A price theory of vertical and lateral integration under two-sided productivity heterogeneity

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September 14, 2016

Abstract

We build on Legros and Newman (2013) by introducing two-sided heterogeneity to analyze the effect of product market competition on the firm boundaries. An enterprise consisting of two supplier units may choose to stay separate or integrate by delegating the control rights to a third party. The equilibrium of the market exhibits positive assortative matching, i.e., more productive supplier units match together to form enterprises. The equilibrium ownership structure is monotone – more productive enterprises integrate, while less productive suppliers stay separate. Moreover, an increase in the product market price may lead to less integration, and hence, the (organizationally augmented) industry supply may be backward-bending.

1 Introduction

There is ample evidence of firm productivity heterogeneity within an industry, which is also correlated with firm organizational variation (e.g. Gibbons, 2010; Syverson, 2010). In this paper, we study the interaction of the product market with organizational decisions (firm boundaries) and the role of ex-ante firm productivity heterogeneity. We are interested in the following questions. Which firms are more likely to integrate: low or high productivity firms? Does integration increase firm productivity, or are more productive firms more likely to integrate? Does a higher market price always imply higher aggregate output supplied when integration decisions are endogenized?

Organizational industrial organization (OIO), which lies at the intersection of industrial and organizational economics and studies how industrial structure affects firm boundaries and vice versa, is still in its early stages.[1] Within this strand of literature, Legros and Newman (2013) highlighted the important

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[1](See Legros and Newman, 2014) for an excellent survey. As Legros and Newman argue: "Nascent efforts at developing an OIO already suggest that market conditions or industrial structure matter for organization design. At the same time, organizational design will affect the productivity of firms, hence eventually the total industry output, the quality of products and information about this quality for consumers. Organizational design matters for consumers, hence for IO."

role of the market price in a perfectly competitive environment with homogeneous firm productivities. A higher market price increases the foregone revenue when firms remain unintegrated and therefore it increases the incentives for integration. More efficient firms – when firms are allowed to differ *exogenously* in productivity – are more likely to integrate. Furthermore, the "organizationally augmented" industry supply curve–which embodies the organizational structure of the firms in the market and how this structure changes in response to market price–is upward sloping. Finally, when managers are not full revenue claimants (managerial firms), the equilibrium suffers from too little integration.

Our driving force is a second (novel) effect, which we call a *utility effect*, that works in the opposite direction from the market price effect. The utility effect emerges, very naturally, when firms of heterogeneous productivities match with each other and the revenue share arises *endogenously* to ensure a stable market equilibrium. Our main results are as follows:

- The equilibrium of the market exhibits positive assortative matching in the sense that higher productivity suppliers on one side of the market form enterprises with higher productivity suppliers on the other side.
- The equilibrium ownership structure is monotone, i.e., more productive enterprises integrate while the low-productivity ones stay separate.
- A higher market price does not always lead to more integration; on the contrary, market price may be *negatively* correlated with integration.
- The organizationally augmented supply curve can be *backward-bending*, so a higher market price can lower aggregate output via the organizational restructuring it induces.

We develop a model in the spirit of the incomplete contracting tradition (e.g. Grossman and Hart, 1986) with productivity heterogeneity and endogenous matching. Production requires two inputs which are provided by two input suppliers (units) A and B. Each enterprise takes the market price as given and produces output. Each unit can be viewed as a collection of assets run by a manager. The two units can either remain separate or integrate.[2] Each manager's utility depends on firm profits and private benefits. Under non-integration each manager makes a non contractible decision to maximize his utility. Since managers also care about their private benefits firm profits are not maximized. Under integration managers give decision power to a third party who maximizes firm profit but ignores the managers' private benefits. Following Hart and Hölmstrom (2010), neither organizational form is superior: nonintegration results in 'too little' coordination and integration results in 'too much' coordination. Further, we assume that the market consists of a continuum of input suppliers who are endowed with different productivity levels.[3] Each A-supplier matches endogenously with one B-supplier to form an enterprise. After they match they decide whether to remain as separate units or integrate. We assume that A-suppliers have all the bargaining power and make take-it-or-leave-it offers to the B-suppliers, taking into account the endogenously determined utility of the A's. Higher productivity suppliers are more desirable and hence command a higher equilibrium utility. We find the equilibrium matching is positive assortative – high

[2]In the airline industry, for instance, major carriers subcontract portion of their network to regional partners. In some cases the majors own the regional partner, while in other cases the two operate as separate units and a contract governs their relationship, (e.g. Forbes and Lederman, 2010).

[3]For example, in the computer industry, computer systems manufacturers rely on networks of independent component suppliers. These suppliers are of various 'qualities' and produce components that are used as inputs in the production of the final product.

productivity A-suppliers match with high productivity B-suppliers, whereas low productivity A-suppliers match with low productivity B-suppliers.

When the two units are separate they use a contract to govern their relationship. Firm revenue is contractible and the contract stipulates the share of the firm revenues that accrues to each party. The inefficiency under non-integration is that managers make decisions not only with firm revenues in mind but also with their private benefits. The revenue share is used as an imperfect instrument to gauge managerial decisions. As the revenue share a supplier receives increases, the supplier, when making a decision, puts more weight on firm revenue and less weight on private benefits. Unfortunately, this means exactly the opposite for the other supplier, since a higher share for one implies a lower share for the other. Hence, both managers coordinate their non contractible decisions the best when the share is intermediate.[4] Under integration, on the other hand, a third party maximizes firm revenue, which is shared perfectly between the two managers. There is still an inefficiency though due to the fact that the third party does not care about the private benefits of the managers.

Integration is more likely to be the preferred design the higher the market price is, *holding* the revenue share fixed. A higher price makes the foregone firm revenue under non-integration more valuable, so the cost of the two firms when they remain separate increases (*price effect*). However, the revenue share, as we discussed above, also affects integration decisions (*utility effect*). Higher productivity B-suppliers receive, in the equilibrium, a higher share of the revenue, and hence higher utility. Thus, when the utility effect dominates, a greater fraction of the enterprises may choose to stay separate.

The price and utility effects interact with each other to yield interesting predictions. As the market price increases the equilibrium share each B-supplier receives decreases (a higher price mitigates the competition among A-suppliers for high productivity B-suppliers). This has the following implications. As we already know, a higher market price makes integration more likely. Lower share, on the other hand, as an outcome of a higher price, can have an opposing effect on a firm's decision to integrate. If the share is already high (high productivity enterprises), then a lower share means a more efficient outcome under non-integration (because we move closer to intermediate shares which tend to favor non-integration). Overall, and depending on the underlying productivity distribution, there are instances where a higher market price is associated with less integration (when the utility effect dominates the price effect). Since nonintegrated firms produce less output industry supply can be *backward-bending*, which can have surprising implications. For instance, policies that aim at boosting household income, and hence output and consumption, can result in less aggregate output produced, if the market equilibrium is at the decreasing part of the market supply curve.

2 The Model

2.1 Technology and matching

Our model builds on Hart and Hölmstrom (2010) and Legros and Newman (2013) by introducing two-sided productivity heterogeneity. On each side of the market there is a continuum of input suppliers of measure 1. Each supplier is a collection of assets and workers overseen by a manager. Suppliers

[4] Nevertheless, an intermediate share is not mutually beneficial, in the absence of side payments, because each side's utility is an increasing function of its own share of the revenue.

are vertically differentiated.[5] In particular, let $J_A = [0, 1]$ be the set of "A-suppliers" on one side of the market and $J_B = [0, 1]$, the set of "B-suppliers" on the other side. Each supplier $i \in J_A$ is assigned a type or 'ability' $a = a(i) \in A$ and each $j \in J_B$ has an assigned type $b = b(j) \in B$ where the type spaces $A = [a_{min}, a_{max}]$ and $B = [b_{min}, b_{max}]$ are subintervals of $\mathbb{R}_+$. Let $G(a)$ be the fraction of A-suppliers with ability lower than a, i.e., $G(a)$ is the cumulative distribution function of a with the associated density function $g(a) > 0$ for all $a \in A$. Similarly, let $F(b)$ be the distribution function of b with the associated density function $f(b) > 0$ for all $b \in B$.

The production of a homogeneous consumer good requires one A-supplier and one B-supplier who are matched one to one to form an 'enterprise'. All decisions and payoffs of each enterprise will only depend on the types of the two participating units, and hence, a typical enterprise will be denoted by (a, b). Each enterprise (a, b) forms via matching between an A-supplier and a B-supplier. A matching is a one-to-one mapping $\lambda : B \to A$ which assigns to each $b \in B$ a type $a = \lambda(b) \in A$. Such enterprises may include lateral as well as vertical relationships. The stochastic output of an enterprise (a, b) is given by:

$$\tilde{y}(a, b) = \begin{cases} z(a, b) & \text{with probability } \pi(e_A, e_B) \equiv 1 - (e_B - e_A), \\ 0 & \text{otherwise.} \end{cases}$$

The success output $z(a, b)$ can be thought of as the productivity of an enterprise (a, b).[6] We assume that $z(a, b)$ is twice continuously differentiable and strictly increasing on $A \times B$. Moreover $z(a, b)$ is assumed to be symmetric and strictly supermodular in (a, b), i.e., $z_{ab}(a, b) > 0$ for all $(a, b) \in A \times B$.[7] Each supplier must make a non-contractible production decision: $e_A \in [0, 1]$ by an A-supplier and $e_B \in [0, 1]$ by a B-supplier. These decisions can be made by the manager of the assets or by someone else. If the two managers coordinate their decisions and set $e_A = e_B$, then inefficiencies disappear and the enterprise reaches its full potential $z(a, b)$ with probability 1. The manager of each supplier is risk neutral and incurs a private cost of the decision made in his unit.[8] The private cost of an A unit is ce_A^2 and that of a B unit is $c(1 - e_B)^2$ with $c > 0$. Clearly, there is a disagreement about the direction of the decisions, what is easy for one is hard for the other and vice versa.[9] Also, managers with zero cash endowments are protected by limited liability, i.e., their state-contingent incomes must always be nonnegative. The importance of this assumption is that the division of surplus between the managers will affect the organizational structure.

[5] Bloom and van Reenen (2007), using a survey from medium-sized manufacturing firms from four countries, document that management practices are heterogeneous and affect firm performance. Gibbons (2010) offers a more detailed account of various empirical studies that document persistent performance differences (PPDs).

[6] Legros and Newman (2013) assume a quadratic probability of success of the form $\pi(e_A, e_B) \equiv 1 - (e_A - e_B)^2$. We use a linear one instead for tractability. The linear probability of success is everywhere below the quadratic one. It implies that integration is more profitable under the quadratic probability and that is why in that case integration never dominates non-integration for all levels of utility. Nevertheless, the important qualitative features of the model are not affected by this choice.

[7] We use subscripts to denote partial derivatives.

[8] The private cost can represent, for example, job satisfaction or a way to capture different beliefs held by managers and workers about the consequences of strategic choices (see Hart and Hölmstrom, 2010).

[9] For example, as discussed in Hart and Hölmstrom (2010), the two units may want to adopt a common standard, as in the Cisco's acquisition of Stratacom. The benefits for the organization from a common standard can be enormous but the private benefits within the firms may decrease because of the change the new standard introduces. Moreover, there is no agreement between the firms about which 'approach' should be adopted. However, agreeing on a common approach (coordination) boosts firm revenue.

2.2 Ownership structures and contracts

The ownership structure can be contractually determined. We assume two different options, each of which implies a different allocation of decision making power. First, the production units can remain separate firms (the *non-integration* regime, denoted by N). In this case, managers have full control over their decisions. Second, the two input suppliers can integrate, a regime denoted by I, into a single firm, giving control over managerial decisions, e_A and e_B, to a third party who always has enough cash to finance the acquisition. The third party is motivated entirely by income and incurs no costs from the managerial decisions. These costs are still borne by the managers. As argued by Hart and Hölmstrom (2010), integration results in an organization where less weight is placed on private benefits than under non-integration. This, however, is offset by the fact that under integration total profit, rather than individual unit profits, is maximized.

Each enterprise's revenue is contractible. We assume that each A-supplier has all the bargaining power in an arbitrary enterprise (a, b) and makes take-it-or-leave-it offers to the B-supplier. A contract $\gamma = (d, s)$ specifies an organization structure $d \in \{I, N\}$ and a revenue share $s \in [0, 1]$. Consider an arbitrary enterprise (a, b). If the members of this enterprise stay separate, then a revenue-sharing contract is simply a share s of the total revenue that accrues to the B-supplier. As we assume limited liability, the units get nothing in the case of failure.

When the two units integrate, a third party, called the *headquarters*, HQ, buys the assets of the A- and B-suppliers for predetermined prices in exchange for a share contract $\mathbf{s} = (s_A, s_B, s_{HQ}) \in \mathbb{R}^3_+$ with $s_A + s_B + s_{HQ} = 1$. HQs are supplied perfectly elastically with an opportunity cost normalized to zero.

2.3 The product market

The product market is perfectly competitive where consumers and suppliers take the product price P as given. Identical consumers maximize a smooth quasi-linear utility which gives rise to a downward-sloping demand curve $D(P)$. Suppliers correctly anticipate price P when they sign contracts and make their production decisions. Define by $R(a, b) \equiv Pz(a, b)$, the revenue of an enterprise (a, b). We assume $0 \leq R(a, b) \leq 2c$ for all $(a, b) \in A \times B$.

2.4 Timing of events

The economy lasts for four dates, $t = 0, 1, 2, 3$. At date 0, one A-supplier and one B-supplier match one to one to form an enterprise (a, b). At $t = 1$, the supplier units decide whether to integrate or stay separate and the A-supplier offers take-it-or-leave-it revenue-sharing contract to the B-supplier. At date 2, the manager of each unit chooses e_A and e_B. Finally at $t = 3$, the revenue of each enterprise is realized and the agreed upon payments are made. We solve the model by backward induction.

2.5 Equilibrium

An equilibrium of the economy consists of a set of enterprises formed through feasible contracts, i.e., organization structures and corresponding revenue shares, for each enterprise and a market-clearing price.

Recall that there are two possible ownership structures for each enterprise in the economy – integration (I) and separation (N). In general, choice of ownership structures depends on the revenue share that accrues to each member of an enterprise, the output of each enterprise and the market price. As pointed out by Legros and Newman (2013), equilibrium may also consist of singleton coalitions whose payoff would not depend neither on the revenue share nor on the market price. We would normalize the payoff achievable by a singleton coalition to $u_0 \geq 0$, the reservation payoff of its member. An allocation for the market (λ, v, u) specifies a one-to-one matching rule $\lambda : B \to A$, and payoff functions $v : A \to \mathbb{R}_+$ and $u : B \to \mathbb{R}_+$ for the A- and B-suppliers.

Definition 1 (Equilibrium) *Given the type distributions $G(a)$ and $F(b)$, an allocation (λ, v, u) and a product-market price P constitute an equilibrium allocation of the economy if they satisfy the following conditions:*

(a) **Feasibility:** *the revenue shares and the corresponding payoffs to the agents in each equilibrium enterprise are feasible given the output of the enterprise and the equilibrium price P;*

(b) **Optimization:** *Each A-supplier of a given type chooses optimally a B-supplier to form an enterprise, i.e., given $u(b) \in \boldsymbol{u}$ for each $b \in B$,*

$$\lambda^{-1}(a) = argmax_b\ \phi(a, b, u(b); P), \qquad (\mathscr{M})$$
$$v(a) = max_b\ \phi(a, b, u(b); P),$$

for each $a \in A$. The function $\phi(a, b, u(b); P)$ is the bargaining frontier *of the enterprise (a, b), which is the maximum payoff that can be achieved by a type a A-supplier given that the B-supplier of type b consumes $u(b)$ at a given market price P.*

(c) **Supplier market clearing:** *The equilibrium matching function satisfies the following 'measure consistency' condition. For any subinterval $[i_0, i_1] \subseteq J_A$, let $i_k = G(a_k)$ for $k = 0, 1$, i.e., a_k is the ability of the A supplier at the i_k-th quantile. Similarly, for any subinterval $[j_0, j_1] \subseteq J_B$, let $j_h = F(b_h)$ for $h = 0, 1$. If $[a_0, a_1] = \lambda([b_0, b_1])$, then it must be the case that*

$$i_1 - i_0 = G(a_1) - G(a_0) = F(b_1) - F(b_0) = j_1 - j_0. \qquad \text{(MC)}$$

(d) **Product market clearing:** *The total (expected) supply in the industry $Q(P)$ is equal to the demand $D(P)$.*

Definition 1-(b) asserts that each A-supplier chooses her partner optimally. Part (c) of the above definition simply says that one cannot match say two-third of the A-suppliers to one-third of the B-suppliers because the matching is constrained to be one-to-one.

3 Analysis and the main results

We proceed as follows. In Section 3.1, we derive the bargaining frontiers under N and I and establish the optimal organization for a given enterprise, as a function of the exogenously given utilities. In Section 3.2, we allow the units to match endogenously in the market, we endogenize the utilities and we show that the equilibrium matching is positive assortative. In Section 3.3, we derive the equilibrium organizational structures in the market. Finally, in Section 4.2, we derive the *organizationally augmented supply* (OAS) curve.

3.1 Optimal ownership structure for an arbitrary enterprise

We analyze the optimal contract for an arbitrary enterprise (a, b). We assume that the A-supplier possesses all the bargaining power in the relationship and makes take-it-or-leave-it offer to the B-supplier. We first analyze each ownership structure separately.

3.1.1 Non-integration

For the time being we suppress the argument (a, b) from the contract terms. Under this organizational structure the shares affect both the size and the distribution of surplus between the two units (imperfectly transferable utility). An optimal contract for a non-integrated enterprise (a, b) solves the following maximization problem:

$$\max_{\{s, e_A, e_B\}} \quad V_A \equiv \pi(e_A, e_B)(1-s)R(a, b) - ce_A^2, \tag{$\mathscr{P}_N$}$$

$$\text{subject to} \quad U_B \equiv \pi(e_A, e_B)sR(a, b) - c\,(1-e_B)^2 = u, \tag{PC$_B$}$$

$$e_A = \operatorname{argmax}_e \left\{\pi(e, e_B)(1-s)R(a, b) - ce^2\right\} = \frac{(1-s)R(a, b)}{2c}, \tag{IC$_A$}$$

$$e_B = \operatorname{argmax}_e \left\{\pi(e_A, e)sR(a, b) - c(1-e)^2\right\} = 1 - \frac{sR(a, b)}{2c}, \tag{IC$_B$}$$

where $u \geq 0$ is the outside option of the B-supplier. Constraint (PC$_B$) is the *participation constraint* of the B-supplier, whereas constraints (IC$_A$) and (IC$_B$) are the *incentive compatibility constraints* of the A-supplier and the B-supplier, respectively. Note that (IC$_A$) and (IC$_B$) together imply that $\pi(e_A, e_B) = \frac{R(a,b)}{2c}$, and hence, $\pi(e_A, e_B) \in [0, 1]$ under the assumption that $R(a, b) \in [0, 2c]$. The maximum value function $\phi^N(a, b, u; P)$ of the above maximization problem, which is the maximum payoff that accrues to the A-supplier given that the B-supplier consumes u at a given market price P, will be called the *bargaining frontier*. This frontier is the set of all feasible utility pairs (u, v) associated with a given enterprise. The following lemma characterizes the optimal revenue share and the bargaining frontier under non-integration.

Lemma 1 *When the supplier units in an arbitrary enterprise (a, b) stay separate, for a given product market price P, the optimal revenue share (accruing to the B-supplier) and the associated bargaining frontier are respectively given by:*

$$s(a, b, u; P) = 1 - \frac{\sqrt{(R(a, b))^2 - 4cu}}{R(a, b)},$$
$$\phi^N(a, b, u; P) = u - \frac{(R(a, b))^2}{4c} + \frac{R(a, b)}{2c} \cdot \sqrt{(R(a, b))^2 - 4cu}. \tag{1}$$

Moreover, the value function has the following properties:

(a) *It is strictly increasing in a and b, strictly decreasing and concave in u; and*

(b) $\phi_2^N(a, b, u; P)\phi_{31}^N(a, b, u; P) - \phi_3^N(a, b, u; P)\phi_{21}^N(a, b, u; P) > 0$ *for all (a, b, u) and P.*

The participation constraint of the B-supplier determines the optimal revenue share $s(a, b, u; P)$ of each type b B-supplier. Note that the outside option of this supplier unit, u, must lie between 0, which corresponds to $s = 0$, and $u^N_{max}(a, b) \equiv \frac{1}{4c}(R(a, b))^2$, the level corresponding to $s = 1$. The bargaining frontier issymmetric with respect to the 45^0 line (on which $\phi^N(a, b, u; P) = u$ and $s = \frac{1}{2}$). This implies that total surplus is maximized when the shares across the two non-integrated units are equal (see Figure 1). Equal, or more broadly 'balanced', shares yields strong incentives to the managers to better coordinate their decisions.

3.1.2 Integration

When the two supplier units integrate, the enterprise is acquired by HQ who is conferred with the decision making power. Motivated entirely by incomes, HQ will choose e_A and e_B to maximize the expected revenue $\pi(e_A, e_B)R(a, b)$ as long as $s_{HQ} > 0$. This induces $e_A = e_B$. The private costs of managerial actions are still borne by the individual supplier units. The aggregate managerial cost, $c\left[e_A^2 + (1 - e_B)^2\right]$ is minimized when $e_A = e_B = \frac{1}{2}$. The bargaining frontier, i.e., the maximum payoff accruing to the A-supplier given that the B-supplier consumes u in each enterprise (a, b) is given by:

$$\phi^I(a, b, u; P) = R(a, b) - \frac{c}{2} - u. \tag{2}$$

The above frontier is linear in u on $[0, u^I_{max}(a, b)]$ where $u^I_{max}(a, b) \equiv R(a, b) - \frac{c}{2}$, i.e., surplus is fully transferable between the two managers since neither the action taken by HQ nor the costs borne by the managers depends on the revenue share. The bargaining frontier is strictly increasing in a and b, strictly decreasing in u (with slope -1) and symmetric with respect to the 45^0 line. Moreover, we have for all (a, b)

$$\phi^I_2(a, b, u; P)\phi^I_{31}(a, b, u; P) - \phi^I_3(a, b, u; P)\phi^I_{21}(a, b, u; P) = R_{ab}(a, b) > 0. \tag{3}$$

Although surplus is fully transferable between A and B suppliers, this form of organization is not the efficient one as HQ, having a stake in the firm's revenue, puts too little weight on the managers' private costs while maximizing the expected revenue.[10]

3.1.3 Choice of organization and the bargaining frontier

Having analyzed the optimal contract and the corresponding bargaining frontier of an arbitrary enterprise under each organizational structure separately, it is now convenient to analyze the optimal organization and the (combined) bargaining frontier associated with a given enterprise. We use the preference relation symbol $\succ$ to denote organizational preference, i.e., $d \succ d'$ means that the organizational structure d is preferred to d' for $d, d' = I, N$ and $d \neq d'$. At any given price P, the combined bargaining frontier is given by:

$$\phi(a, b, u; P) = \max_u \{\phi^N(a, b, u; P), \phi^I(a, b, u; P)\}.$$

The frontier $\phi(a, b, u; P)$ represents the maximum payoff that accrues to an A-supplier given that the B-supplier consumes u and the enterprise has chosen its organizational structure optimally. The equality

[10]Too much coordination in an integrated firm ignores the private managerial costs, while in a non-integrated firm the managers of the supplier units care too much about the private costs. Therefore, both types of organizations lead to loss of efficiency. Note that the first-best surplus, $\frac{R^2}{2c}$, is strictly higher than $R - \frac{c}{2}$, the surplus accrued to an integrated firm as well as $\frac{3R^2}{8c}$, the maximum surplus in a non-integrated firm (which corresponds to $s = \frac{1}{2}$).

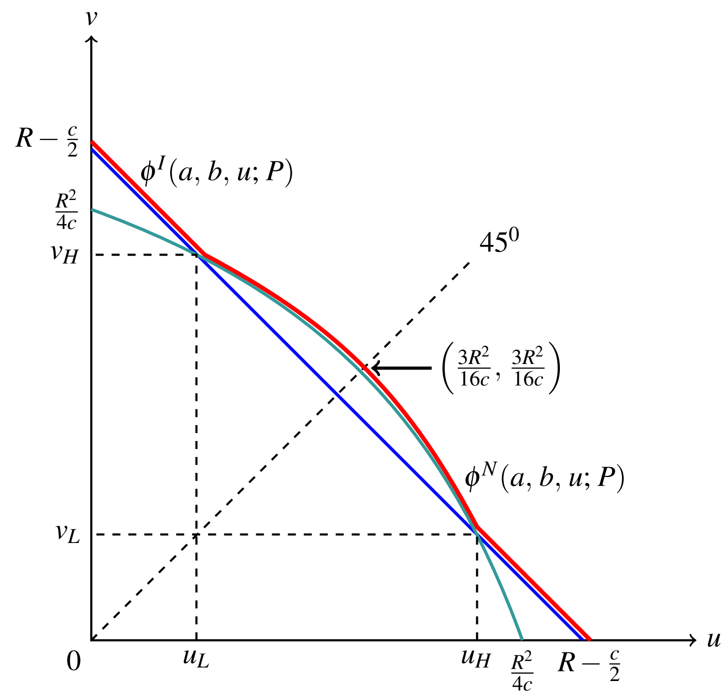

Figure 1: The two bargaining frontiers, ϕ^N and ϕ^I, when $R(a,b) \in [R^-, R^+]$. For intermediate values of the B supplier utility u, $N \succ I$, while for high or low values of u, $I \succ N$. The combined frontier is the upper envelope of ϕ^N and ϕ^I.

between the two frontiers, $\phi^N(a,b,u;P)$ and $\phi^I(a,b,u;P)$, gives two cut-off levels $u_L(a,b)$ and $u_H(a,b)$ of the utility of the B-supplier with $0 < u_L(a,b) \leq u_H(a,b)$, which are given by:

$$u_L(a,b) = \frac{1}{8}\left[4R(a,b) - 2c - \frac{R(a,b)}{c} \cdot \sqrt{(R(a,b)-2c)(3R(a,b)-2c)}\right], \tag{4}$$

$$u_H(a,b) = \frac{1}{8}\left[4R(a,b) - 2c + \frac{R(a,b)}{c} \cdot \sqrt{(R(a,b)-2c)(3R(a,b)-2c)}\right]. \tag{5}$$

In an arbitrary enterprise (a,b), the optimal choice of the organizational structure depends on the revenue of the enterprise, $R(a,b)$. One can find two threshold values $R^- \equiv (2-\sqrt{2})c$ and $R^+ \equiv \frac{2c}{3}$ of $R(a,b)$ with $0 < R^- < R^+ < 2c$ such that when $R(a,b) < R^-$ the managers of the supplier units prefer to stay separate because in this case $\phi^N(a,b,u;P) > \phi^I(a,b,u;P)$ for all $(a,b,u;P)$. On the other hand, the suppliers prefer to integrate when $R(a,b) > R^+$. Interestingly, when $R^- \leq R(a,b) \leq R^+$ there is no clear dominance of one organizational structure over the other. This case is depicted in Figure 1 where $\phi^N(a,b,u;P)$ is the strictly concave frontier and $\phi^N(a,b,u;P)$ is the linear frontier. Clearly, they intersect twice at $u_L(a,b)$ and $u_H(a,b)$. Therefore, for $R^- \leq R(a,b) \leq R^+$, non-integration is chosen by each enterprise (a,b) if $u \in U^N(a,b) \equiv [u_L(a,b), u_H(a,b)]$. By contrast, integration is preferred if $u \in U^I(a,b) \equiv [0, u_L(a,b)) \cup (u_H(a,b), u^I_{max}(a,b)]$. Note that when $R(a,b) = R^+$, we have $u_L(a,b) = u_H(a,b)$. The combined frontier is given by the upper envelope of $\phi^N(a,b,u;P)$ and $\phi^I(a,b,u;P)$. Thus to summarize,

Proposition 1 (Bargaining frontier of a given enterprise) *In a given enterprise* (a, b)*, there exist two threshold values* R^- *and* R^+ *of the enterprise revenue* $R(a, b)$ *with* $0 < R^- < R^+ < 2c$ *such that*

(a) *When* $R(a, b) < R^-$*, the enterprise chooses non-integration over integration. The bargaining frontier is given by* $\phi(a, b, u; P) = \phi^N(a, b, u; P)$*;*

(b) *When* $R(a, b) \in [R^-, R^+]$*, non-integration is preferred if* $u \in U^N(a, b)$*, and integration is chosen by the enterprise if* $u \in U^I(a, b)$*. The bargaining frontier is given by*

$$\phi(a, b, u; P) = \begin{cases} \phi^N(a, b, u; P) & \text{if } u \in U^N(a, b), \\ \phi^I(a, b, u; P) & \text{if } u \in U^I(a, b); \end{cases}$$

(c) *When* $R(a, b) > R^+$*, the enterprise chooses integration over non-integration. The bargaining frontier is given by* $\phi(a, b, u; P) = \phi^I(a, b, u; P)$*.*

Low revenue (or low productivity), i.e., $R < R^-$ implies that an organization puts more emphasis on private benefits relative to the benefits accruing from coordination and chooses non-integration over integration for all levels of u. On the other hand, for high revenue (or productivity) enterprises, $R > R^+$, more weight is placed on coordination and revenue maximization, and hence, integration is the optimal choice for all u. For intermediate productivity enterprises, $R \in [R^-, R^+]$, either organizational structure may be optimal, depending on the level of u, or the share s. For intermediate values of u, $N \succ I$ because the corresponding shares s are balanced and so a high level of coordination among the two units can be achieved without being integrated. But for the extreme values of u, either high or low, $I \succ N$ because the shares are tilted in favor of one unit and the incentives for revenue maximization under N are weak.

3.2 Equilibrium matching

In this section, we analyze the equilibrium matching function $a = \lambda(b)$ and show that the matching is positive assortative (PAM), i.e., $\lambda'(b) \geq 0$. The property that the equilibrium exhibits PAM relies on the properties of the (combined) bargaining frontier $\phi(a, b, u(b); P)$. In the enterprise formation stage at date 0, each type a A-supplier solves the program ($\mathscr{M}$) to choose a B-supplier. The first-order condition is given by the following ordinary differential equation (ODE):

$$u'(b) = -\frac{\phi_2(a, b, u(b); P)}{\phi_3(a, b, u(b); P)}. \tag{6}$$

The above differential equation is similar to the *local downward incentive constraint* in an optimal screening problem, which is the equality between the marginal earnings of a type b B-supplier and his marginal contribution to the match surplus. Next, we determine the equilibrium matching function $a = \lambda(b)$. The sign of $\lambda'(b)$ is determined from the second-order condition of program ($\mathscr{M}$), which is given by:

$$\Phi(a, b, u(b); P) \equiv \phi_{22} + \phi_{23}u'(b) + u'(b)[\phi_{32} + \phi_{33}u'(b)] + \phi_3 u''(b) \leq 0. \tag{7}$$

Differentiating the first-order condition (6) with respect to a along the equilibrium path, i.e., at $a = \lambda(b)$, we obtain

$$\phi_{21}(a, b, u(b); P) + u'(b)\phi_{31}(a, b, u(b); P) = -\lambda^{-1'}(a)\Phi(a, b, u(b); P).$$

Since $a = \lambda(b)$, the bargaining frontier is strictly downward sloping, i.e., $\phi_3 < 0$, $\Phi \leq 0$ by (7) and $\phi_2 + u'(b)\phi_3 = 0$ by (6), the above equation implies that

$$[\phi_2(a,b,u(b);P)\phi_{31}(a,b,u(b);P) - \phi_3(a,b,u(b);P)\phi_{21}(a,b,u(b);P)]\lambda'(b) \geq 0. \tag{8}$$

Therefore, a sufficient condition for the equilibrium matching to be a *positive assortative matching* (PAM), i.e., $\lambda'(b) \geq 0$ is that

$$\phi_2(a,b,u;P)\phi_{31}(a,b,u;P) - \phi_3(a,b,u;P)\phi_{21}(a,b,u;P) > 0 \tag{SCP}$$

Thus,

Proposition 2 *The bargaining frontier $\phi(a,b,u;P)$ of each enterprise (a,b) satisfies condition (SCP), and hence, in the equilibrium of the supplier market, more a able A-supplier forms enterprise with a more able B-supplier following a positive assortative matching pattern.*

When an enterprise (a,b) with low revenue chooses to stay separate in equilibrium, the bargaining frontier is given by $\phi(a,b,u;P) = \phi^N(a,b,u;P)$. Therefore, part (b) in Lemma 1 is same as the condition (SCP), and hence, there is PAM for the low-revenue enterprises. On the other hand, for the high-revenue enterprises, condition (3) is equivalent to (SCP), and hence, these is also PAM for the high-revenue enterprises. But for the enterprises with intermediate levels of revenue there is no clear dominance of one organizational structure over the other. Therefore, proving (SCP) for such an enterprise is not trivial. In the Appendix, we prove this property of the (combined) bargaining frontier $\phi(a,b,u;P)$.

The equilibrium matching pattern is driven by the fact that the enterprise revenue $R(a,b)$ is supermodular in the types of the supplier units, i.e., the types are complementary in each enterprise. Notice that condition (SCP) implies a *single-crossing property* of the equilibrium bargaining frontier $\phi(a,b,u(b);P)$. Let $v(a \mid b)$ be the equilibrium payoff function of the A-supplier evaluated at any b. Then applying the Envelope theorem to the maximization problem we get $v'(a \mid b) = \phi_1(a,b,u(b);P)$, and hence,

$$\frac{\partial v'(a \mid b)}{\partial b} = -\frac{\phi_2(a,b,u(b);P)\phi_{31}(a,b,u(b);P) - \phi_3(a,b,u(b);P)\phi_{21}(a,b,u(b);P)}{\phi_3(a,b,u(b);P)} > 0.$$

The above implies that, for any b' and b with $b' > b$, $v(a \mid b')$ is steeper than $v(a \mid b)$, and hence they intersect each other only once. Therefore, the more able A-suppliers are matched with the more able B-suppliers.[11]

The equilibrium matching function must satisfy the measure consistency condition (MC) which implies that

$$G(\lambda(b)) = F(b) \iff \lambda(b) = G^{-1}(F(b)) \quad \text{for } a = \lambda(b).$$

Therefore,

$$\lambda'(b) = \frac{f(b)}{g(\lambda(b))} \quad \text{for } a = \lambda(b).$$

For example, if a and b follow the same distribution on $[0,x]$, then the equilibrium matching function is linear with slope equal to 1. Since densities are local measures of dispersion, the equilibrium matching function is steeper (flatter) at a given b if the A suppliers are more dispersed (concentrated) at $\lambda(b)$ relative to the B-suppliers at b.

[11] It is worth noting that condition (SCP) is weaker than the supermodularity of the frontier $\phi(a,b,u;P)$ for any given P which requires that ϕ_{12}, ϕ_{23} and ϕ_{31} are all positive. Moreover, the proof does not rely on the fact that each individual frontier is supermodular.

3.3 Equilibrium organizational choices

In this subsection, we intend to determine the equilibrium organizational pattern in the supplier market. In Section 3.1, we have analyzed the optimal contract of each enterprise (a,b) for given levels of enterprise revenue $R(a,b)$ and the utility of each B-supplier. In the market equilibrium, both revenue and utility are endogenized through the equilibrium matching function $a = \lambda(b)$. In what follows, we will demonstrate that higher-productivity B-suppliers induce greater revenue and command higher utility, which in turn affect the choice of organizations.

Consider first an enterprise $(\lambda(b), b)$ along the equilibrium path whose productivity is given by $\tilde{z}(b) \equiv z(\lambda(b), b)$. Since $z_a, z_b > 0$ and the equilibrium exhibits PAM, we have $\tilde{z}'(\cdot) > 0$ and hence, an inverse function $\tilde{z}^{-1}$ exists. Therefore, for a given level of revenue R of enterprise $(\lambda(b), b)$, we may write

$$b = \tilde{z}^{-1}\left(\frac{R}{P}\right) \equiv Z(R). \tag{9}$$

We further assume that $R_{min} \equiv P\tilde{z}(b_{min}) \in (0, R^-)$. This is equivalent to saying that the minimum ability is low enough, i.e., $b_{min} < b^- \equiv Z(R^-)$, which ensures that the low-productivity enterprises (with productivity close to b_{min}) choose N in the market equilibrium. Since the stand-alone utility u_0 of all the B-suppliers is the outside option of a B-supplier with type b_{min}, in equilibrium we must have $u(b_{min}) = u_0$. From now on we assume that $u_0 > 0$.

Using (A.15) and (A.16) for N and (2) for I, the ODE (6) can be expressed as follows

$$u'(b) = \begin{cases} Pz_b(\lambda(b), b) & \text{if } u \in U^I(\lambda(b), b), \\ \frac{P^2}{2c}\left[\frac{1}{s(b;P)} + s(b;P) - 1\right] z(\lambda(b), b) z_b(\lambda(b), b) & \text{if } u \in U^N(\lambda(b), b) \end{cases} \tag{10}$$

where the equilibrium share function $s(b;P) \equiv s(\lambda(b), b, u(b); P)$ is given in (1).

According to the Picard-Lindelöf Theorem (e.g. Birkhoff and Rota, 1989) a unique solution to the ODE exists (at least in the neighborhood of the initial condition) and is given (implicitly) by

$$u(b) = u_0 + \int_{b_{min}}^{b} u'(\tau) d\tau \tag{11}$$

provided that $u'(b)$ is bounded, Lipschitz continuous in u and continuous in b. In the I region the ODE assumes a simple form; all these properties are satisfied and an analytical solution can be easily obtained. In the N region, however, the ODE is much more complicated and an analytical solution does not exist. In this region, we require to establish the existence and uniqueness of a solution. Our assumptions ensure that it is continuous in b, because b enters u' through $\tilde{z}(b)$ which is continuous because $z(\cdot)$ is assumed to be a continuous function. The term u enters through the share. If a differentiable function has a derivative that is bounded everywhere by a real number, then the function is Lipschitz continuous, which holds in this case. Moreover, u' is bounded above as long as $u > 0$. If $u = 0$, then $s = 0$ and u' becomes unbounded. Hence, we require that u_0 to be strictly positive. Given that a closed form solution for $u(b)$ does not exist, it is not possible to come up with clean conditions that guarantee a monotone organizational pattern, i.e., along the equilibrium path the organizational structure changes from one regime to the other only once. Nevertheless, we can derive useful insights as we explain next.

In what follows, we analyze the organizational choices in the equilibrium of the supplier market. In Figure 2, a point (b,u) in the 'type-utility' space represent the type of a B-supplier and the utility consumed by this type in an enterprise $(\lambda(b), b)$ which has been formed in the equilibrium. Let $b^- = Z(R^-)$ and $b^+ = Z(R^+)$. Clearly, an enterprise $(\lambda(b), b)$ would strictly prefer to stay separate if $b < b^-$ and would integrate if $b > b^+$ irrespective of the utility allocations. But whenever $b \in [b^-, b^+]$, there is no clear dominance of one ownership structure over the other. In Figure 2, the parabola-shaped curve is such that any point (b, u) on it implies that the enterprise $(\lambda(b), b)$ is indifferent between the two ownership structures.[12] Therefore, the segment on the vertical line $b = b^-$ on which $u \geq \tilde{u}_H(b^-)$ and the non-linear together define the *indifference locus* $\tilde{u}(b)$ which partitions the 'type-utility space' into two disjoint regions in which one ownership structure is preferred to the other by all enterprises $(\lambda(b), b)$ formed in equilibrium.

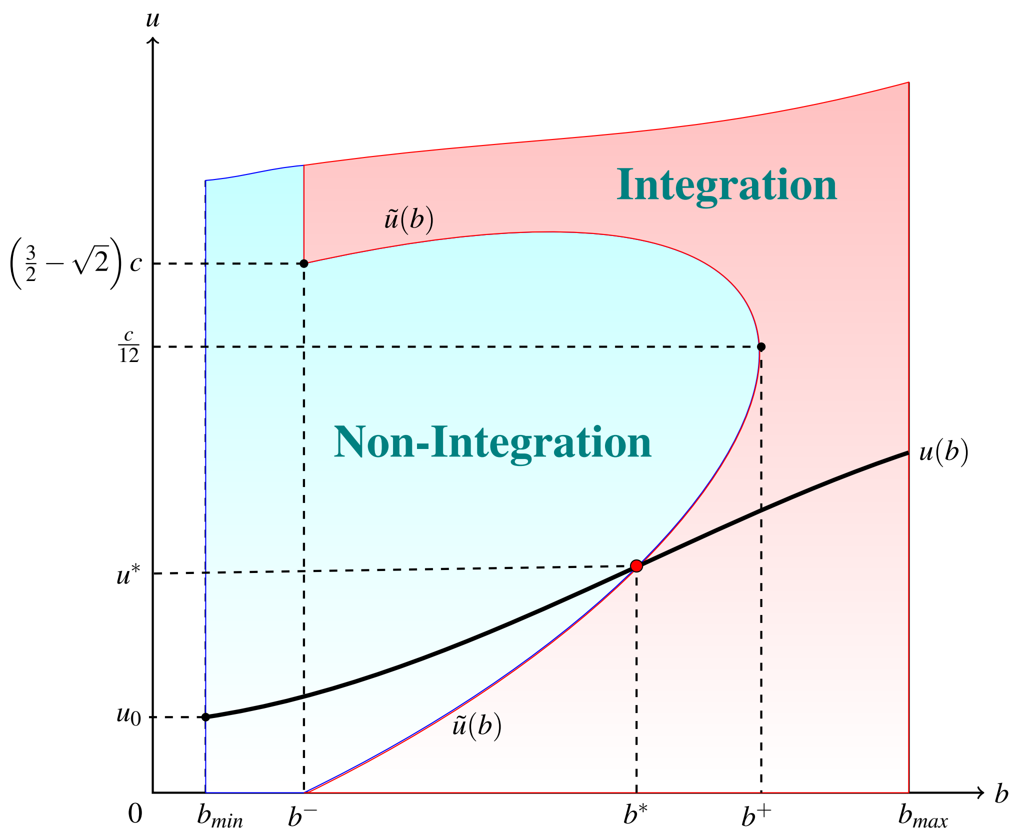

Figure 2: Monotonic organizational structures in the market equilibrium. Enterprises with $b < b^*$ stay separate, whereas those with $b \geq b^*$ integrate.

[12]Define $\tilde{u}_L(b) \equiv u_L(\lambda(b), b)$ and $\tilde{u}_H(b) \equiv u_H(\lambda(b), b)$ where u_L and u_H are given by (4) and (5). Note that $\tilde{u}_L(b^+) = \tilde{u}_H(b^+) = \tilde{u}(b^+) = \frac{c}{12}$. This allows us to define the parabola-shaped curve on which an enterprise $(\lambda(b), b)$ is indifferent between N and I whenever $b \in [b^-, b^+]$. At this juncture, it is worth noting that if one uses a quadratic probability of success function $1 - (e_A - e_B)^2$ instead (e.g. Legros and Newman, 2013), then $\tilde{u}_L(b)$ and $\tilde{u}_H(b)$ are such that

$$\tilde{u}_H(b) - \tilde{u}_L(b) = \frac{R(\lambda(b), b)}{R(\lambda(b), b) + 1},$$

which is strictly positive and increasing on B, and hence, the two curves $\tilde{u}_L(b)$ and $\tilde{u}_H(b)$ never intersect each other.

Our goal is to provide conditions under which the equilibrium ownership structure is monotonic in types, i.e., enterprises comprising of low-ability B-suppliers, and hence, low-ability A-suppliers since the equilibrium matching is PAM, would choose to stay separate, while high-ability enterprises would prefer integration. The monotonicity of ownership structure crucially depends of the value of u_0 and the marginal productivities of the A- and B-suppliers which in turn determines the slope of the equilibrium utility function $u(b)$. The equilibrium ownership structures are described in the following proposition.

Proposition 3 *There exists a unique threshold ability $b^* \in (b_{min}, b_{max})$ of the B-suppliers such that an equilibrium enterprise $(\lambda(b), b)$ chooses to stay separate (integrate) if and only if $b < (>) b^*$ if one of the following three conditions hold.*

(c) *$u(b)$ is very steep;*

(b) *$u_0 < \left(\frac{3}{2} - \sqrt{2}\right) c$ and $r(a,b) \equiv \frac{f(b)z_a(a,b)}{g(a)z_b(a,b)} \in [\underline{r}, \bar{r}]$ for all (a,b) where $\underline{r} \approx 0.414$ and $\bar{r} \approx 2.414$;*

(c) *$u_0 > \left(\frac{3}{2} - \sqrt{2}\right) c$ and $r(a,b) > \hat{r}$ where for all (a,b) where $\hat{r} \approx ?$.*

The intersection between the equilibrium utility $u(b)$ and the indifference locus $\tilde{u}(b)$ determines the equilibrium organizational choices. The slope of $u(b)$, which depends critically on the marginal productivity of B suppliers R_b, determines whether $u(b)$ intersects $\tilde{u}(b)$ only once, i.e., b^* is unique. When $u(b)$ is very steep, it intersects the vertical part of the indifference locus only once, and hence, $b^* = b^-$. Part (b) asserts rules out the fact that, for low values of u_0, the equilibrium utility $u(b)$ cannot be too flat at the beginning and too steep at the end so that there are more than one intersections between $u(b)$ and the indifference locus. Finally, part (c) asserts that if u_0 is very high, then $u(b)$ cannot be too flat to intersect $\tilde{u}(b)$.

4 Effect of price changes on the equilibrium

4.1 Incidence of integration

We now examine how a change in the product market price P affects the fraction of integrated firms in the market equilibrium. We will identify two countervailing effects, namely a *price effect* and a *utility effect*. The price effect is similar to the one in Legros and Newman (2013), which we describe in Lemma 2. The utility effect is novel and arises because the utility of each supplier is endogenized through the equilibrium matching. Recall that $(\lambda(b^*), b^*)$ is the enterprise indifferent between integrate and stay separate (see Figure 2). The above two effects thus refer to how the threshold ability b^* responds to a change in the product market price, i.e., $b^* = b^*(P)$.

In order to analyze the price effect, note that when the product market price increases from P to P', both b^- and b^+ decrease. As a result, the indifference locus $\tilde{u}(b)$ shifts in such a way that the initial and the final loci cross each other at a point (b^0, u^0) with $b^0 \in (b^-(P), b^+(P'))$ and $u^0 > \left(3/2 - \sqrt{2}\right) c$. This is depicted in Figure 3.

Lemma 2 (Price effect) *Suppose that, following a change in the product market price, the equilibrium utility $u(b)$ of each type b B-supplier remains unaltered.*

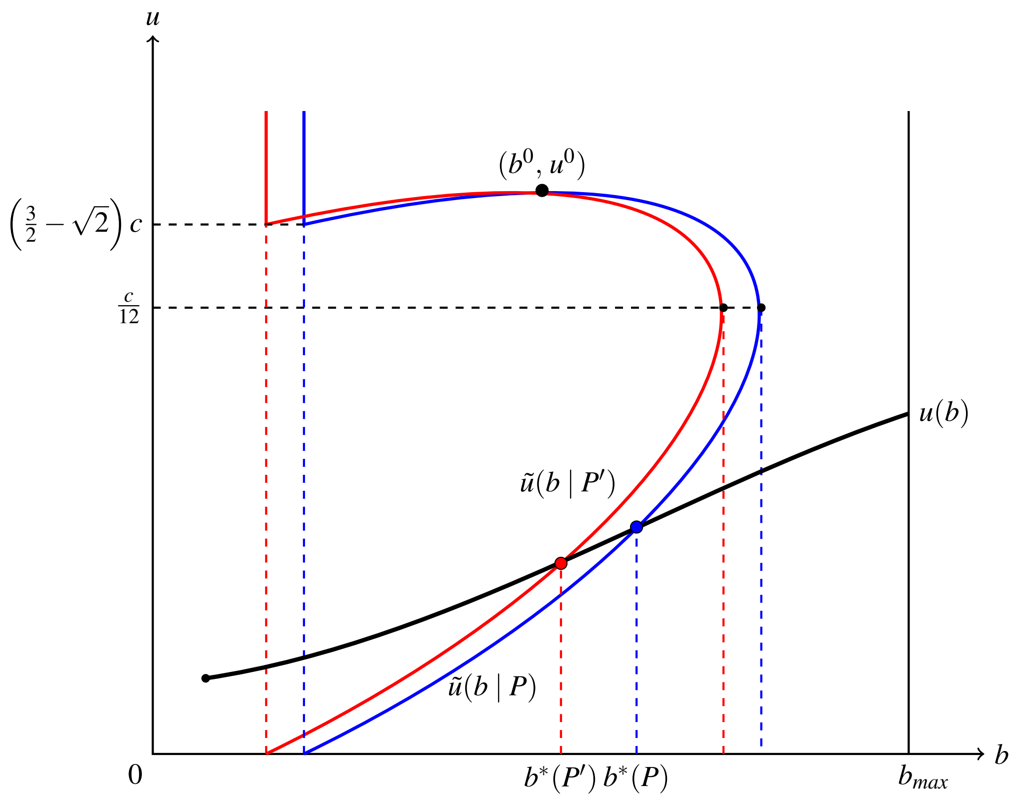

Figure 3: Price effect. Greater fraction of enterprises strictly prefer to integrate, i.e., $b^*(P') < b^*(P)$.

(a) *A greater fraction of enterprises integrate following an increase in P if $u(b)$ intersects both the initial and final indifference loci (i) either at their vertical segments, or (ii) If $u(b)$ is steep enough so that it intersects neither with $\tilde{u}_L(b)$ nor with $\tilde{u}_H(b)$, then a ;*

(b) *Otherwise, more enterprises integrate following an increase in P if and only if (b^*, u^*) is either on $\tilde{u}_L(b)$, or on $\tilde{u}_H(b)$ but b^* is sufficiently close to b^+.*

From Proposition 3, we know that there is a unique enterprise $(\lambda(b^*), b^*)$ that is indifferent in equilibrium between N and I. Moreover, from (9), we can write $b = b(P)$. Notice that b^- and b^+ are both decreasing in P. When $u(b)$ is steep enough so that it intersects neither $\tilde{u}_L(b)$ nor $\tilde{u}_H(b)$, we have $b^* = b^-$. Therefore, an increase in P implies a greater fraction of enterprises integrate in the new equilibrium, i.e., b^- decreases. When $u(b)$ intersects the indifference locus only once, the price effect is not so straightforward. Figure **??** depicts this case. A higher product market price shifts the $\tilde{u}_L(b)$ curve up for all $b \in [b^-, b^+]$, but the effect is asymmetric on the $\tilde{u}_H(b)$ curve. It is easy to see that there is a unique $b^0 \in (b^-, b^+)$ such that $\tilde{u}_H(b)$ is higher (lower) following an increase in P for $b < (>)\, b^0$. Therefore, if $u(b)$ intersects $\tilde{u}_L(b)$, then b^* decreases unambiguously, and hence, more enterprises integrate. The same happens if $u(b)$ intersects $\tilde{u}_H(b)$ but $b^* > b^0$. On the other hand, if $u(b)$ intersects $\tilde{u}_L(b)$ at (b^*, u^*) so that $b^* < b^0$, then the fraction of enterprises that choose N goes up in the new equilibrium. Part (b) of Lemma 2 rules out such an intersection. How the critical thresholds change with P depends on how the two bargaining frontiers, ϕ^I and ϕ^N shift following an increase in P, relative to each other. Therefore, when the critical u increases the enterprise that was indifferent between integration and non-integration prefers now to integrate if it is on $\tilde{u}_L(b)$ or on the portion of $\tilde{u}_H(b)$ that shifts down, but prefers to remain separate if it is on the portion of $\tilde{u}_H(b)$ that shifts up.[13]

Lemma 2 assumes that the equilibrium utility does not change even when the market price changes. But the utility, being endogenously determined through the equilibrium matching, will change. This is our key difference with Legros and Newman (2013) which pegs the utility of all B-suppliers at zero. In the following lemma we analyze the effect of a marginal increase in the product market price on the B-supplier utility.

Lemma 3 *The equilibrium utility $u(b)$ is an increasing function of the market price P for any $b > b_{min}$. Moreover, $du(b)/dP$ is an increasing function of b.*

Since in equilibrium $u(b_{min}) = u_0$, both in the old and new equilibria the utility of any B-supplier of type b_{min} is unaffected by a change in the product market price. Higher market price benefits all the B suppliers (except the lowest-productivity ones) and more so the higher types. The utility effect refers to the change in the fraction of integrated enterprises when the price changes that is attributed to the increase in the equilibrium utility.

Lemma 4 (Utility effect) *Let the product market price increases from P to P' and suppose that the $\tilde{u}_L(b)$ and $\tilde{u}_H(b)$ curves do not change.*

(a) *If both the old and new equilibrium utilities $u(b)$ intersect the $\tilde{u}_L(b)$ curve, then $b^*(P) < b^*(P')$, i.e., fewer enterprises will integrate in the equilibrium;*

(b) *If both the old and new equilibrium utilities $u(b)$ intersect the $\tilde{u}_H(b)$ curve, then $b^*(P) > b^*(P')$, i.e., a greater fraction of the enterprises will integrate in the equilibrium;*

Since $u(b)$ is increasing in P, higher utility, and hence a higher share for B suppliers, makes integration more profitable when the movement is in the direction of more balanced shares. This happens when the indifferent enterprise is on u_L. The opposite is true when the indifferent enterprise is in u_H.

The price and utility effects are opposing, unless the indifferent enterprise is on the u_H curve and sufficiently close to $z(\tilde{b}) = \frac{2c}{3P}$. Suppose b^* is on u_L. Then the price effect suggests more integrated enterprises but the utility effect points in the other direction. The reverse holds when $\tilde{b}$ is on u_H. The net result depends on the strength of these two effects. Because a closed form solution for $u(b)$ does not exist it is impossible to come up with clean conditions to characterize the net effect of P on the incentives to integrate. The next two numerical examples reveal that either effect can dominate.

Example 1 (higher prices lead to lesser integration) *We assume that the production function takes the Cobb-Douglas form $z(a,b) = a^{\alpha} b^{\beta}$, where $\alpha = 0.0001$ and $\beta = 0.575$. Note that the marginal productivity of B suppliers, relative to that of A suppliers, is very low. This makes the $u(b)$ curve steep and so it intersects with u_H. We set $c = 0.2$, and the price increases from $P = 0.55$ to 0.56. Furthermore, we assume that the productivity densities are the same across the A and B suppliers so that $\lambda(b) = b$. The*

[13] The corresponding u_L and u_H curves when the probability of success is quadratic, as in Legros and Newman (2013), never intersect and shift up as P increases. So, the implication is that a higher price increases the fraction of integrated enterprises if and only if the indifferent enterprise is on u_L. The prediction from Lemma 2 is not very different qualitatively.

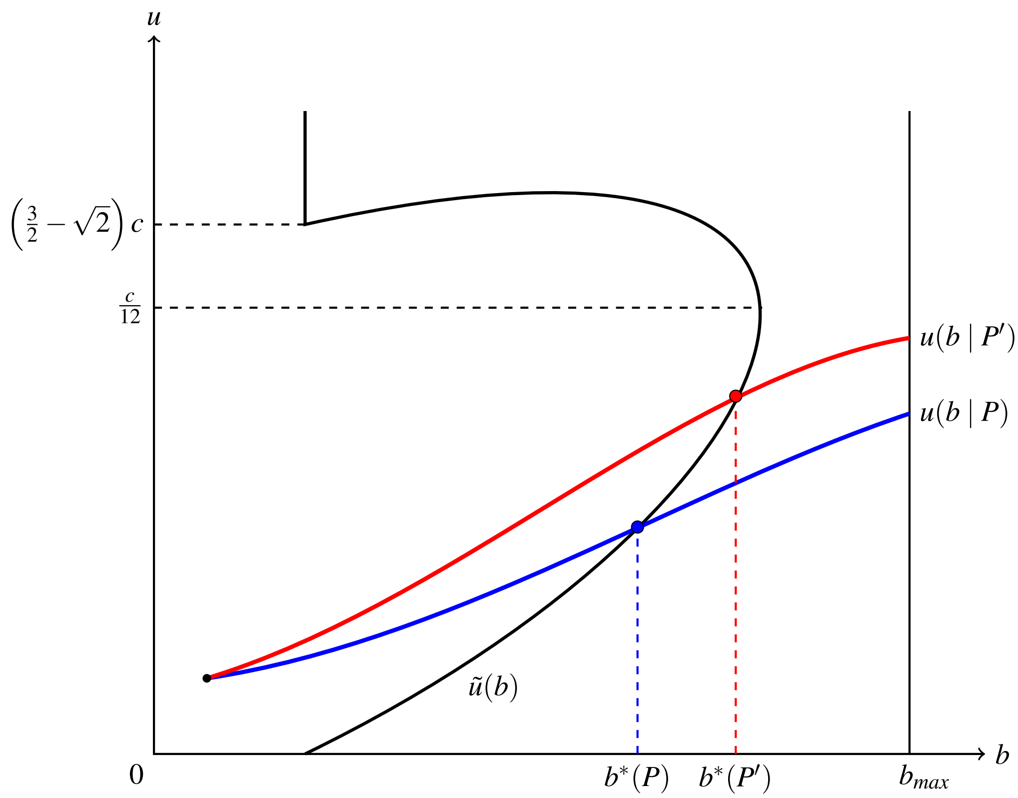

Figure 4: Utility effect. Greater fraction of enterprises strictly prefer to stay separate, i.e., $b^*(P') > b^*(P)$.

productivity support is $[0.05, 0.1]$ *and* $u(0.05) = 0.012$. *We solve the ODE numerically using the Maple software. Figure* **??** *depicts the Maple-generated graph. When* $P = 0.55$, *for* $b \in [0.05, 0.0702]$ *the choice is N, while for* $b \in [0.0702, 0.1]$ *the choice is I. At the higher price* $P = 0.56$, *for* $b \in [0.05, 0.07183]$ *the choice is N, while for* $b \in [0.07183, 0.1]$ *the choice is I. So,* $\tilde{b}(P)$ *is increasing in P, i.e., less integration as price increases.*

Example 2 (higher prices lead to more integration) *We assume that the production function takes the Cobb-Douglas form* $z(a,b) = a^{\alpha}b^{\beta}$, *where* $\alpha = 0.4$ *and* $\beta = 0.4$. *We set* $c = 0.5$, *and the price increases from* $P = 1$ *to* 1.05. *Furthermore, we assume that the productivity densities are the same across the A and B suppliers so that* $\lambda(b) = b$. *The productivity support is* $[0.1, 0.3]$ *and* $u(0.05) = 0.001$. *We solve the ODE numerically using the Maple software. Figure* **??** *depicts the Maple-generated graph. As it can be clearly discerned from the graph, more enterprises choose I at the higher price.*

4.2 Organizationally augmented industry supply and the product market equilibrium

We derive now the industry supply curve (OAS) as a function of the product market price. Consider an arbitrary enterprise (a, b). If this enterprise stays separate, then its expected output is given by:

$$q^N(a, b, P) = \pi(e_A(a, b), e_B(a, b))z(a, b) = \frac{R(a, b)}{2c} \cdot z(a, b) = \frac{P[z(a, b)]^2}{2c}.$$

The output of enterprise (a, b) is strictly increasing in the product market price P. On the other hand, if this enterprise integrates, its expected output is given by:

$$q^I(a, b, P) = \pi(e_A(a, b), e_B(a, b))z(a, b) = z(a, b).$$

The output of an integrated enterprise does not depend on the product market price. Clearly, an integrated enterprise produces greater expected output because $R(a, b) \leq 2c$ for all (a, b). The organizationally augmented industry supply is the expected output aggregated across all the enterprises in equilibrium, which is given by:

$$Q(P) = \int_{b_{min}}^{b^*(P)} q^N(\lambda(b), b, P)dF(b) + \int_{b^*(P)}^{b_{max}} q^I(\lambda(b), b, P)dF(b). \qquad \text{(OAS)}$$

A change in the product market price P affects $Q(P)$ via two channels – a rise in P (a) augments the output q^N of each non-integrated enterprise, but leaves the integrated output q^I unaltered and (b) changes the fraction of integrated enterprises by changing the threshold productivity $b^*(P)$ of the indifferent enterprise. Therefore, these two effects are countervailing. Whether the augmented industry supply curve $Q(P)$ is increasing or decreasing in P depends on the sign of $db^*(P)/dP$. Clearly, if the values of P are close to zero, then non-integration is the preferred ownership structure for all the enterprises because $b^-(P)$ is arbitrarily high. Therefore, $Q(P)$ must be increasing for low product market prices. But for high values of P if the threshold productivity $b^*(P)$ increases, the reduction in the number of integrated enterprises, and hence, the aggregate expected integration output may outweigh the increase in the aggregate expected non-integration output. Consequently, $Q(P)$ may decrease.

Proposition 4 *For low product market prices the organizationally augmented industry supply $Q(P)$ is increasing in P. However, $Q(P)$ may be backward-bending for high prices. A necessary condition for the backward-bending supply curve is a negative correlation between the product market price and integration, i.e., $\frac{db^*(P)}{dP} > 0$.*

In the following example we show that the industry supply curve may be backward-bending, which is built on Example 1.

Example 3 Let $z(a,b) = a^\alpha b^\beta$, where $\alpha = 0.0001$ and $\beta = 0.575$. Note that the marginal productivity of B suppliers, relative to that of A suppliers, is very low. This makes the $u(b)$ curve steep and so it intersects once with $\tilde{u}_H(b)$. We set $c = 0.2$ and $u_0 = 0.012$. Furthermore, we assume that both a and b are uniformly distributed on $[0.05, 0.1]$, and hence, $\lambda(b) = b$.

Using the above data we solve the ODE (10) numerically using the Maple. The two product market prices are $P = 0.55$ and $P' = 0.56$. It follows from (OAS) that $Q(P) = 0.1662150603$ and $Q(P') = 0.1616694501$. ■

In the above numerical example, a 1% increase in the product market price P implies an approximate decrease in the industry supply $Q(P)$ by 1.5%. To close our model, the equilibrium product market price P^* is determined by the intersection of the OAS $Q(P)$ and the demand curve $D(P)$.

5 Conclusions

The present paper analyzes the interaction between product market prices and firm boundary decisions in a market where the organizations are generically heterogeneous. Being consistent with the recent empirical evidence, our model implies that high-productivity enterprises tend to integrate, while the low-productivity ones stay separate and share surplus through contingent contracts. A decrease in the product market price level may imply a greater incidence of integration in a given economy due to the dominance of a novel utility effect over the traditional price effect. This finding helps explain why highly competitive economies such as the US witness the existence of pervasive integration (both vertical and lateral) in the highly productive industries such as cement.

As a consequence of the negative correlation between the market price and integration, a price increase may imply a reduction in the aggregate output. In other words, the organizationally augmented industry supply curve may be backward-bending. This has interesting testable implications. A favorable preference shock that exogenously shifts demand may imply lower market price, higher aggregate output and yet greater degree of integration if the old and the new demand intersect the OAS in its backward-bending part. This is due to an interaction between the product and input markets which generates a novel general equilibrium effect. Exogenous variations in the distributions of supplier productivity may have similar consequences which would be an interesting research agenda for the future.

A Appendix: Proofs of Lemmas and Propositions

Proof of Lemma 1

Substituting for e_A and e_B from the incentive compatibility constraints (IC_A) and (IC_B), the optimal contracting problem in an arbitrary enterprise (a, b) reduces to:

$$\max_{s\in[0,1]} V_A(s;R) \equiv \frac{R^2}{4c}(1-s^2), \tag{$\mathscr{M}'$}$$

$$\text{subject to } U_B(s;R) \equiv \frac{R^2}{4c}s(2-s) = u. \tag{PC'_B}$$

From (PC'_B) it follows that

$$s(a,b,u;P) = 1 - \frac{\sqrt{(R(a,b))^2 - 4cu}}{R(a,b)}.$$

We ignore the other root since it is strictly larger than 1. The value function is given by:

$$\phi^N(a,b,u;P) = \frac{R^2}{4c}\left[1-(s(a,b,u;P))^2\right] = u - \frac{(R(a,b))^2}{4c} + \frac{R(a,b)}{2c}\cdot\sqrt{(R(a,b))^2-4cu}. \tag{A.12}$$

Differentiating (PC'_B) with respect to $\theta = a, b$ and u, respectively we obtain

$$s_\theta(a,b,u;P) = -\frac{s(2-s)}{1-s}\cdot\frac{R_\theta}{R} < 0 \ \text{ for } \ \theta = a, b, \tag{A.13}$$

$$s_u(a,b,u;P) = \frac{2c}{R^2(1-s)} > 0. \tag{A.14}$$

Differentiating (A.12) with respect to a, b and u, we obtain

$$\phi_1^N(a, b, u; P) = \frac{1-s+s^2}{1-s} \cdot \frac{RR_a}{2c} > 0,$$

$$\phi_2^N(a, b, u; P) = \frac{1-s+s^2}{1-s} \cdot \frac{RR_b}{2c} > 0, \tag{A.15}$$

$$\phi_3^N(a, b, u; P) = -\frac{s}{1-s} < 0. \tag{A.16}$$

Form the above it follows that

$$\log \phi_2^N = \log R_b + \log R - \log(2c) + \log(1-s+s^2) - \log(1-s).$$

Differentiating the above with respect to a we get

$$\frac{\phi_{21}^N}{\phi_2^N} = \frac{R_{ab}}{R_b} + \frac{R_a}{R} + \left(\frac{2s-1}{1-s+s^2} + \frac{1}{1-s}\right) s_a = \frac{R_{ab}}{R_b} + \frac{1-3s+s^3}{(1-s)^2(1-s+s^2)} \cdot \frac{R_a}{R}. \tag{A.17}$$

On the other hand,

$$\log \phi_3^N = \log s - \log(s-1),$$

and hence,

$$\frac{\phi_{31}^N}{\phi_3^N} = -\frac{2-s}{(1-s)^2} \cdot \frac{R_a}{R}. \tag{A.18}$$

Therefore, equations (A.17) and (A.18) together imply

$$\frac{\phi_{21}^N}{\phi_2^N} - \frac{\phi_{31}^N}{\phi_3^N} = \frac{R_{ab}}{R_b} + \left[\frac{1-3s+s^3}{(1-s)^2(1-s+s^2)} + \frac{2-s}{(1-s)^2}\right] \frac{R_a}{R} = \frac{R_{ab}}{R_b} + \frac{3}{1-s+s^2} \cdot \frac{R_a}{R} > 0.$$

The above is true because $R_\theta = Pz_\theta > 0$ for $\theta = a, b$, $R_{ab} = Pz_{ab} > 0$ and $s \in [0, 1]$. Note also that

$$\frac{\phi_{21}^N}{\phi_2^N} - \frac{\phi_{31}^N}{\phi_3^N} > 0 \implies \phi_2^N \phi_{31}^N - \phi_3^N \phi_{21}^N > 0$$

since $\phi_2^N > 0$ and $\phi_3^N < 0$. This completes the proof of the Lemma.

Proof of Proposition 1

First, consider the case when non-integration completely dominates integration in a given enterprise (a, b). In this case the minimum non-integration surplus must be strictly higher than the maximum surplus under integration, i.e.,

$$\frac{R^2}{4c} > R - \frac{c}{2} \iff \left[R - c\left(2-\sqrt{2}\right)\right]\left[R - c\left(2+\sqrt{2}\right)\right] > 0$$

Since $R \leq 2c$, the above holds for $R < \left(2-\sqrt{2}\right)c \equiv R^-$. Next, consider the case when integration completely dominates non-integration in a given enterprise (a, b). In this case the minimum non-integration surplus must be strictly higher than the maximum surplus under non-integration, i.e.,

$$R - \frac{c}{2} > \frac{3R^2}{8c} \iff \left(R - \frac{2c}{3}\right)(R-2c) < 0$$

Since $R \le 2c$, the above holds for $R > \frac{2c}{3} \equiv R^+$. This completes the proofs of Parts (a) and (c). To show Part (b), note first that $\phi^N(a, b, u; P)$ intersects the linear frontier $\phi^I(a, b, u; P)$ exactly twice since both are symmetric with respect to the 45^0-line and the non-linear frontier is strictly concave. The two intersection points are given by:

$$u_L = \frac{1}{8}\left[4R - 2c - \frac{R}{c}\cdot\sqrt{(R-2c)(3R-2c)}\right],$$
$$u_H = \frac{1}{8}\left[4R - 2c + \frac{R}{c}\cdot\sqrt{(R-2c)(3R-2c)}\right].$$

Note that $u_H = R - \frac{c}{2} - u_L$. Thus, for the existence of the two intersection points it suffices to show that $u_L \ge 0$, which holds if $R \le -\frac{(2+\sqrt{10})c}{3}$ or $R^- \le R \le R^+$ or $2c \le R \le (2+\sqrt{2})c$. Given that $0 \le R \le 2c$, this case occurs when $R^- \le R \le R^+$. This completes the proof of the proposition.

Proof of Proposition 2

The bargaining frontier of each enterprise (a, b) has been derived for each P, which is a shift parameter. Therefore, we drop the argument P from $\phi(a, b, u; P)$ for the time being. Following Proposition 1, when $R(a, b) < R^-$, the frontier is given by $\phi_1^I(a, b, u; P)$, and hence, by condition (3) this property holds good. When $R(a, b) > R^+$, the frontier is given by $\phi_1^N(a, b, u; P)$. Therefore by Lemma 1, this property is satisfied. Finally, when $R^- \le R(a, b) \le R^+$, the bargaining frontier of (a, b) is given by:

$$\phi(a, b, u; P) = \begin{cases} \phi^I(a, b, u; P) & \text{if } u \in U^I(a, b), \\ \phi^N(a, b, u; P) & \text{if } u \in U^N(a, b); \end{cases}$$

Therefore,

$$\frac{d\phi}{db}(a, b, u(b)) = \begin{cases} \phi_2^I(a, b, u(b)) + u'(b)\phi_3^I(a, b, u(b)) & \text{if } u(b) \in U^I(a, b), \\ \phi_2^N(a, b, u(b)) + u'(b)\phi_3^N(a, b, u(b)) & \text{if } u(b) \in U^N(a, b), \end{cases}$$

Thus, the first order condition of the above maximization problem ($\mathscr{M}$) is given by $d\phi/db = 0$ which implies

$$u'(b) = \begin{cases} -\frac{\phi_2^I(a,b,u(b))}{\phi_3^I(a,b,u(b))} > 0 & \text{if } u(b) \in U^I(a, b), \\ \\ -\frac{\phi_2^N(a,b,u(b))}{\phi_3^N(a,b,u(b))} > 0 & \text{if } u(b) \in U^N(a, b). \end{cases} \tag{FOC}$$

Since ϕ^I and ϕ^N are strictly increasing in b, and hence, so is ϕ, the bargaining frontier is differentiable almost everywhere with respect to b. The second-order condition is given by:

$$\frac{d^2\phi}{db^2}(a, b, u(b)) = \begin{cases} \phi_{22}^I + u'(b)\phi_{23}^I + u'(b)[\phi_{32}^I + u'(b)\phi_{33}^I] + u''(b)\phi_3^I \le 0 & \text{if } u(b) \in U^I(a, b), \\ \\ \phi_{22}^N + u'(b)\phi_{23}^N + u'(b)[\phi_{32}^N + u'(b)\phi_{33}^N] + u''(b)\phi_3^N \le 0 & \text{if } u(b) \in U^N(a, b). \end{cases} \tag{SOC}$$

Differentiating (FOC) with respect to a along the equilibrium path $a = \lambda(b)$ we get:

$$\begin{cases} [\phi^I_{21} + u'(b)\phi^I_{31}]\lambda'(b) + \underbrace{[\phi^I_{22} + u'(b)\phi^I_{23} + u'(b)[\phi^I_{32} + u'(b)\phi^I_{33}] + u''(b)\phi^I_3]}_{\text{negative by (SOC)}} = 0 & \text{if } u(b) \in U^I(a,b), \\ [\phi^N_{21} + u'(b)\phi^N_{31}]\lambda'(b) + \underbrace{[\phi^N_{22} + u'(b)\phi^N_{23} + u'(b)[\phi^N_{32} + u'(b)\phi^N_{33}] + u''(b)\phi^N_3]}_{\text{negative by (SOC)}} = 0 & \text{if } u(b) \in U^N(a,b). \end{cases}$$

Therefore,

$$\phi^d_{21} + u'(b)\phi^d_{31} = \phi^d_{21} - \frac{\phi^d_2}{\phi^d_3} \cdot \phi^d_{31} > 0$$
$$\iff -\frac{1}{\phi^d_3}[\phi^d_2\phi^d_{31} - \phi^d_3\phi^d_{21}] > 0 \iff \phi^d_2\phi^d_{31} - \phi^d_3\phi^d_{21} > 0 \text{ whenever } u(b) \in U^d(a,b) \text{ for } d = I, N$$

is a sufficient condition for PAM, i.e., $\lambda'(b) \geq 0$, which follows from Lemma 1 and condition (3). Note that, at $u_L(a,b)$ and $u_H(a,b)$, we have $\phi^d(a,b,u(b)) \equiv \phi(a,b,u(b))$ for all (a,b), and hence, (SCP) holds at these intersection points for all (a,b). This completes the proof of the Proposition.

Proof of Proposition 3

We derive sufficient conditions for a single-crossing between $u(b)$ and either u_L or u_H.

First, we assume that the first intersection is between $u(b)$ and u_L. We differentiate u_L, as is given in (4), with respect to b on the equilibrium path. This yields

$$\frac{du_L}{db} = \frac{du_L}{dR}\left(R_a\lambda'(b) + R_b\right),$$

where $\frac{du_L}{dR}$ is 0.7071067810 at R^- and $+\infty$ at R^+ and increasing in that interval.

When the first crossing occurs, at $\hat{b}$, it must be

$$\frac{du_L}{dR}\left(R_a\lambda'(b) + R_b\right) > R_b. \tag{A.19}$$

A sufficient condition for a second crossing not to happen is the term $R_a\lambda'(b)$ does not become very small for b's higher than $\hat{b}$.

Next, we assume that the first intersection is between $u(b)$ and u_H. We differentiate u_H, as is given in (5), with respect to b on the equilibrium path. This yields

$$\frac{du_H}{dR}\left(R_a\lambda'(b) + R_b\right) < R_b. \tag{A.20}$$

where $\frac{du_H}{dR}$ is 0.2928932190 at R^- and $-\infty$ at R^+ and increasing in that interval. A sufficient condition for a second crossing not to happen is the term $R_a\lambda'(b)$ does not become very large for b's higher than $\hat{b}$.

Combining (A.19) and (A.20), and using the lowest slope for u_L and the highest for u_H, the sufficient condition for a single-crossing is $R_a\lambda'(b) \in [0.414213563R_b, 2.414213560R_b]$ on the equilibrium path for $b > \hat{b}$.

Proof of Lemma 3

The equilibrium utility is given by

$$u(b) = u(b_{min}) + \int_{b_{min}}^{b} u'(\tau) d\tau,$$

where

$$u'(\tau) = \left(\frac{1}{s} + s - 1\right) \frac{P^2 z_b z}{2c}$$

and $s = 1 - \frac{\sqrt{P^2 z^2 - 4cu}}{Pz}$ depends on P directly (negatively) and indirectly through u (it depends positively on u). Let $A \equiv \left(\frac{1}{s} + s - 1\right)$. Then we have

$$\frac{du(b)}{dP} = \int_{b_{min}}^{b} P \frac{z z_b}{2c} \left[P \frac{dA}{ds} \left(\frac{ds}{dP} + \frac{ds}{du} \frac{du(\tau)}{dP} \right) + 2A \right] d\tau, \tag{A.21}$$

where $dA/ds < 0$, $ds/dP < 0$ and $ds/du > 0$.

First, note that $du(b_{min})/dP = 0$. This is intuitive since the lowest utility is given exogenously and is not a function of the price P. Second, for b's arbitrarily close to b_{min}, $du(b)/dP$ is positive. Since in this case $du(\tau)/dP$ is arbitrarily close to zero, and the other terms in the integrand of (A.21) are positive, $du(b)/dP$ cannot be negative for b's in the neighborhood of b_{min}.

Suppose by way of contradiction that $du(b)/dP$ becomes negative for some b. Consider the b, denoted by $\tilde{b}$, at which $du(\tilde{b})/dP = 0$. Given, as we showed above, that the integrand of (A.21) is positive initially, then at $\tilde{b}$ it must be negative (so that the positive and negative areas cancel each other out in the integration). But the integrand of (A.21) evaluated at $\tilde{b}$ is strictly positive, contradiction.

Given that the integrand of (A.21) is always positive, it follows that $du(b)/dP$ is increasing in b.

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