Soft Information Production and Investment in Specific Assets

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Abstract

We analyse how soft information about specific assets invested in a project contributes to the expanded financing of socially valuable projects that require investment in both general and specific assets. We develop a model that captures important aspects of the lender-borrower relationship in project financing and allows for an interpretation of degree of softness of information with the lender. We emphasize that soft information is the output of a costly "information mechanism." The lender uses the soft information about a specific asset to improve recall decisions prior to the revelation of the return on the project. Soft information is not necessarily verifiable but it will induce investment in specific assets if the project manager knows that the lender employs the costly information mechanism. Higher quality of soft information strengthens the incentive to invest in the specific asset.

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1. Introduction

The objective of this paper is to analyse how soft and costly information about assets invested in a project contribute to the expanded financing of socially valuable projects that require investment in both general and specific assets. We develop a model that captures important aspects of the lender-borrower relationship in project financing including the possibility of loan recall based on soft information generated during the life of the project.

Soft information and its role from informational and value perspectives has been analysed within several contexts. The review of literature on soft information below shows that the exact specification of soft information tends to depend on the role information plays in a particular context. Soft information often refers to the uncertainty and verifiability of a particular piece of information that is being used as an input in a decision. We view the soft information as the output of a process that combines pieces of information within a subjective model that allows the pieces of information to be interpreted for a particular purpose. In our context this purpose is for a lender to use the soft information to possibly recall a loan before the project return is realized. The value of the soft information does not depend on its verifiability but on the alignment of the incentives of the lender and the manager-entrepreneur controlling the project.

In our setting the lender finances the investment in a general asset while the manager-entrepreneur invests in a specific asset. The final project return depends on the returns on both general and project specific assets. During the course of the project, information about the general asset becomes freely available while soft information about the specific asset requires that the lender incurs costs of an "information mechanism." Using this costly mechanism, the lender may want to recall the loan before project returns are realized. By incurring greater costs, the soft information can be improved and, thereby, reduce the expected costs of Type 1 and Type 2 errors from the recall decision.

The information mechanism (IM) makes it possible for the lender to process any possibly value-relevant publicly or privately available hard and soft pieces of information. Thereby, the mechanism produces a signal that provides a range of possible values for the specific asset and the project as a whole. This range defines the softness of the information. It is subjective because the information mechanism uses the lender's knowledge or model for interpretation of pieces of information.

If the manager-entrepreneur has invested in the specific asset, the soft signal generated by the IM has value for the lender, ex ante, if it improves the recall decision and, thereby, the lender's expected return. The manager-entrepreneur's incentive to invest in the specific asset depends on the lender's information mechanism.

The soft information can be compared to a credit rating for the loan. Such ratings are generally produced by external agencies and the costs are borne by the borrowers. Most loans are not rated, which means that lenders must produce their own ratings. It makes little difference whether costs are borne by the borrower or the lender / bank. The soft information in our model can also be thought of as a Level 2 or 3 Fair Value of a specific asset.[4] Fair Values are typically produced by means of proprietary and subjective models. Both ratings and Fair Values require input information about the project and its management from accounting as well as other sources. Whether we talk about ratings or Fair Values we can think of them as soft signals that imply a range for true values. In our context, the lender produces a soft signal during the course of project.

The rest of the paper proceeds as follows. Literature on soft information is reviewed in Section 2. Thereafter, in Section 3 we provide an overview of the model. The lender's recall decision is first analysed in Section 4 under the condition that the recall must be based only on hard information about the general asset. In Section 5 the lender has soft information about the value of the specific asset as well. We analyse the lender's recall policy taking the quality of the soft information as given. The quality of the soft information signal is made endogenous in Section 6 where we take into account the trade-off between the cost of the information mechanism and the benefits in terms of improved recall decisions. The lender's decision on whether to invest in the information mechanism and on whether to make the loan are analysed in Section 7. The manager/entrepreneur is brought

[4] A Level 1 Fair Value is an observable market value while Level 2 and Level 3 Fair Values are estimates produced by a comparison with a similar asset's market value or a theoretical model for the market value. In both cases there is subjectivity in the estimate.

into the analysis in Section 8 where we ask whether the lender's incentive to produce soft information is consistent with the project manager's incentive to invest in the specific asset. We show that if the lender has only hard information as basis for its recall policy, the incentives of the manager to invest in the specific asset are weak or non-existent. If the lender invests in the information mechanism to produce soft information, the recall policy based on both hard and soft information strengthens the project manager's incentive to invest in the specific asset. An important factor increasing the incentive to produce soft information is a relatively low expected value the general asset, which can be valued with only hard information and recovered in liquidation. Thus, if the investment in the general asset is relatively risky, there are strong incentives to spend resources to reduce the risk associated with evaluation of the specific asset. We conclude in Section 9.

2. Related literature on soft information[5]

Several papers analyze the role of soft information for valuation in decentralized asset markets. In Bertomeu and Marinovic (2013) managers can disclose hard and soft value-relevant information to external stakeholders and choose the level of disclosure for each type of information. Audited information is hard from the point of view of external stakeholders while non-audited information can be made hard or less soft through a costly certification process similar to out information mechanism. Without costly certification non-audited information can be manipulated. It is shown that more aggressive disclosure of soft information is associated with misrepresentation. There is a trade-off between release of hard information and the quality of soft information. We do not introduce managers' manipulation of available information explicitly but we argue that one aspect of the lender's costs of producing a soft signal is to evaluate incentives and effects of manipulation when incentives exist.

Another paper showing that increased disclosure of soft information can lead to an information distortion is Edmans, Heinle and Huang (2013). Since intellectual property

[5] There is a literature on optimal contracting and information disclosure, which we will not review here, since it does not explicitly focus on softness of information. (See, for example, Aghion and Bolton,1992, and Sridhar and Magee, 1997.) We take contractual terms as given in order to focus on the recall decision and information production.

cannot be assessed without soft information, increased disclosure of hard information can lead to underinvestment in intellectual property. Although there is a strong similarity with the results in Bertomeu and Marinovic (2013), Edmans et al assume that soft information cannot be credibly disclosed and, therefore, not affect the market valuation of a firm.[6] Increased disclosure is distorting because the relative amounts of hard and soft information affect market valuation and managers' choice of investment in assets requiring different kinds of information.

Demurs and Vega (2010) provide an empirical analysis of the information value of pieces of soft information included in managers' statements in connection with earnings reports. Soft information is defined as qualitative and verbal pieces of information in the statements. Linguistic analysis is used to identify soft information in terms of optimism and certainty. Thereafter the authors analyze the role of soft information in the price formation process. The key result is that soft information plays a relatively large role in the settings where the hard signal is relatively noisy. Thus, hard and soft information substitute for each other in the sense that investors rely more on soft information when hard information is less informative. This result indicates that soft information has some credibility and not just "cheap talk." The empirical results in Demurs and Vega support the prediction of the previous papers that increased disclosure of hard information may reduce the quality of soft information. In our model the hard information about the general asset is costlessly available but the contents of this information affect the lenders incentive to produce soft information.

The original "cheap talk" model in Crawford and Sobel (1982) predicts that managers provide credible information as long as the interests of managers and investors are aligned. The alignment of incentives plays an important role in our analysis as well. Dye and Sridhar (2004), on the other hand, emphasize that verifiability by outside observers is a key mechanism for inducing truthful information revelation. In their model managers release both hard and soft information and market prices of firms depend on both. The two types of information are complementary in this paper because increased disclosure of hard information can enhance the credibility of the soft information.

[6] Edmans et al follow Stein (2002) in their conceptualization of soft information.

Another context wherein soft information plays a role is incentive contracting. Rajan and Reichelstein (2006) ask whether soft information can be used to compensate a manager and, thereby, provide efficient incentives. Soft information is defined as non-verifiable for contracting purposes. The authors show that the compensation contract can be based on a subjective signal if it has a degree of credibility.[7] This credibility can be achieved within a bonus pool wherein one person's reduce compensation based on a negative signal corresponds to increased compensation for others.

In the lender-borrower relationship described in our model soft information is produced at a cost during the course of a project. Rajan (1992) argues that relationship lending enables banks to gather and evaluate a variety of hard and soft information about a project's and a project manager's quality while the project is on-going. This advantage of relationship lending requires that the bank has the right to recall the loan before the completion of the project. We analyse how the recall decision as well as the decision to invest in a specific asset depends on soft information with endogenous quality and the remuneration contract of the manager-entrepreneur.

3. Model

In period -1, the entrepreneur-manager decides whether to invest in a project-specific asset. Should she choose to invest in the project-specific asset, she bears the cost, θ, of acquiring the project-specific asset. It is convenient to think about the specific asset in the model as non-marketable intellectual property controlled by a manager/entrepreneur.

In period 0, the bank / lender decides whether to provide the financing for a project investment in a general asset, I, that generates a payoff, $\tilde{x}$, in period 2. General assets are traded in well-functioning markets where prices can be observed while specific assets have no value outside the project.[8]

[7] Thereby, the signal satisfies Holmstrom's proposition (1979) that an additional signal is valuable for contracting purposes only if it is informative in a statistical sense given the initial signal.
[8] Many assets have a general as well as a specific component. In accounting terms an asset is specific to the extent its replacement value is higher than its liquidation or market value.

The general asset obtains a high value, $\tilde{x} = \bar{x}$ with probability q and a low value, $\tilde{x} = \underline{x}$, with probability $1 - q$. If an entrepreneur-manager has invested in a project-specific asset in period -1, this asset adds $\tilde{b}$ to the project return. The total return in period 2 then becomes $\tilde{y} = \tilde{x} + \tilde{b}$. Note that the project *must* continue to period 2 for the stochastic value-augmentation $\tilde{b}$ to bear fruition. The stochastic value-augmentation $\tilde{b}$ lies in an interval $[-\varepsilon, \varepsilon]$, with distribution $f(\tilde{b})$, with cumulative distribution $F(\tilde{b})$ and with $E(\tilde{b}) \geq 0$, where E is the expectations operator in period 1. At this point we do not make specific assumptions about the distribution except that its density for $b>0$ is greater than the density for $b<0$ in order to make its expected value greater than zero.[9]

If the bank / lender decides to provide financing in period 0, it may additionally also choose to invest in an information mechanism (IM) that *will* generate soft information about the specific asset. The cost of producing improved quality of the soft information is described by the following cost function for the IM:

A.1*:* $C'(e) > 0, C''(e) > 0, C(0) = 0$,

where an increasing e between 0 and 1 represents increased quality of information. The IM is associated with a cost in period 0 because the lender must devote resources to be able to analyse and interpret value-relevant information about the firm and the manager to generate a signal, $\sigma(b)$ about the value of the specific asset. The information may be disclosures provided by the firm's accounting system, information obtained in direct relationships between lender and borrower, publicly available information as well as knowledge about models for interpretation of value-relevant information.

In period 1, the lender receives an accounting signal about $\tilde{x}$. This constitutes a piece of hard information for the lender. If the lender had chosen to invest in the information mechanism in period 0, then it also receives a signal $\sigma(b)$ about the value of the specific asset. This constitutes the lender's soft information. For every b, the mechanism generates

[9] We could have assumed a normal distribution for the return on the specific asset with a mean greater than zero but we want correspondence between the distribution for the return on the asset and the specification of signal quality with respect to the same asset below. The return distribution and signal quality are specified below. See also fn 10 below.

in period 1 a value, $\sigma(b)$ with a certain quality, e. Once the signal with quality e is obtained the lender knows that the true b will lie within a range, $\left[l(b,e),u(b.e)\right]$. The following assumption shows how the quality of the signal affects the lender's perception in period 1 about the range of the value of the specific asset:

A.2: (i) $-\varepsilon \leq l(b,e) \leq b \leq u(b,e) \leq \varepsilon$

(ii) $\lim_{e\to 1} l(b,e) = \lim_{e\to 1} u(b,e) = b$

(iii) $\lim_{e\to 0} l(b,e) = -\varepsilon, \lim_{e\to 0} u(b,e) = \varepsilon$

(iv) $\frac{\partial l(.)}{\partial e} \geq 0, \frac{\partial u(.)}{\partial e} \leq 0, 0 < e < 1$

(v) $\frac{\partial l(.)}{\partial b} \geq 0, \frac{\partial u(.)}{\partial b} \geq 0, -\varepsilon < b < \varepsilon$

The first expression says that the IM searches for a value of b spread over a smaller range between l and u than what one uses when there is no soft information and that, this smaller range still contains b. The second expression is a formalization of the notion that if the IM were perfect ($e = 1$), it would correctly predict the true value. The third expression states that if the signal has no information, $e=0$, the range is equal to the full range of possible b-values. For the IM to be informative, the range it provides should be smaller than the original range of possible values. The fourth expression assumes that the band limits change smoothly with signal quality. Both the upper limit and the lower limit of the band approach the true value (from different directions) as the signal's quality, e, increases. The final expression (v) means that the range of possible signals about the true value of b moves in the same direction as b.

The soft signal is not perfect but informative in a sense dictated by the need of the lender to apply its recall rights. Consider the situation where the lender wants to know if a particular entrepreneur who has invested in, for example, skill has a value $b \geq \hat{b}$. Without the IM, the lender knows that b lies within the range $[-\varepsilon, +\varepsilon]$ and that the probability of $b \geq \hat{b}$ is simply $[1 - F(\hat{b})]$. The quality of the signal obtained by the IM about $b \geq \hat{b}$ is measured by the parameter $e, 0 \leq e \leq 1$. The higher is the value of this parameter, the greater is the signal's quality. Any imperfect mechanism, $e < 1$, trying to predict whether the true value of b is greater than $\hat{b}$ or not, can make two types of errors. It may say that

$b \geq \hat{b}$ when it is not so (Type 1 error), and it may say it is not so when it actually is (Type 2 error). A perfect mechanism, $e = 1$, will not make any of these two types of errors.

Now, based on the information it has available in period 1, the lender decides whether to recall the loan. If $\tilde{x} = \overline{x}$ and the lender recalls its loan in period 1, then the lender receives its outstanding debt claim, denoted by D. However, if $\tilde{x} = \underline{x}$ and the lender recalls its loan in period 1, then the lender receives only $\underline{x}$. Recall that the stochastic value-augmentation $\tilde{b}$ bears fruition only if the project is continued till period 2. If the project is liquidated in the interim period 1, then $b=0$ and the project simply yields the value of the general asset $\tilde{x}$.

The debt claim D is such that if the low value of the general asset is realized, the debt obligation cannot be repaid in full. More specifically, we assume

A.3*:* $\overline{x} > D > \underline{x} > \varepsilon > 0$

The outstanding debt claim, D, is defined as the loan amount plus a risk-premium to compensate the lender for expected losses. The risk-free interest rate is assumed to be zero.

(3.1) $D=I+rp$

The risk-premium and, thereby, the debt claim, D, is set so that the lender's expected profit is zero if there is no investment in the specific asset.

We assume that, if the lender is indifferent between recalling and rolling over the loan in period 1, it will roll over the loan and allow the project to continue since there is a potential private benefit to the manager-entrepreneur as we will see below.[10] If the project continues to period 2, the manager-entrepreneur extracts some private benefit and the lender receives the maximum of its debt claim, D. More specifically, if specific investment was made in period -1 and loan was not recalled in period 1, lender gets $D,\ if\ \ \tilde{y} \geq D$ and gets $\tilde{y},\ if\ \ \tilde{y} < D$. If specific investment was not made in period -1 and loan was not recalled in period 1, lender gets $D,\ if\ \tilde{x} = \overline{x}$ and gets $\tilde{x},\ if\ \ \tilde{x} = \underline{x}$.

[10] In Aghion and Bolton (1992) an efficient contract must resolve the trade-off between costs to the lender of a manager's desire to prolong a project in order to extract private benefits, and costs of insufficient investments in project-specific managerial assets.

The manager's remuneration scheme (denoted by π_M) depends on the project's return and the lender's recall policy in the following way:

(3.2) $\pi_M = (b-(D-x)) + z$, if $(b-(D-x)) \geq 0$

$= z$, if $(b-(D-x)) < 0$

Where $x=\bar{x}, \underline{x};$

The first term in (3.2) represents monetary remuneration while z represents the manager's private non-monetary remuneration if the project continues to period 2. There is no private benefit if the loan is recalled in period 1.

The monetary remuneration represents a fraction, k, of what remains of the project return after the debt claim has been repaid. If the manager-entrepreneur is the only shareholder based on the investment in the specific asset, then k=1. The condition (3.2) can be thought of as limited liability of the manager/entrepreneur and other shareholders, if they exist, since the remuneration does not become negative even if b turns out to be negative.

With probability q, x=$\bar{x}$. The manager receives $k(\bar{x}$-D) if the project is recalled in period 1. With probability $(1-q)$, x=$\underline{x}$. In this case the manager does not receive any remuneration if the loan is recalled in period 1. If the loan is not recalled the manager still does not receive remuneration if the return on the specific asset is insufficient for full repayment of the debt claim.

4. The lender's pay-off and recall policy with only hard information

In this section we analyse the recall policy in period 1 assuming that the manager-entrepreneur invests in the project-specific asset in period -1 and that the lender knows this. We also assume that the lender makes the loan in period 0 and chooses not to invest in the IM in period 0. Now, the recall policy in period 1 must be based only on hard information about the general asset. This recall policy is specified by Proposition 1.

Proposition 1. Assume that the manager-entrepreneur invests in the project-specific asset in period -1 and that the lender knows this. Assume also that the lender makes the loan in period 0 and chooses not to invest in the IM in period 0. Then, if $\tilde{x} = \bar{x}$, the lender will always recall in period 1. If $\tilde{x} = \underline{x}$, the lender will not recall only if $(D - \underline{x}) - \int_{-\varepsilon}^{D-\underline{x}} F(\tilde{b})db > 0$.

Proof in Appendix.

If the lender only has the hard information made available through the accounting system about $\tilde{x}$, it has to make the recall decision in period 1 based on the observed signal about the value of $\tilde{x}$ and the known distribution of $\tilde{b}$. Figure 1 describes the payoff to the project and the lender in period 2 for the two possible realizations of $\tilde{x}$. The horizontal axis shows realizations of $\tilde{b}$ while the vertical axis shows the total project return $\tilde{y} = \tilde{x} + \tilde{b}$ and the debt payment, D.

Assume first that $\tilde{x} = \bar{x}$. In this case the lender can recall in period 1 and liquidate the asset at a value greater than the debt claim and be fully repaid. If the lender does not recall it expects to be repaid in full only if $\tilde{b} \geq D - \bar{x}$**.** If the value of the specific assets turns out to be $\tilde{b} < D - \bar{x}$, the lender will receive $(\bar{x} + \tilde{b}) < D$, where $\tilde{b}$ is negative**.** Since there is a positive probability of $\tilde{b} < D - \bar{x}$**,** the lender will always recall in period 1 if $\tilde{x} = \bar{x}$ in the absence of updated information about *b*.

Figure 2 shows the costs of Type I and Type II errors associated with the recall decision. Continue with the case $\tilde{x} = \bar{x}$. If the lender does not recall (allows continuation) there is a risk of making a Type 1 error as shown in Figure 2. The expected value in period 1 of this Type 1 error is:

$$E\,\langle Type\ 1 \,|\, \tilde{x} = \bar{x} \rangle = \int_{-\varepsilon}^{D-\bar{x}} (D - (\bar{x} + b))F(\widetilde{b}) > 0.$$

If the lender recalls (does not allow continuation) there is no possibility of a Type 2 error as shown in Figure 2. Thus, as already noted, the lender will always choose to recall in order not to risk a Type 1 error for very low realizations of *b*. The pay-off for the lender in case of recall can be expressed as $E(\pi_B|\overline{x}) = D$.

Now assume that $\tilde{x} = \underline{x}$. Then for $\tilde{b} > 0$ the lender is better off with no recall while if $\tilde{b} < 0$, recall in period 1 would have been the best choice. In period 1 with information only that $\tilde{x} = \underline{x}$ the lender faces the possibility of both Type 1 errors, if it does not recall, and Type 2 errors if it recalls. As shown in Figures 1 and 2 errors of Type 1 occur if the lender chooses no recall (continues) and *b* turns out to be less than 0. Type 2 errors occur if recall has been chosen in period 1 and *b* turns out to be positive. The magnitude of the cost associated with the Type 2 error depends on whether *b* is large enough to repay the whole loan or not.

We can see in Figures 1 and 2 that if $0 \leq b < D-\underline{x}$ the loss associated with recall is *b*. In the range $b \geq D-\underline{x}$, the loss associated with recall is $D-\underline{x}$. Thus the expected costs associated with the errors are:

(4.2) $E\left\langle Type\ 1 \middle| \tilde{x} = \underline{x} \right\rangle = \int_{-\varepsilon}^{0} \tilde{b}\, dF(\tilde{b})$

(4.3) $E\left\langle Type\ 2 \middle| \tilde{x} = \underline{x} \right\rangle = \int_{0}^{D-\underline{x}} \tilde{b}\, dF(\tilde{b}) + \int_{D-\underline{x}}^{\varepsilon} (D-\underline{x})\, dF(\tilde{b})$

Summing up these expected costs of error and noting that the lender will not recall if and only if $E\langle Type\ 2 | \tilde{x} = \underline{x} \rangle > E\langle Type\ 1 | \tilde{x} = \underline{x} \rangle$, we arrive at our condition for no recall in Proposition 1:

(4.5) $\left(D - \underline{x}\right) - \int_{-\varepsilon}^{D-\underline{x}} F\left(\tilde{b}\right) db > 0$

This expression says that the lender will choose No Recall if $D - \underline{x}$, which can be considered ‘Loss Given Recall’, is greater than a value that depends on the probability that the outcome for *b* falls below ‘Loss Given Recall’. Under this condition the lender will choose Recall if $\tilde{x} = \bar{x}$ and No recall if $\tilde{x} = \underline{x}$. The lender will choose to always Recall if (4.5) does not hold, i.e if $D - \underline{x}$, is relatively small. In other words, if the loss from recalling the loan is expected to be small even if the general asset obtains its low value, the lender will never allow the project with specific assets to continue to its completion if the lender has access to only hard information. Clearly, there is little incentive for the manager-entrepreneur to invest in the specific asset under these circumstances. This issue is analysed in more detail in Section 8.[11]

[11] For convenience we work with $D - \underline{x}$ as exogenous although we expressed D as a function of $\underline{x}$ through the risk-premium in Section 3. No results are affected as long as $\underline{x} < I$ and $q > 0$.

5. The information mechanism (IM) and the lender's recall policy with soft information

In this section we add an information mechanism (IM), which produces soft information for the lender about the value of the specific asset in period 1 when the recall decision is made. The soft information is additional to the hard information about the general asset. We still maintain the assumptions that the manager-entrepreneur invests in the project-specific asset in period -1 and that the lender makes the loan in period 0. Further, we assume that the lender chooses to invest in the IM in period 0. Now, the recall policy in period 1 will be based on both hard information about the general asset and soft information about the specific asset. This recall policy is specified by Proposition 2.

Proposition 2. Assume that the manager-entrepreneur invests in the project-specific asset in period -1, that the lender makes the loan in period 0 and also chooses to invest in the IM in period 0. Then, if $\tilde{x} = \bar{x}$, the lender will recall in period 1 only if the signal $\sigma(b)$ implies an $l < D - \bar{x}$. If $\tilde{x} = \underline{x}$, then the lender will recall in period 1 only if either (i) or (ii) is satisfied.

(i) the signal $\sigma(b)$ is such that $l < 0, u < 0$

(ii) the signal $\sigma(b)$ is such that $l < 0, u > 0$ and the signal $\sigma(b)$ is lower than the signal that makes the expected cost of Type 1 error at the lower bound equal to the expected cost of Type 2 error at the upper bound

Proof in Appendix.

The intuition for Proposition 2 hinges on how the expected values of Type 1 and Type 2 errors in period 1 depend on observations of $\tilde{x}$ and the soft signal. In terms of Figures 1 and 2 we can think of the signal, $\sigma(b)$, as a certain number, $\hat{b}$, as well as a range for the true b between l and u is given by e. Thus, when the signal value is observed the lender also knows the lower and upper bounds for the actual b-values.[12]

[12] There are two reasons for specifying the signal quality as a range rather than as a normal distribution. First, bank's behavior is better described this way without the restrictive assumption about a normal distribution of forecast errors. Second, if the normal distribution were assumed the lender would always want to recall in case the high value for the hard

The quality of the signal, e, going to zero means that the mechanism generates no valuable information since then, the lower bound $l = -e$ and the upper bound $u = +e$, and so, the probability with which the IM calls out a value of $b ³ \hat{b}$ is simply $[1- F(\hat{b})]$. Alternatively, if $e = 1$, then $l = u = b$ and, by choosing a value of b from within this (degenerate) range, the IM reveals the true value of b.

We use the notation *G(b)* for the distribution of *b* between the lower and upper limits.

(5.1) $G(b) = \frac{F(b)-F(l(b,e))}{F(u(b,e)-F(l(b,e)}$

Therefore, $G(l) = 0$ *and* $G(u) = 1$.

We can now define how expected values of Type 1 and Type 2 errors depend on available hard and soft information. As shown in Figures 1 and 2 the critical values for Type 1 and Type 2 errors depend on whether $\tilde{x} = \bar{x}$ or $\tilde{x} = \underline{x}$. In addition expected Type 1 and Type 2 errors depend on the lower and upper bounds relative to critical values. Thus, the recall policy will be conditional on observations of hard and soft information in period 1. At a given signal quality, *e*, observation of the soft signal implies observations of the lower and upper bounds for *b*.

Figure 1 shows that the recall policy will depend on information in the following way:

For $\tilde{x} = \bar{x}$:

(i) $-\varepsilon < l < u < D - \bar{x} < \varepsilon$

Both the lower and the upper range for actual *b* is lower than the value of $D - \bar{x}$, below which costly Type 1 errors occur. Thus, the lender's choice in this case is to recall and to receive the debt claim D.

(ii) $-\varepsilon < l < D - \bar{x} < u < \varepsilon$

In this case there is a positive probability that a Type 1 error may occur if the lender does not recall. If b turns out to be greater than $D - \bar{x}$ the lender is indifferent between recall and no recall. By assumption, when the lender is indifferent, she will not recall. In this case, she will receive D.

information is realized because there would always be a non-zero probability that the true value of b is below a certain level as in Section 2.

(iii) -ε < $D - \bar{x}$ < l < u < ε

There is no possibility of Type 1 errors. The lender is indifferent between recall and no recall and by assumption, she will not recall.

If $\tilde{x} = \bar{x}$ at the time of the recall decision case (iii) is the only case when the lender will not recall with soft information. Therefore, the simple recall rule is: Recall if the signal implies an $l < D - \bar{x}$. Following this rule the lender knows that if $\tilde{x} = \bar{x}$ the debt claim will always be repaid in full. In Section 2, we found that the lender will always recall when $\tilde{x} = \bar{x}$ and there is only hard information. The soft information allows the lender to choose no recall if it indicates a sufficiently high lower bound for the value of the specific asset, *b*.

For $\tilde{x} = \underline{x}$.

Figures 1and 2 show that the critical value for Type 1 and Type 2 errors is *b=0*. In addition the magnitude of the Type 2 errors depends on whether *b* is greater or smaller than $D - \underline{x}$. This is the maximum loss the lender has to accept in period 1 if it recalls. As before we call this value Loss Given Recall in period 1.

There are three cases to consider.

(i) -ε < $l < 0 < \left(D - \underline{x}\right) < u <$ ε (*See Figure 1*)

The expected costs of error in this case are given by:

(5.2) Expected Cost of Type I error: $-\int_{l}^{0} b \, dG(\tilde{b})$

(5.3) Expected Cost of Type II error*:* $\int_{0}^{D-\underline{x}} b \, dG(\tilde{b}) + \int_{D-\underline{x}}^{u(b,e)} (D - \underline{x}) \, dG(\tilde{b})$

The lender minimizes the sum of expected errors with respect to *b* under the assumption that the lower and upper bounds (*l* and *u*) for the actual b are revealed by the signal in period 1. After some algebraic simplification, the first-order condition with respect to *b* yields:

$$\frac{-l(b,e)}{D-\underline{x}} = \frac{g_b(u(b,e))}{g_b(l(b,e))} \quad (5.5)$$

where $g_b(u(b,e)$ refers to the derivative of G with respect to b at the upper limit (*u*), which is a function of *b* and *e*. The upper limit, *u*, depends on the realized *b* and the quality of the IM.

The recall policy based on this condition implies that the lender will recall in period 1 up to a point when the signal indicates that the expected decrease in the cost of Type 1 error at *l* equals the expected increase in the cost of Type 2 error at *u*. The value of b at the lower bound (in absolute value) times its probability equals $(D - \underline{x})$ times the probability of *b* being equal to the upper bound above $(D - \underline{x})$. The maximum loss at the upper bound is $(D - \underline{x})$.

The second order condition for a minimum must also be satisfied. In words, the expected sum of the costs of errors must first fall and then increase as the signal for b, along with upper and lower bounds, increase. The second order condition is given by:

$$(5.6) \quad l(b,e).\,g'_b\left(l(b,e)) + (D - \underline{x}).\,g'_b\big(u(b,e)\big)\right) > -g_b\big(l(b,e)\big) < 0$$

This condition implies restrictions on the shapes of *F(b)* and *G(b).* Note that the right hand side is negative and that the left hand side is the weighted average (with *l* being negative and *u* positive) of the change in frequencies as *b* increases at the lower and the upper bounds.

(ii) $-\varepsilon < l < 0 < u < (D - \underline{x}) < \varepsilon$ (See Figure 1)

The expected costs of error in this case are given by:

(5.7) Expected Cost of Type I error: $-\int_l^0 b\; dG(\tilde{b})$

(5.8) Expected Cost of Type II error*:* $\int_0^{u(b,e)} b\; dG(\tilde{b})$

Once again, the lender minimizes the sum of expected errors with respect to *b* under the assumption that the lower and upper bounds (*l* and *u*) for the actual b are revealed by the signal in period 1. After some algebraic simplification, the first-order condition with respect to *b* yields:

$$(5.10) \quad l(b,e)g_b\big(l(b,e)\big) + u(b,e)g_b\big(u(b,e)\big) = 0$$

The interpretation of this condition is similar to the interpretation of condition (5.5). The expected decrease in the cost of Type 1 error at *l* equals the expected increase in the cost of Type 2 error at u. In this case ($u < (D - \underline{x})$) the value of b at the lower bound (in absolute value) times its probability equals b at the upper bound times its probability.

The second order condition for a minimum is given by:

$$(5.11)\quad l(b,e)g'_b\big(l(b,e)\big) + u(b,e)g'_b\big(u(b,e)\big) > -\big[g_b\big(l(b,e)\big) + g_b\big(u(b,e)\big)\big] < 0$$

The interpretation is similar to the condition in the previous case.

This condition implies restrictions on the shapes of *F(b*) and *G(b).* It is less restrictive than condition (5.6) above for the case when the maximum Type 2 cost was $(D - \underline{x})$. As in (5.6) the left hand side is a weighted sum of the changes in probabilities at the two bounds while the right hand side is the negative of the sum of the probabilities at the two bounds. (5.11) is less restrictive than (5.6). Thus, the existence of a minimum becomes more likely when the signal quality improves and, thereby, the upper bound is more likely to fall between 0 and $(D - \underline{x})$. Furthermore, for relatively small absolute values of the lower and upper bounds the second order condition (5.11) is more likely to be satisfied, in particular, if the frequencies *g(l)* and *g(b)* on the right hand side of (5.11) are relatively high for small absolute values of *l* and *u*.

(iii) $\varepsilon < 0 < l < u < \varepsilon$

In this case the lender will not recall since the signal indicates that the specific asset will contribute positively to the return on the project in period 2 and, therefore, to the repayment of the debt claim.

The recall policy can then be summarized as in Proposition 2.

Corollaries 1 – 2 explore how the incentive to recall depends on the value of the general asset through $D - \bar{x}$ and $D - \underline{x}$ in comparison to the case when only hard information was available.

Corollary 1. Soft information increases the likelihood the lender will choose no recall when x=$\bar{x}$ and this likelihood is increasing in $\bar{x}$.

Proof. If the high value of the general asset is realized the recall occurs if $l < D - \bar{x} < 0$. A higher value of the general asset (lower $D - \bar{x}$) implies that the lower bound below which recall is triggered becomes lower (more negative). Thus, the range of b-values that trigger recall becomes smaller. The range for no recall becomes larger as $\bar{x}$ increases. In the case with only hard information and x=$\bar{x}$ in Section 4 the lender chose to always recall as a result of a positive probability that b would be so low that the loan could not be repaid in full. Thus, **soft information increases the likelihood the lender will choose no recall and this likelihood is increasing in $\bar{x}$.**

Q.E.D.

Corollary 2. An increase in $\underline{x}$ reduces the cost of recalling and strengthens the incentive to choose recall.

Proof. For the case when $\tilde{x} = \underline{x}$ and $-\varepsilon < l < u < D - \bar{x} < \varepsilon$, the first order condition 5.5 shows that when $(D - \underline{x})$ increases ($\underline{x}$ falling) there is an increase in the cost of making Type 2 errors (cost of recall) while if $-\varepsilon < l < 0 < u < (D - \underline{x}) < \varepsilon$ the first order condition (5.10) is independent of $(D - \underline{x})$. In the latter case, recall policy is independent of $\underline{x}$ while in the former case the expected cost of recalling becomes smaller as $\underline{x}$ increases. Also. the range over which $(D\text{-}\underline{x})$ is relevant declines when $\underline{x}$ increases. We can observe in Figure 2 that when $\underline{x}=D$ the expected cost of recall is zero. Thus, **an increase in $\underline{x}$ reduces the cost of recalling and strengthens the incentive to choose recall**.

Q.E.D.

In Section 4 it was shown that with only hard information the lender's recall decision depended only on what value the general asset took and on $(D\text{-}\underline{x})$. The lender will choose No Recall only if $\underline{x}$ is relatively low. **With soft information No Recall may be chosen when x=$\bar{x}$ as well as when x=$\underline{x}$.**

6. The lender's choice of soft information production

We continue in this section with the assumption that the lender knows that the manager has invested in the specific asset in period -1 and that the lender has made the loan in period 0.

The questions now are what quality of soft information (e*) will be produced in period 0, and what factors determine this quality, i.e. the investment by the lender in IM?

The lender knows the distribution of the return on the specific asset, $f(\tilde{b})$. Recall that the per Assumption A.2, the spread between the lower limit for *b (l)* and the upper limit for *b (u)* for any given signal, σ, decreases from both directions. In other words *dl/de*>0 and *du/de*<0. We also assume that *dl/de = -du/de*.

The benefit to the lender in period 0 of improving the IM can be expressed as the decline in the expected sum of Type 1 and Type 2 errors over the range of possible values for *b*. The lender knows how the recall policy depends on both the hard and the soft information it will receive in period 1 as described in Section 5. Improved quality of information will affect both the probability that a particular recall policy will be implemented and the expected costs associated with each policy. The task is to determine the quality of information, *e*, that makes the expected marginal benefit of increased quality, *e*, equal to the marginal cost of increased *e* as defined by A.3.

In period 0 the lender knows that it will follow the rule for recall described in Section 5. If $\tilde{x} = \bar{x}$ and the lender plans to follow the recall rule described above the lender will always be repaid fully independent of the realized b and the choice of information quality, *e*. However, if $\tilde{x} = \underline{x}$ and the lender plans to follow the recall rules described above there will be costs of Type 1 or Type 2 errors as described in Section 5. The firm will invest in the IM to determine an information quality, *e**, that minimizes the sum of expected costs of Type 1 and Type 2 errors, and the costs of the IM:

(6.1) Min. (Prob $-\varepsilon < l < 0 < (D - \underline{x}) < u < \varepsilon$) (Cost Type 1+Cost Type 2 in range) +(Prob $-\varepsilon < l < 0 < u < (D - \underline{x}) < \varepsilon$)(Cost Type 1 + Cost Type 2 in range) – C(e)

where expression (5.4) describes the Costs in range 1 and (5.8) describes costs in range 2.

Proposition 3 states the argument that there is an IM that gives an e* between 0 and 1 that makes the marginal benefit of improving the quality of soft information equal to its marginal cost.

Proposition 3. If $\frac{\delta l(.)}{\delta(e)} = -\frac{\delta u(.)}{\delta(e)}$, then there exists an optimal quality of information e*.

Proof in Appendix.

Intuitively, the expected sum of costs of Type 1 and Type 2 errors declines as the range between the lower and upper bounds for b at a given true b. Furthermore, the probabilities that $l < 0$ and $u > 0$ at the same time decline as the range between l and u narrows. It is clearly possible that e^* is either 0, if the marginal cost of obtaining any soft information of value is high, or 1, if the marginal cost remains relatively low even when information is perfect.

It can be noted that the incentive to produce soft information is declining in q, the probability that the general asset obtains its high value, since information has no value in this case. It is also clear that an increase in the cost of producing better soft information will reduce the equilibrium level, e^*. The distribution function affects costs of errors as well but we will not explore this issue further.

In section 8 we ask whether soft information also affects the incentive of the project manager to invest in the specific asset and if an increase in the quality of soft information increases this incentive.

7. Lender's period 0 decisions

This section analyses the decisions the lender makes in period 0. There are two decisions the lender makes in period 0: first, whether to make a loan and second, should she choose to make a loan, then whether to invest in the IM. We will analyse the second decision first. Assume that the manager-entrepreneur invests in the specific asset in period -1 and that the lender chooses to make the loan in period 0. Then, the decision of whether to invest in the IM hinges on a comparison of the lender's expected payoff with and without the IM.

If she chooses not to invest in the IM, then her recall decision is based only on hard information (as in Section 4). Per Proposition 1, if $x = \overline{x}$, the lender will always recall in period 1. If $x = \underline{x}$,

the lender will not recall only if $\left(D - \underline{x}\right) - \int_{-\varepsilon}^{D-\underline{x}} F\left(\tilde{b}\right) db > 0$. Therefore, the lender's expected payoff is given by:

$$(7.1)\ E_0(\pi_B|No\ IM) = qD + (1-q)Prob\left(\left(D - \underline{x}\right) - \int_{-\varepsilon}^{D-\underline{x}} F\left(\tilde{b}\right) db < 0\right)\underline{x}$$

$$+(1-q)Prob\left(\left(D - \underline{x}\right) - \int_{-\varepsilon}^{D-\underline{x}} F\left(\tilde{b}\right) db > 0\right)\min\left(D, \underline{x} + E_0\left(\tilde{b}\right)\right)$$

Note that $x = \overline{x}$ with probability q and per Assumption A.3, $\overline{x} > D$ so that a recall in period 1 on getting a signal of $\overline{x}$ will yield a payoff of D for the lender. This is noted in the first term on the right-hand side of expression (7.1). Now, $x = \underline{x}$ with probability (1 – q) and per Assumption A.3, $\underline{x} < D$ so that a recall in period 1 on getting a signal of $\underline{x}$ will yield a payoff of $\underline{x}$ for the lender. This combined with the fact that the lender recalls only if $\left(D - \underline{x}\right) - \int_{-\varepsilon}^{D-\underline{x}} F\left(\tilde{b}\right) db < 0$ gives us the second term on the right-hand side of expression (7.1). Note again that $x = \underline{x}$ with probability (1 – q) and that on getting a signal of $\underline{x}$, should the lender choose not to recall, then in period 2, its payoff will be the minimum of D and $\underline{x} + \tilde{b}$. This combined with the fact that the lender does not recall only if $\left(D - \underline{x}\right) - \int_{-\varepsilon}^{D-\underline{x}} F\left(\tilde{b}\right) db > 0$ gives us the third term on the right-hand side of expression (7.1).

If the lender chooses to invest in the IM, she incurs a cost of $C(e)$. The lender's recall decision in period 1is based on both hard and soft information (as in Section 5). Per Proposition 2, if $x = \overline{x}$, the lender will recall only if the signal $\sigma(b)$ implies an $l < D - \bar{x}$ and this ensures that a signal of $\overline{x}$ will yield a payoff of D for the lender. If $x = \underline{x}$, then the recall policy specified by Proposition 2 implies two sets of costs of errors. First, if $l < 0 < \left(D - \underline{x}\right) < u$, then the expected cost of type I error is $-\int_{l}^{0} b\ dG(\tilde{b})$ and the expected cost of type II error is $\int_{0}^{D-\underline{x}} b\ dG(\tilde{b}) + \int_{D-\underline{x}}^{u(b,e)} (D - \underline{x})\ dG(\tilde{b})$. The expected payoff in this case is given by D minus the sum of the expected costs of Type I and Type II errors. Second, if $l < 0 < u < \left(D - \underline{x}\right)$, then the expected cost of type I error is $-\int_{l}^{0} b\ dG\left(\tilde{b}\right)$ and the expected cost of type II error is $\int_0^{u(b,e)} b\, dG(\tilde{b})$. Once again, the expected payoff is given by D minus the sum of the expected costs of Type I and Type II errors. Putting these together, the lender's expected payoff is given by:

$$E_0(\pi_B|IM) = qD - C(e) + (1-q)\big(Prob\ l < 0 < (D - \underline{x}) < u\big)$$
$$\left\{D - \left(\int_0^{D-\underline{x}} b\, dG(\tilde{b}) + \int_{D-\underline{x}}^{u(b,e)} (D - \underline{x})\, dG(\tilde{b}) - \int_l^0 b\, dG(\tilde{b})\right)\right\}$$
$$+(1-q)\left(Prob\ l < 0 < u < (D - \underline{x})\right)\left\{D - \left(\int_0^{u(b,e)} b\, dG(\tilde{b}) - \int_l^0 b\, dG(\tilde{b})\right)\right\}$$

Some algebraic simplification yields:

(7.2) $E_0(\pi_B|IM) = qD - C(e)$

$$+(1-q)\big(Prob\ l < 0 < (D - \underline{x}) < u\big)\left\{D - \left(\int_{l(b,e)}^{0} G(\tilde{b})db + (D - \underline{x}) - \int_0^{D-\underline{x}} G(\tilde{b})db\right)\right\}$$
$$+(1-q)\left(Prob\ l < 0 < u < (D - \underline{x})\right)\left\{D - \left(\int_{l(b,e)}^{0} G(\tilde{b})db + u(b,e) - \int_0^{u(b,e)} G(\tilde{b})db\right)\right\}$$

A comparison of the expected payoffs given by expressions (7.1) and (7.2) solves the lender's decision of whether to invest in the IM. The lender invests in the IM only if $E_0(\pi_B|IM) > E_0(\pi_B|No\ IM)$. This, in turn, implies that the lender invests in IM only if the cost $C(e)$ of investing in the IM is less than the incremental benefit ($E_0(\pi_B|IM) - E_0(\pi_B|No\ IM)$) of investing in the IM. This is stated more formally in Proposition 4.

Proposition 4. Assume that the manager-entrepreneur invests in the specific asset in period -1 and that the lender knows this. Further, assume $\frac{\delta l(.)}{\delta(e)} = -\frac{\delta u(.)}{\delta(e)}$ (so that an optimal e* exists) and that the lender has chosen to make a loan to the manager-entrepreneur in period 0. Then, there exists a threshold μ_0 such that the lender chooses to invest in the IM only if $C(e) < \mu_0$.

Proof in Appendix.

We now turn to the lender's decision of whether to make a loan in period 0. If the lender's expected payoff from investing in the IM exceeds her expected payoff from not investing in the IM, then by Proposition 4, she will invest in the IM. In this case, she is better off making the loan only if her expected payoff from investing in the IM exceed the nominal value of the debt, *D*. On the contrary, if the lender's expected payoff from not investing in the IM exceeds her expected payoff from investing in the IM, then by Proposition 4, she will invest in the IM. In this case, she is better off making the loan only if her expected payoff from not investing in the IM exceed the nominal value of the debt, *D*. More formally, we have Proposition 5.

Proposition 5. Assume that the manager-entrepreneur invests in the specific asset in period -1 and that the lender knows this. Further, assume $\frac{\delta l(.)}{\delta(e)} = -\frac{\delta u(.)}{\delta(e)}$ (so that an optimal e* exists). Then, there exist thresholds μ_1 and μ_2 such that if $\mu_1 > \mu_2$, the lender makes a loan only if $D \leq \frac{\mu_1 - C(e)}{1-q}$. And, if $\mu_1 < \mu_2$, the lender makes a loan only if $D \leq \frac{\mu_2 - C(e)}{1-q}$.

Proof in Appendix.

8. Alignment of incentives for producing soft information and investment in the specific asset

Throughout Sections 4-7 we had assumed that the project manager has invested in the specific asset in period -1. We now turn to the question of whether it makes sense for the manager to invest in specific assets, given what we already know about the lender's decision-making problem. The production of IM in period 0 does not take place if the manager has not invested in the specific asset in period -1. The incentive of the manager/entrepreneur to invest is likely to depend on his or her knowledge about the lender's recall policy and, therefore on the incentive to produce soft information, as well as on the remuneration scheme for the manager. The alignment of incentives also requires that the manager can infer that the lender will spend resources on the IM. Consistency of incentives is a necessary condition for the alignment of incentives.

We analyse first whether the incentive of the manager/entrepreneur to invest in the specific asset is **consistent** with the incentive of the lender to invest in the IM and to improve the quality of the soft information. The manager/entrepreneur will invest in the specific asset in period -1 only if the expected remuneration exceeds the cost, θ, of acquiring the specific asset and if the expected remuneration with the specific asset is greater than the expected remuneration without the specific asset. We define the incentive to invest in the specific asset, I, as:

(8.1) $I= E[\pi_M| \theta]- E[\pi_M| \theta=0]- \theta$

For simplicity, the cost of the specific asset is a constant. We ask first whether this incentive is positive when there is only hard information, $I|\bar{x}, \underline{x}$. Then we ask whether this incentive is positive when there is soft information as well as hard information, $I|\bar{x}, \underline{x}, e$. The additional incentive to invest in the specific asset with soft information is denoted AI in the following.

(8.2) $AI=I|\bar{x}, \underline{x}, e - I|\bar{x}, \underline{x}$

The incentives to produce soft information and to invest in the specific asset are consistent if AI >0 and if $\delta I|\bar{x}, \underline{x}, e/\delta e>0$. The first condition is necessary for soft information to contribute to the incentive to invest in the specific asset. Without this condition soft information has no value for the manager-entrepreneur. The second condition implies that the incentive is increasing in the quality of the soft information. We also analyse how these incentives depend on other factors in the model. In particular, we ask how the incentive to invest in the specific asset depends on upside and downside risk with respect to the value of the hard asset.

8.1 Hard information only

In Section 4 we showed that the lender will always recall in period 1 if $x=\bar{x}$ and the investment in the specific asset has taken place. If $\tilde{x} = \underline{x}$, the lender will not recall only if $(D-\underline{x}) - \int_{-\varepsilon}^{D-\underline{x}} F(\tilde{b})db > 0$. That is, the probability of No Recall in this case is either 1 or 0 depending on the sign of condition $(D-\underline{x}) - \int_{-\varepsilon}^{D-\underline{x}} F(\tilde{b})db$. For a relatively small $D-\underline{x}$ the lender will always recall. As a result, there is no incentive for the manager entrepreneur to invest in the specific asset and the lender will not set up the costly IM.

For a sufficiently large $D-\underline{x}$ the lender will never recall if $x=\underline{x}$. In this case the incentive to invest in the specific asset is the following:

(8.3) $I|\bar{x}, \underline{x} = E[\pi_M|\bar{x}, \underline{x}, \theta]- E[\pi_M|\bar{x}, \underline{x}, \theta=0]- \theta =$

$$= -z\left[1-(1-q)(Prob\ No\ Recall\ |\underline{x})\right]$$

$$+ (1-q)(Prob\ No\ Recall\ |\underline{x}) \int_{D-\overline{x}}^{\varepsilon} k\left(b-\left(D-\underline{x}\right)\right) f(\tilde{b}) db - \theta$$

where the manager/entrepreneur's expected compensation with the specific asset in place is compared to the expected compensation without the specific asset in place. In the latter case the lender will never recall.

The incentive in (8.3) can be positive or negative (zero). The private benefit of No Recall, *z*, is a source of disincentive to invest in the specific asset since the manager knows that the lender will never recall if there is no investment in it. Also, a higher likelihood, *q*, of ($x=\bar{x}$) creates a disincentive.

We can conclude that with only hard information about the general asset in period 1, the manager may invest in the specific assets but only if $\mathrm{D}-\underline{x}$ is relatively large. More formally, we have the following proposition.

Proposition 6. Assume that the lender will not invest in the IM in period 0 and that the lender will make the loan in period 0 and that the manager-entrepreneur knows these in period -1. If $\left(\mathrm{D}-\underline{\mathrm{x}}\right) > \int_{-\varepsilon}^{\mathrm{D}-\underline{\mathrm{x}}} \mathrm{F}(\tilde{\mathrm{b}})\mathrm{db}$, then the manager-entrepreneur invests in the specific asset only if the cost of investing in the specific asset, $\theta < -zq + (1-q) \int_{D-\underline{x}}^{\varepsilon} k\big(\tilde{b} - (D-\underline{x}\big) f\big(\tilde{b}\big) db\big)$. If $\left(\mathrm{D}-\underline{\mathrm{x}}\right) < \int_{-\varepsilon}^{\mathrm{D}-\underline{\mathrm{x}}} \mathrm{F}(\tilde{\mathrm{b}})\mathrm{db}$, then the manager-entrepreneur does not invest in the specific asset.

Proof in Appendix.

8.2 Hard and soft information

We assume now that the manager knows that the lender will incur the costs, $C(e)$, of creating a soft information signal with quality *e* using its information mechanism, IM. In this case the incentive of the manager to invest in the specific asset depends on the lender's recall policy in the following way:

(8.4) $I|\overline{x}, \underline{x}, e = E[\pi_M|\overline{x}, \underline{x}, e, \theta] - E[\pi_M|\overline{x}, \underline{x}, e, \theta=0] - \theta =$

$$qk\left(\bar{x}-D\right)\left[1-\left(\text{prob } \ell>(D-\bar{x})\right)\right]+z\left[(\text{prob } \ell>\left(D-\bar{x}\right))q+\left(\text{prob no recall }|\underline{\text{x}},\text{e}\right)(1-q)-1\right]$$
$$+k\left[(\text{prob } \ell>\left(D-\bar{x}\right))q\int_{D-\bar{x}}^{\varepsilon}\left(\tilde{b}-\left(D-\bar{x}\right)\right)f\left(\tilde{b}\right)db+\left(\text{prob no recall }|\underline{\text{x}},\text{e}\right)(1-q)\int_{D-\underline{x}}^{\varepsilon}\left(\tilde{b}-\left(D-\underline{x}\right)\right)f\left(\tilde{b}\right)db\right]-\theta$$

This expression is derived in the proof for Proposition 7. It will be discussed in more detail below but it can be noted that a disincentive arises from high private benefits, z, from No recall in the second term. The explanation is that without investments in the specific asset there will never be recalls when x=$\bar{x}$ while if the investment occurs, a high value of the general asset favors recall.

Proposition 7. Assume that the lender will invest in the IM in period 0 and that the lender will make the loan in period 0 and that the manager-entrepreneur knows these in period -1. Further, assume $\frac{\delta l(.)}{\delta(e)}=-\frac{\delta u(.)}{\delta(e)}$ (so that an optimal e* exists). Then, there exists a threshold μ_3 such that the manager-entrepreneur invests in the specific asset only if the cost of investing in the specific asset, $\theta<\mu_3$.

Proof in Appendix.

8.3 Additional incentive from having access to soft information

We focus next on the additional incentive to invest in the specific asset from the IM. There are two cases: if $\mathrm{D}-\underline{x}$ is sufficiently small (Proposition 6) there is never an incentive to invest in the specific asset without soft information. The additional incentive to invest in the specific asset with soft information is therefore equal to the incentive described in expression 8.4.

If $\mathrm{D}-\underline{x}$ is larger (specifically, $\left(\mathrm{D}-\underline{\mathrm{x}}\right)>\int_{-\varepsilon}^{\mathrm{D}-\underline{\mathrm{x}}}\mathrm{F}(\tilde{\mathrm{b}})\mathrm{db}$), then the incentive to invest in the specific asset may be positive with or without soft information generated by the IM. In this case the additional incentive, AI, can be obtained as the difference between expressions (8.4) and expression (8.3). The following expression is derived in Proposition 8:

$$(8.5)\text{AI} = -qk\left(\text{prob } \ell > D-\bar{x}\right)\left[\left(\bar{x}-D\right) - \int_{D-\underline{x}}^{\varepsilon}\left(\tilde{b}-\left(D-\underline{x}\right) f\left(\tilde{b}\right)\right)db\right]$$

$$+z\left[q\left(\text{prob } \ell > \left(D-\bar{x}\right)\right)+\left(1-q\right)\left[\left(\text{prob no recall }|\underline{\text{x}},\text{e}\right)-1\right]\right]$$

$$+\left(1-q\right)k\left[\left(\text{prob no recall }|\underline{\text{x}},\text{e}\right)-1\right]\int_{D-\underline{x}}^{\varepsilon}\left(\tilde{b}-\left(D-\underline{x}\right) f\left(\tilde{b}\right)\right)db - qk\left(\bar{x}-D\right)-\theta$$

The manager has incentive to invest in the specific asset if AI>0 if the manager also knows that the lender will produce soft information using the IM to achieve information quality e.

The only clearly negative impact on the additional incentive from soft information comes from the first term in (8.4). This term depends on the likelihood that the lender may recall with soft information when (x=$\bar{x}$) with probability q and investment in the specific asset has taken place. This disincentive comes from the absence of recall if there is no specific asset. The impact of the private benefit, *z*, is not negative if the probability of recall with soft information is less than the probability of recall with only hard information. It can also be observed that the likelihood of a negative incentive increases with a high value of the hard asset relative to the loan amount, $\boldsymbol{\bar{x} - D}$.

We can conclude that the manager's incentive to invest in the specific asset is positive if the information mechanism is in place with sufficient quality and if the expected value of the general asset is not too high relative to the loan amount.

Proposition 8. Assume that the lender will make the loan in period 0 and that the manager-entrepreneur knows this in period -1. Further, assume $\frac{\delta l(.)}{\delta(e)} = -\frac{\delta u(.)}{\delta(e)}$ (so that should the lender choose to invest in an IM, then an optimal e* exists). Then, there exist thresholds μ_4 and μ_5 such that if $\left(\mathrm{D}-\underline{\mathrm{x}}\right) > \int_{-\varepsilon}^{\mathrm{D}-\underline{\mathrm{x}}} \mathrm{F}(\tilde{\mathrm{b}})\mathrm{db}$, the manager-entrepreneur has a positive additional incentive to invest in the specific asset only if $\theta < \mu_4$. And, if $\left(\mathrm{D}-\underline{\mathrm{x}}\right) < \int_{-\varepsilon}^{\mathrm{D}-\underline{\mathrm{x}}} \mathrm{F}(\tilde{\mathrm{b}})\mathrm{db}$, the manager-entrepreneur has a positive additional incentive to invest in the specific asset only if $\theta < \mu_5$.

Proof in Appendix.

8.4 Consistency of incentives for soft information production and for specific asset investment

To analyse how the quality of soft information affects the incentive to invest in the specific asset given that the additional incentive is positive as described above, it is sufficient to analyse how expression (8.4) for $\mathrm{I}|\bar{x}, \underline{x}, e$ depends on information quality, e. Note that the expression for $\mathrm{I}|\bar{x}, \underline{x}, e$ is valid for relatively large as well as relatively small values of $\mathrm{D} - \underline{x}$. In the proof for Proposition 9, we derive the derivative of this incentive with respect to information quality, e.

The derivative of this incentive with respect to the quality of information, e, is most interesting from the point of view of consistency between the lender's incentive to produce soft information and the manager's incentive to invest in the specific asset. The expression for this derivative in Proposition 9 shows that the derivative is positive as long as the lender's incentive to recall when ($\mathrm{x}=\bar{x}$) with probability q is not too high. The only negative incentive effects of improved soft information occur for a relatively high expected return on the general asset as a result of a high q or a high $\bar{\boldsymbol{x}}$. Improved soft information strengthens the incentive to invest in the specific asset with higher values for z and k while a lower value of the general asset, $\underline{x}$. (larger value of $\mathrm{D} - \underline{x}$) has a positive impact on the incentive if improved information has a sufficiently strong impact on the probability of No recall.

Conditional on a positive incentive to invest in the specific asset, this incentive decreases with an increase in the low value of the general asset, $\underline{x}$, as well as with an increase in the high value of the general asset, $\bar{\boldsymbol{x}}$. Thus, an increase in the expected value of the general asset reduces the incentives to invest in the specific asset. This incentive is generally consistent with the incentive of the lender to invest in the production of soft information.

As a general conclusion of this section we can state that the minimum benefit required by the manager to invest in the specific asset depends strongly on the lender's production of soft information about the specific asset. The likelihood of investment in the specific asset depends positively on the quality of the soft information and, therefore negatively on the cost of information.

Proposition 9. Assume that the lender will make the loan in period 0 and that the manager-entrepreneur knows this in period -1. Further, assume $\frac{\delta l(.)}{\delta(e)} = -\frac{\delta u(.)}{\delta(e)}$ (so that an optimal e* exists) and that the additional incentive to invest in the specific asset is positive (per Proposition 8). Then, there exists a quantity μ_6 such that the incentives for soft information production and for specific asset investment will be consistent only if $\mu_6 > 0$.

Proof in Appendix.

9. Conclusions and implications

We have emphasized that a lender's valuation of the assets employed in a project relies on a mechanism that produces soft information with a quality that can be increased at a cost. This view implies that there is a degree of softness to almost any asset valuation except in the case when market prices perfectly reflect the economic value of a general asset.

The soft information in our model has potential value in combination with the right to recall the loan based on both hard and soft information prior to the completion of a project. The value of the information depends also on the effect of the recall policy on the manager-entrepreneur's incentive to invest in a specific asset.

A recall policy and associated information production will occur under a wider range of circumstances whether the specific asset exists or must be induced by the lender's recall policy. In the latter case the lender's incentive to produce costly soft information must be aligned with the manager-entrepreneur's incentive to invest in the specific asset. This alignment of incentives implies that the lender can trust that the investment in the specific asset has taken place when the project is initiated. At the same time the manager-entrepreneur must trust that the lender will produce soft information about the specific asset with a certain quality. In particular, the manager-entrepreneur's investment in the specific asset before the lender decides to lend and produce soft information requires knowledge of factors that determine the lender's decision with respect to quality of soft information that will be produced during the course of the project and determine the recall policy. It lies in the mutual interest of the lender and the project manager that the relevant information is available to both parties at the time the loan is given and the investment in the specific asset must be made.

We derive the equilibrium spending on an information mechanism and, therefore, the signal quality. The assignment of the cost of the information mechanism to the lender is not essential. The cost of providing and signal credible information to the lender could have been borne by the project or an outside "ratings agency." Either way the quality of the signal must be related to the cost of the information mechanism in a known way.

One variable of importance for the value of soft information is the distribution of the expected return on the general asset relative to the amount of the loan. A high expected return reduces the value of the soft information while a low expected return increases the value of the soft information. The possible low return on the general asset is particularly important since it determines the maximum loss to the lender if it chooses to recall the loan. A relatively large potential loss on the general asset increases both the incentive to produce soft information and the incentive to invest in the specific asset. In this sense there is a substitutability between the expected value of the general asset and the value of soft information about the specific asset. Other papers as, for example, Bertomeu and Marinovic (2013) show in a different context that there is substitutability between the value of a costly certification process for soft, non-audited information and the availability of hard, audited information.

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Table 1. Actions and information

Period	Action	Information
-1	- Entrepreneur-manager decides whether to invest in project-specific asset at a cost of θ	- $F(\tilde{b})$ - θ
0	-Lender decides whether to make a loan -Lender decides whether to invest in an Information Mechanism (IM) at a cost $C(e)$	- $\bar{x}, \underline{x}$ - q - $C(e)$
1	- Lender decides whether to recall the loan; - If loan is recalled, Lender gets $\begin{cases} D, & if \quad \tilde{x} = \bar{x} \\ \underline{x}, & if \quad \tilde{x} = \underline{x} \end{cases}$	- Accounting signal about $\tilde{x}$; - Manager knows realized $\tilde{b}$ if specific investment was undertaken; - Lender obtains signal σ if investment in IM was made in period 0
2	- Manager extracts private benefit z if loan was not recalled in period 1; - If specific investment was made in period -1 and loan was not recalled in period 1, Lender gets $\begin{cases} D, & if \quad \tilde{y} \geq D \\ \tilde{y}, & if \quad \tilde{y} < D \end{cases}$ - If specific investment was not made in period -1 and loan was not recalled in period 1, Lender gets $\begin{cases} D, & if \quad \tilde{x} = \bar{x} \\ \underline{x}, & if \quad \tilde{x} = \underline{x} \end{cases}$	- $\tilde{x}$; - $\tilde{b}$ if specific investment was made in period -1

Figure 1 – Lender's payoffs with and without recall. Recall decisions at $x = \overline{x}$ and $x = \underline{x}$

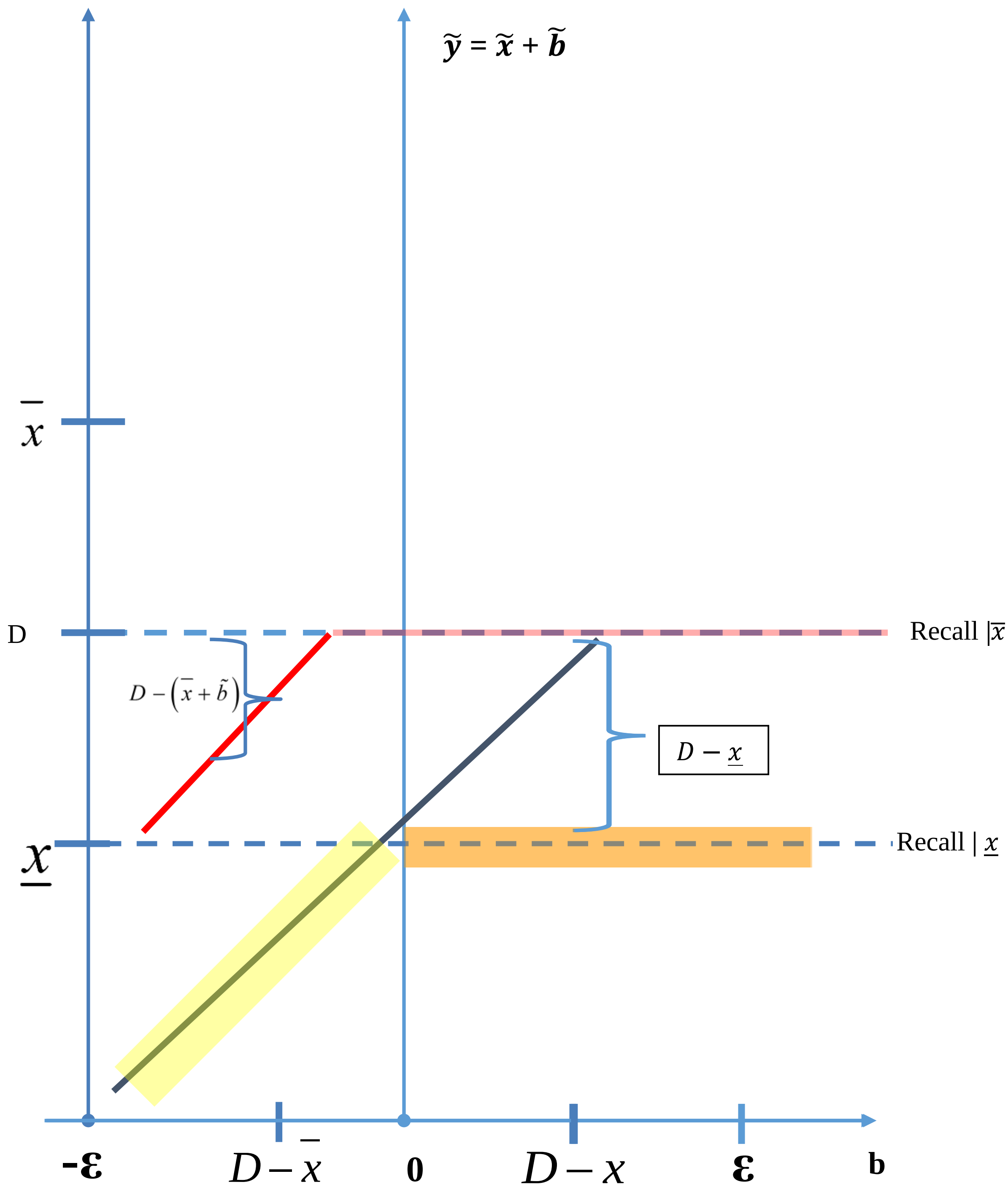
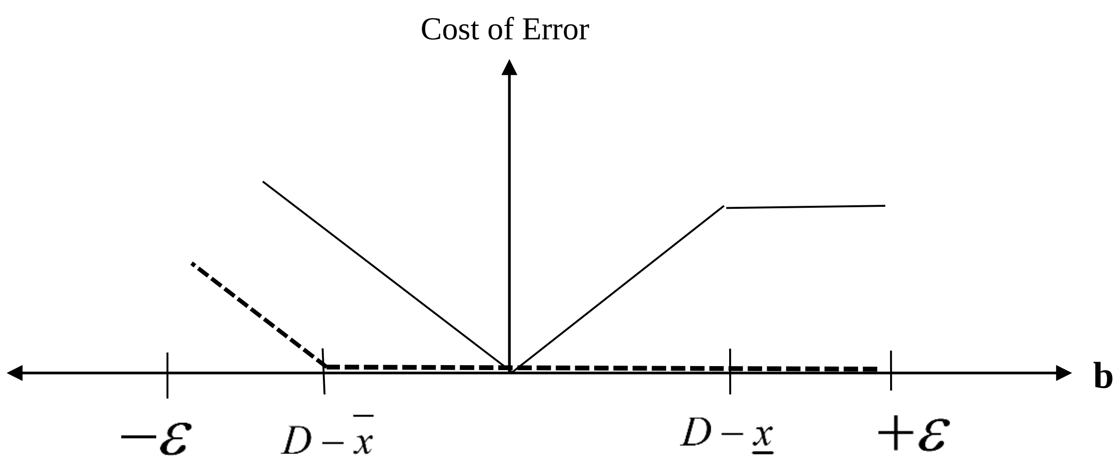


Figure 2

Cost of Type 1 (No Recall) and Type 2 (Recall) errors for $x = \bar{x}$ and $x = \underline{x}$

----------- $x = \bar{x}$ ——— $x = \underline{x}$



Type 1 for $x = \underline{x}$ ⟵ | ⟶ Type 2 for $x = \underline{x}$ after recall

Type 1 for $x = \bar{x}$ ⟵ |

Appendix

Proof of Proposition 1. The lender's objective is to maximize the expected value of its debt claim, $E(\pi_B)$. It may equivalently be expressed as the objective of minimizing the sum of expected costs of Type 1 and Type 2 errors given the debt claim, D.

(4.1) Max $E(\pi_B) =$ Min [E(*Type 1*) +E(*Type 2*)]

Consider $\tilde{x} = \overline{x}$. If the lender does not recall (allows continuation) there is a risk of making a Type 1 error. The expected value in period 1 of this Type 1 error is:

$E\left\langle Type\ 1 \middle| \tilde{x} = \bar{x} \right\rangle = \int_{-\varepsilon}^{D-\bar{x}} (D - (\bar{x} + b))F(\widetilde{b}) > 0.$

If the lender recalls (does not allow continuation) there is no possibility of a Type 2 error.

$E\left\langle Type\ 2 \middle| \tilde{x} = \bar{x} \right\rangle = 0$

Thus, the sum of expected costs of Type 1 and Type 2 errors is minimized by always choosing to recall.

Now consider $\tilde{x} = \underline{x}$, then for $\tilde{b} > 0$ the lender is better off with no recall while if $\tilde{b} < 0$, recall in period 1 would have been the best choice. In period 1 with information only that $\tilde{x} = \underline{x}$ the lender faces the possibility of both Type 1 error, if it does not recall, and Type 2 error if it recalls. The magnitude of the cost associated with the Type 2 error depends on whether b is large enough to repay the whole loan or not. If $0 \leq b < \mathrm{D} - \underline{x}$ the loss associated with recall is b. In the range $b \geq \mathrm{D} - \underline{x}$, the loss associated with recall is $\mathrm{D} - \underline{x}$. Thus the expected costs associated with the errors are:

(4.2) $E\left\langle Type\ 1 \middle| \tilde{x} = \underline{x} \right\rangle = \int_{-\varepsilon}^{0} \tilde{b}\, dF(\widetilde{b})$

(4.3) $\mathrm{E}\left\langle Type\ 2 \middle| \tilde{x} = \underline{x} \right\rangle = \int_{0}^{\mathrm{D}-\underline{x}} \tilde{b}\ d\mathrm{F}(\widetilde{b}) + \int_{\mathrm{D}-\underline{x}}^{\varepsilon} (\mathrm{D} - \underline{x}) d\mathrm{F}(\widetilde{b})$

Summing up the expected values of Type I and Type 2 errors given $\underline{x}$, the lender will not recall if

(4.4) $E\left\langle Type\ 2 \middle| \tilde{x} = \underline{x} \right\rangle > E\left\langle Type\ 1 \middle| \tilde{x} = \underline{x} \right\rangle$

Insert (4.2) and (4.3) in (4.4). The condition for no recall can then be expressed as:

$$\int_{0}^{\mathrm{D}-\underline{x}} \tilde{b}\ d\mathrm{F}(\tilde{b}) + \int_{\mathrm{D}-\underline{x}}^{\varepsilon} (\mathrm{D} - \underline{x}) d\mathrm{F}(\tilde{b}) - \int_{-\varepsilon}^{0} \tilde{b}\, dF(\tilde{b}) > 0$$

where, $\int_{0}^{\mathrm{D}-\underline{x}} \tilde{b}\ d\mathrm{F}(\tilde{b}) = [\tilde{b}F(\tilde{b})]_{0}^{\mathrm{D}-\underline{x}} - \int_{0}^{\mathrm{D}-\underline{x}} \mathrm{F}(\tilde{b})\ db$

Therefore, No Recall if

$$[\tilde{b}F(\tilde{b})]_{0}^{D-\underline{x}} - \int_{0}^{D-\underline{x}} F(\tilde{b})\ db - [\tilde{b}F(\tilde{b})]_{-\varepsilon}^{0} + \int_{-\varepsilon}^{0} F(\tilde{b})\ db + (D-\underline{x})[F(\tilde{b})]_{D-\underline{x}}^{\varepsilon} > 0$$

which simplifies to –

$$(4.5)\ (D-\underline{x}) - \left[\int_{0}^{D-\underline{x}} F(\tilde{b})\ db + \int_{-\varepsilon}^{0} F(\tilde{b})\ db\right] > 0$$

Or, $(D-\underline{x}) - \int_{-\varepsilon}^{D-\underline{x}} F(\tilde{b})\ db > 0$

Q. E. D.

Proof of Proposition 2. The lender's objective is to maximize the expected value of its debt claim, $E(\pi_B)$. It may equivalently be expressed as the objective of minimizing the sum of expected costs of Type 1 and Type 2 errors given the debt claim, D and the quality of soft information, *e*.

Consider $\tilde{x} = \bar{x}$. There are three cases to consider here.

(i) $-\varepsilon < l < u < D - \bar{x} < \varepsilon$

The sum of costs of error is minimized here by choosing to recall.

(ii) $-\varepsilon < l < D - \bar{x} < u < \varepsilon$

If $b < -\bar{x}$, the sum of costs of error is minimized by choosing to recall. If $b > -\bar{x}$, the lender is indifferent between recall and no recall. By assumption, when the lender is indifferent, she will choose no recall.

(iii) $-\varepsilon < D - \bar{x} < l < u < \varepsilon$

There is no possibility of Type 1 errors. The lender is indifferent between recall and no recall. By assumption, when the lender is indifferent, she will choose no recall.

The three cases put together imply that the lender will recall in period 1 if the signal $\sigma(b)$ implies an $l < D - \bar{x}$.

Now consider $\tilde{x} = \underline{x}$. We use the notation $G(b)$ for the distribution of b between the lower and upper limits.

$$(5.1)\quad G(b) = \frac{F(b) - F(l(b,e))}{F(u(b,e) - F(l(b,e))}$$

Note that $F(-\varepsilon) = 0, F(\varepsilon) = 1$.

Therefore, $G(l) = 0\ and\ G(u) = 1$.

Once again, there are three cases to consider here.

(i) $l < 0 < (D - \underline{x}) < u$

(5.2) Expected Cost of Type I error:

$$-\int_{\ell(b,e)}^{0} bg(\tilde{b})db$$

$$= \ell(b,e)G(\ell(b,e)) + \int_{\ell(b,e)}^{0} G(\tilde{b})db$$

$$= \int_{\ell(b,e)}^{0} G(\tilde{b})db, \text{since} \, G(\ell(b,e)) = 0$$

(5.3) Expected Cost of Type II error:

$$\int_{0}^{D-\underline{x}} bg(\tilde{b})db + \int_{D-\underline{x}}^{u(b,e)} (D-\underline{x})g(\tilde{b})db$$

$$= (D-\underline{x})G(u(b,e)) - \int_{0}^{D-\underline{x}} G(\tilde{b})db$$

$$= (D-\underline{x}) - \int_{0}^{D-\underline{x}} G(\tilde{b})db, \text{since} \, G(u(b,e)) = 1$$

Therefore, the sum of expected costs of errors:
(5.4)

$$(D-\underline{x}) + \int_{\ell(b,e)}^{0} G(\tilde{b})db - \int_{0}^{D-\underline{x}} G(\tilde{b})db$$

since $G(u(b,e)=1$. The lender minimizes this sum of expected errors with respect to *b* under the assumption that the lower and upper bounds (*l* and *u*) for the actual b are revealed by the signal in period 1.

(5.5) First-order condition with respect to *b* yields: $\dfrac{-\ell(b,e)}{D-\underline{x}} = \dfrac{g_b(u(b,e))}{g_b(\ell(b,e))}$

where $g_b(u(b,e)$ refers to the derivative of G with respect to b at the upper limit (*u*), which is a function of *b* and *e*. The upper limit, *u*, depends on the realized *b* and the quality of the IM. The recall policy based on this condition implies that the lender will recall in period 1 up to a point when the signal indicates that the expected decrease in the cost of Type 1 error at *l* equals the expected increase in the cost of Type 2 error at *u*.

The second order condition for a minimum must also be satisfied. In words, the expected sum of the costs of errors must first fall and then increase as the signal for b, along with upper and lower bounds, increase. The second order condition with respect to *b* yields:

$$l'_b(b,e).\,g_b\big(l(b,e)\big) + l(b,e).\,g'_b\big(l(b,e)\big) + \big(D - \underline{x}\big).\,g'_b\big(u(b,e)\big) > 0$$

Note that $l'_b(b,e) = u'_b(b,e) = 1$

The second-order condition can, therefore, be re-written as:

$$(5.6) \quad l(b,e).\,g'_b\Big(l(b,e)) + \big(D - \underline{x}\big).\,g'_b\big(u(b,e)\big)\Big) > -g_b\big(l(b,e)\big) < 0$$

This condition implies restrictions on the shapes of *F*(*b*) and *G*(*b*).

(ii) $l < 0 < u < \big(D - \underline{x}\big)$

(5.7) Expected Cost of Type I error:

$$-\int_{\ell(b,e)}^{0} bg\left(\tilde{b}\right)db$$

$$= \ell\left(b,e\right)G\left(l\left(b,e\right)\right) + \int_{\ell(b,e)}^{0} G\left(\tilde{b}\right)db$$

$$= \int_{\ell(b,e)}^{0} G\left(\tilde{b}\right)db$$

(5.8) Expected Cost of Type II error:

$$\int_{0}^{u(b,e)} bg\left(\tilde{b}\right)db$$

$$= u\left(b,e\right)G\left(u\left(b,e\right)\right) - \int_{0}^{u(b,e)} G\left(\tilde{b}\right)db$$

$$== u\left(b,e\right) - \int_{0}^{u(b,e)} G\left(\tilde{b}\right)db$$

The sum of the expected costs of errors is:

(5.9)

$$u(b,e)+\int_{\ell(b,e)}^{0} G(\tilde{b})db-\int_{0}^{u(b,e)} G(\tilde{b})db$$

(5.10) First-order condition:

$$l(b,e)g_b\big(l(b,e)\big)+u(b,e)g_b\big(u(b,e)\big)=0$$

The interpretation of this condition is similar to the interpretation of condition (5.5). The expected decrease in the cost of Type 1 error at *l* equals the expected increase in the cost of Type 2 error at *u*.

(5.11) Second-order condition:

$$l'_b(b,e)g_b\big(l(b,e)\big)+u'_b(b,e)g_b\big(u(b,e)\big)+\ l(b,e)g'_b\big(l(b,e)\big)+u(b,e)g'_b\big(u(b,e)\big) > 0$$

Note that $l'_b(b,e)=1, u'_b(b,e)=1$ *and* $g_b\big(l(b,e)\big)>0, g_b\big(u(b,e)\big)>0$

The second order condition can, therefore, be re-written as:

(5.11) $l(b,e)g'_b\big(l(b,e)\big)+u(b,e)g'_b\big(u(b,e)\big)>-\big[g_b\big(l(b,e)\big)+g_b\big(u(b,e)\big)\big] < 0$

This condition implies restrictions on the shapes of *F(b*) and *G(b).*

(iii) ε $< 0 < l < u <$ ε

In this case the lender will not recall since the signal indicates that the specific asset will contribute positively to the return on the project in period 2 and, therefore, to the repayment of the debt claim.

Q. E. D.

Proof of Proposition 3. In period 0 knowing only $F(\tilde{b})$, the lender decides on the quality of information (0 < *e* <1) as defined by a signal σ_b in period 1 with upper limit and lower limit $l(b,e)$ for the true *b*. Note that $\frac{\delta l(e,\sigma_b)}{\delta e}>0$ and $\frac{\delta u(e,\sigma_b)}{\delta e}<0$. At time 0, the distribution of σ_b is identical to *f*($\tilde{b}$). Thus, $l(\sigma_b,e)=l(b,e)$ and $u(\sigma_b,e)=u(b,e)$.

Assume that $\frac{\delta l(.)}{\delta(e)}=-\frac{\delta u(.)}{\delta(e)}$

Minimize the expected sum of cost of Type I and Type 2 errors plus the cost of the IM, $C(e)$ assuming that $x = \underline{x}$. Note that if $x = \overline{x}$, no costs of error are expected since the lender will receive *D* if it follows its recall policy.

$$\underset{e}{Min}\left(Prob\ l < 0 < \left(D - \underline{x}\right) < u\right)\left[\int_{l(b,e)}^{0} G\left(\tilde{b}\right)db + \left(D - \underline{x}\right) - \int_{0}^{D-\underline{x}} G\left(\tilde{b}\right)db\right]$$

$$+\left(Prob\ l < 0 < u < \left(D - \underline{x}\right)\right)\left[\int_{l(b,e)}^{0} G\left(\tilde{b}\right)db + u(b,e) - \int_{0}^{u(b,e)} G\left(\tilde{b}\right)db\right] + C(e)$$

First bracket = [1] >0; second bracket = [2]>0. Note that [1] = expression (5.4) and [2]= expression (5.8). These expressions represent the sum of expected costs of errors within the two ranges.

First-order condition:

$$[1]\frac{\partial}{\partial e}\left(Prob\ l < 0 < \left(D - \underline{x}\right) < u\right) + [2]\frac{\partial}{\partial e}\left(Prob\ l < 0 < u < \left(D - \underline{x}\right)\right) + \frac{\partial}{\partial e}C(e) = 0$$

Note that $\frac{\partial}{\partial \mathrm{e}}[1] = 0$ and $\frac{\partial}{\partial \mathrm{e}}[2] = 0$.

For the first-order condition to be satisfied, we need that

$$\frac{\partial}{\partial e}\left(Prob\ l < 0 < \left(D - \underline{x}\right) < u\right) < 0\ and\ \frac{\partial}{\partial e}\left(Prob\ l < 0 < u < \left(D - \underline{x}\right)\right) < 0$$

Note also that

$$\lim_{e \to 1}\left(prob\ell < 0 < D - \underline{x} < u\right) = 0$$

$$\lim_{e \to 1}\left(prob\ell < 0 < u < \left(D - \underline{x}\right)\right) = 0$$

because for $e = 1, \ell = u = \hat{b}$.

Consider now $0 < e < 1$ and the specific case when $D - \underline{x} = 0$. In the case, there are only type I errors for $\ell < 0$. Prob $\ell < 0$ must be decreasing in e.

At the other extreme $(D-\underline{x})=\varepsilon$. Then $\delta\left(prob\ell<0<u<(D-\underline{x})\right)/\delta e<0$ since for any *b*, the probability that ℓ and *u* are on different sides of *b* is decreasing when the range ℓ *to u* is decreasing.

For the same reason, when $0<D-\underline{x}<\varepsilon$ the sum of $prob(0<D-\underline{x}<u)$ and $prob(0<u<D-\underline{x})=prob\ (u>\varepsilon)$ must be decreasing *e*. There is a range where an increase in *e* may increase prob $(u<D-\underline{x})$ at the expense of a decrease in $prob(D-\underline{x}<u)$. The sum of the changes in the probabilities is always negative, however. Taking into account the expected cost savings from the changes in the probabilities confirms that the first order condition is satisfied since cost of errors are higher when $(D-\overline{x})<u$ than when $u<(D-\underline{x})$.

Second-order condition:

$$\left[\int_{l(b,e)}^{0} G(\tilde{b})db+(D-\underline{x})-\int_{0}^{D-\underline{x}} G(\tilde{b})db\right]\frac{\partial^2}{\partial e^2}\left(Prob\ l<0<(D-\underline{x})<u\right)$$

$$-G(l(b,e))\frac{\partial}{\partial e}\left(Prob\ l<0<(D-\underline{x})<u\right)+\frac{\partial^2}{\partial e^2}C(e)$$

$$+\left[\int_{l(b,e)}^{0} G(\tilde{b})db+u(b,e)-\int_{0}^{u(b,e)} G(\tilde{b})db\right]\frac{\partial^2}{\partial e^2}\left(Prob\ l<0<u<(D-\underline{x})\right)$$

$$+\left[-G(l(b,e))+\frac{\partial}{\partial e}u(b,e)-G(u(b,e))\right]\frac{\partial}{\partial e}\left(Prob\ l<0<u<(D-\underline{x})\right)$$

We need to show that the second-order condition > 0 (since we are minimizing the sum of costs of errors).

Q.E.D.

Proof of Proposition 4. Define $\mu_0\ =$

$$(1-q)\left(Prob\ l<0<(D-\underline{x})<u\right)\left\{D-\left(\int_{l(b,e)}^{0} G(\tilde{b})db+(D-\underline{x})-\int_{0}^{D-\underline{x}} G(\tilde{b})db\right)\right\}$$

$$+(1-q)\left(Prob\ l<0<u<(D-\underline{x})\right)\left\{D-\left(\int_{l(b,e)}^{0} G(\tilde{b})db+u(b,e)-\int_{0}^{u(b,e)} G(\tilde{b})db\right)\right\}$$

$$-(1-q)Prob\left(\left(D-\underline{x}\right)-\int_{-\varepsilon}^{D-\underline{x}}F\left(\tilde{b}\right)db<0\right)\underline{x}$$

$$-(1-q)Prob\left(\left(D-\underline{x}\right)-\int_{-\varepsilon}^{D-\underline{x}}F\left(\tilde{b}\right)db>0\right)\min\left(D,\underline{x}+E_0\left(\tilde{b}\right)\right)$$

Since $\frac{\delta l(.)}{\delta(e)}=-\frac{\delta u(.)}{\delta(e)}$ (by assumption), therefore, by proposition 3 should the lender choose to invest in am IM, it is feasible for it to do so (that is, an optimal e^* exists). If the lender chooses to invest in the IM, then its expected payoff is given by –

$$E_0(\pi_B|IM)=qD-C(e)$$
$$+(1-q)\left(Prob\ l<0<\left(D-\underline{x}\right)<u\right)\left\{D-\left(\int_{l(b,e)}^{0}G\left(\tilde{b}\right)db+\left(D-\underline{x}\right)-\int_{0}^{D-\underline{x}}G\left(\tilde{b}\right)db\right)\right\}$$
$$+(1-q)\left(Prob\ l<0<u<\left(D-\underline{x}\right)\right)\left\{D-\left(\int_{l(b,e)}^{0}G\left(\tilde{b}\right)db+u(b,e)-\int_{0}^{u(b,e)}G\left(\tilde{b}\right)db\right)\right\}$$

If the lender chooses not to invest in the IM, then its expected payoff is given by –

$$E_0(\pi_B|No\ IM)=qD+(1-q)Prob\left(\left(D-\underline{x}\right)-\int_{-\varepsilon}^{D-\underline{x}}F\left(\tilde{b}\right)db<0\right)\underline{x}$$
$$+(1-q)Prob\left(\left(D-\underline{x}\right)-\int_{-\varepsilon}^{D-\underline{x}}F\left(\tilde{b}\right)db>0\right)\min\left(D,\underline{x}+E_0\left(\tilde{b}\right)\right)$$

The lender is better off investing in the IM only if $E_0(\pi_B|IM)>E_0(\pi_B|No\ IM)$. Inserting the expressions for $E_0(\pi_B|IM)$ and for $E_0(\pi_B|No\ IM)$ in the above inequality and simplifying, we get that the lender is better off investing in the IM only if $C(e)<\mu_0$.

Q.E.D.

Proof of Proposition 5. Define $\mu_1=$

$$(1-q)\left(Prob\ l<0<\left(D-\underline{x}\right)<u\right)\left\{D-\left(\int_{l(b,e)}^{0}G\left(\tilde{b}\right)db+\left(D-\underline{x}\right)-\int_{0}^{D-\underline{x}}G\left(\tilde{b}\right)db\right)\right\}$$

$$+(1-q)\left(Prob\ l < 0 < u < \left(D-\underline{x}\right)\right)\left\{D-\left(\int_{l(b,e)}^{0} G\left(\tilde{b}\right)db + u(b,e) - \int_{0}^{u(b,e)} G\left(\tilde{b}\right)db\right)\right\}$$

And, $\mu_2 = (1-q)Prob\left(\left(D-\underline{x}\right) - \int_{-\varepsilon}^{D-\underline{x}} F\left(\tilde{b}\right)db < 0\right)\underline{x}$

$$+(1-q)Prob\left(\left(D-\underline{x}\right) - \int_{-\varepsilon}^{D-\underline{x}} F\left(\tilde{b}\right)db > 0\right)\min\left(D, \underline{x} + E_0\left(\tilde{b}\right)\right) + C(e)$$

Since $\frac{\delta l(.)}{\delta(e)} = -\frac{\delta u(.)}{\delta(e)}$ (by assumption), therefore, by proposition 3 should the lender choose to invest in am IM, it is feasible for it to do so (that is, an optimal e^* exists). If the lender chooses to invest in the IM, then its expected payoff is given by –

$$E_0(\pi_B|IM) = qD - C(e)$$

$$+(1-q)\left(Prob\ l < 0 < \left(D-\underline{x}\right) < u\right)\left\{D-\left(\int_{l(b,e)}^{0} G\left(\tilde{b}\right)db + \left(D-\underline{x}\right) - \int_{0}^{D-\underline{x}} G\left(\tilde{b}\right)db\right)\right\}$$

$$+(1-q)\left(Prob\ l < 0 < u < \left(D-\underline{x}\right)\right)\left\{D-\left(\int_{l(b,e)}^{0} G\left(\tilde{b}\right)db + u(b,e) - \int_{0}^{u(b,e)} G\left(\tilde{b}\right)db\right)\right\}$$

And, if the lender chooses not to invest in the IM, then its expected payoff is given by –

$$E_0(\pi_B|No\ IM) = qD + (1-q)Prob\left(\left(D-\underline{x}\right) - \int_{-\varepsilon}^{D-\underline{x}} F\left(\tilde{b}\right)db < 0\right)\underline{x}$$

$$+(1-q)Prob\left(\left(D-\underline{x}\right) - \int_{-\varepsilon}^{D-\underline{x}} F\left(\tilde{b}\right)db > 0\right)\min(D, \underline{x} + E_0\left(\tilde{b}\right))$$

First, consider $\mu_1 > \mu_2$. In this case, $E_0(\pi_B|IM) > E_0(\pi_B|No\ IM)$ and should the lender choose to make a loan, it will also by proposition 4, choose to invest in the IM. Therefore, the lender is better off making the loan only if –

$E_0(\pi_B|IM) \geq D$

Or, $\mu_1 + qD - C(e) \geq D$, since $E_0(\pi_B|IM) = \mu_1 + qD - C(e)$

Or, $D \leq \frac{\mu_1 - C(e)}{1-q}$

Now consider, $\mu_1 < \mu_2$. In this case, $E_0(\pi_B|IM) < E_0(\pi_B|No\ IM)$ and should the lender choose to make a loan, it will by proposition 4, choose not to invest in the IM. Therefore, the lender is better off making the loan only if –

$E_0(\pi_B|No\ IM) \geq D$

Or, $\mu_2 + qD - C(e) \geq D$, since $E_0(\pi_B|No\ IM) = \mu_2 + qD - C(e)$

Or, $D \leq \frac{\mu_2 - C(e)}{1-q}$

Q.E.D.

Proof of Proposition 6. The manger's expected compensation is given by:

$$E[compensation] = qE[compensation\ |\overline{x}] + (1-q)E[compensation\ |\underline{x}]$$

In addition, the manager enjoys the private benefit z, if there is no recall.

Recall that if there is no investment in specific asset (and the lender knows this), the lender never recalls and therefore $E[compensation\ |\theta = 0] = qk(\overline{x} - D) + z$.

Now, consider the case where the manager invests in the specific asset and the lender knows this. From Proposition 1, if $x = \overline{x}$, the lender always recalls and if $x = \underline{x}$, the lender recalls only if $(D - \underline{x}) < \int_{-\varepsilon}^{D-\underline{x}} F(\tilde{b})db$. Therefore, if $x = \overline{x}$, the manager's expected compensation is given by $E[compensation\ |\overline{x}\ and\ \theta > 0] = qk(\overline{x} - D)$. To analyse the case of $x = \underline{x}$, we will consider two sub-cases.

Consider first the case of $\boldsymbol{(D - \underline{x}) > \int_{-\varepsilon}^{D-\underline{x}} F(\tilde{b})db}$.

$E[compensation\ |\underline{x}\ and\ \theta > 0]$

$= (1-q)(Prob\ No\ Recall)\left[\int_{D-\underline{x}}^{\varepsilon} k\left(\tilde{b} - (D - \underline{x})\right) f(\tilde{b})db + z\right]$

$= (1-q)\left[\int_{D-\underline{x}}^{\varepsilon} k\left(\tilde{b} - (D - \underline{x})\right) f(\tilde{b})db + z\right]$, since in this case the Prob No Recall = 1

Thus, the incentive to invest in θ is given by:

(8.3.1) $I|\overline{x},\underline{x} = \; qk(\overline{x}-D) + (1-q)\left[\int_{D-\underline{x}}^{\varepsilon} k\left(\tilde{b}-\left(D-\underline{x}\right)\right) f\left(\tilde{b}\right)db + z\right] -$

$qk(\overline{x}-D) - z - \theta = -zq + (1-q)\int_{D-\underline{x}}^{\varepsilon} k\left(\tilde{b}-\left(D-\underline{x}\right)\right) f\left(\tilde{b}\right)db - \theta$

Thus, the manager-entrepreneur will invest in the specific asset only if $\theta < -zq +$ $(1-q)\int_{D-\underline{x}}^{\varepsilon} k\left(\tilde{b}-\left(D-\underline{x}\right)\right) f\left(\tilde{b}\right)db$.

Now, consider the case of $\boldsymbol{\left(D-\underline{x}\right) < \int_{-\varepsilon}^{D-\underline{x}} F(\tilde{b})db}$.

$E[compensation \; |\underline{x} \; and \; \theta > 0]$

$= (1-q)(Prob\ No\ Recall)\left[\int_{D-\underline{x}}^{\varepsilon} k\left(\tilde{b}-\left(D-\underline{x}\right)\right) f\left(\tilde{b}\right)db + z\right]$

$= 0$, since in this case the Prob No Recall = 0

Thus, the incentive to invest in θ is given by:

(8.3.2) $I|\overline{x},\underline{x} = \; qk(\overline{x}-D) - qk(\overline{x}-D) - z - \theta = -z - \theta$

The incentive to invest in the specific asset is negative. Thus, if there is no information about the specific asset and $\left(D-\underline{x}\right)$ is relatively small, the lender will always recall and there is no incentive to invest in the specific asset.

Q.E.D.

Proof of Proposition 7. Define $\mu_3 =$

$$qk\left(\overline{x}-D\right)\left[1-\left(\text{prob } \ell > (D-\overline{x})\right)\right] + z\left[(\text{prob } \ell > \left(D-\overline{x}\right))q + \left(\text{prob no recall } |\underline{\text{x}},\text{e}\right)(1-q) - 1\right]$$

$$+k\left[(\text{prob } \ell > \left(D-\overline{x}\right))q\int_{D-\overline{x}}^{\varepsilon}\left(\tilde{b}-\left(D-\overline{x}\right)\right) f\left(\tilde{b}\right)db + \left(\text{prob no recall } |\underline{\text{x}},\text{e}\right)(1-q)\int_{D-\underline{x}}^{\varepsilon}\left(\tilde{b}-\left(D-\underline{x}\right)\right) f\left(\tilde{b}\right)db\right] - \theta$$

Recall policy for $x = \overline{x}$: recall with $Prob(l < (D-\overline{x}))$,

no recall with $Prob(l > (D-\overline{x}))$.

Recall policy for $x = \underline{x}$: $Prob\ Recall \mid \underline{x}, e$

Denote by $I|\overline{x},\underline{x},e$ the incentive to invest in specific asset. Then, $I|\overline{x},\underline{x},e =$ $E[\pi_M|\overline{x},\underline{x}\ e,,\theta]$ - $E[\pi_M|\overline{x},\underline{x},e,\theta=0]$ - θ, where $\theta=0$ denotes that there is no investment in the specific asset. The project manager is assumed to know that the lender has the information mechanism in place.

$$I \mid \bar{x}, \underline{x}, e = q\left\{\left(\text{prob } \ell < D-\bar{x}\right)k\left(\bar{x}-D\right)+\left(\text{prob } \ell > \left(D-\bar{x}\right)\left[z+k\int_{D-\bar{x}}^{\varepsilon}\left(\tilde{b}-\left(D-\bar{x}\right)\right)f\left(\tilde{b}\right)db\right]\right)\right\}$$

$$+\left(1-q\right)\left\{\left(\text{prob no recall} \mid \underline{\text{x}},\text{e}\right)\left[z+k\int_{D-\underline{x}}^{\varepsilon}\left(\tilde{b}-\left(D-\underline{x}\right)\right)f\left(\tilde{b}\right)db\right]\right\}-qk\left(\bar{x}-D\right)-z-\theta$$

Note that when there is no specific asset $(\theta = 0)$ the lender will never recall as in the case with only hard information. We obtain after noting that $\text{prob } \ell < \left(D-\bar{x}\right) = 1 - \text{prob } \ell > \left(D-\bar{x}\right)$:

$$(8.4) I \mid \bar{x}, \underline{x}, e = qk\left(\bar{x}-D\right)\left[1-\left(\text{prob } \ell > (D-\bar{x})\right)\right]$$

$$+z\left[(\text{prob } \ell > \left(D-\bar{x}\right))q+\left(\text{prob no recall} \mid \underline{\text{x}},\text{e}\right)(1-q)-1\right]$$

$$+k\left[(\text{prob } \ell > \left(D-\bar{x}\right))q\int_{D-\bar{x}}^{\varepsilon}\left(\tilde{b}-\left(D-\bar{x}\right)\right)f\left(\tilde{b}\right)db+\left(\text{prob no recall} \mid \underline{\text{x}},\text{e}\right)\left(1-q\right)\int_{D-\underline{x}}^{\varepsilon}\left(\tilde{b}-\left(D-\underline{x}\right)\right)f\left(\tilde{b}\right)db\right]-\theta$$

That is, the manager-entrepreneur invests in the specific asset only if $\theta < \mu_3$.

Q.E.D.

Proof of Proposition 8. Define μ_4 and μ_5 as:

$$\mu_4 = -qk\left(\text{prob } \ell > D-\bar{x}\right)\left[\left(\bar{x}-D\right)-\int_{D-\underline{x}}^{\varepsilon}\left(\tilde{b}-\left(D-\underline{x}\right)f\left(\tilde{b}\right)\right)db\right]$$

$$+z\left[q\left(\text{prob } \ell > \left(D-\bar{x}\right)\right)+\left(1-q\right)\left[\left(\text{prob no recall} \mid \underline{\text{x}},\text{e}\right)-1\right]\right]$$

$$+\left(1-q\right)k\left[\left(\text{prob no recall} \mid \underline{\text{x}},\text{e}\right)-1\right]\int_{D-\underline{x}}^{\varepsilon}\left(\tilde{b}-\left(D-\underline{x}\right)f\left(\tilde{b}\right)\right)db-qk\left(\bar{x}-D\right)$$

$$\mu_5 = I \mid \bar{x}, \underline{x}, e = qk\left(\bar{x}-D\right)\left[1-\left(\text{prob } \ell > (D-\bar{x})\right)\right]+z\left[(\text{prob } \ell > \left(D-\bar{x}\right))q+\left(\text{prob no recall} \mid \underline{\text{x}},\text{e}\right)(1-q)-1\right]$$

$$+k\left[(\text{prob } \ell > \left(D-\bar{x}\right))q\int_{D-\bar{x}}^{\varepsilon}\left(\tilde{b}-\left(D-\bar{x}\right)\right)f\left(\tilde{b}\right)db+\left(\text{prob no recall} \mid \underline{\text{x}},\text{e}\right)\left(1-q\right)\int_{D-\underline{x}}^{\varepsilon}\left(\tilde{b}-\left(D-\underline{x}\right)\right)f\left(\tilde{b}\right)db\right]$$

Define AI as <u>the additional incentive to invest in specific asset with soft information.</u> In the case when $\boldsymbol{(D-\underline{x}) > \int_{-\varepsilon}^{D-\underline{x}} F(\tilde{b})db}$, Prob No Recall | $\underline{\mathrm{x}}$ = 1. The investment in the specific asset may occur even if there is only hard information.

$$\mathrm{AI} = I \mid \bar{x}, \underline{x}, e - I \mid \bar{x}, \underline{x},$$

$$q\left\{\left(\text{prob } \ell < D-\bar{x}\right)k\left(\bar{x}-D\right)+\left(\text{prob } \ell > \left(D-\bar{x}\right)\left[z+k\int_{D-\bar{x}}^{\varepsilon}\left(b-\left(D-\bar{x}\right)\right)f\left(\tilde{b}\right)db\right]\right)\right\}$$

$$+\left(1-q\right)\left\{\left(\text{prob no recall } |\underline{\mathrm{x}},\mathrm{e}\right)\left[z+k\int_{D-\bar{x}}^{\varepsilon}\left(b-\left(D-\underline{x}\right)\right)f\left(\tilde{b}\right)db\right]\right\}$$

$$-2qk\left(\bar{x}-D\right)-z-\theta-\left(1-q\right)\left\{\left(\text{prob no recall } |\underline{\mathrm{x}}\right)\left[z+k\int_{D-\bar{x}}^{\varepsilon}\left(b-\left(D-\bar{x}\right)\right)f\left(\tilde{b}\right)db\right]\right\}$$

Re-grouping terms, we have:

$$AI = qk\left[\left(\text{prob } \ell < \left(D-\bar{x}\right)\right)\left(\bar{x}-D\right)+\left(\text{prob } \ell > \left(D-\bar{x}\right)\right)\int_{D-\bar{x}}^{\varepsilon}\left(b-\left(D-\bar{x}\right)-f\left(\tilde{b}\right)db\right)\right]$$

$$+z\left[q\left(\text{prob } \ell > \left(D-\bar{x}\right)\right)+\left(1-q\right)\left[\left(\text{prob no recall } |\underline{\mathrm{x}},\mathrm{e}\right)-\left(\text{prob no recall } |\underline{\mathrm{x}}\right)\right]\right]$$

$$+\left(1-q\right)k\left[\left(\text{prob no recall } |\underline{\mathrm{x}},\mathrm{e}\right)-\left(\text{prob no recall } |\underline{\mathrm{x}}\right)\right]\int_{D-\underline{x}}^{\varepsilon}\left(\tilde{b}-\left(D-\underline{x}\right)f\left(\tilde{b}\right)\right)db$$

$$-2qk\left(\bar{x}-D\right)-\theta$$

Insert Prob No Recall | x = 1 and we have:

$$AI = qk\left[\left(\text{prob } \ell < \left(D-\bar{x}\right)\right)\left(\bar{x}-D\right)+\left(\text{prob } \ell > \left(D-\bar{x}\right)\right)\int_{D-\bar{x}}^{\varepsilon}\left(b-\left(D-\bar{x}\right)-f\left(\tilde{b}\right)db\right)\right]$$

$$+z\left[q\left(\text{prob } \ell > \left(D-\bar{x}\right)\right)+\left(1-q\right)\left[\left(\text{prob no recall } |\underline{\mathrm{x}},\mathrm{e}\right)-1\right]\right]$$

$$+\left(1-q\right)k\left[\left(\text{prob no recall } |\underline{\mathrm{x}},\mathrm{e}\right)-1\right]\int_{D-\underline{x}}^{\varepsilon}\left(\tilde{b}-\left(D-\underline{x}\right)f\left(\tilde{b}\right)\right)db-2qk\left(\bar{x}-D\right)-\theta$$

Use $\left(\text{prob } \ell < \left(D-\bar{x}\right)\right)=\left(1-\text{prob}\,\ell > \left(D-\bar{x}\right)\right)$ in the expression for AI:

$$\text{AI} = -qk\left(\overline{x} - D\right)\left(\text{prob } \ell > D - \overline{x}\right) + qk\left(\text{prob } \ell > \left(D - \overline{x}\right)\right) \int_{D-\underline{x}}^{\varepsilon} \left(\tilde{b} - \left(D - \underline{x}\right) f\left(\tilde{b}\right)\right) db$$

$$+z\left[q\left(\text{prob } \ell > \left(D - \overline{x}\right)\right) + (1-q)\left[\left(\text{prob no recall } |\underline{\text{x}},\text{e}\right) - 1\right]\right]$$

$$+(1-q)k\left[\left(\text{prob no recall } |\underline{\text{x}},\text{e}\right) - 1\right] \int_{D-\underline{x}}^{\varepsilon} \left(\tilde{b} - \left(D - \underline{x}\right) f\left(\tilde{b}\right)\right) db - qk\left(\overline{x} - D\right) - \theta$$

Re-grouping terms, we have:

$$(8.5)\text{AI} = -qk\left(\text{prob } \ell > D - \overline{x}\right)\left[\left(\overline{x} - D\right) - \int_{D-\underline{x}}^{\varepsilon} \left(\tilde{b} - \left(D - \underline{x}\right) f\left(\tilde{b}\right)\right) db\right]$$

$$+z\left[q\left(\text{prob } \ell > \left(D - \overline{x}\right)\right) + (1-q)\left[\left(\text{prob no recall } |\underline{\text{x}},\text{e}\right) - 1\right]\right]$$

$$+(1-q)k\left[\left(\text{prob no recall } |\underline{\text{x}},\text{e}\right) - 1\right] \int_{D-\underline{x}}^{\varepsilon} \left(\tilde{b} - \left(D - \underline{x}\right) f\left(\tilde{b}\right)\right) db - qk\left(\overline{x} - D\right) - \theta$$

That is, the manager-entrepreneur has a positive additional incentive to invest in the specific asset only if $\theta < \mu_4$.

Now, consider the case of $\boldsymbol{(D - \underline{x}) < \int_{-\varepsilon}^{D-\underline{x}} F(\tilde{b})db}$. Then, Prob No Recall | x = 0. Investments in the specific asset will not happen if there is only hard information. Thus, the additional incentive (AI) from soft information equals the incentive for investing in the specific asset with soft information in Proposition 7.

$$AI = I \mid \overline{x}, \underline{x}, e = qk\left(\overline{x} - D\right)\left[1 - \left(\text{prob } \ell > (D - \overline{x})\right)\right] + z\left[(\text{prob } \ell > \left(D - \overline{x}\right))q + \left(\text{prob no recall } |\underline{\text{x}},\text{e}\right)(1-q) - 1\right]$$

$$+k\left[(\text{prob } \ell > \left(D - \overline{x}\right))q \int_{D-\overline{x}}^{\varepsilon} \left(\tilde{b} - \left(D - \overline{x}\right)\right) f\left(\tilde{b}\right) db + \left(\text{prob no recall } |\underline{\text{x}},\text{e}\right)(1-q) \int_{D-\underline{x}}^{\varepsilon} \left(\tilde{b} - \left(D - \underline{x}\right)\right) f\left(\tilde{b}\right) db\right] - \theta$$

That is, the manager-entrepreneur has a positive additional incentive to invest in the specific asset only if $\theta < \mu_5$.

Q.E.D.

Proof of Proposition 9. Define

$$\mu_6 = \left[\delta\left(\text{prob } \ell > \left(D - \overline{x}\right)\right) / \delta e\right] q \left[-k\left(\overline{x} - D\right) + z + k \int_{D-\overline{x}}^{\varepsilon} \left(\tilde{b} - \left(D - \overline{x}\right)\right) f\left(\tilde{b}\right) db\right]$$

$$+\left[\delta\left(\text{prob no recall } |\underline{\text{x}},\text{e}\right) / \delta e\right](1-q)\left[z + k \int_{D-\underline{x}}^{\varepsilon} \left(\tilde{b} - \left(D - \underline{x}\right)\right) f\left(\tilde{b}\right) db\right]$$

Recall that

$$I\mid\bar{x},\underline{x},e = qk\left(\bar{x}-D\right)\left[1-\left(\text{prob }\ell>(D-\bar{x})\right)\right]+z\left[(\text{prob }\ell>\left(D-\bar{x}\right))q+\left(\text{prob no recall }|\underline{\text{x}},\text{e}\right)(1-q)-1\right]$$

$$+k\left[(\text{prob }\ell>\left(D-\bar{x}\right))q\int_{D-\bar{x}}^{\varepsilon}\left(\tilde{b}-\left(D-\bar{x}\right)\right)f\left(\tilde{b}\right)db+\left(\text{prob no recall }|\underline{\text{x}},\text{e}\right)\left(1-q\right)\int_{D-\underline{x}}^{\varepsilon}\left(\tilde{b}-\left(D-\underline{x}\right)\right)f\left(\tilde{b}\right)db\right]-\theta$$

Then, the derivative of $I\mid\bar{x},\underline{x},e$ with respect to *e* is given by:

$$\delta I\mid\bar{x},\underline{x},e\,/\,\delta e$$

$$=\left[\delta\left(\text{prob }\ell>\left(D-\bar{x}\right)\right)/\,\delta e\right]q\left[-k\left(\bar{x}-D\right)+z+k\int_{D-\bar{x}}^{\varepsilon}\left(\tilde{b}-\left(D-\bar{x}\right)\right)f\left(\tilde{b}\right)db\right]$$

$$+\left[\delta\left(\text{prob no recall }|\underline{\text{x}},\text{e}\right)/\,\delta e\right](1-q)\left[z+k\int_{D-\underline{x}}^{\varepsilon}\left(\tilde{b}-\left(D-\underline{x}\right)\right)f\left(\tilde{b}\right)db\right]$$

The only negative incentive effect is caused by a high *q* and a high $\bar{x}$, given D. The incentive effect of *e* is strengthened by *D, z* and *k* if $q\bar{x}$ is not too high. Loss given default $\left(D-\underline{x}\right)$ has an ambiguous effect on the derivative. Now, the incentives for soft information production and for specific asset investment will be consistent only if $\delta I\mid\bar{x},\underline{x},e\,/\,\delta e>0$. That is , only if $\mu_6 > 0$.

Q.E.D.