicit function theorem on (23):

$\{y''_{B^{ma}}(B^{ma}_{t+1})\frac{\partial B^{ma}_{t+1}}{\partial e_t^a}[g_e'(B_t^c, e_t^c) - q'(e_t^c)n_t]\overline{c}^{a\prime} + y'_{B^{ma}}(B^{ma}_{t+1})[g_e'(B_t^c, e_t^c) - q'(e_t^c)n_t]\overline{c}^{a\prime\prime} +$
$\gamma n_t y_e''(B_t^a, e_t^a) y'_{B^a}(B^a_{t+1})[q'(e_t^c) - \overline{c}^{c\prime}]\}de_t^a$
$+\{y''_{B^{ma}}(B^{ma}_{t+1})\frac{\partial B^{ma}_{t+1}}{\partial e_t^c}[g_e'(B_t^c, e_t^c) - q'(e_t^c)n_t]\overline{c}^{a\prime} + y'_{B^{ma}}(B^{ma}_{t+1})[g_e''(B_t^c, e_t^c) - q''(e_t^c)n_t]\overline{c}^{a\prime} +$
$\gamma n_t y_e'(B_t^a, e_t^a) y''_{B^a}(B^a_{t+1})\frac{\partial B^a_{t+1}}{\partial e_t^c}[q'(e_t^c) - \overline{c}^{c\prime}] + \gamma n_t y_e'(B_t^a, e_t^a) y''_{B^a}(B^a_{t+1})\frac{\partial B^a_{t+1}}{\partial e_t^c}[q''(e_t^c) - \overline{c}^{c\prime\prime}]\}de_t^c$
$+\{y''_{B^{ma}}(B^{ma}_{t+1})\frac{\partial B^{ma}_{t+1}}{\partial n_t}\overline{c}^{a\prime}[g_e'(B_t^c, e_t^c) - q'(e_t^c)n_t] + \gamma y_e'(B_t^a, e_t^a) y'_{B^a}(B^a_{t+1})[q'(e_t^c) - \overline{c}^{c\prime}]\}dn_t$

$+\{y''_{B^{ma}}(B^{ma}_{t+1})\frac{\partial B^{ma}_{t+1}}{\partial B^a_t}\overline{c}^{a\prime}[g'_e(B^c_t,e^c_t)-q'(e^c_t)n_t]+y'_{B^{ma}}(B^{ma}_{t+1})[g'_e(B^c_t,e^c_t)-q'(e^c_t)n_t]\frac{\partial \overline{c}^{a\prime}}{\partial B^a_t}+$

$\gamma n_t y''_{e,B^a}(B^a_t,e^a_t)y'_{B^a}(B^a_{t+1})[q'(e^c_t)-\overline{c}^{c\prime}]\}dB^a_t$

$=0$

or,

$$\Psi_1 de^a_t+\Psi_2 de^c_t+\Psi_3 dn_t+\Psi_4 dB^a_t=0 \tag{25}$$

Using the assumptions (15) - (17) and (**??**), it is straightforward to ascertain that $\Psi_1>0,\Psi_2<0$. The signs of Ψ_3 and Ψ_4 are ambiguous.

Let us examine $\Psi_3\equiv y''_{B^{ma}}(B^{ma}_{t+1})\frac{\partial B^{ma}_{t+1}}{\partial n_t}\overline{c}^{a\prime}[g'_e(B^c_t,e^c_t)-q'(e^c_t)n_t]+\gamma y'_e(B^a_t,e^a_t)y'_{B^a}(B^a_{t+1})[q'(e^c_t)-\overline{c}^{c\prime}]$ more closely. Substituting $\gamma y'_e(B^a_t,e^a_t)y'_{B^a}(B^a_{t+1})[q'(e^c_t)-\overline{c}^{c\prime}]=-(1/n_t)y'_{B^{ma}}(B^{ma}_{t+1})\overline{c}^{a\prime}[g'_e(B^c_t,e^c_t)-q'(e^c_t)n_t]$ from (23), we get:

$\Psi_3\equiv y''_{B^{ma}}(B^{ma}_{t+1})\frac{\partial B^{ma}_{t+1}}{\partial n_t}\overline{c}^{a\prime}[g'_e(B^c_t,e^c_t)-q'(e^c_t)n_t]-(1/n_t)y'_{B^{ma}}(B^{ma}_{t+1})\overline{c}^{a\prime}[g'_e(B^c_t,e^c_t)-q'(e^c_t)n_t]$

$=\overline{c}^{a\prime}[g'_e(B^c_t,e^c_t)-q'(e^c_t)n_t](1/n_t)y'_{B^{ma}}(B^{ma}_{t+1})\{y''_{B^{ma}}(B^{ma}_{t+1})\frac{\partial B^{ma}_{t+1}}{\partial n_t}\frac{n_t}{y'_{B^{ma}}(B^{ma}_{t+1})}-1\}$

$=\overline{c}^{a\prime}[g'_e(B^c_t,e^c_t)-q'(e^c_t)n_t](1/n_t)y'_{B^{ma}}(B^{ma}_{t+1})\left[\frac{\partial y'_{B^{ma}}}{\partial B^{ma}_{t+1}}\frac{B^{ma}_{t+1}}{y'_{B^{ma}}}\times\frac{\partial B^{ma}_{t+1}}{\partial n_t}\frac{n_t}{B^{ma}_{t+1}}-1\right]$

$=\overline{c}^{a\prime}[g'_e(B^c_t,e^c_t)-q'(e^c_t)n_t](1/n_t)y'_{B^{ma}}(B^{ma}_{t+1})[\zeta_{y'_{B^{ma}}}\zeta_{B^{ma}_{t+1}}-1]$, where $\zeta_{y'_{B^{ma}}},\ \zeta_{B^{ma}_{t+1}}<0$ are elasticity of $y'_{B^{ma}}$ w.r.t. B^{ma}_{t+1}, and that of B^{ma}_{t+1} w.r.t. n_t, respectively.

Thus, $\Psi_3\gtreqless 0$ *iff* $\zeta_{y'_{B^{ma}}}\zeta_{B^{ma}_{t+1}}\gtreqless 1$.

Now total differentiating (24) with respect to e^a_t,e^c_t,n_t,and B^a_t :

$0de^a_t$

$+\{\theta n_t y''_{B^a}(B^a_{t+1})\frac{\partial B^a_{t+1}}{\partial e^c_t}[q'(e^c_t)-\overline{c}^{c\prime}][g'_n(n_t)-q(e^c_t)]+\theta n_t y'_{B^a}(B^a_{t+1})[q''(e^c_t)-\overline{c}^{c\prime\prime}][g'_n(B^c_t,n_t)-q(e^c_t)]$

$-\gamma[g''_e(B^c_t,e^c_t)-q''(e^c_t)n_t][y(B^a_{t+1},e^a_{t+1})+g(B^c_{t+1},e^c_{t+1},n_{t+1})-qn_{t+1}]-\gamma[g'_e(B^c_t,e^c_t)-$

$q'(e^c_t)n_t]y'_{B^a}(B^a_{t+1})\frac{\partial B^a_{t+1}}{\partial e^c_t}\}de^c_t$

$+\{\theta y'_{B^a}(B^a_{t+1})[q'(e^c_t)-\overline{c}^{c\prime}][g'_n(B^c_t,n_t)-q(e^c_t)]+\theta n_t y'_{B^a}(B^a_{t+1})[q'(e^c_t)-\overline{c}^{c\prime}]g''_n(B^c_t,n_t)]\}dn_t$

$+0dB^a_t$

$=0$

or,

$$\Phi_1 de^a_t+\Phi_2 de^c_t+\Phi_3 dn_t+\Phi_4 dB^a_t=0 \tag{26}$$

Note that $\Phi_1 = \Phi_4 = 0$. Using the assumptions (15) - (17) and (**??**), it is straightforward to ascertain that $\Phi_2, \Phi_3 > 0$.

The system can be represented in the following matrix form:

$$\begin{bmatrix} \Psi_1 & \Psi_2 & \Psi_3 & \Psi_4 \\ 0 & \Phi_2 & \Phi_3 & 0 \end{bmatrix} \begin{bmatrix} de_t^a \\ de_t^c \\ dn_t \\ dB_t^a \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

or, $\begin{bmatrix} \Psi_2 & \Psi_3 \\ \Phi_2 & \Phi_3 \end{bmatrix} \begin{bmatrix} de_t^c \\ dn_t \end{bmatrix} = \begin{bmatrix} -\Psi_1 & -\Psi_4 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} de_t^a \\ dB_t^a \end{bmatrix}$

Assuming $\Psi_3 \geqslant 0$,the determinant $\Delta = \Psi_2\Phi_3 - \Psi_3\Phi_2 < 0$.

Using Cramer's rule:

$\frac{de_t^c}{de_t^a} = \frac{1}{\Delta} \begin{vmatrix} -\Psi_1 & \Psi_3 \\ 0 & \Phi_3 \end{vmatrix} > 0$; $\frac{dn_t}{de_t^a} = \frac{1}{\Delta} \begin{vmatrix} \Psi_2 & -\Psi_1 \\ \Phi_2 & 0 \end{vmatrix} < 0$;

Now $\frac{dn_t}{de_t^c} = \frac{dn_t}{de_t^a} / \frac{de_t^c}{de_t^a} < 0$; and $\frac{dB_{t+1}^a}{dn_t} = \frac{dB_{t+1}^a}{de_t^c} \div \frac{dn_t}{de_t^c} > 0$. Hence there is no quantity-quality trade-off for the high-PAL workers.

Proof of Proposition 2:

Recall, for non high-PAL workers $\frac{\partial B_{t+1}^{ma}}{\partial e_t^a} = \frac{\partial B_{t+1}^{ma}}{\partial e_t^c} = 0$, but $\frac{\partial B_{t+1}^a}{\partial e_t^c} < 0$ by (18). Given e_t^a they choose e_t^c and n_t.

Therefore, (13) and (14) are rewritten as:

$$(1-\gamma)u_t'[g_e'(e_t^c, n_t) - q'(e_t^c)n_t] + \gamma\beta u_{t+1}' n_t y_{B^a}'(B_{t+1}^a)\frac{\partial B_{t+1}^a}{\partial e_t^c} = 0 \tag{27}$$

$$(1-\gamma)u_t'[g_n'(e_t^c, n_t) - q(e_t^c)] + \gamma\beta u_{t+1}'\widetilde{y}(B_{t+1}^a) = 0 \tag{28}$$

Combining Eqs. (27) and (28), and neglecting terms g_{n,e^c}'' and $g_{e^c,n}''$:

$\theta n_t y_{B^a}'[q'(e^c) - \overline{c}^{c\prime}][g_n'(B^c, n) - q(e^c)] - [g_e'(B^c, e^c) - q'(e^c)n]\widetilde{y}(B_{t+1}^a) = 0$

Now total differentiating the above with respect to e_t^c and n_t :

$\{\theta^2 n_t y_{B^a}''[q'(e^c) - \overline{c}^{c\prime}]^2[g_n'(B^c, n) - q(e^c)] + \theta n_t y_{B^a}'[q''(e_t^c) - \overline{c}^{c\prime\prime}][g_n'(B^c, n_t) - q(e_t^c)]$

$$-\theta n_t y'_{B^a}[q'(e_t^c) - \overline{c}^{c\prime}]q'(e_t^c) - [g''_e(B^c, e_t^c) - q''(e_t^c)n_t]\widetilde{y}(B_{t+1}^a)$$
$$-[g'_e(B^c, e_t^c) - q'(e_t^c)n_t]y'_{B^a}[q'(e_t^c) - \overline{c}^{c\prime}]\}de_t^c +$$
$$\{\theta y'_{B^a}[q'(e_t^c) - \overline{c}^{c\prime}][g'_n(B^c, n_t) - q(e_t^c)] + \theta n_t y'_{B^a}[q'(e_t^c) - \overline{c}^{c\prime}]g''_n(B^c, n_t)$$
$$+q'(e^c)\left[y(B_{t+1}^a) + g(e_{t+1}^c, n_{t+1}) - q(e_{t+1}^c)n_{t+1}\right]\}dn_t = 0$$

or,

$$\Gamma_1 de_t^c + \Gamma_2 dn_t = 0 \tag{29}$$

where, the sign of Γ_1 is ambiguous, while $\Gamma_2 > 0$ unambiguously. A sufficient condition for $\Gamma_1 > 0$ is $g''_e(B^c, e_t^c) - q''(e_t^c)n_t \leq 0$. Hence, $\frac{dn_t}{de_t^c} > 0$ if $n_t \leq \frac{g''_e(B^c, e_t^c)}{q''(e_t^c)}$.