The Power of the Agenda Setter: A Dynamic Legislative Bargaining Model^{*}

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Abstract

We consider an infinitely repeated legislative bargaining model in the spirit of Baron and Ferejohn (1989), with three agents dividing a dollar in every period. The status quo evolves endogenously over time, as agents can approve new proposals by a majority of two votes. One agent has veto power and must approve any proposal that changes the status quo. Our key parameter of interest is the veto player's agenda setting power, defined as the probability (p) that she is randomly selected as the proposer in a given period. We characterize and show existence of a symmetric Markov Perfect Equilibrium for all possible primitives (i.e. the initial status quo, the discount factor, and p). Our main result is that the veto player's equilibrium welfare is non-monotonic in her agenda setting power. If p is low, the veto player can offer alternating bribes to her opponents, which leads to an equilibrium with *full surplus extraction* by the veto player in the long run (exactly analogous to Nunnari, 2014). However, once p exceeds a critical threshold, we show existence of a new equilibrium with only *partial surplus extraction*, because the non-veto players can form a blocking coalition which is immune to such "minimumwinning" bribery schemes. The stark contrast with Nunnari (2014) stems from the non-veto players having inherently different but self-enforcing beliefs about equilibrium play. For a large set of primitives, this new equilibrium guarantees strictly positive and identical shares to the non-veto players in the long run, and makes them both strictly better off than the full extraction equilibrium. We argue how adding an initial "cheap talk" stage can rule out the full extraction equilibrium from being selected whenever it is dominated. Finally, we show that the non-veto players can sustain larger shares in the long run if their initial shares are allocated more equitably, or if agents are more patient. In the limit case where patience and the veto player's agenda setting power grow very large, our model also replicates the findings of Diermeier, Egorov and Sonin (2013) for the three-player case.

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1 Introduction

Motivation Basic insights of game theory suggest that the institutional rules underlying a bargaining game or a committee decision are important in determining the outcome. It has been well established in the literature on group decision making that having the ability to set the agenda can be used strategically to influence the outcome of the bargaining process. For example, in a simple voting game with three voters, three alternatives and preference orderings that exhibit a Condorcet cycle, any selected agenda setter can always implement her favorite alternative by first asking for a vote between her two least preferred alternatives, and then have the winner take it up against her most preferred alternative. In a more general decision-theoretic framework, Plott and Levine (1977) show that the agenda setter can influence committee decisions in her favor, and find support for their findings through a laboratory experiment. More recently, Knight (2005) considers empirical evidence from bargaining episodes in Congress, concerning the distribution of transportation projects. He finds support for the qualitative prediction that members on the transportation committee with proposer power secure more project spending than members from other districts, suggesting that having proposer power can be very valuable. In a setting that is more related to our paper, Bernheim, Rangel and Rayo (2006) consider a legislative bargaining game where the status quo evolves endogenously. They find that when amendments to the status quo can be made sequentially by different players, then the final proposer has near-dictatorial power under fairly weak conditions. This evidence suggests that it should always be beneficial for an agent to have more agenda setting power, since that creates more frequent opportunities to propose amendments to the current status quo, and therefore to shape the outcome of the bargaining process.

Main result In this paper, we provide a simple and tractable theoretical framework where the common intuition that "having more agenda setting power helps" does *not* hold. More specifically, we prove existence of a symmetric Markov Perfect Equilibrium in which an increase of the agenda setting power of any given player may be detrimental to that player's equilibrium welfare. This equilibrium is characterized by an infinite and monotonically increasing sequence of critical threshold values of agenda setting power. At every threshold, a marginal increase in the agenda setting power of a given agent will trigger a discontinuous drop in that agent's welfare. This creates a potential micro-foundation for real-life phenomena where an agent may deliberately try to curb her own agenda setting power, since this could increase her welfare in equilibrium. In a zero-sum game, this implies that the welfare of the other agents is reduced, even though they can now propose more often. We will provide a motivating example after briefly describing the model setup first.

Basic setup We consider a legislative bargaining game in the spirit of Baron and Ferejohn (1989). We extend their benchmark model to a dynamic context where the status quo policy evolves endogenously over time. There are three players who bargain in an infinite number of periods over the division of a fixed budget, and the allocation implemented at the end of every period becomes the default allocation in the next. Importantly, the model diverges from Baron and Ferejohn (1989) in that one of the players is uniquely and permanently endowed with veto power throughout the game, allowing her to unilaterally reject amendments to the

status quo proposed by other players. Every period, the veto agent is randomly selected with probability p to propose an alternative allocation. Each non-veto player has symmetric agenda setting power denoted by $\frac{1-p}{2}$. We consider a so-called "closed rule" where a motion proposed by one player is voted immediately against the status quo, as opposed to an "open rule" where amendments can be made sequentially by other players in a known order, as in Bernheim, Rangel and Rayo (2006). Importantly, we allow for recognition rules that are asymmetric, such that the veto and non-veto players may have a different probability of being selected as the proposer in any given period.

Example Our main motivating example is one of a country that has three political players and a fixed amount of annual fiscal revenues to divide among those players every year. The three players represent a Monarch with veto power, a Nobility, and a Bourgeoisie. This is characteristic of constitutional monarchies that have upper and lower houses of parliament. The upper house and lower house represent different strata of society, but are largely similar in terms of their powers. Throughout the paper, we treat the two non-veto players as symmetric in terms of agenda setting power, although they may have different budget shares in the initial status quo. Each institution would like to extract as much of the budget as possible, and cares about the future generations of their "type". The Monarch, while having veto power, is constrained in that she requires the support of either the Bourgeoisie or the Nobility in order to change the status quo. Conversely, if either the Nobility or the Bourgeoisie make a new proposal, it must always be acceptable by the Monarch. Our main result establishes a potential micro-foundation for why the Monarch may want to commit to permanently lower her agenda setting power, in order to secure a larger share of the pie for herself or her future descendants. By relinquishing some of her legislative power and seemingly empowering the two houses of parliament, she effectively increases competition for resources between them, prevents them from forming credible blocking coalitions with each other, and ends up stealing more of the total surplus in the long run.

Contribution to the literature The two papers most closely related to ours are Diermeier, Egorov and Sonin (2013), and Nunnari (2014). Both papers consider a similar dynamic legislative bargaining environment with at least one veto agent, but they find dramatically different results in terms of the expropriation power of the veto player(s). In Diermeier et al. (2013), the non-veto players form endogenous coalitions to protect each other's property rights, which precludes full expropriation by the veto player(s). In Nunnari (2014), however, the veto player asymptotically extracts the full surplus in the long run, leaving the non-veto players defenseless and exploitable. Interestingly, the model in our paper is able to predict both of these contrasting outcomes, depending on the primitives of the model. However, our setup differs from both Diermeier et al. (2013) and Nunnari (2014) in a number of dimensions.

Diermeier et al. (2013) consider a general environment where three or more legislators (at least one of whom has veto power) bargain over a discrete policy space, with a general voting rule, whereas we consider a baseline model with only three agents and a simple majority rule. However, Diermeier et al. (2013) assume that (1) only the veto player(s) can propose new

allocations, and (2) legislators are *extremely* patient.¹ In comparison, our model deliberately treats the distribution of agenda setting power and the agents' common discount factor as two key primitives of the model, and we show how the equilibrium changes as a function of these two primitives and the initial status quo allocation.

In Section 3, we show how the equilibrium with partial surplus extraction exists for a vast set of primitives. In Section 5, we show that it converges to that of Diermeier et al. (2013) in the limiting case where the veto player has all the proposal power and players become infinitely patient. This suggests that (1) our model nests that of Diermeier et al. (2013) for the three-player case, and (2) the assumption of having discrete surplus divisions in Diermeier et al. (2013) is not driving the stark contrast with the full expropriation result obtained by Nunnari (2014), where allocations are assumed to be continuous.

Nunnari (2014) considers a baseline model with an underlying symmetric structure and equilibrium concept which are identical to ours, and shows that the veto player can fully expropriate her two opponents (asymptotically) for a reasonably large set of primitives.² For the remaining set of primitives (i.e high levels of patience and a fairly powerful veto player), no equilibrium is defined. Furthermore, Nunnari (2014) considers various extensions of the baseline model which show robustness of the full expropriation equilibrium to games with more than three players and more general voting rules, which further contrasts his results with those of Diermeier et al. (2013). In comparison, our paper shows the existence of a symmetric MPE for *all* possible primitives, which we can roughly partition into three regions.

In the first region, comprising all the primitives for which Nunnari (2014) does not define an equilibrium, we show existence of an equilibrium with *partial surplus extraction* by the veto player, where the non-veto players can sustain strictly positive long-run shares.³

In the second region, Nunnari's (2014) equilibrium with full surplus extraction exists, but we prove existence of a different equilibrium with only *partial* expropriation by the veto player. We provide conditions under which our equilibrium Pareto-dominates Nunnari's (2014) equilibrium from the perspective of the non-veto players.⁴ The stark contrast between these respective equilibria stems from the non-veto players having inherently different but self-enforcing beliefs about equilibrium play. In this region, if both non-veto players believe that the other non-veto player would *accept* any allocation where the veto player offers one opponent a large enough bribe and the other opponent nothing, then accepting these "minimum-winning" bribes becomes a mutual best response. Under these beliefs, the

³Note that we are not claiming equilibrium uniqueness in this region.

¹More specifically, Diermeier et al. (2013) require that the discount factor $\delta \in (\delta_0, 1)$, where $\delta_0 = \left(\frac{|A|}{1+|A|}\right)^{\frac{1}{|A|}}$ and A is the set of all feasible (discrete) allocations that exhaust the budget. In a setup with 3 players and a discrete budget $b \in \mathbb{N}$, this lower bound δ_0 approaches 1 very fast as b increases. For example, if b = 1, then $A = \{(1,0,0), (0,1,0), (0,0,1)\}$, and hence $\delta_0 = 0.9086$. Similarly, for b = 3, |A| = 10 and $\delta_0 = 0.9905$. In the limit case where $b \in \mathbb{N}$ grows unboundedly (or, analogously, if allocations become continuous), their results require $\delta \approx 1$, which significantly complicates the equilibrium analysis due to stationarity concerns and classic folk theorem arguments.

²More specifically, the author shows that, irrespective of the initial status quo, the full expropriation equilibrium exists whenever $\delta \leq \frac{1+3p-\sqrt{1+6p-7p^2}}{4p^2}$, where δ is the common discount factor and p is the recognition probability of the veto player. For example, if $p = \frac{3}{4}$, this requires $\delta \leq \frac{8}{9}$.

⁴This is still work in progress, along with the extension of our model where pre-game cheap talk is introduced as a potential equilibrium selection mechanism in Section 4.

equilibrium of Nunnari (2014) obtains where at most one non-veto player holds a positive share after the initial period, and where the veto player holds the total share in the long run. However, if both non-veto players start the game with the belief that they will *not* accept such bribes, but instead block any proposal which fully expropriates either one of them, then this becomes a mutual best response as well. This alternative set of beliefs is internally consistent with the *partial surplus extraction*. To resolve this multiplicity, we introduce a "cheap talk" stage as a potential equilibrium selection mechanism in Section 4.

Finally, in the third region, we do not find an equilibrium with partial surplus extraction, and the only long-run stable point is where the veto player holds the total surplus. This case typically holds when the veto player is relatively weak and players are impatient, making it easier for the veto player to offer feasible and acceptable bribes, and to form alternating minimum-winning coalitions. However, even though both our and Nunnari's (2014) equilibrium imply full expropriation by the veto player in the long run, equilibrium play in the initial period (and hence welfare levels) may still be different. In particular, our equilibrium does not exhibit any mixing behavior on behalf of the veto player, whereas Nunnari's (2014) does.

Overview In Section 2, we present the basic model and work out a simple example of how the players' beliefs about equilibrium play will affect their belief-consistent continuation values, strategies and long-run outcomes.

In Section 3, we characterize the equilibrium, and provide conditions on the primitives under which the non-veto players can sustain strictly positive long-run shares.

Section 4 contains an extension of the model with an initial "cheap talk" phase in which players can communicate costlessly. This allows us to rule out the full extraction equilibrium from being played whenever there exists a dominating partial extraction equilibrium.⁵

Section 5 presents comparative statics and some insightful graphs for the equilibrium with partial surplus extraction. First, we show how the veto player's welfare may be non-monotonic in her agenda setting power, which follows from the analysis in Section 3. Then, we consider the impact of a change in the level of patience, and the effect of a change in the level of inequality in the initial status quo. Perhaps unsurprisingly, the non-veto players will be more likely to resist bribes by the veto player and form a blocking coalition if they are either more patient or more equal in terms of initial shares. Moreover, we show how our equilibrium converges to the one found by Diermeier et al. (2013) in the limit, when the veto player acquires all the agenda setting power and players become infinitely patient.

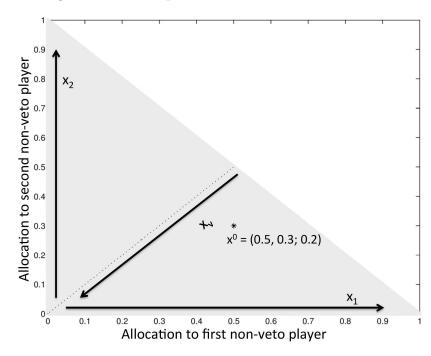
Section 6 concludes and hints at future research. We hope to extend our current threeplayer setting towards a more general framework, in order to assess the robustness of our results and potentially apply them to a wider range of real-world scenarios.

2 Model

Set-up Consider an infinitely repeated bargaining game between three agents indexed by $i \in \{1, 2, v\}$, where v denotes the veto player. In every period $t \ge 1$, the agents have

 $^{^5\}mathrm{We}$ are still working on formalizing this notion, and we may consider alternative equilibrium selection mechanisms.

Figure 1: The simplex Δ - set of feasible allocations



to divide a dollar among themselves in every, by collectively choosing an allocation vector $\mathbf{x}^t = (x_1^t, x_2^t, x_v^t)$. An allocation \mathbf{x}^t is feasible if $x_i^t \ge 0$ for all $i \in \{1, 2, v\}$ and $\sum_i x_i^t = 1$. Denote the set of feasible allocations by Δ . As shown in Figure 1, any feasible allocation $\mathbf{x} = (x_1, x_2, x_v)$ can be represented by a point in \mathbb{R}^2_+ with Cartesian coordinates (x_1, x_2) , such that the veto player receives a residual share $x_v = 1 - x_1 - x_2$. Then, the origin corresponds to the allocation $\mathbf{x} = (0, 0, 1)$, where the veto player v holds the full surplus.

Stage game In every period $t \ge 1$, all agents vote between the status quo policy \mathbf{x}^{t-1} and a new allocation $\mathbf{y} \in \Delta$ that is proposed by one of the agents. The initial status quo \mathbf{x}^0 is exogenously given. We consider simply majority voting. Moreover, agent v is uniquely endowed with veto power throughout the game. This implies that any proposal \mathbf{y} can be unilaterally blocked by agent v, even if the other two agents vote in favor of \mathbf{y} over \mathbf{x}^{t-1} . The bargaining protocol is as follows. Let $p \in (0, 1]$ denote the (common knowledge) probability with which the veto player is recognized as the proposer in any given period t. Each non-veto player i = 1, 2 is selected with (symmetric) probability $\frac{1-p}{2}$.⁶ Hence, p captures the relative agenda setting power of the veto player. At the start of every period t, a single agent is randomly selected (according to p) to propose an allocation \mathbf{y} which will compete against \mathbf{x}^{t-1} . There are no amendments by other agents.⁷ If \mathbf{y} gets the support of the veto player and at least one other agent i = 1, 2, the proposal gets implemented and becomes the new

⁶Note that p = 1/3 corresponds to the symmetric baseline scenario considered by Nunnari (2014). However, he also considers heterogeneous recognition probabilities in an extension of the model.

⁷Although this simple setup is common in the literature, other alternatives can be thought of. For example, Bernheim, Rangel and Rayo (2006) consider multiple proposers who move sequentially in a known order, in what is essentially a one-shot game. Another alternative would be to rotate (or randomize) both the veto power as well as the proposer power, or to endogenize the proposer power as in Cotton (2012).

status quo, $\mathbf{x}^t = \mathbf{y}$. Otherwise, the previous status quo \mathbf{x}^{t-1} stays in place until the next period. Instantaneous payoffs \mathbf{x}^t are consumed after they realize, and there is no borrowing or saving.⁸ Agents maximize the present discounted value of their lifetime utility at every time t, given the implemented allocation \mathbf{x}^t :

$$U_i(\mathbf{x}^t) = x_i^t + \sum_{s=t+1}^{+\infty} \delta^{s-t} x_i^s$$

where $i \in \{1, 2, v\}$ and $\delta \in [0, 1)$ denotes the common discount factor.

Equilibrium concept We focus on symmetric Markov Perfect equilibria where the single relevant state variable is \mathbf{x}^{t-1} , the status quo implemented in the previous period. The strategy of agent *i* can then be described by a pair

$$\sigma_i(\mathbf{x}^{t-1}) = \{\mu_i(\mathbf{y}|\mathbf{x}^{t-1}), A_i(\mathbf{x}^{t-1})\}\$$

where $\mu_i(\mathbf{y}|\mathbf{x}^{t-1})$ denotes the new allocation(s) that player *i* proposes, conditional on being selected as the agenda setter and the status quo \mathbf{x}^{t-1} , and $A_i(\mathbf{x}^{t-1}) \subseteq \Delta$ is the set of allocations for which agent *i* would vote in favor of if the current status quo is \mathbf{x}^{t-1} . Note that the acceptance set A_i does not depend on the identity of the player who makes the proposal, since players only care about their own payoff, and because the current status quo is a sufficient statistic for the continuation values of every player. Without loss of generality, we assume that indifferent agents always vote in favor of a proposal. Note that the proposal strategies can be pure or mixed, allowing agents to randomize between proposing different allocations with positive probability.

Comparison with Nunnari (2014) So far, our setup has been identical to the one in Nunnari (2014), who establishes the stark result that the veto player is able to fully expropriate the two non-veto players asymptotically. Although we derive an equilibrium with fundamentally different outcomes; since our models are so closely related, we first present two of the main results from Nunnari (2014):

Lemma 1 (Nunnari (2014)) Assume a symmetric recognition rule where $p = \frac{1}{3}$. Then, there exists a symmetric MPE in which, irrespective of the discount factor (δ) and the initial division of the dollar (\mathbf{x}^0), the status quo eventually gets arbitrarily close to the veto player's ideal point. Formally: $\forall \epsilon > 0$, $\exists T \ s.t. \ \forall t \geq T$, $x_v^t \geq 1 - \epsilon$.

The next result generalizes this by relaxing the assumption that $p = \frac{1}{3}$, although existence of the MPE now requires some primitives to be excluded:

⁸We do not allow agents to borrow or save for two main reasons. Firstly, this would imply that the object being bargained over can be stored or transferred for future use or lending, which is not always feasible (e.g. plots of land, perishable goods, livestock). Secondly, access to credit markets would allow agents to offer and implement negative allocations, which alters the strategy space. However, our main results should go through as long as the feasible allocations are reasonably bounded from above and below, such that the veto player cannot offer infinitely large bribes.

Lemma 2 (Nunnari (2014)) Assume a general recognition rule where $p \in (0, 1]$. Then, there exists a symmetric MPE in which, irrespective of the initial division of the dollar (\mathbf{x}^0) , the status quo policy eventually gets arbitrarily close to the veto player's ideal point, as long as 1) $p \in [0, \frac{1}{2}]$ and $\delta \in [0, 1)$, or 2) p = 1 and $\delta \in [0, 1)$, or 3) $p \in (\frac{1}{2}, 1)$ and $\delta \leq \overline{\delta}(p) = \frac{1+3p-\sqrt{1+6p-7p^2}}{4p^2}$.

Although Nunnari (2014) does not claim uniqueness, the robustness of this equilibrium still suggests there is little or no hope for the non-veto players; for the majority of primitives, they will be fully expropriated in the long run. The main reason why we find an alternative equilibrium is because we consider the possibility of sustaining cooperation among non-veto players. This is inherently different from the beliefs implicitly assumed by Nunnari (2014). In each equilibrium, the agents share a certain set of common *beliefs* about equilibrium play which are self-sustaining and consistent (i.e. incentive compatible) with respect to the equilibrium strategies.

In Nunnari (2014), both non-veto players believe that if the veto player were to propose an allocation which fully expropriates one non-veto player and offers a sizable bribe to the other, then that proposal will be approved by the non-veto player who receives the bribe. The veto player's equilibrium strategy where she keeps offering a "minimum-winning" bribe to the poorest (or weakest) non-veto player at the expense of the other non-veto player is consistent with the common belief that the non-veto players are willing to sell each other out and accept such bribes.

As we will illustrate in the example below, our equilibrium is fundamentally different because both non-veto players believe that if the veto player were to offer such a "minimumwinning" bribe to either non veto-player at the expense of the other, then they would both reject that proposal and hence maintain the current status quo for another period. Under these beliefs, the non-veto players effectively form a blocking coalition which is immune to certain bribes. In the spirit of Diermeier et al. (2013), this endogenous coalition formation implies that the veto player *cannot* obtain the entire surplus in the long run. The next example and subsequent analysis in Section 3 will hopefully make clear what the driving forces (i.e. primitives) are behind the non-veto players' (in)ability to enforce strictly positive long-run shares for themselves.

Throughout the example and the rest of the paper, we will refer to the beliefs underlying Nunnari's (2014) equilibrium with full surplus extraction as *non-cooperative beliefs*, and the beliefs underlying our equilibrium with partial surplus extraction as *cooperative beliefs*. However, it is important to keep in mind that the structure of the underlying strategic game does not change.

Example Let the initial status quo be $\mathbf{x}^0 = (\frac{1}{2}, \frac{1}{2}, 0)$ and let p = 1, i.e. the veto player is the sole agenda setter, but she starts out with no initial wealth. Let $\delta \in [0, 1)$ be the common discount factor. In this simple setting, the only relevant strategies are the veto player's proposals and the non-veto players' (symmetric) acceptance sets.

Case 1: Non-cooperative beliefs

- At t = 1, consider the strategy where the veto player proposes either $\mathbf{y} = (\frac{1}{2-\delta}, 0, 0)$ or $\mathbf{z} = (0, \frac{1}{2-\delta}, 0)$, bribing each non-veto player with 50% chance.
- We now check incentive compatibility for the non-veto player who is offered the bribe. Assuming (wlog) that player 1 gets offered the bribe, we need to verify that $V_1^{Accept}(\mathbf{y}) \geq V_1^{Reject}(\mathbf{y})$ for all $\delta \in [0, 1)$.
- If player 1 *accepts* the bribe and implements $\mathbf{x}^1 = \mathbf{y}$ at t = 1, then the veto player will propose $\mathbf{x}^t = (0, 0, 1)$ for all $t \ge 2$. Player 2 votes in favor due to indifference; he receives 0 no matter what he does. Hence, the continuation value of both non-veto players is 0, and $V_1^{Accept}(\mathbf{y}) = \frac{1}{2-\delta}$.
- If player 1 *rejects* the bribe and implements $\mathbf{x}^1 = \mathbf{x}^0$ at t = 1, then given the veto player's equilibrium mixing strategy at t = 2, this yields a value $V_1^{Reject}(\mathbf{y}) = \frac{1}{2} + \delta[\frac{1}{2}\frac{1}{2-\delta}]$. Crucially, this continuation value incorporates the belief that the other non-veto player would *accept* the same bribe $(\frac{1}{2-\delta})$ when offered.
- It is easily verified that $V_1^{Accept}(\mathbf{y}) = V_1^{Reject}(\mathbf{y})$ for all feasible values of $\delta \in [0, 1)$. By symmetry, this implies that both non-veto players will accept the one-time bribe when offered, and receive 0 forever after.
- Hence, when p = 1, the veto player acquires the full surplus after just two rounds, even when the non-veto players are patient and start out sharing the full surplus equally. This example demonstrates the significance of the non-cooperative belief system implicitly assumed in Nunnari (2014).

Case 2: Cooperative beliefs

- Now, we evaluate player 1's incentive compatibility constraint when he considers an infinite-period rejection of the veto player's bribe attempts, rather than just a oneperiod rejection as in Case 1. Under the belief that player 2 would also *reject* any bribe offered by the veto player, the continuation values along this path look fundamentally different from the previous case. Consistent with their beliefs, the non-veto players jointly enforce that $\mathbf{x}^t = \mathbf{x}^0$ for all $t \ge 1$, so that $V_1^{Reject} = V_2^{Reject} = \frac{1}{2(1-\delta)}$.
- Now suppose that the veto player, realizing that his opponents are collaborating by refusing any deviation from the initial status quo, tries to offer the largest feasible bribe to either non-veto player, in an attempt to break the endogenous coalition between her opponents. However, if player 1 ever accepts $\mathbf{y} = (1,0,0)$ or if player 2 ever accepts $\mathbf{z} = (0,1,0)$, then the status quo will again become $\mathbf{x}^t = (0,0,1)$ for all $t \ge 2$. This follows because the veto player cannot credibly commit to any other future path once one non-veto player is fully expropriated. Therefore, accepting a one-time bribe yields a maximal value of $V_1^{Accept}(\mathbf{y}) = V_2^{Accept}(\mathbf{z}) = 1$ to either non-veto player.
- It is easily verified that $V_1^{Reject} > V_1^{Accept}(\mathbf{y})$ and $V_2^{Reject} > V_2^{Accept}(\mathbf{z})$ whenever $\delta \in (\frac{1}{2}, 1)$. If $\delta \in [0, \frac{1}{2}]$, the veto player can offer a feasible bribe equal to $\frac{1}{2(1-\delta)}$ and achieve the full surplus after two rounds.

• Therefore, the status quo $\mathbf{x}^0 = (\frac{1}{2}, \frac{1}{2}, 0)$ can be sustained as a Markov Perfect Equilibrium whenever the non-veto players are reasonably patient $(\delta > \frac{1}{2})$.

Note that this simple example already highlights two important features of our equilibrium. First of all, it shows that the result of endogenous cooperation (in which the non-veto players can sustain strictly positive shares in the long run) in Diermeier et al (2013) is robust to a much larger range of patience than was considered in that paper. Second, it demonstrates how the beliefs of the non-veto players may have a tremendous impact on their equilibrium welfare levels. In Section 4, we will propose a potential equilibrium selection mechanism to deal with this multiplicity, by allowing the players to coordinate on a set of beliefs through a cheap talk stage before the bargaining game starts.

3 Equilibrium analysis

In this section, we show existence of the equilibrium for all possible primitives $(\mathbf{x}^0, \delta, p) \in \Omega = \Delta \times [0, 1) \times (0, 1]$, assuming that the non-veto players share *cooperative* beliefs about each other, i.e. they will form a blocking coalition whenever that is incentive compatible. As shown by the previous example, these beliefs will not always be sustainable for all primitives. In these cases (e.g. when δ is low), the veto player will eventually obtain the full surplus.

Before analyzing the case with a general status quo $\mathbf{x}^0 \in \Delta$, we start with the special case where the status quo lies on one of the two axes. Define this set as $\overline{\Delta} = \{\mathbf{x} \in \Delta : \min(x_1, x_2) = 0\}$. This will be useful to fix ideas about what happens if one of the non-veto players ever accepts a "minimum-winning" bribe offered by the veto player.

Then, we derive conditions on the primitives in Ω under which partial surplus extraction by the veto player is an equilibrium. We show that if these conditions are not met, the status quo will reach the set $\overline{\Delta}$ after one period.

3.1 A simple sufficient condition for full surplus extraction

The set $\overline{\Delta}$ contains all allocations which can be implemented by a minimal winning coalition, i.e. the veto player and one non-veto player. Our first result states that the veto player can always obtain the full surplus asymptotically if the status quo reaches this set after some time $t \ge 0$. In other words, if any non-veto players is ever fully expropriated, then full expropriation of *both* non-veto players will inevitably follow in the long run, irrespective of the discount factor δ and the distribution of agenda setting power p. Because the non-veto players cannot sustain any advantageous blocking coalition in this region, this is essentially the same result as the one obtained by Nunnari (2014) in Lemma 1 and 2.

Lemma 3 Let the primitives be $(\mathbf{x}^0, \delta, p) \in \overline{\Delta} \times [0, 1) \times (0, 1]$. Then, there exists a Markov Perfect Equilibrium where the veto player (asymptotically) extracts the full surplus. That is, $\forall \epsilon > 0$, there exists T such that $\forall t \geq T$, the veto player's payoff in the status quo satisfies $x_v^t > 1 - \epsilon$.

Appendix A contains the proof, deriving and analyzing the equilibrium strategies and corresponding value functions of each player in detail.

Equilibrium structure In equilibrium, every agent proposes an allocation that gives a positive share to a minimum winning coalition, where one non-veto player is fully expropriated. This makes $\overline{\Delta}$ an absorbing set. Since the veto player's vote is always required to pass an allocation, she will always receive a positive share in every period. Moreover, she will never receive less than her current status quo share, since she could always invoke her veto power and maintain the status quo. Let $\overline{x} \equiv \max(x_1, x_2)$ be the share held by the wealthier non-veto player. Since $\mathbf{x}^0 \in \overline{\Delta}$, the other non-veto player has a share of 0. The equilibrium strategies are simple and intuitive.

If a non-veto player is selected as the proposer, he will offer the veto player exactly her current share $(1 - \overline{x})$ and will keep the remainder of the surplus (\overline{x}) for himself. Under the standard tie-breaking rule, the veto player accepts the new proposal when kept indifferent.

If the veto player is selected as the proposer, she will offer a bribe to the non-veto player who has a share of 0, since that is the cheapest vote to buy. In equilibrium, the optimal bribe demanded by this non-veto player takes the following form:

$$d(\overline{x}, \delta, p) = \frac{\delta(1-p)}{2-\delta(1+p)}\overline{x}$$

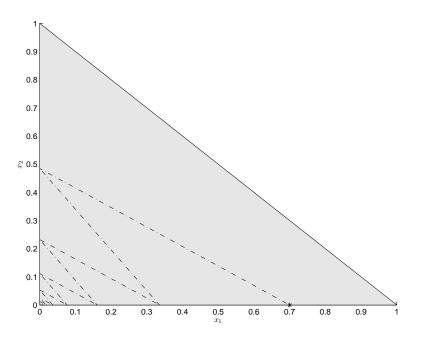
This bribe keeps the poorest non-veto player indifferent between either accepting it and altering the current status quo, or rejecting it and sticking to \mathbf{x}^0 , which currently gives him 0. By the tie-breaking rule, he will accept the bribe at the expense of the other non-veto player. Note that as long as the players are not infinitely patient ($\delta < 1$), this bribe is always strictly smaller than the share of the other non-veto player, \overline{x} . This implies that the veto player can steal a strictly positive amount ($\overline{x} - d$) whenever she proposes, which happens infinitely often as long as p > 0. Hence, the veto player's share displays a "ratchet effect", and converges to the only stable outcome, $\mathbf{x}^{\infty} = (0, 0, 1)$.⁹ In the extreme case where either the agents are fully myopic ($\delta = 0$) or the veto player is the only agenda setter (p = 1), the demand $d(\overline{x}, \delta, p) = 0$. Intuitively, the non-veto player who currently has nothing is indifferent between accepting a share of 0 or maintaining the current status quo. Hence, the veto player can always propose and implement $\mathbf{y} = (0, 0, 1)$ without delay.

Figure 2 shows an example of the evolution of the status quo if the initial allocation is $\mathbf{x}^0 = (0.7, 0, 0.3)$, with $\delta = 0.95$ and p = 0.3. For simplicity, the graph plots only the first 25 allocations proposed by the veto player, in order to show the steady convergence towards the veto player's ideal point, $\mathbf{x}^{\infty} = (0, 0, 1)$. Whenever a non-veto player proposes, the status quo would flip along the 45-degree line.

Comparative statics Once the status quo lies in the absorbing set $\overline{\Delta}$, the veto player's continuation value strictly increases with her agenda setting power p, for any discount factor δ . This happens for two reasons. First of all, a higher value of p allows the veto player to propose, and hence steal some part of the surplus, more frequently. Secondly, the size of the share that she is able to steal each time she proposes, $(\overline{x} - d)$, strictly increases with p. This follows because an increase in p reduces the non-veto players' chances of proposing in any future period. Hence, it becomes less likely that they will be able to form minimal winning coalitions with the veto player and steal the share of the other non-veto player. This reduces

⁹The main results of Nunnari (2014) stem from the asymptotic nature of this process of convergence.

Figure 2: Evolution of the status quo once $\overline{\Delta}$ is reached, for p = 0.50, $\delta = 0.90$



the non-veto players' continuation values, which in turn allows the veto player to offer them lower bribes, and reach the origin faster.

Finally, any increase in the discount factor δ will increase the bribe required to make the poorer non-veto player indifferent. This slows down the convergence rate of the status quo towards the origin.

3.2 Conditions for full surplus extraction

We have shown that full (asymptotic) surplus extraction by the veto player is always a feasible equilibrium if the status quo $\mathbf{x}^t \in \overline{\Delta}$ for some $t \ge 0$. Now, we seek to find more general conditions on the primitives under which the status quo will reach that absorbing set $\overline{\Delta}$, even if both non-veto players start out with strictly positive shares (i.e. $\mathbf{x}^0 \notin \overline{\Delta}$).

Let the initial status quo be given by $\mathbf{x}^0 = (x_1, x_2, 1 - x_1 - x_2)$, and let $\overline{x} \equiv \max(x_1, x_2)$ and $\underline{x} \equiv \min(x_1, x_2)$. The main result can then be stated as follows.

Proposition 1 Let the primitives be $(\mathbf{x}^0, \delta, p) \in \Delta \times [0, 1) \times (0, 1]$. If $2 - 3\delta(1+p) + \delta^2(1+p+2p^2) < 0$, then there exists a symmetric Markov Perfect Equilibrium where, irrespective of the initial division of the dollar (\mathbf{x}^0) , the status quo policy eventually gets arbitrarily close to the veto player's ideal point. Moreover, the status quo reaches the absorbing set $\overline{\Delta}$ after at most one period, i.e. $\min(x_1^t, x_2^t) = 0 \ \forall t \geq 1$.

The detailed proof is given in Appendix **B**.

Before giving some interpretation and comparative statics of these thresholds, we discuss how the equilibrium is set up.

Equilibrium structure The proof in the Appendix shows in more detail how for a given δ and p, the set of possible allocations can be partitioned into four different subsets of the space Δ , which we call Δ^A , Δ^B , Δ^C , and Δ^D . The difference between them lies in two factors. First, that the veto player is willing to accept a bribe of $d_v = 0$ in regions Δ^B and Δ^C , and not in regions Δ^A and Δ^D . Second, that the veto player plays a mixed strategy while bribing the non-veto players in regions Δ^C and Δ^D , and not in regions Δ^A and Δ^B . Suppose that nonveto player 1 is selected as the proposer, he will optimally form a minimal winning coalition with the veto player and propose an allocation $\mathbf{x}^1 = (1 - d_v, 0, d_v)$, where $d_v \geq 0$ is the bribe offered to the veto player. Symmetrically, player 2 would propose $\mathbf{x}^1 = (0, 1 - d_v, d_v)$. Optimality requires that the proposer does not offer anything more to the veto player than strictly necessary to induce her to support \mathbf{x}^1 . Now, recall from Proposition 1 that as soon as one non-veto player gets fully expropriated at some point, then the veto player will be able to extract the full surplus in the long run. Hence, accepting the new proposal \mathbf{x}^1 gives her a guaranteed high continuation value. This, in turn, might make her willing to accept negative bribes $d_v < 0$, which are infeasible (e.g. due to her credit constraints). This will be the case if the veto player has a relatively low initial share to begin with. Graphically, Δ^B and Δ^C correspond to status quos \mathbf{x}^0 that are far away from the origin $(x_1, x_2) = (0, 0)$, where the veto player owns the full surplus. In these cases, she will accept a bribe $d_v = 0$. Conversely, if she has a relatively large initial share, then the veto player will only accept a strictly positive amount $d_v > 0$. These will be the cases Δ^A and Δ^D .

Solving for d_v and rewriting the constraint that $d_v > 0$ allows us to derive bound that partitions off the subsets Δ^A and Δ^D .

Also, Δ^C and Δ^D correspond to status quos \mathbf{x}^0 that are closer to the 45 degree line which corresponds to an equal allocation to both non-veto players. In these cases, the veto player will offer a bribe to either non-veto player according to a probability calculated so as to keep the non-veto players indifferent between accepting or rejecting that bribe. Conversely, if there is more inequality among non-veto players in the initial allocation, then the veto player will only bibe the poorer non-veto player. These will be the cases Δ^A and Δ^B . Solving for μ (the probability of bribing the poorer non-veto player) and rewriting the constraint that $\mu < 1$ allows us to derive bound that partitions off the subsets Δ^C and Δ^D .

Based on the nature of the equilibrium play in the game described so far, we derive a boundary such that cooperation among non veto players is sustainable. When $2 - 3\delta(1 + p) + \delta^2(1 + p + 2p^2) > 0$, Δ contains a fifth sub-region called Δ^E where the non-veto players can sustain cooperation to ensure that each of them receives a positive allocation in the long run.

3.3 Conditions for partial surplus extraction

For given values of p and δ , this region is defined as the set of status quos that satisfy:

$$\underline{x} > \left(\frac{(2 - \delta(1 + p))^2}{2(2 - \delta(1 - p))(1 - \delta p)}\right)$$

where we define $\underline{x} \equiv \min(x_1^0, x_2^0)$ as the share of the poorer non-veto player, respectively.¹⁰

¹⁰Note that this set is empty if $2 - 3\delta(1+p) + \delta^2(1+p+2p^2) < 0$

It is easily checked that these two sets form a proper partition of the set Δ . We have already derived the optimal strategies (offers and voting) and the corresponding value functions if the initial status quo $\mathbf{x}^0 \in \Delta \setminus \Delta^E$. However, we need to define it for the residual region Δ^E as well. First, we shall show that for non-veto players, there will be no bribe which could induce them to leave Δ^E . Then we shall invoke the symmetry of payoffs to argue that Δ^E can itself be partitioned into five sub-regions $\{\Delta^{A_2}, \Delta^{B_2}, \Delta^{C_2}, \Delta^{D_2}, \Delta^{E_2}\}$. This partitioning can be iterated indefinitely with each Δ^{E_J} region being partitioned into three sub-regions. In each sub-region of type Δ^{i_J} (i = A, B, C, D), convergence shall take place in the way described before to a convergence point that is the "origin" for region $\Delta^{E_{J-1}}$.

Proposition 2 Let the primitives $(\mathbf{x}^0, \delta, p) \in \Omega$ be such that $2-3\delta(1+p)+\delta^2(1+p+2p^2) > 0$, and define $J \geq 0$ such that $\mathbf{x}^0 \in \Delta^{E_J} \setminus \Delta^{E_{J+1}}$. Then, any L-shaped set $\Delta^{E_J} \setminus \Delta^{E_{J+1}}$ must be absorbing, and the status quo will converge to the allocation $(x^{\star J}, x^{\star J}, 1-2x^{\star J})$.¹¹ Moreover, the discounted lifetime value of each non-veto player i = 1, 2 is bounded from below by

$$V_i(\mathbf{x}^0) > \frac{x^{\star J}}{1-\delta}$$

Proof: Consider the status quo $\mathbf{x}^{\mathbf{0}} \in \Delta^{E_J} \setminus \Delta^{E_{J+1}}$. In Appendix C, we show that any status quo that originates in Δ^{E_1} will stay lead to all future allocations being from within that region. The fact that payoffs within any Δ^{E_J} region are simply a shift of the origin and change in scale from a corresponding $\Delta^{E_{J-1}}$ region means that we can extrapolate and iterate the result to the following two conditions which will hold in equilibrium:

(1) Both non-veto players are strictly better off by staying anywhere within the *L*-shaped set $\Delta^{E_J} \setminus \Delta^{E_{J+1}}$ than by deviating towards the adjacent *L*-shaped set $\Delta^{E_{J-1}} \setminus \Delta^{E_J}$ situated on the lower-left. Therefore, they will jointly block any attempt by the veto player to move the status quo towards any *L*-shaped set that lies closer to the origin.

(2) The veto player is strictly better off by staying within the *L*-shaped set $\Delta^{E_J} \setminus \Delta^{E_{J+1}}$ than by deviating towards the adjacent *L*-shaped set $\Delta^{E_{J+1}}$, situated on the upper right. Therefore, she will block any attempt by the non-veto players to move the status quo towards any *L*-shaped set that lies farther away from the origin.

By (1) and (2), any *L*-shaped set $\Delta^{E_J} \setminus \Delta^{E_{J+1}}$ must be absorbing. The equilibrium dynamics imply that the status quo will converge to the lower-left corner of this set (defined as $(x^{\star J}, x^{\star J}, 1 - 2x^{\star J})$) at a rate that depends on the primitives (δ, p) .

We could also add the following interesting corollary that follows from our above claims (1) and (2):

Corollary 1 The equilibrium dynamics imply that following incentive constraints must hold $\forall i = 1, 2, \forall j \ge 1, \forall \mathbf{x} \in \Delta^{E_j} \setminus \Delta^{E_{j+1}}, \forall \mathbf{y} \in \Delta^{E_{j-1}} \setminus \Delta^{E_j}, \forall \mathbf{z} \in \Delta^{E_{j+1}}$:

$$\begin{cases} V_i(\mathbf{z}) > V_i(\mathbf{x}) > V_i(\mathbf{y}) \\ V_v(\mathbf{y}) > V_v(\mathbf{x}) > V_v(\mathbf{z}) \end{cases}$$

¹¹Although it will never actually reach this allocation, since it has to stay on the interior of Δ^{E_J} .

Basically, this says that, for any allocation in given *L*-shaped set $\Delta^{E_j} \setminus \Delta^{E_{j+1}}$, both the non-veto players are strictly better off by staying inside that set forever than by moving outside of it. This means e.g. that the first non-veto playr would strictly prefer starting out from $\mathbf{x}^0 = (\frac{1}{3}, \frac{2}{3}, 0)$ instead of a much more advantageous initial status quo $\mathbf{y}^0 = (\frac{4}{5}, \frac{1}{5}, 0)$, if this latter allocation lies in a lower *L*-shaped set. Similarly, the veto player could be strictly worse off (in terms of lifetime value) starting off with a high initial endowment (e.g. $\mathbf{x}^0 = (\frac{1}{5}, \frac{1}{5}, \frac{3}{5})$) than with an alternative $\mathbf{y}^0 = (1, 0, 0)$, because the latter allocation allows her to steal everything in the long run while the former might lie in a higher *L*-shaped curve and offer lower future discounted value.

Since both non-veto players can trust each other no never propose or accept any allocation outside of Δ^E , if the game starts with a status quo within the region, all future allocations must lie within that region. This means that the payoffs of both non-veto players in every period must be at least x^* , where:

$$x^* = \left(\frac{(2 - \delta(1 + p))^2}{2(1 - \delta p)(2 - \delta(1 - p))}\right)$$

This reduces the "effective budget" to $1 - 2x^*$, other than that, other things are exactly the same as in Δ . Therefore, Δ^E would be partitioned just as Δ was, and the strategies in equilibrium would be the same as calculated earlier. The only differences will be the shift in origin, and reduction in "effective budget". Now, we can take this argument one step further and argue that such partitioning would take place iteratively indefinitely.

Iterative partitioning of Δ We define the space of primitives $\Omega = \{(\mathbf{x}^0, \delta, p) | \mathbf{x}^0 \in \Delta, \delta \in [0, 1]\}$ where $\Delta = \{\mathbf{x}^0 = (x_1^0, x_2^0, x_v^0) | \sum_i x_i^0 = 1\}$ is the set of all possible allocations and $x_v^0 = 1 - x_1^0 - x_2^0$ denotes the share of the veto player. Then, depending on the exogenously given values of (δ, p) , we partition Δ into five subsets $\Delta^i \subsetneq \Delta$ ($i \in \{A_1, B_1, C_1, D_1, E_1\}$), which we define as follows:

$$\mathbf{x}^{0} \in \Delta^{A_{1}} \Leftrightarrow \begin{cases} \overline{x} < 1 - \underline{x} \left(\frac{2-\delta(1-p)}{2-\delta(1+p)}\right) \\ \overline{x} \geq \underline{x} \left(\frac{2-\delta(1-p)}{2-\delta(1+p)}\right) \\ \underline{x} \leq \left(\frac{(2-\delta(1+p))^{2}}{2(1-\delta p)(2-\delta(1-p))}\right) \end{cases}$$
$$\mathbf{x}^{0} \in \Delta^{B_{1}} \Leftrightarrow \begin{cases} \overline{x} \geq 1 - \underline{x} \left(\frac{2-\delta(1-p)}{2-\delta(1+p)}\right) \\ \overline{x} \geq \left(\frac{\delta^{2}p(1-p)}{(1-\delta p)(2-\delta(1-p))}\right) + \underline{x} \left(\frac{2-\delta(1+p)-\delta^{2}p(1-p)}{(1-\delta p)(2-\delta(1+p))}\right) \\ \underline{x} \leq \left(\frac{2-\delta(1+p)}{2-\delta(1-p)}\right) \\ \underline{x} \leq \left(\frac{2-\delta(1+p)}{2-\delta(1-p)}\right) \\ \underline{x} \leq \left(\frac{(2-\delta(1+p))^{2}}{2(1-\delta p)(2-\delta(1-p))}\right) + \underline{x} \left(\frac{2-\delta(1+p)-\delta^{2}p(1-p)}{(1-\delta p)(2-\delta(1+p))}\right) \\ \overline{x} \leq \left(\frac{\delta^{2}p(1-p)}{(1-\delta p)(2-\delta(1-p))}\right) + \underline{x} \left(\frac{2-\delta(1+p)-\delta^{2}p(1-p)}{(1-\delta p)(2-\delta(1+p))}\right) \\ \overline{x} \leq \left(\frac{2-\delta(1+p)}{2-\delta(1-p)}\right) + \underline{x} \left(\frac{2-\delta(1+p)-\delta^{2}p(1-p)}{(1-\delta p)(2-\delta(1+p))}\right) \\ \overline{x} \leq \left(\frac{2-\delta(1+p)}{2-\delta(1-p)}\right) \left(\frac{2-\delta p}{1-\delta p}\right) - \underline{x} \\ \underline{x} \leq \left(\frac{(2-\delta(1+p))^{2}}{2(1-\delta p)(2-\delta(1-p))}\right) \end{cases}$$

$$\mathbf{x}^{0} \in \Delta^{D_{1}} \Leftrightarrow \begin{cases} \overline{x} < \frac{2-\delta}{2-\delta(1-p)} - \underline{x} \\ \overline{x} < \underline{x} \left(\frac{2-\delta(1-p)}{2-\delta(1+p)}\right) \\ \underline{x} \leq \left(\frac{(2-\delta(1+p))^{2}}{2(1-\delta p)(2-\delta(1-p))}\right) \end{cases}$$
$$\mathbf{x}^{0} \in \Delta^{E_{1}} \Leftrightarrow \underline{x} > \left(\frac{(2-\delta(1+p))^{2}}{2(1-\delta p)(2-\delta(1-p))}\right)$$

where we define $\overline{x} \equiv \max(x_1^0, x_2^0)$ and $\underline{x} \equiv \min(x_1^0, x_2^0)$ as the shares of the wealthier and poorer non-veto player, respectively. It is easily checked that these three sets form a proper partition of the set Δ .

By iterating this process, we can keep partitioning the residual Δ^{E} -regions ad infinitum, until we obtain a sequence of collections of sets $\{\Delta^{A_j}, \Delta^{B_j}, \Delta^{C_j}, \Delta^{D_j}, \Delta^{E_j}\}$ for j = 1, 2, 3, ...that satisfy the following properties:

$$\begin{cases} \Delta^{i_j} \cap \Delta^{k_j} = \emptyset & \forall j \ge 1, \forall i \ne k, i, k \in \{A, B, C, D, E\} \\ \Delta^{A_j} \cup \Delta^{B_j} \cup \Delta^{C_j} \cup \Delta^{D_j} \cup \Delta^{E_j} = \Delta^{E_{j-1}} & \forall j \ge 1 \\ \lim_{j \to \infty} \Delta^{E_j} = \{(\frac{1}{2}, \frac{1}{2}, 0)\} \end{cases}$$

The first two conditions imply that the sets generated in every iteration $j \geq 1$ form a proper partition of the set $\Delta^{E_{j-1}}$ generated in the previous iteration, where we define $\Delta^{E_0} \equiv \Delta$. The third condition (which we prove later on) ensures convergence of the Δ^{E_j} region towards the singleton allocation $\mathbf{x} = (\frac{1}{2}, \frac{1}{2}, 0)$. The speed of convergence will depend on the other primitives, δ and p. We will show that this convergence result holds if and only if $2 - 3\delta(1 + p) + \delta^2(1 + p + 2p^2) > 0$. If p and δ do not satisfy this condition, then the sets $\Delta^{E_j} = \emptyset$ for all $j \geq 1$, and the convergence result fails. The intuitive reason is that if the non-veto players are relatively impatient (i.e. low δ given $p \in [0, 1]$), or if they get to propose more often (i.e. high p given $\delta \in [0, 1)$ they are willing to accept full expropriation in the future, and their optimal demand will never exceed the total budget. In this case, our previous results for regions $\{\Delta^{A_1}, \Delta^{B_1}, \Delta^{C_1}, \Delta^{D_1}\}$ show that $\mathbf{x} = (0, 0, 1)$ is the unique stable allocation for any initial status quo $\mathbf{x}^0 \in \Delta$. Notice that for $\mathbf{x}^0 \in \Delta^{A_1} \cup \Delta^{B_1} \cup \Delta^{C_1} \cup \Delta^{D_1}$ our results boil down to those derived by Nunnari (2014), which considered the symmetric case where $p = \frac{1}{3}$.

Conversely, if the veto player's agenda setting power (p) is high enough, and the non-veto players are patient enough, to reject any feasible bribe offered to them by the veto player (i.e. $2 - 3\delta(1+p) + \delta^2(1+p+2p^2) > 0$), then we will show that, depending on the initial status quo \mathbf{x}^0 , they can sustain strictly positive (and symmetric) shares for themselves in the long run. To characterize these limiting stable allocations, we first introduce a sequence of cutoff values $\{x^{\star j}\}_{j\geq 1}$ that is directly linked to the sequential partitions of Δ^{E_j} from before:

$$\mathbf{x} = (x_1, x_2, 1 - x_1 - x_2) \in \Delta^{E_j} \Leftrightarrow \min(x_1, x_2) > x^{\star j}$$

Hence, for every $j \ge 1$, the cutoff value $x^{\star j}$ defines an upper right triangle in the set of possible allocations Δ , where the allocation $(x^{\star j}, x^{\star j}, 1 - 2x^{\star j})$ corresponds to the lower-left corner of this (open) set Δ^{E_j} . For completeness, we define $x^{\star 0} = 0$ as the lower-left left corner

of the (open) set $\Delta^{E_0} = \Delta \setminus \overline{\Delta}$, which excludes all allocations on either the horizontal or vertical axis. This sequence of cutoffs $\{x^{\star j}\}_{j>0}$ is defined as follows:

$$x^{\star j} = \frac{1}{2} \left[1 - \gamma^j \right] \quad \text{if} \quad j \ge 0$$

where the factor γ satisfies:

$$\gamma = \left(1 - 2\left(\frac{(2-\delta(1+p))^2}{2(1-\delta p)(2-\delta(1-p))}\right)\right) \text{ if } \delta \in [0,1)$$

It is easily checked that $\gamma \in [0, 1)$ whenever $2 - 3\delta(1 + p) + \delta^2(1 + p + 2p^2) > 0$. Under this condition on (δ, p) , the sequence of cutoffs $\{x^{\star j}\}_{j\geq 0}$ is strictly increasing and converges towards $\frac{1}{2}$. The corresponding sequence of nested sets $\{\Delta^{E_j}\}_{j\geq 0}$ is strictly decreasing and converges towards the singleton $(\frac{1}{2}, \frac{1}{2}, 0)$. Conversely, if $2 - 3\delta(1 + p) + \delta^2(1 + p + 2p^2) \leq 0$, then γ will be negative. For status quos

Conversely, if $2 - 3\delta(1+p) + \delta^2(1+p+2p^2) \leq 0$, then γ will be negative. For status quos \mathbf{x}^0 in that case, the origin (0,0,1) is the unique stable outcome and the values of $\{x^{\star j}\}_{j\geq 1}$ do not exist. For these cases, we have already characterized the optimal strategies (demands and voting) for every player and the corresponding value functions, depending on whether \mathbf{x}^0 lies in Δ^{A_1} , Δ^{B_1} , Δ^{C_1} , or Δ^{D_1} . Given these values for $x^{\star j}$, we can now characterize the sets $\{\Delta^{A_j}, \Delta^{B_j}, \Delta^{C_j}, \Delta^{D_j}, \Delta^{E_j}\}$ for j = 1, 2, 3, ...

$$\mathbf{x}^{0} \in \Delta^{A_{J}} \Leftrightarrow \begin{cases} \overline{x} < 1 - 2x^{*J-1} - (\underline{x} - x^{*J-1}) \left(\frac{2 - \delta(1-p)}{2 - \delta(1+p)}\right) \\ \overline{x} \geq x^{*J-1} + (\underline{x} - x^{*J-1}) \left(\frac{2 - \delta(1-p)}{2 - \delta(1+p)}\right) \\ \underline{x} > x^{*J-1} \\ \underline{x} \leq x^{*J} \end{cases}$$

$$\mathbf{x}^{0} \in \Delta^{B_{J}} \Leftrightarrow \begin{cases} \overline{x} \geq 1 - 2x^{*J-1} - (\underline{x} - x^{*J-1}) \left(\frac{2 - \delta(1-p)}{2 - \delta(1+p)}\right) \\ \overline{x} \geq (1 - 2x^{*J-1}) \left(\frac{\delta^{2} p(1-p)}{(1 - \delta p)(2 - \delta(1-p))}\right) + (\underline{x} - x^{*J-1}) \left(\frac{2 - \delta(1+p) - \delta^{2} p(1-p)}{(1 - \delta p)(2 - \delta(1+p))}\right) \\ \underline{x} \leq \left(\frac{2 - \delta(1+p)}{2 - \delta(1-p)}\right) (1 - 2x^{*J-1}) \\ \underline{x} \geq x^{*J-1} \\ \underline{x} \leq x^{*J} \end{cases}$$

$$\mathbf{x}^{0} \in \Delta^{C_{J}} \Leftrightarrow \begin{cases} \overline{x} \geq (1 - 2x^{*J-1}) \frac{2-\delta}{2-\delta(1-p)} - (\underline{x} - x^{*J-1}) \\ \overline{x} < (1 - 2x^{*J-1}) \left(\frac{\delta^{2}p(1-p)}{(1-\delta p)(2-\delta(1-p))} \right) + (\underline{x} - x^{*J-1}) \left(\frac{2-\delta(1+p)-\delta^{2}p(1-p)}{(1-\delta p)(2-\delta(1+p))} \right) \\ \overline{x} \leq (1 - 2x^{*J-1}) \left(\frac{2-\delta(1+p)}{2-\delta(1-p)} \right) \left(\frac{2-\delta p}{1-\delta p} \right) - (\underline{x} - x^{*J-1}) \\ \underline{x} > x^{*J-1} \\ \underline{x} \leq x^{*J} \end{cases}$$

$$\mathbf{x}^{0} \in \Delta^{D_{J}} \Leftrightarrow \begin{cases} \overline{x} < (1 - 2x^{*J-1})\frac{2-\delta}{2-\delta(1-p)} - (\underline{x} - x^{*J-1}) \\ \overline{x} < (\underline{x} - x^{*J-1}) \left(\frac{2-\delta(1-p)}{2-\delta(1+p)}\right) \\ \underline{x} > x^{*J-1} \\ \underline{x} \le x^{*J} \\ \mathbf{x}^{0} \in \Delta^{E_{J}} \Leftrightarrow \underline{x} > x^{*J} \end{cases}$$

Where x^{*J} is caluculated using the formula described above.

Thus, for any initial status quo \mathbf{x}^0 such that $\min(x_1^0, x_2^0) > 0$, there exists a unique integer $J \in \{0, 1, 2, ...\}$ such that $x^{\star J} < \min(x_1^0, x_2^0) \le x^{\star J+1}$. In other words, the initial status quo $\mathbf{x}^0 \in \Delta^{E_J} \setminus \Delta^{E_{J+1}}$, which we call an "L-shaped" set. We now move to the major results of this paper.

4 Interesting results

4.1 The veto player's lifetime valuation is non-monotonic in p

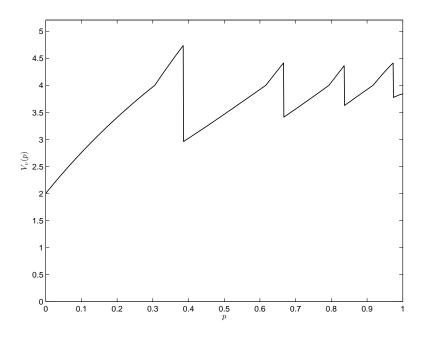
This is the main takeaway from this paper. The "Monarch" or "President" who is already endowed with veto power could be made worse off if she has higher proposal power (p). Figure 3 shows the veto player's lifetime valuation for a given status quo and δ as p is increased:

The graph in Figure 3 is endogenous to the status quo, and depends on the level of patience. However, for all status quos such that $min\{x_1^0, x_2^0\} > 0$ and $\delta > 0.5$ there exists a threshold value of proposal power p beyond which the veto player suffers a discrete reduction in her lifetime valuation.

First, suppose that the veto player can credibly commit to any agenda setting policy. More specifically, we assume that the veto player can unilaterally choose a parameter $p \in [0, 1]$ at time t = 0, after observing the initial status quo \mathbf{x}^0 . Once p is chosen, it will remain fixed forever and the game proceeds exactly as before. Since the veto player's value function is strictly increasing in p below the threshold that we derived in Section 3, it will be optimal to commit to a value of p that is exactly equal to that threshold. This will maximize the speed at which the veto player can extract the surplus of her opponents.

Although this case seems trivial, it leads to a surprising conclusion. Consider our previous analogy where the veto player represents a Monarch, attempting to expropriate the Bourgeoisie and the Nobility. Assume that the primitives are such that the Monarch cannot fully expropriate the wealth of her citizens (i.e. $\underline{x} > \left(\frac{(2-\delta(1+p))^2}{2(2-\delta(1-p))(1-\delta p)}\right)$ holds) where \underline{x} represents the wealth of the poorer Bourgeoisie. If the Monarch has the ability to credibly lower her agenda setting power by delegating some to her opponents, she certainly will do so. This implies that certain democratic reforms, such as the founding of houses of parliament for both the Nobility and the Bourgeoisie, might actually increase the wealth of the Monarch over time to the expense of her citizens. Indeed, it would allow the Monarch to "divide and conquer", by setting the Nobility and the Bourgeoisie up against each other, while slowly stealing some of their surplus along the way in a slow, controlled manner. Note that all

Figure 3: Comparing Equilibrium Value Functions for Veto Player $v: V_v(p, \delta, \mathbf{x}^0)$, for $\mathbf{x}^0 = (0.5, 0.3, 0.2), \delta = 0.90$



citizens should rationally oppose such transitions towards a more democratic system, if they are able to foresee that it will ultimately lead to their ultimate demise if the Monarch is never overthrown, or if her veto power never diminishes in the future.

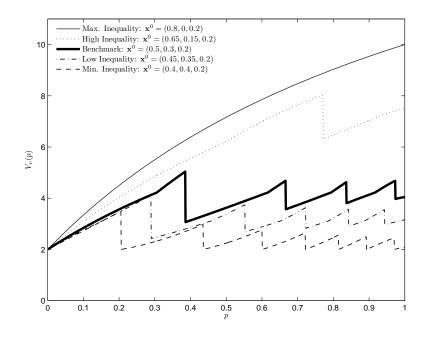
4.2 Inequality matters

Inequality between the non-veto players matters; at times, much more than their collective wealth. The mechanism the veto players uses to appropriate wealth is akin to *dividing and* ruling. When the non-veto players are more equal in terms of their wealth, they have more of an incentive to work together to block the veto player from usurping either of their allocations. The greater equality moves the status quo into a Δ^{E_J} region with a higher J.

Figure 4 shows this effect of increasing equality on the Veto player's payoff:

As we noted earlier, both non-veto players would be better off if they could move to a Δ^{E_J} region from a $\Delta^{E_{J-1}}$ region. If we modified the game to allow non-veto players to transfer wealth to each other (without any strings attached) before the game begins, then the richer non-veto player would prefer to transfer some of her wealth to the poorer non-veto player (so as to mover to a Δ^{E_J} region). This result displays a preference for equality without using any other regarding preferences. However, the veto player would use her power to bock any such movement toward greater equality.

Figure 4: The Effect of Varying Initial Inequality on the Equilibrium Value Function of Veto Player v, for $\delta = 0.90$



4.3 The result converges to Diermeier et al (2013)'s

In Diermeier et al (2013), p = 1, and the result claimed is that if δ is sufficiently high, then for each status quo, the point directly below, or to the left of it on the 45 degree line will be a stable point. In continuous allocations and with three players, as δ and pincrease to values close to 1, we see several absorbing *L*-shaped regions compress to form *L*-lines (almost) the vertices of which form the points of convergence for each region. This closely approximates the result of Diermeier et al (2013).

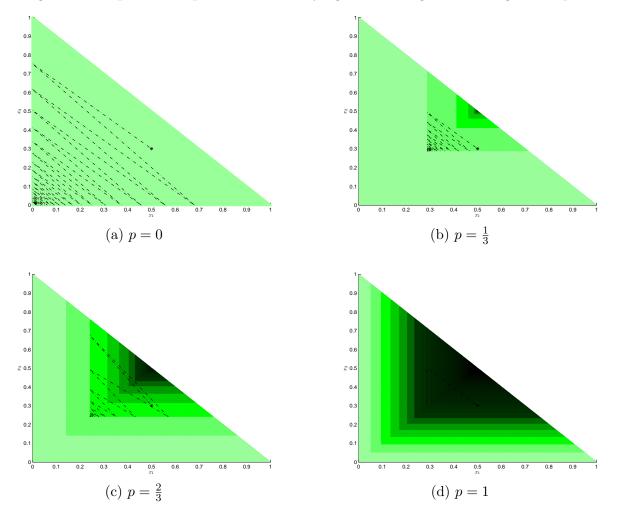


Figure 5: Cooperative Equilibrium for Varying levels of Agenda-Setting Power p

Notes: This panel shows the shape of the simplex and the evolution of the initial status quo for various values of p. For comparison, we keep the discount factor $\delta = 0.95$ and the initial status quo $\mathbf{x}^0 = (0.5, 0.3, 0.2)$ in all panels. As p increases from 0 to 1, the Veto Player gradually obtains more agenda-setting power, but may become worse off in the long run. In panel (a), both Non-Veto Players are eventually expropriated, since cooperation cannot be sustained for p = 0. In panel (b), the initial status quo lies in the 2nd L-shaped region, defined by its vertex $x_2^*(\delta, p) = 0.288$. Hence, the status quo will converge to (0.288, 0.288, 0.424). In panel (c), the initial status quo lies in the 3rd L-shaped region, defined by its vertex $x_3^*(\delta, p) = 0.242$. Hence, the status quo will converge to (0.242, 0.242, 0.516). Finally, in panel (d), the initial status quo lies in the 9th L-shaped region, defined by its vertex $x_9^*(\delta, p) = 0.285$. Hence, the status quo will converge to (0.285, 0.285, 0.430). These examples illustrate how the long-run share held by each player evolve nonmonotonically with p. This is in sharp contrast with the non-cooperative equilibrium obtained by Nunnari (2016), where the Veto Player will obtain the full surplus in the long run for any primitives (\mathbf{x}^0, δ, p) for which the equilibrium exists.

5 Resolving multiplicity of equilibria

This is an environment where two Markov perfect equilibria coexist. Nunnari (2014) finds one of those equilibria, we find another. In this section, we shall describe a method of choosing between multiple equilibria and reaching uniqueness. First, we describe a modified game with a pre-play cheap talk to select one of the equilibria. Then, we discuss the results and implications of this modified game.

5.1 Modified game

We modify the environment defined in section 2 by adding a period 0 for pre-game communication. Players will be able to see the primitives of the game $(\mathbf{x}^0, p, \delta)$ before deciding on which equilibrium would get played. The timing of the modified game will be:

- Players observe the primitives: $(\mathbf{x}^0, p, \delta)$
- Players discuss playing one of the possible equilibria
- If two players weakly prefer an equilibrium, that equilibrium is enforced. The third player will be forced to best respond

Period 1 onwards:

- At each t, one agent is selected to make a proposal **y**
- All agents vote between \mathbf{x}^{t-1} and \mathbf{y}
- If y gets 2 votes (including the veto), then $\mathbf{x}^t = \mathbf{y}$, otherwise $\mathbf{x}^t = \mathbf{x}^{t-1}$
- \mathbf{x}^t is allocated

5.2 Results of the modified game

The modification of the game described above implies that if any two players are better off in a particular equilibrium, they will be able to coordinate on it in period 0.

In Figure 6, we colour the simplex according to which equilibrium gets selected in period 0. The yellow region corresponds to a subset of primitives where all agents are indifferent between selecting either equilibrium. In the green region, both non-veto players prefer to play our equilibrium rather than the one decribed in Nunnari (2014). Irrespective of the primitives, there is no region in which the Veto player is strictly better off in our equilibrium, nor is there any primitive such that any two players are better off in the equilibrium described in Nunnari (2014).

We can see this result more clearly in Figures 7, 8, and 9. Here, we note that for a large range of primitives, both non-veto players are strictly better off in our equilibrium and would therefore be able to coordinate to enforce it in period 0 of the modified game.

Figure 6: Comparing Equilibria for $p = 0.50, \delta = 0.90$

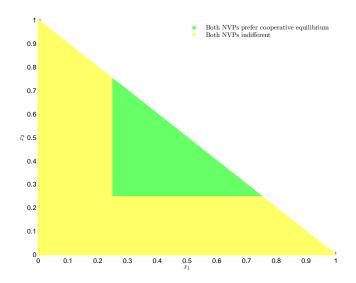
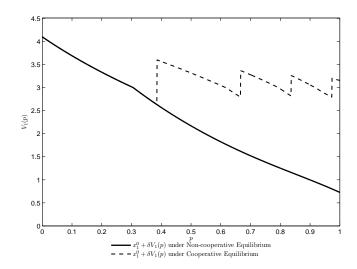


Figure 7: Comparing Equilibrium Values for Non-Veto Player 1: $x_1^0 + \delta V_1(p, \delta, \mathbf{x}^0)$, for $\mathbf{x}^0 = (0.5, 0.3, 0.2), \delta = 0.90$



6 Conclusion

We consider an infinitely repeated legislative bargaining game with endogenous status quos. We find a Markov perfect equilibrium which differs from the one found by Nunnari (2014). We show that in our equilibrium, it may be optimal for the veto player to commit to lowering her proposal power, in order to secure a larger share of the surplus in the long run. If the veto player cannot credibly commit to reducing her proposal power, then her opponents may prefer to protect each other's property rights by forming a blocking coalition, inhibiting the veto player from stealing the full surplus. We then argue that inequality in the initial allocation

Figure 8: Comparing Equilibrium Values for Non-Veto Player 2: $x_2^0 + \delta V_2(p, \delta, \mathbf{x}^0)$, for $\mathbf{x}^0 = (0.5, 0.3, 0.2), \delta = 0.90$

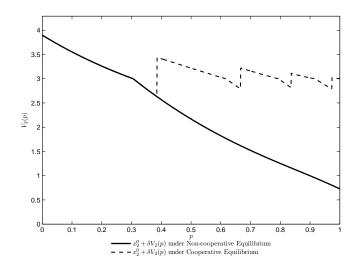
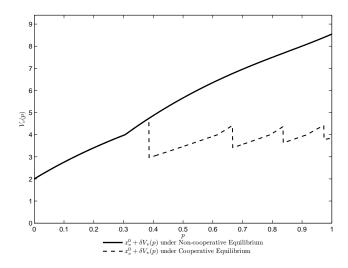


Figure 9: Comparing Equilibrium Values for Veto Player $v: x_v^0 + \delta V_v(p, \delta, \mathbf{x}^0)$, for $\mathbf{x}^0 = (0.5, 0.3, 0.2), \delta = 0.90$



matters much more so than the sum of allocations available to the non-veto players. So much so, that the richer non-veto player would strictly benefit by transferring some of her money to the poorer non-veto player (even at a cost). This result is particularly interesting since we do not have other-regarding preferences here. However, since inequality among the non-veto players allows the veto player to expropriate, she would block any such transfer. We find support for our equilibrium in the findings of Diermeier et al (2013) and show that their results are robust to a large range of proposal power and discount values, as well as continuous

allocations (in stead of only discrete allocations). Finally, we propose a modified game in order to choose between the equilibrium found in Nunnari (2014) and our equilibrium. We find that for large subset of primitives, our equilibrium is strictly preferred by the two non-veto players and would be enforced by them in period 0. For all other primitives, the two equilibria are identical and either one could be enforced.

Appendices

A Proof of Lemma 3

Consider a status quo allocation $\mathbf{x} = (x_1, x_2, 1 - x_1 - x_2) \in \overline{\Delta}$. Define $\overline{x} \equiv \max(x_1, x_2)$, and note that $\min(x_1, x_2) = 0$. Denote the poorest non-veto player by index i (i = 1, 2) and the wealthier non-veto player by j $(j \neq i)$. Consider the following (pure) proposal strategies for the three agents: (1) agent 1 proposes $\mathbf{y}_1 = (1 - d_v, 0, d_v)$, (2) agent 2 proposes $\mathbf{y}_2 =$ $(0, 1 - d_v, d_v)$ and (3) veto agent v proposes $\mathbf{y}_v = (d_i, 0, 1 - d_i)$ if i = 1 and $\mathbf{y}_v = (0, d_i, 1 - d_i)$ if i = 2, respectively. The offered bribes d_k (k = 1, 2, v) will be functions of the primitives $(\overline{x}, \delta, p)$ in equilibrium. For this to constitute an equilibrium with asymptotic full extraction by the veto player, we need to check that the following conditions hold:

(1) The veto player weakly prefers accepting d_v over rejecting it and sticking to **x**.

(2) The veto player strictly prefers to bribe the poorest non-veto player i.

(3) Agent *i* weakly prefers accepting d_i over rejecting it and sticking to **x**.

(4) Agent j, who gets expropriated if agent v is the proposer, would not be willing to accept the bribe d_i .

(5) The bribing scheme is budget feasible and allows asymptotic full extraction by the veto player.

Note that conditions (2) and (4) ensure that the veto player follows a pure strategy. If the veto player would instead follow a mixed strategy where she offers the bribe to either player i or j with positive probability, then both non-veto players should be indifferent between accepting or rejecting the bribe, which requires a higher bribe and would be suboptimal for the veto player. Optimality requires that the *IC* constraint of (poorer) player i binds in equilibrium, while the *IC* constraint of (wealthier) player j should be violated when evaluated at d_i . Condition (5) requires that $d_i + d_v < 1$, so the veto player can steal a strictly positive share of the surplus whenever he proposes, ensuring asymptotic convergence towards $\mathbf{x}^{\infty} = (0, 0, 1)$, the unique stable outcome.

First, we show that there exist demand (or bribe) functions (d_1, d_2, d_v) that constitute a symmetric equilibrium. Denote by $V_i(\mathbf{x})$ the expected continuation value of player $i \in \{1, 2, v\}$ when the status quo is \mathbf{x} . Then, the optimal offered bribes must make the receiver indifferent between accepting and rejecting. This implies that the following incentive constraints must hold for all $\mathbf{x} \in \overline{\Delta}$:

where $V_v(1 - d_v, 0, d_v) = V_v(0, 1 - d_v, d_v)$, since the veto player does not care about the identity of the non-veto players when she is offered a bribe d_v . By conditions (1) and (3), the first three incentive constraints $(IC_1) - (IC_3)$ must bind in equilibrium. The strict inequalities (IC_4) and (IC_5) correspond to condition (4). Condition (2) is incorporated in the continuation values stated below, and condition (5) will be checked later. Given the recognition probabilities of each player and the proposed strategies of all agents, we can write out the continuation values for the non-veto players:

$$\begin{split} V_1(0,\overline{x},1-\overline{x}) &= \frac{1-p}{2} \left(1-d_v + \delta V_1(1-d_v,0,d_v)\right) + \frac{1-p}{2} \left(0 + \delta V_1(0,1-d_v,d_v)\right) + \\ &\quad p \left(d_1 + \delta V_1(d_1,0,1-d_1)\right) \\ V_1(\overline{x},0,1-\overline{x}) &= \frac{1-p}{2} \left(1-d_v + \delta V_1(1-d_v,0,d_v)\right) + \frac{1-p}{2} \left(0 + \delta V_1(0,1-d_v,d_v)\right) + \\ &\quad p \left(0 + \delta V_1(0,d_2,1-d_2)\right) \\ V_2(0,\overline{x},1-\overline{x}) &= V_1(\overline{x},0,1-\overline{x}) \\ V_2(\overline{x},0,1-\overline{x}) &= V_1(0,\overline{x},1-\overline{x}) \end{split}$$

where d_1 , d_2 and d_v are endogenous functions of (\bar{x}, δ, p) yet to be determined. Similarly, the continuation values for the veto player are given by

$$V_{v}(\overline{x}, 0, 1 - \overline{x}) = \frac{1-p}{2} (d_{v} + \delta V_{v}(1 - d_{v}, 0, d_{v})) + \frac{1-p}{2} (d_{v} + \delta V_{v}(0, 1 - d_{v}, d_{v})) + p (1 - d_{2} + \delta V_{v}(0, d_{2}, 1 - d_{2}))$$

$$V_{v}(0, \overline{x}, 1 - \overline{x}) = V_{v}(\overline{x}, 0, 1 - \overline{x})$$

Since all agents have linear utilities and the two non-veto players have the same agenda setting power, we solve this system by guessing a solution which is symmetric and linear in the share of the wealthier non-veto player, \overline{x} :

$$d_1(\overline{x}, \delta, p) = d_2(\overline{x}, \delta, p) = A\overline{x} + B; \ d_v(\overline{x}, \delta, p) = C\overline{x} + D;$$

$$V_1(0, \overline{x}, 1 - \overline{x}) = V_2(\overline{x}, 0, 1 - \overline{x}) = E\overline{x} + F; \ V_1(\overline{x}, 0, 1 - \overline{x}) = V_2(0, \overline{x}, 1 - \overline{x}) = G\overline{x} + H;$$

$$V_v(\overline{x}, 0, 1 - \overline{x}) = V_v(0, \overline{x}, 1 - \overline{x}) = I\overline{x} + J.$$

It is easy to see that if $\overline{x} = \max(x_1, x_2) = 0$, then the veto player will maintain that status quo and keep the full surplus forever. This implies that B = F = H = 0, D = 1 and $J = \frac{1}{1-\delta}$. Moreover, we know that a proposing non-veto player can never offer the veto player something strictly less than her current share, since that would be unilaterally blocked. However, the (wealthier) non-veto player will also never offer the veto player something strictly larger than her current share, because then it would be profitable to deviate and propose $\mathbf{x}' = \mathbf{x}$ instead, which will give the veto player her current share irrespective of her vote. If poorer non-veto player can propose, he will also keep the veto player indifferent by offering her her current share $(1-\overline{x})$, and take \overline{x} for himself. Hence, $d_v(\overline{x})$ is equal to the veto player's current share $(1-\overline{x})$, using the continuation value functions and the condition that the value functions must sum up to $\frac{1}{1-\delta}$. The optimal demands are given by

$$\begin{aligned} d_{i=1,2}(\overline{x},\delta,p) &= \frac{\delta(1-p)}{2-\delta(1+p)}\overline{x} \\ d_v(\overline{x},\delta,p) &= 1-\overline{x} \end{aligned}$$

Plugging these demand (or bribe) functions back into the value functions allows us to verify that the veto player's value is increasing in her status quo payoff, and that the non-veto players' payoffs are both increasing in their combined share \overline{x} :

$$V_1(\overline{x}, 0, 1 - \overline{x}) = V_2(0, \overline{x}, 1 - \overline{x}) = \frac{(1-p)\left(2\delta^2 p - (1+3p)\delta + 2\right)}{2(1-\delta)(1-\delta p)(2-\delta(1-p))}\overline{x}$$

$$V_1(0, \overline{x}, 1 - \overline{x}) = V_2(\overline{x}, 0, 1 - \overline{x}) = \frac{(1-p)(2-\delta(1-p))}{2(1-\delta)(1-\delta p)(2-\delta(1-p))}\overline{x}$$

$$V_v(\overline{x}, 0, 1 - \overline{x}) = V_v(0, \overline{x}, 1 - \overline{x}) = \frac{1}{1-\delta} - \frac{(2-\delta)(1-p)}{(1-\delta)(2-\delta(1-p))}\overline{x}$$

It is easily checked that the equilibrium conditions (1) - (5) are satisfied. The veto player can steal a positive amount whenever she can propose, since $d_i(\bar{x}, \delta, p) < \bar{x}$ for all p > 0 and $\delta < 1$. In other words, the veto player's bribing scheme not only keeps the (poorest) non-veto player in her minimal winning coalition indifferent between accepting or rejecting, but it also satisfies $d_{i=1,2} + d_v < 1$, leaving some positive rent to be extracted each time the veto player can propose. Since p > 0, this rent extraction happens infinitely often as $t \to \infty$, such that the status quo converges to $\mathbf{x}^{\infty} = (0, 0, 1)$. Moreover, if $\delta = 0$ or p = 1, the convergence happens in finite time, and \mathbf{x}^{∞} is reached after two periods. This completes the proof of Lemma 3. \Box

B Proof of Proposition 1

Consider an initial status quo $\mathbf{x}^0 = (x_1, x_2, 1 - x_1 - x_2) \in \overline{\Delta}$. Define $\overline{x} \equiv max(x_1, x_2)$ and $\underline{x} \equiv min(x_1, x_2)$. Denote the poorest non-veto player by index i (i = 1, 2) and the wealthier non-veto player by j $(j \neq i)$. Since the non-veto players have equal proposer power, the demands and value functions will be symmetric for the non-veto players, and everything can be written as a function of the primitives $(\overline{x}, \underline{x}, \delta, p)$. To save on notation, we will suppress the last two parameters (δ, p) , and write all demands and value functions as a function of (x_1, x_2) . Let $V_1(\overline{x}, \underline{x})$ denote the value for agent 1 if he is the wealthier player (i.e. $x_1 = \overline{x}$), and $V_1(\underline{x}, \overline{x})$ the value if he is the poorer player (i.e. $x_1 = \underline{x}$). The value functions are symmetric for player 2, such that $V_2(\overline{x}, \underline{x}) = V_1(\underline{x}, \overline{x})$ and $V_2(\underline{x}, \overline{x}) = V_1(\overline{x}, \underline{x})$ as his demand when he is the poorer player, and $d_1(\overline{x}, \underline{x})$ as his demand when he is the wealthier player. For player 2, symmetry implies that $d_2(\overline{x}, \underline{x}) = d_1(\underline{x}, \overline{x})$ and $d_2(\underline{x}, \overline{x}) = d_2(\overline{x}, \underline{x})$. Finally, the veto player demands $d_v(\overline{x}, \underline{x}) = d_v(\underline{x}, \overline{x})$, since she does not care about the identities of the two non-veto players. Finally, define $\mu \in [0, 1]$ as the probability with which the veto player offers a bribe to the poorest non-veto player. In equilibrium, μ will also be a function of $(\overline{x}, \underline{x})$.

We also add the following boundary on the values of p and δ :

$$2 - 3\delta(1+p) + \delta^2(1+p+2p^2) < 0$$

We note that this condition is sufficient, while not being necessary to ensure that the bribe scheme described is budget feasible. As will become clearer in the next section, the above boundary condition provides the range of proposal power (p) and patience (δ) for which cooperation among non-veto players cannot be unconditionally sustained.

Below, we restate Nunnari's (2016) result (in Proposition 5) which states that, depending on the primitives of the model, the veto player may either adopt a mixed strategy where she offers a bribe to each non-veto player with positive probability, or she will follow a pure strategy and bribe the poorest player with probability 1 (as in Proposition 1). Moreover, if a non-veto player is selected as the proposer, the veto player may demand either a strictly positive amount d_v , or may be willing to accept $d_v = 0$ (negative amounts are not allowed). Thus, the primitives $(\bar{x}, \underline{x}, \delta, p)$ are denoted by \mathbf{x}^0 and can be partitioned into 4 different subsets $\Delta^i \subseteq \Omega$ ($i \in \{A, B, C, D\}$). As in Nunnari (2016), we now consider each separate case. There is a fifth region, Δ^E , however, it is not present when $2 - 3\delta(1+p) + \delta^2(1+p+2p^2) < 0$

B.1 Case A: $\mathbf{x}^0 \in \Delta^A$

In this case, assume that (1) whenever a non-veto player proposes, then the veto player demands a positive amount $d_v \geq 0$ in order to vote Yes, and (2) if the veto player proposes, she adopts a pure strategy of bribing the poorest non-veto player with probability $1.^{12}$ Hence, the incentive compatibility constraints of both the poorest non-veto player as well as the veto player must bind in equilibrium, and we must also verify that this is indeed optimal within some range Δ^A . Guessing demands and value functions that are linear in \overline{x} and \underline{x} , we obtain a system of 10 unknowns and 10 (non-redundant) equations, similar to the proof in Appendix A. We obtain the following optimal demands and value functions:

Note that if $\underline{x} = 0$, our results simplify to the ones in the simple case where $\mathbf{x}^0 \in \overline{\Delta}$ in Proposition 1. This implies that $\overline{\Delta} \in \Delta^A$. Now, in order to find the range of primitives for which these strategies are optimal, we must verify our assumptions that (1) $d_v^A > 0$ and (2) that the veto player always wants to bribe the poorer non-veto player (or $\mu^A = 1$), which implies that the IC constraint of the wealthier non-veto player must be violated when evaluated at the equilibrium bribe level offered to the poorer non-veto player. Combined, these conditions imply the following bounds on the primitives:

$$\mathbf{x}^{0} \in \Delta^{A} \Leftrightarrow \begin{cases} \overline{x} < 1 - \underline{x} \left(\frac{2 - \delta(1-p)}{2 - \delta(1+p)} \right) \\ \overline{x} \geq \underline{x} \left(\frac{2 - \delta(1-p)}{2 - \delta(1+p)} \right) \\ \underline{x} \leq \left(\frac{(2 - \delta(1+p))^{2}}{2(1 - \delta p)(2 - \delta(1-p))} \right) \end{cases}$$

Finally, it is easily verified that the demands satisfy $d_{1,2}^A + d_v^A \leq 1$ irrespective of the primitives. Hence, the bribing scheme is always feasible.

B.2 Case B: $\mathbf{x}^0 \in \Delta^B$

In this case, the veto player still bribes the poorer non-veto player with probability 1. The difference with the previous case is that the feasibility constraint that $d_v \ge 0$ will now be

 $^{^{12}}$ It is easily shown that it can never be optimal for the veto player to bribe the wealthiest non-veto player with probability 1, since the other non-veto player would be willing to compete.

binding in equilibrium, since the veto player would be willing to accept negative amounts in order to reach an allocation on the frontier, since this will allow her to steadily appropriate all the surplus in the future. Intuitively, this will be the case if the veto player has a low initial endowment, and is willing to sacrifice her full share in order to bribe a non-veto player and reach a new allocation in the absorbing set $\overline{\Delta}$. Therefore, (IC_3) will be slack in equilibrium. We obtain the following solution:

$$\begin{array}{rcl} d_1^B(\underline{x},\overline{x}) &=& d_2^B(\overline{x},\underline{x}) &=& \frac{2(1-\delta)(2-\delta(1-p))}{(2-\delta(1+p))^2}\underline{x} + \frac{\delta(1-p)}{2-\delta(1+p)} \\ d_v^B(\underline{x},\overline{x}) &=& d_v^B(\overline{x},\underline{x}) &=& 0 \\ V_1^B(\overline{x},\underline{x}) &=& V_2^B(\underline{x},\overline{x}) &=& \frac{\delta p(1-p)}{(1-\delta p)(2-\delta(1+p))}\underline{x} + \frac{(1-p)(2\delta^2 p - (1+3p)\delta + 2)}{2(1-\delta)(1-\delta p)(2-\delta(1-p))} \\ V_1^B(\underline{x},\overline{x}) &=& V_2^B(\overline{x},\underline{x}) &=& \frac{p}{1-\delta p}\underline{x} + \frac{(1-p)(2-\delta(1+p))}{2(1-\delta)(1-\delta p)(2-\delta(1-p))} \\ V_v^B(\overline{x},\underline{x}) &=& V_v^B(\underline{x},\overline{x}) &=& \frac{2p}{(1-\delta)(2-\delta(1-p))} - \frac{2p}{2-\delta(1+p)}\underline{x} \end{array}$$

In equilibrium, optimality and feasibility for this case require (1) that the *IC* constraint for the veto player is slack at $d_b^B = 0$, (2) that the *IC* constraint of the wealthier non-veto player is violated when evaluated at the demand offered to the poorer non-veto player (or $\mu^B = 1$), and (3) that $d_{1,2}^B + d_v^B \leq 1$. Combined, these three constraints imply the following bounds on the initial status quo:

$$\mathbf{x}^{0} \in \Delta^{B} \Leftrightarrow \begin{cases} \overline{x} \geq 1 - \underline{x} \left(\frac{2 - \delta(1-p)}{2 - \delta(1+p)} \right) \\ \overline{x} \geq \left(\frac{\delta^{2} p(1-p)}{(1-\delta p)(2-\delta(1-p))} \right) + \underline{x} \left(\frac{2 - \delta(1+p) - \delta^{2} p(1-p)}{(1-\delta p)(2-\delta(1+p))} \right) \\ \underline{x} \leq \left(\frac{2 - \delta(1+p)}{2 - \delta(1-p)} \right) \\ \underline{x} \leq \left(\frac{(2 - \delta(1+p))^{2}}{2(1-\delta p)(2-\delta(1-p))} \right) \end{cases}$$

B.3 Case C: $\mathbf{x}^0 \in \Delta^C$

In the remaining two cases, the veto player will not adopt a pure strategy to bribe the poorest non-veto player. Instead, she will mix and offer a bribe to either player 1 or player 2 with some probability. However, in this case the veto player is still willing to accept an amount $d_v^C = 0$, as in Case **B**. Let $\mu^C \in (0, 1)$ be the probability that the veto player offers the bribe to the poorest non-veto player. If μ^C is interior, the veto player must be indifferent between bribing either non-veto player, which implies she must offer them the same amount irrespective of their current shares (x_1, x_2) . This implies that at the optimal bribe level $d_1^C = d_2^C$, both non-veto players must have the same value of either accepting the bribe and moving to region $\overline{\Delta} \in \Delta^A$ (and getting the same continuation value), or rejecting the bribe and sticking to the status quo:

$$d_1^C + \delta V_1^A(d_1^C, 0) = x_1 + \delta V_1^C(\mathbf{x}^0, \mu^C) = x_2 + \delta V_2^C(\mathbf{x}^0, \mu^C)$$

If this were not the case, then the veto player would strictly prefer to bribe the agent with the lowest status quo share, since he is willing to accept a lower amount than the other player. However, $\mu^{C} = 1$ would then no longer be interior. Competition between the non-veto players would then induce the veto player to start mixing again. The continuation values for the non-veto players can be summarized by a vector $(\bar{x}, \underline{x}, \mu^C)$:

$$\begin{split} V_1^C(\overline{x},\underline{x},\mu^C) &= V_2^C(\underline{x},\overline{x},\mu^C) \\ V_1^C(\overline{x},\underline{x},\mu^C) &= V_2^C(\overline{x},\overline{x},\mu^C) \\ V_1^C(\underline{x},\overline{x},\mu^C) &= V_2^C(\overline{x},\underline{x},\mu^C) \\ &= V_2^C(\overline{x},\underline{x},\mu^C) \\ &= V_2^C(\overline{x},\underline{x},\mu^C) \\ &= \frac{1-p}{2} \left(b - d_v + \delta V_1^A(1-d_v,0,d_v) \right) + \frac{1-p}{2} \left(0 + \delta V_1^A(0,1-d_v,0) \right) + \frac{1-p}{2} \left(0 + \delta V_1^A(0,1-d_v,$$

We now solve this system of equations for the optimal strategies (d_1^C, μ^C) . By rewriting the optimality conditions from before, we can express the optimal mixing probability μ^C as a function of the primitives and the optimal demand d_1 :

$$\mu^{C} = \frac{1}{2} \left(1 + \frac{\overline{x} - \underline{x}}{d_{1}^{C}} \left(\frac{1 - \delta p}{\delta p} \right) \left(\frac{2 - \delta(1 - p)}{2 - \delta(1 + p)} \right) \right)$$

It is easy to see that this is always weakly greater than $\frac{1}{2}$, since $\overline{x} \geq \underline{x}$. In other words, the veto player always mixes in favor of the poorest non-veto player to equalize the status quo values of both non-veto agents. By plugging this (non-linear) expression for μ back into the value functions $V_i^C(\overline{x}, \underline{x}, \mu^C)$ and $V_i^C(\underline{x}, \overline{x}, \mu^C)$ (for i = 1, 2), it can be checked that, conditional on having a linear demand function $d_1^C (= d_2^C)$, the value functions of the non-veto players will also be linear in \overline{x} and \underline{x} . After substituting out μ^C , we can solve for the optimal demands and value functions by using a similar linear "guess and verify" method. The optimal demands are given by:

$$\begin{array}{lcl} d^C_{i=1,2}(\overline{x},\underline{x}) & = & \frac{(1-\delta)(1-\delta p)(2-\delta(1-p))}{(2-\delta(1+p))(2-\delta-2\delta p+\delta^2 p^2)}(\overline{x}+\underline{x}) + \frac{\delta(1-p)(1-\delta p)}{2-\delta-2\delta p+\delta^2 p^2} \\ d^C_v(\overline{x},\underline{x}) & = & 0 \end{array}$$

The associated value functions are given by:

The mixing probability μ is nonlinear in $(\overline{x}, \underline{x})$ and can be found by plugging in the optimal bribe $d_1^C(\overline{x}, \underline{x})$ in the condition for μ^C we derived before. In order for the equilibrium to be optimal and feasible, we need (1) that (IC_3) is slack when evaluated at $d_v^C = 0$, (2) that $\mu^C < 1$ and (3) that the optimal demands satisfy $d_{1,2}^C + d_v^C \leq b$. These three conditions imply the following bounds:

$$\mathbf{x}^{0} \in \Delta^{C} \Leftrightarrow \begin{cases} \overline{x} \geq \frac{2-\delta}{2-\delta(1-p)} - \underline{x} \\ \overline{x} < \left(\frac{\delta^{2}p(1-p)}{(1-\delta p)(2-\delta(1-p))}\right) + \underline{x} \left(\frac{2-\delta(1+p)-\delta^{2}p(1-p)}{(1-\delta p)(2-\delta(1+p))}\right) \\ \overline{x} \leq \left(\frac{2-\delta(1+p)}{2-\delta(1-p)}\right) \left(\frac{2-\delta p}{1-\delta p}\right) - \underline{x} \\ \underline{x} \leq \left(\frac{(2-\delta(1+p))^{2}}{2(1-\delta p)(2-\delta(1-p))}\right) \end{cases}$$

As in Case B, the third (feasibility) condition rules out some primitives. For example, in the extreme case where $\underline{x} = \overline{x} = \frac{1}{2}$, feasibility requires that $\delta > \overline{\delta} = \frac{1+3p-\sqrt{1+6p-7p^2}}{4p^2}$. This corresponds to the condition derived by Nunnari (2014) in his Appendix of Proposition 5. We note that the condition is satisfied for all p and δ when $2 - 3\delta(1+p) + \delta^2(1+p+2p^2) < 0$. Note finally that the first and third conditions are parallel lines with slope -1. It can be shown that, for all feasible values of (δ, p) , there is always a non-empty region of allocations which satisfies both constraints. In other words, region C always exists irrespective of the primitives.

B.4 Case D: $\mathbf{x}^0 \in \Delta^D$

This case is analogous to the previous one, except that the veto player now demands a positive amount $d_v^D \ge 0$, so his *IC* constraint must bind. The continuation values for all players are analogous to before. Then, we can solve for symmetric linear equilibrium by imposing the (IC_3) constraint and the condition that $d_v = 0$ when $(\overline{x}, \underline{x})$ is exactly at the boundary for case *C* (where $\overline{x} = \frac{2-\delta}{2-\delta(1-p)} - \underline{x}$). Beyond this bound, then by construction of case *D*, the veto player demands $d_v^C \ge 0$. Moreover, since the veto player is again mixing, the optimal bribe offered to each non-veto player must again satisfy $d_1^C = d_2^C$, since both non-veto players must have the same value of either accepting the bribe and moving to region $\overline{\Delta} \in \Delta^A$ (and getting the same continuation value), or rejecting the bribe and sticking to the status quo:

$$d_1^D + \delta V_1^A(d_1^D, 0) = x_1 + \delta V_1^D(\mathbf{x}^0, \mu^D) = x_2 + \delta V_2^D(\mathbf{x}^0, \mu^D)$$

Solving the system of equations yields the following equilibrium demands for Case D:

$$\begin{array}{lcl} d^D_{i=1,2}(\overline{x},\underline{x},\delta,p) & = & \frac{(1-\delta p)(2-\delta(1-p))}{(2-\delta)(2-\delta(1+p))}(\overline{x}+\underline{x}) \\ d^D_3(\overline{x},\underline{x},\delta,p) & = & 1 - \frac{2-\delta(1-p)}{2-\delta}(\overline{x}+\underline{x}) \end{array}$$

The corresponding value functions are given by

$$V_1^D(\overline{x},\underline{x}) = V_2^D(\underline{x},\overline{x}) = \frac{2-\delta(1+p)}{2\delta(2-\delta)(1-\delta)}(\overline{x}+\underline{x}) - \frac{\overline{x}}{\delta}$$
$$V_1^D(\underline{x},\overline{x}) = V_2^D(\overline{x},\underline{x}) = \frac{2-\delta(1+p)}{2\delta(2-\delta)(1-\delta)}(\overline{x}+\underline{x}) - \frac{\overline{x}}{\delta}$$
$$V_v^D(\underline{x},\overline{x}) = V_v^D(\overline{x},\underline{x}) = \frac{1}{1-\delta} - \frac{2-\delta-p}{(2-\delta)(1-\delta)}(\overline{x}+\underline{x})$$

Feasibility and optimality for this case require that (1) $d_v^D > 0$ and (2) $\mu^D < 1$, which imply the following boundary conditions for case D:

$$\mathbf{x}^{0} \in \Delta^{D} \Leftrightarrow \begin{cases} \overline{x} < \frac{2-\delta}{2-\delta(1-p)} - \underline{x} \\ \overline{x} < \underline{x} \left(\frac{2-\delta(1-p)}{2-\delta(1+p)}\right) \\ \underline{x} \leq \left(\frac{(2-\delta(1+p))^{2}}{2(1-\delta p)(2-\delta(1-p))}\right) \end{cases}$$

Finally, it is easy to check that $d_1^D + d_v^D \leq 1$ for all primitives within the range Δ^D .

As long as $2 - 3\delta(1+p) + \delta^2(1+p+2p^2) < 0$, there will only be four regions; namely: Δ^A , Δ^B , Δ^C , and Δ^D . When $2 - 3\delta(1+p) + \delta^2(1+p+2p^2) \ge 0$, we will have a fifth region in which there will be partial (not full) surplus extraction. That region, Δ^E follows:

$$\mathbf{x}^{0} \in \Delta^{E} \Leftrightarrow \underline{x} > \left(\frac{(2-\delta(1+p))^{2}}{2(1-\delta p)(2-\delta(1-p))}\right)$$

This is the boundary condition stated in Proposition 2. This completes the proof. \Box

C Proof of Proposition 2

In order to prove that the non-veto players will never have an incentive to move outside of region $E(\Delta^E)$, we show that both non-veto players are better off being within Δ^E than at any point outside it. In our equilibrium, they trust each other to realise this and therefore block any move by the veto player to move outside Δ^E .

It is easily shown that in the L-shaped region of $\Delta \setminus \Delta^E$ the point that offers the first non-veto player the highest lifetime continuation value is (1, 0, 0).

We therefore compare the valuation for the first non-veto player at the *worse than worst* case scenario in region E - i.e. to stay at $(x^*, x^*, 1 - x^*)$ forever

 to

The valuation of the first non-veto player at (1, 0, 0) which moves the new status quo to region A

If the first non-veto player finds the bribe acceptable, cooperation among the two non-veto players will break down and the veto player will be able to asymptotically expropriate both non-veto players. Note the following values:

$$x^* = \left(\frac{(2 - \delta(1 + p))^2}{2(2 - \delta(1 - p))(1 - \delta p)}\right)$$
(1)

The value to the first non-veto player from staying at $(x^*, x^*, 1 - x^*)$ forever is given by:

$$Value = \frac{1}{1 - \delta} \left(\frac{(2 - \delta(1 + p))^2}{2(2 - \delta(1 - p))(1 - \delta p)} \right)$$
(2)

The valuation for the first non-veto player from accepting the bribe to move to (1, 0, 0)and converging to the origin thereafter is given by: $1 + \delta V_1^A(1, 0)$

$$Value = 1 + \delta \left[\frac{(1-p)(2\delta^2 p - (1+3p)\delta + 2)}{2(1-\delta)(1-\delta p)(2-\delta(1-p))} \right]$$
(3)

We then compare this with the value to the first non-veto player from staying at $(x^*, x^*, 1-x^*)$ forever, and find that the first non-veto player is at least as well off staying at $(x^*, x^*, 1-x^*)$ forever. We find an analogous result for the second non-veto player.

Since both non-veto players would rather stay at the *worse than worst* point in Δ^E than accept the best bribe that the veto player could possibly offer either one of them, we can be assured that they would trust each other to remain within Δ^E and collectively block any attempt by the veto player to move to $\Delta \setminus \Delta^E$.

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