Bargaining with Uncertain Commitment: On the Limits of Disagreement *

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Abstract

This paper examines commitment in a two stage bargaining setting using global games arguments. The object is to study the possibility of disagreement. Earlier work such as Crawford(1982) assumed that the cost of revoking a commitment attempt was private information and showed that not only can disagreement occur but when there is little probability of a high revoking cost, any equilibrium must entail the possibility of disagreement. Here I examine the symmetric information case where the revoking costs become publicly known following incompatible demands. This is the natural environment when revoking costs are in the form of audience costs in international negotiations or labor disputes. When the revoking cost is drawn from a binary distribution that is either zero or larger than the size of the pie, disagreement is an equilibrium outcome, even if both players face the same uncertain cost. However, with continuous distributions and global game perturbations, disagreement is possible only if the independent distributions of revoking costs fail to stochastically dominate the uniform distribution. Both players facing the same uncertain cost never leads to disagreement. The sharp contrast with the symmetric information disagreement results of Ellingsen and Miettinen(2008) is shown to stem from deriving the success probability of a commitment attempt from equilibrium behavior instead of assuming it to be exogenous.

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1 Introduction

Bargaining impasses entail significant costs. Whether they manifest as strikes, lockouts or war, the bargaining parties end up at a highly inefficient outcome. One explanation for the existence of such disagreement relies on the ability of rational bargaining agents to commit themselves to aggressive demands. An agent who credibly commits herself to an aggressive demand can force an uncommitted opponent to concede. The ability to commit arises from a (revoking)cost which rational agents must pay to back down from their stated demand. Uncertainty regarding the revoking cost results in uncertain commitment ability. Both players may then attempt commitment to aggressive demands hoping that they themselves face a high revoking cost while their opponent faces a low (or no) cost. Simultaneous attempts to commit to aggressive demands yield disagreement. This leads to the question that this paper formally addresses: When does the ability to attempt commitment to aggressive demands lead to disagreement in bargaining between two rational agents, given that the success of the commitment attempt is ex ante uncertain?

The above question has been answered in the asymmetric information environment by Crawford(1982). This paper extends the basic model of Crawford(1982) to analyze the symmetric information case. In particular, I study a two stage game with two players bargaining over a pie of size 1. In stage 1 the two players announce their demands simultaneously. If these demands are compatible (add up to no more than 1) then each agent gets her own demand and half the remaining surplus, if any. If the demands are incompatible a second stage simultaneous move game is played. Each player can either *stick* to her demand or *accept* the other's offer. If one player sticks to her demand while the other player concedes (Accept), the former gets her first stage demand while the latter only gets what was offered by the former. In addition, the conceding player must pay his revoking cost. If both players concede then both get their opponents offer, pay their respective revoking costs and split in half the excess of the surplus over the sum of their offers. Both players sticking to their incompatible demands results in disagreement with a resulting payoff of 0 to both. When making their demands the two players only know the distribution of the revoking costs. These costs become commonly known only after the demand stage but before the second stage game. This feature gives rise to the uncertain commitment ability of players.

I study this basic model under two sets of informational assumptions. In the first, as in Crawford(1982), I assume that the revoking costs can take values of either 0 or some number greater than 1 (henceforth referred to as binary distributions). If the players face revoking costs which have independent and identical binary distributions then I find that disagreement can always be supported in equilibrium, irrespective of the particular probability of facing the high cost, q. Further, if facing a high cost is less probable, $0 < q < 1/2$, any equilibrium must involve disagreement. Disagreement continues to be supported in equilibrium even if the revoking cost distribution functions are identical and perfectly positively correlated (the two players face an identical but uncertain cost). These results are collected in *Proposition 2*, showing the pervasiveness of disagreement in the presence of binary distributions.

In the second set of informational assumptions, players *do not* believe that intermediate revoking costs are impossible. In particular, the density functions for the revoking costs are assumed to be strictly positive and continuous over an interval between and including 0 and some value greater than 1.[1] In addition it is assumed that before the second stage game each player gets to know the realized values of the revoking costs but with a small amount of noise. The equilibrium predictions of this model are analyzed for the limit case when the amount of noise is made arbitrarily small. *Proposition 3* shows that if the revoking cost distribution functions are identical and perfectly positively correlated, disagreement cannot be supported in equilibrium, *irrespective of the particular distribution function considered.* If the distribution functions are independent and First Order Stochastically Dominate the uniform distribution then two results hold. First, the efficient profile of each party demanding half the surplus can be supported in equilibrium. Second, disagreement cannot be supported in equilibrium.

Symmetric Information: The study of symmetric information environments in this paper is motivated by the observation that in bargaining settings where such commitment tactics are available the revoking costs often end up becoming (almost) commonly known before concession decisions are made. For example, in international or domestic political disputes revoking costs take the form of "audience costs" as discussed in Fearon(1994). The two leaders make public announcements of their demand while the domestic audiences assess the performance of the leadership. Backing down may entail a revoking cost in the form of a significantly lower chance of reelection. The particular cost is determined by the relevance of a particular negotiation to the domestic audience's assessment. While uncertain when the demands are made, these costs can be easily ascertained by all parties soon after.

A recent movement in India, for example, involved Anna Hazare and the Indian gov-

[1]Such density functions made the problem intractable in the asymmetric information setting of Crawford(1982).

ernment making incompatible demands regarding the contents of an anti corruption bill to be passed in parliament. Given the unconstitutional nature of the Hazare demand on the one hand and the ineffective past anti corruption role of the government on the other, it was by no means certain which way public opinion would swing. The Hazare movement ended up with an unprecedented level of public support. Hazare's high realized revoking cost consisted of losing credibility in front of such a large group of supporters. The Indian Government garnered less sympathy and therefore stood to lose less by backing down. News outlets, opinion polls and visible public rallies made the costs apparent to all soon after the demands had been made public. Eventually the Indian Government backed down.

Similar examples can be found in the the debt ceiling debates of the Obama and Clinton administrations. In such instances lobbying groups are an important source of revoking costs for elected leaders. Importantly, in all these cases, the uncertainty regarding revoking costs when demands are made gets resolved almost entirely before the concession decisions are made. Many more illuminating examples of such bargaining instances are discussed in detail in Schelling(1960), Martin(1993) and Fearon(1994).

Ellingsen and Miettinen(2008)(henceforth EM) also analyze symmetric information settings, but with findings that contrast sharply with this paper. EM show that the presence of uncertain commitment *always* results in disagreement, with both parties demanding the entire surplus in equilibrium. In EM bargaining agents have access to independent random commitment devices, using which, following incompatible demands, an agent is forced to either back down or stick (*achieve commitment*) to her demand with exogenously fixed probabilities. The key modeling difference in the present paper is that *achieving commitment* is required to be the result of equilibrium behavior in the second stage game, as in Crawford(1982). An agent must *choose* to play *Stick* in order to *achieve commitment*. This modeling difference leads to very distinct implications. In particular, in EM, the probability of a successful commitment attempt is independent of the demands made. By contrast, in this paper, with continuous densities and noisy signals, equilibrium play results in a systematic dependence of second stage concession behavior on first stage demands. The particular dependence, so established, often eliminates the possibility of disagreement. In such a setting, demanding the entire surplus can never be supported in equilibrium.

Demands and Concession Behavior: The analysis of binary distributions in this paper gives results that are similar to EM. In particular, disagreement is shown to always be supportable in equilibrium. The reason for this lies in the existence of equilibria in these models in which the probability with which a player backs down in the second stage does not depend upon the first stage demands. Notice that a player has no option but to stick to her demand when her revoking cost is greater than 1. So, if one player faces a cost of 0 and the other faces the high cost, the dominance solvable outcome involves the latter playing *Stick* while the former plays *Accept*. The existence of multiple equilibria in the second stage game, when both player face 0 costs, makes supporting disagreement essentially a question of selecting an appropriate equilibrium. Making the particular equilibrium selection independent of first stage demands makes supporting disagreement in equilibrium possible. In other words, such equilibria behave *as if* the probability of a successful commitment attempt were exogenous.

The analysis of continuous distributions with noisy signals, however, limits the possibility of disagreement considerably. To understand the intuition behind these results it will help to spell out the counteracting forces involved in the model. Disagreement arises if both parties make high demands that are incompatible, since there is always a state of the world where neither player can back down following such a demand profile. Player 1's incentive to make a higher demand is driven by the possibility that following incompatible demands she will face a high revoking cost (and therefore *achieve commitment*), while player 2 faces a low cost and is therefore better off conceding. The opposite scenario works as a disincentive for making higher demands. A second disincentive arises from the possibility that both face high costs and are unable to back down resulting in the loss of the entire surplus.

These features are present in both the binary and continuous distribution models. The continuous distribution models along with the global games information structure, by making concession behavior dependent on first stage demands, gives rise to another disincentive to making higher demands. A higher demand systematically makes it more difficult for one's opponent to concede thereby conferring a greater probability of success to the latter's commitment attempt. This in turn reduces the payoff an agent can hope to get by making the higher demand. It is the addition of this disincentive that results in the lack of disagreement in the continuous density models. Importantly, it is not merely the use of continuous densities that yields the agreement results. The presence of noise is critical for generating the global games argument. Section 4.2 gives an example of disagreement with continuous, identical and perfectly correlated density functions in the absence of noise.

The global games structure results in the risk dominant outcome of the second stage being played as a result of iterated elimination of dominated strategies whenever there would otherwise be multiple equilibria. This argument is especially acute for the case where both agents face the same (but uncertain) revoking cost. Given an incompatible demand profile, in equilibrium, if one player makes a (sufficiently) higher demand than the other, then in the second stage either both players stick to their demands (when the cost is high enough) or the player with the higher demand backs down while the one with the lower demand gets her way. So, in equilibrium, conditional on making incompatible demands, each player would want to make the smaller demand. Consequently there is always some player who wishes to deviate from an incompatible demand profile. When the distributions are independent, the players weigh the benefits of making a higher demand against the subsequent shrinking of the risk dominant region (of the state space) where she actually gets her demand. This systematic relationship between the probability of a successful commitment attempt and first stage demands makes the results of this analysis different from those with binary distributions or exogenous commitment probabilities.

The paper proceeds as follows. Section 2 discusses the related literature. Section 3 presents the disagreement results in informational settings involving binary distributions. Section 4 considers the continuous density case where both parties face an identical but uncertain cost. Section 5 deals with the independent continuous density case. Section 6 concludes. Proofs are collected in the appendix.

2 Related Literature

Commitment and Reputation in Bargaining: The basic framework of the present analysis is almost identical to that of a symmetric information version of Crawford(1982). The only difference is the payoffs that result following incompatible demands if both player's choose to back down. In Crawford(1982) the payoff is given by an exogenously set compromise payoff, while in the present model each player gets what the other offered and half the remaining surplus. This assumption is also made in Kambe(1999), Abreu and Gul(2000) and Compte and Jehiel(2002). To show that this difference preserves the arguments leading to disagreement in the asymmetric information model in Crawford(1982), the latter's disagreement results are replicated using the present model in Section 3.1. Given that the analysis gets rid of an additional parameter (the compromise solution), the disagreement result of Crawford(1982) can in fact be seen in a simpler setting.

While specific arguments regarding the role of commitment tactics in bargaining can be traced back to Schelling(1960), Crawford(1982) was the first to analyze this issue in a formal game theoretic setting. A number of papers have extended the asymmetric information model of Crawford(1982) in a way closely related to the notion of reputation. Kambe(1999) replaces the second stage one-shot game with an infinite horizon counterpart where players may either stick to their demand or lower it, giving rise to a war of attrition game. While focussing on binary distributions, the analysis rules out the possibility of delay. Wolitzky(2011) considers the same model as Kambe(1999), but focusses on minmax profiles and payoffs as opposed to sequential equilibria. The goal here is to characterize the highest payoff a player can guarantee herself by announcing a bargaining posture, with the only assumptions being that her opponent is rational and believes that she will be committed to her posture (face the high cost) with some given probability. In Myerson(1991), Abreu and Gul(2000) and Compte and Jehiel(2002), the irrational or obstinate types are given exogenously, and rational players attempt to increase their shares by mimicking these types. This is in contrast with the earlier papers where following the choice of any demand, the player could become obstinate with a given probability(the probability of facing the high revoking cost). Abreu and Gul(2000) show the possibility of delay when with positive probability a player could be an obstinate type. Compte and Jehiel(2002) show that the existence of outside options in this setting may cancel out the effects of these obstinate types.

Relation to the Global Games Literature: A few comments regarding the global games information structure, critical for the results in this paper, are in order. Firstly, while the paper heavily uses the methods developed in Carlsson and Van Damme(1993) (henceforth CvD), it is not possible to directly apply the results of CvD in the present setting. In CvD it is shown that for a certain kind of perturbation to a *fixed* complete information strategic game with multiple strict equilibria, as the perturbation is made arbitrarily small, the unique rationalizable strategy profile corresponds to the risk dominant profile. In the present paper multiplicity of equilibria is a potential problem in the second stage game. However, the second stage game is itself generated endogenously by the choice of demands in the first stage. In such a case it is by no means self evident that for a sufficiently small amount of noise, in equilibrium only the risk dominant profiles will be played in *all* second stage games. Indeed, the latter statement is false for any positive amount of noise. The crucial part comprises in proving that the class of games where the multiplicity is unresolved for a small enough amount of noise, has a sufficiently negligible effect on the choices made in the first stage. The non trivial nature of such an extension of the equilibrium selection argument to endogenously determined games in the global games literature along with a general result in this regard can be found in Chassang(2008). Unfortunately the particular game studied in this paper does not satisfy the required conditions of Chassang(2008) and must therefore be studied separately.

Secondly the equilibrium selection result implicit in this paper is not one involving the perturbation of a perfect information game. The original game in this study is already one of incomplete information. The equilibrium selection argument in this case applies to subgame perfect strategy profiles of the incomplete information game. Consequently the criticism of Weinstein and Yildiz(2007) does not apply in this case. The limit results in this paper involve the amount of private noise becoming arbitrarily small. The common uncertainty (public noise) regarding revoking costs shared by both players in the first stage is held fixed since it is an intrinsic part of the strategic environment studied here and not itself a perturbation of some complete information game. Any concern regarding the generality of the class of perturbations considered here would then have to do with the class of densities considered for private noise. The generality of this class can be assessed by evaluating assumptions *A2* and *A3* in Section 4.1.

3 Binary distributions and pervasive disagreement

This section shows that if the revoking costs are drawn from binary distributions, either 0 or some value greater than the size of the entire pie, then there always exist equilibria which result in a positive probability of disagreement. This is true irrespective of whether the revoking costs become known privately(asymmetric case) or publicly(symmetric case), following incompatible demands.

For the rest of the section the following basic model applies. Each subsection will add a different set of assumptions to this framework. Two players, 1 and 2, play a two stage game. In what follows, a generic player will be denoted as player i where $i \in \{1,2\}$, with j being the other player, $j \in \{1,2\}, j \neq i$. In the first stage player i makes a demand $z_i \in [0,1]$. If the demands are compatible, $z_1 + z_2 \leq 1$, the game ends and the payoffs are given by (y_1, y_2) where $y_i = z_i - d$ with $d = (z_1 + z_2 - 1)/2$. If the demands are incompatible, $z_1 + z_2 > 1$, the payoffs for the players are determined by the outcome of the following game.

	Accept	*Stick*
Accept	$1 - z_2 + d - k_1, 1 - z_1 + d - k_2$	$1 - z_2 - k_1, z_2$
Stick	$z_1, 1 - z_1 - k_2$	$0, 0$

Table 1: Payoffs following incompatible demands

3.1 Asymmetric information case

The informational assumptions of this subsection are identical to that of Crawford(1982). The only modeling difference lies in the payoff specification when both players simultaneously concede following incompatible demands. In Crawford(1982) these payoffs are given exogenously, while it is endogenously determined here. The results below show that the disagreement results of Crawford(1982) are *not* weakened by this change. Moreover, in the absence of additional parameters representing exogenous compromise payoffs, the disagreement results can be seen more transparently.

Add to the game defined above, the assumption that players in the first stage do not know the value of k_i. They only know that they are independent random variables with $Pr(k_i > 1) = q$ and $Pr(k_i = 0) = 1 - q$. Following incompatible demands and before playing the second stage game, players get to know their own but not their opponent's revoking cost, k_i. Given these assumptions the following results hold.

Proposition 1. *(a) For any value of* $q \in (0, 1)$ *there exists an equilibrium with a positive,* q^2*, probability of disagreement.*
(b) If $0 < q < \frac{1}{2}$ *then any equilibrium must entail a positive probability of disagreement.*

Proposition 1(a) may seem like a stronger result than the disagreement result in Crawford(1982). In the latter paper it was shown that disagreement can be supported in equilibrium if q is small. The possibility of disagreement with high values of q was indeterminate. Proposition 1, on the other hand, shows that even if, ex ante, the probability of commitment is arbitrarily high (close to 1), the players may still choose incompatible demands and therefore lose the surplus with near certainty. However, Gori and Villanacci(2011) have shown that disagreement can be supported in the Crawford(1982) model even when q is large.

To understand the rationale behind Proposition 1(a), notice first that following incompatible demands if a player faces the high revoking cost her strictly dominant action (irrespective of the demands made) is to play *Stick*. Suppose player i plays *Accept* when her cost is 0. Then the two second stage choices available to j yield exactly the same payoff if both players made a demand of $z = \frac{q+1}{2}$. Further if player i makes a demand higher than z while still playing *Accept* when her cost is 0, player j must then optimally choose *Stick* when her cost is 0. Following a demand profile (z, z) each player can therefore play *Stick* with a high cost and *Accept* with a low cost in equilibrium. A higher demand by player i can be dissuaded by player j playing *Stick* irrespective of the cost, forcing i to concede when the cost is 0, resulting in a payoff loss. The strategies for the second stage Bayesian game with demands (z, z)) continue to be in equilibrium if one of the players makes a lower but still incompatible demand, giving the latter a lower payoff.

Given such a second stage strategy profile and initial demands of $z = \frac{q+1}{2}$ each, no player has an incentive to deviate. Such a demand profile, being incompatible, leads to disagreement with probability q^2. It may seem surprising that players would not want to deviate to simply making a compatible demand, especially when q is very high. Notice, though, that when q is really high, the share being offered by the other player is also sufficiently low, $\frac{1-q}{2}$. This low offer makes it a strictly better alternative for a player to make the higher incompatible demand and rely on the small probability with which she gets her stated demand.

Proposition 1(b) is driven by the fact that when $q < \frac{1}{2}$, if some player deviates from compatible demands to making a higher demand, the probability with which the entire surplus is lost, q^2, is less than the probability with which the deviating player gets her demand $q(1-q)$. If the deviating player's increase in demand is small enough, she can ensure that there is still enough room for the other player to back down upon facing a 0 cost. Given a compatible demand profile, the deviating player would be the one with the smaller of the two compatible demands.

3.2 Symmetric information case

Asymmetric information has been shown to give rise to inefficiency in numerous bargaining models. In studying the role of commitment it is important to ascertain if the disagreement results are an artifact of asymmetric information. Ellingsen and Miettinen(2008) have shown that, even without asymmetric information, when the probability of a successful commitment attempt is exogenous(and commonly known), disagreement is an immediate outcome. This subsection studies the symmetric information scenario by making the revoking costs publicly known following incompatible demands. However, the probability of a successful commitment attempt is derived endogenously from equilibrium behavior in the second stage game. The results below show that when the revoking costs are drawn from binary distributions, there always exist equilibria that support disagreement. This is true even if the players know for sure that they will face the *same* revoking cost in the second stage but are unsure about its value when making their demands.

In this subsection, in addition to the basic model outlined earlier, it is assumed that while the costs of backing down are uncertain to both players at the demand stage, they become common knowledge following incompatible demand profiles. In particular, in the first stage it is common knowledge that player i faces cost k_i which takes a value greater than 1 with probability q while $Pr(k_i = 0) = 1 - q$.

Two settings are analyzed. In the first, the distribution functions for k_1 and k_2 are assumed to be independent. In the second it is assumed that both players face identical revoking costs, $Pr(k_1 = k_2) = 1$. Following incompatible demands the true values of k_1 and k_2 are made common knowledge before the second stage game is played. The departure from Section 3.1 lies in the elimination of asymmetric information in the second stage game. In this symmetric information setup the following results hold.

Proposition 2. *If the distribution functions for k_1 and k_2 are independent,*
(a) For $0 < q < 1$, the incompatible demand profile $(1, 1)$ can be supported in equilibrium, resulting in disagreement with probability q^2.
(b) For $0 < q < 1/2$, no efficient equilibrium exists.
If the players face the same revoking cost, $Pr(k_1 = k_2) = 1$,
(c) For $0 < q < 1$ the incompatible demand profile $(1, 1)$ can be supported in equilibrium, resulting in disagreement with probability q^2.

The disagreement results in Proposition 2 depend heavily on the multiplicity of Nash Equilibria in the second stage games following incompatible demands. The multiplicity allows for the construction of equilibria in which the probability with which a player backs down in the second stage *does not* depend upon the particular demands made in the first stage. It is this independence of second stage behavior from first stage demands that makes disagreement supportable in equilibrium.

Consider the setting with independent revoking costs. Following incompatible demands, three of the possible four second stage games are dominance solvable. If both players face high costs the unique profile is *(Stick, Stick)*. If player i faces the high cost and j the low cost, the dominance solvable profile involves i playing *Stick* and j playing *Accept*. If both players face 0 costs, however, there exist two strict pure strategy Nash Equilibria. The disagreement result of Proposition 2(a) relies on the appropriate equilibrium selection in these second stage games, following different incompatible demand profiles. In the subgame perfect equilibrium constructed to support the profile $(1, 1)$, the choice of second stage Nash Equilibrium for the case of $k_1 = k_2 = 0$ is entirely *independent of the first stage incompatible demands.* In particular, Player 1 plays *Stick* while Player 2 plays *Accept* following any incompatible profile when they both face a cost of 0. Player 2 cannot, for instance, force Player 1 to concede by making a lower demand since the second stage behavior is independent of the particular incompatible demand profile.

Proposition 2(c) further highlights the acuteness of the second stage multiplicity problem. In this case both players know that they will face identical revoking costs in the second stage. So the incentive to making a higher demand that arises from the possibility that one will find it too costly to back down while one's opponent wont simply does not exist. Disagreement is again supported by making appropriate equilibrium selection in the second stage games, independent of the first stage demands. If player 1 never backs down, irrespective of the revoking cost, then player 2 can do no worse by playing *Accept* when the cost is 0. Further if both players demand the entire pie, making a compatible offer does not help either. The rationale behind the non existence of efficient equilibria when the probability of facing a high revoking cost is low, as established in Proposition 2(b), is very similar to that for Proposition 1(b). Deviating from a compatible profile yields a gain with probability $q(1-q)$ and a loss of the entire surplus with probability q^2. When q is small, deviating to a demand of 1 results in a gain that outweighs the loss.

Interestingly, both players demanding the entire pie cannot be supported in the asymmetric information environment of Section 3.1. The second stage multiplicity in the symmetric information setting, in fact, makes it easier to support disagreement. As argued earlier, disagreement is easy to support if the probability of a successful commitment attempt can be made independent of the first stage demands. In the strategic environments described in Sections 4 and 5, it is precisely this independence of second stage behavior from first stage demands that collapses. Further, the particular dependence that is established overturns the disagreement results of this section.

4 Identical revoking costs with continuous density functions

This section studies the bargaining game in settings where the revoking cost can take values from an interval containing the points 0 and 1. The idea captured in this assumption is that players do not believe that intermediate values of revoking costs are impossible. The probability attached to such values, however, can be arbitrarily small.

Two players, 1 and 2, play a two stage game. In what follows, a generic player will be denoted as player i where $i \in \{1,2\}$, with j being the other player, $j \in \{1,2\}, j \neq i$. In the first stage player i makes a demand $z_i \in [0,1]$. If the demands are compatible, $z_1 + z_2 \leq 1$, the game ends and the payoffs are given by (y_1, y_2) where $y_i = z_i - d$ with

$d = (z_1 + z_2 - 1)/2$. If the demands are incompatible, $z_1 + z_2 > 1$, the payoffs for the players are determined by the outcome of the following game.

	Accept	*Stick*
Accept	$1 - z_2 + d - k, 1 - z_1 + d - k$	$1 - z_2 - k, z_2$
Stick	$z_1, 1 - z_1 - k$	$0, 0$

Table 2: Payoffs following incompatible demands

4.1 Noisy signals and agreement

In the first stage, when choosing their demands, players' prior regarding the cost of backing down k is given by a random variable K which takes values in $[0, \bar{k}]$ where $\bar{k} > 1$. Having announced their demands, each player i gets a noisy signal, k_i^ϵ about k before playing the simultaneous move game. In particular, player i observes a realization of the random variable K_i^ϵ that is defined by

$$K_i^\epsilon = K + \epsilon E_i, \qquad i = 1, 2$$

where E_i is a random variable taking values in $\mathbb{R}$ and $\epsilon > 0$ serves as the scale parameter for the noise. A strategy for player i, comprises of a demand $z_i \in [0, 1]$ and a measurable function $s_i(z_1, z_2)$ for every incompatible demand profile, that gives the probability of playing *Accept* as a function of the the observed cost of backing down k_i^ϵ. So, $s_i(z_1, z_2) : [-\epsilon, \bar{k} + \epsilon] \to [0, 1]$. Γ^ϵ is used to denote this two stage game for a particular value of ϵ.

The following assumptions are made on the parameters of the model.

A1. *K admits a density h that is continuously differentiable on $(0, \bar{k})$, strictly positive, continuous and bounded on $[0, \bar{k}]$.*

A2. *The vector (E_1, E_2) is independent of K and admits a density φ.*

A3. *The support of each E_i is contained in the interval $[-1, 1]$ in $\mathbb{R}$ and φ is continuous on $[-1, 1] \times [-1, 1]$.*

As a result of these assumptions the model acquires the structure of a global game as studied in CvD. I am interested in the perfect equilibrium prediction of Γ^ϵ for small values of ϵ. To this effect the following proposition holds.

Proposition 3. *Given $A1, A2, A3$, and for sufficiently small $\epsilon > 0$, if players use pure strategies for their first stage demands, there is never any disagreement in any perfect equilibrium of the game Γ^{ϵ}.*

The impossibility of disagreement in this setting is in sharp contrast with Proposition 2(c) which showed that disagreement can be supported in equilibrium irrespective of the revoking cost probability function. Notice that the assumptions for Proposition 3 allow for density functions that can arbitrarily approximate the two point random variables considered in Section 3.

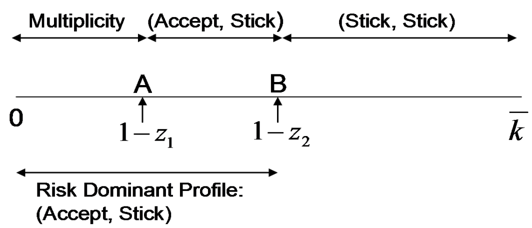

Figure 1: Second stage equilibrium behavior: Common Cost

To get some intuition for Proposition 3 consider Figure 1. Suppose player 1 makes the higher demand in an incompatible demand profile (z_1, z_2). The $0\bar{k}$ line represents the state space for the revoking cost. In the absence of noise ($\epsilon = 0$), the second stage game following the incompatible profile (z_1, z_2) would be one of complete information and would depend on the realized value (k) of the revoking cost, K. Now for all realizations of K in the $B\bar{k}$ region the dominant strategy for both players would be to play *Stick* since backing down would incur a cost strictly greater than the share received by playing *Accept*. The unique NE in the second stage for such values of K would thus be (*Stick*, *Stick*). If K takes a value in AB, then Player 2 has a strictly dominant action in *Stick* since it would be too costly for her to back down. Conditional on Player 2 backing down, the optimal choice for Player 1 is to play *Accept*, since the revoking cost is not higher than the share she would get by conceding. The unique NE for all such k in AB is thus (*Accept*, *Stick*). K taking a value in $0A$, however results in multiplicity. Both (*Accept*, *Stick*) and (*Stick*, *Accept*) are pure NE of the second stage game for such values of K. Since the revoking cost is low enough relative to the amount received by both players upon concession, the problem now becomes one of coordination. In the absence of noise, the choice of Nash Equilibrium in this region can be entirely arbitrary.

It turns out, however, that for all values of K in the region $0B$ the unique risk dominant profile is (*Accept*, *Stick*). In the presence of a small amount of noise the setting becomes a global game. Iterated elimination of strictly dominated strategies in the resulting Bayesian games results in the players coordinating on the risk dominant profile for every realization of K. This in turn implies that while (*Stick*, *Stick*) is played for all realizations in the region $B\bar{k}$, (*Accept*, *Stick*) would be played for all realizations in the $0B$ region.

Player 1 receiving a noisy signal sufficiently in the interior of $B\bar{k}$ would know for sure that the true state of the world is in fact in $B\bar{k}$ and would therefore play her strictly dominant action *Stick* for such observations. Similarly player 2 would play *Stick* following any observation sufficiently in the interior of $A\bar{k}$. Given that player 2 plays *Stick* for observations in $A\bar{k}$, player 1 upon observing a value sufficiently in the interior of AB would infer that player 2 must have observed a value greater than A. It would then be conditionally dominant for Player 1 to play *Accept* for such observations. So there emerges an interval where the profile (*Accept*, *Stick*) is played. The question now is what is the left limit of this interval. In other words, what is the highest observed value of K when one of these players choose to switch their actions from the (*Accept*, *Stick*) profile. For a small enough value of ϵ it turns out that this left limit cannot be greater than 0, resulting in the profile (*Accept*, *Stick*) being played for all values of K when earlier there was multiplicity.

A crucial part of this argument is the existence of a sufficiently large (with respect to ϵ) region AB. So if Player 1 makes a sufficiently larger demand than Player 2, given an incompatible profile, whenever some player does back down it must be Player 1. Since backing down always pays less than simply accepting the other parties offer, Player 1 would be better off making a compatible demand in the first stage. More importantly this shows that conditional on an incompatible demand being made each party would want to make the lower demand and force the other to concede. This applies to the case when the region AB is not that large. In this case one of the players would have a strict incentive to lower her demand marginally and force a concession from the other whenever the cost is low. Such a deviation may not be possible if lowering ones demand essentially leads to a compatible profile. However it is shown that for an incompatible demand profile that makes deviation to compatible positions unprofitable, it must be that both players are making sufficiently high demands. This in turn ensures the possibility of lowering ones demand and still make it incompatible.

The result, therefore, relies on these two features of equilibrium strategies in this game. Firstly for a given incompatible profile, if no player wants to simply deviate to a compatible demand then the original demands must be sufficiently high. Secondly, conditional on making incompatible demands that are sufficiently high, each player has a strict incentive to make a lower demand and force the other player to concede most of the time. These two features make the existence of an incompatible demand profile and consequently disagreement, in equilibrium, an impossibility.

It should be pointed out that the assumptions $A1, A2, A3$, are slightly weaker than the corresponding assumptions made for the one-dimensional case in CvD. In particular the noise density function is allowed to be discontinuous at the boundary points of its support in the present study, while this is ruled out by the assumptions in CvD.[2]

The outline of the proof is as follows. *Lemma 1* establishes a result that is crucial for the global game arguments used for the result. In particular the distribution of player 1's observation conditional on player 2's observation is symmetric to the distribution of player 2's observation conditional on player 1's observation, in the sense that they add up arbitrarily close to 1. *Lemma 2* establishes a continuity result. It shows that for a given profile of measurable strategies,$(s_i)_{i\in\{1,2\}}$, and for any incompatible demand profile, the probability with which player i chooses *Accept* and the expected value of the true revoking cost, k, conditional on player j making an observation,k_j, is continuous in player j's observation. *Lemma 3* shows how following an incompatible demand profile if player i observes a cost sufficiently larger than $1 - z_j$ her dominant action is to play *Stick*. It is then argued in *Lemma 4* that following incompatible demands (z_1, z_2) if z_i is sufficiently larger than z_j, then there will always be observation values for which the unique dominance solvable outcome would involve i backing down while j plays *Stick*. *Lemmas 5 - 7* then show that following such an incompatible demand profile, either for all lower observations i will continue to back down with j playing *Stick*, or there will be two observation values particularly close to each other where the two players will switch their actions. *Lemma 8*, the critical part of the proof, then shows that if z_i is sufficiently larger than z_j, such switch points cannot exist and therefore player i will continue to back down with j playing *Stick*. This result is a consequence of the global games information structure that appears in the model for small enough $\epsilon > 0$. *Lemma 8* relies heavily on the properties of symmetry and continuity established in *Lemmas* 1 and 2. This result allows for a complete characterization of equilibrium second stage strategies and payoffs following incompatible demand profiles and is stated in *Lemma 9*.

[2]Indeed, the motivating example in CvD involves noise with a uniform density, and does not satisfy the assumptions of their paper. However the discontinuity at the boundary points merely requires a little more work as is done in *Lemma 2*, and does not endanger the equilibrium selection argument in CvD. I thank Hans Carlsson for helping me with my doubts regarding this issue.

I then consider the choice of first stage demands. It can be easily seen that demands that add up to less than 1 always allow for deviations. *Lemma 10*, in addition, also shows that incompatible profiles with one player making a sufficiently higher demand than the other cannot be supported. This is a natural implication of *Lemma 8* where the player with the higher demand was shown to always be the one to concede. Making a compatible demand would do strictly better than making such a high incompatible demand. Next, *Lemma 11* establishes a lower bound that the sum of the demands must satisfy to be an incompatible profile from which neither player wants to deviate to a compatible profile. Finally it is shown that if an incompatible profile of demands involves z_1 and z_2 that do not differ much in value (no demand is sufficiently greater than the other as in *Lemma 8*) but sum up to greater than the bound mentioned in *Lemma 11*, then there is always a player i who could strictly improve her payoff by making a lower but still incompatible demand. This lower demand by i forces j, in equilibrium, to always be the one backing down in the second stage. These arguments together exhaust the possible set of incompatible demand profiles. Consequently it is shown that equilibria involving pure strategies in the first stage cannot involve incompatible demands, thereby eliminating the possibility of disagreement.

First I define a few terms for the game Γ^ϵ that allow the use of *Lemma 4.1* in Carlsson and van Damme(1993), henceforth (CvD). Let $F_i^\epsilon(k_j|k_i)$ and $f_i^\epsilon(k_j|k_i)$ be the distribution and density functions, respectively, of K_j^ϵ conditional on $K_i^\epsilon = k_i$. Let φ^ϵ be the joint density of $(\epsilon E_1, \epsilon E_2)$. Then,

$$f_i^\epsilon(k_j|k_i) = \frac{\int h(k)\varphi^\epsilon(k_1 - k, k_2 - k)dk}{\int\int h(k)\varphi^\epsilon(k_1 - k, k_2 - k)dk_j dk} \tag{1}$$

The following lemma is the one dimensional version of *Lemma 4.1* in *CvD* that applies to the present model. This symmetry result is critical for the proof of *Lemma 8*.

Lemma 1 (CvD). *Let $k_1, k_2 \in [-\epsilon, \bar{k} + \epsilon]$. Then there exists a constant $\kappa > 0$ such that for sufficiently small $\epsilon > 0$,*

$$|F_1^\epsilon(k_2|k_1) + F_2^\epsilon(k_1|k_2) - 1| \leq \kappa\epsilon \tag{2}$$

Next, it is shown that for a pair of measurable second stage strategies, player i's expectation regarding the true value of k and the probability with which j plays *Accept*, conditional on observing k_i^ϵ are continuous functions of k_i^ϵ. Given j's second stage strategy s_j, let the probability with which i, conditional on observing k_i^ϵ, expects that j will play

Accept be denoted by $Pr(A_j|k_i^\epsilon, s_j)$.[3] So,

$$Pr(A_j|k_i^\epsilon, s_j) = \int s_j(k_j) f_i^\epsilon(k_j|k_i^\epsilon) dk_j \tag{3}$$

Also, let i's expectation of k given her observation k_i^ϵ be denoted as $E^\epsilon(k|k_i^\epsilon)$.

Lemma 2. *For a given incompatible demand profile (z_1, z_2) and strategies s_i, s_j, $Pr(A_j|k_i^\epsilon, s_j)$ and $E^\epsilon(k|k_i^\epsilon)$ are continuous in player i's observation k_i^ϵ.*

Equilibrium behavior in the second stage game following an incompatible demand profile is considered next. The payoffs specified in Table 2 make it evident that if the observed cost is high enough the player would strictly prefer to play *Stick*. The following lemma captures this immediate but useful implication of observing such high costs of backing down.

Lemma 3. *In equilibrium, following an incompatible demand profile (z_1, z_2), conditional on observing $k_i^\epsilon > 1 - z_j + \epsilon$,* Stick *is the strictly dominant action for player i.*

Lemma 3 shows that for high enough observation values (i.e. greater than $1 - \min\{z_1, z_2\} + \epsilon$) the unique dominance solvable outcome in the second stage game is (*Stick, Stick*).

The next lemma shows that if the higher of the two incompatible demands is sufficiently larger than the lower demand, there will be an interval of observations that would always lead to a unique dominance solvable outcome in the second stage game where the player with the higher demand plays *Accept* while the other plays *Stick*. This is the crucial *dominance solvable region* in CvD that has a remote influence on the rest of the state space.

Lemma 4. *For an incompatible demand profile (z_1, z_2) such that $z_i - z_j > 4\epsilon$, the unique dominance solvable outcome of the second stage game following both players making an observation in $(1 - z_i + 3\epsilon, 1 - z_j - \epsilon)$, involves i playing* Accept *and j playing* Stick.

Given an equilibrium of Γ^ϵ and a pair of incompatible demands (z_1, z_2) where $z_i - z_j > 4\epsilon$, let $k_i^{\epsilon*}$ denote the highest observation value k_i^ϵ below $1 - z_i + 3\epsilon$ for which i chooses to play *Stick*. Similarly let $k_j^{\epsilon*}$ denote the highest observation value k_j^ϵ below $1 - z_i + 3\epsilon$ for which j chooses to play *Accept*. It is assumed that if i following some observation

[3] The dependence of s_j on the demand profile (z_1, z_2) is suppressed for notational convenience, but it should be noted that the arguments are for a given pair of incompatible demands.

strictly greater than $-\epsilon$ is indifferent between her actions she chooses to play *Stick* while when j is indifferent he plays *Accept*. The next lemma shows that $k_i^{\epsilon *}$ and $k_j^{\epsilon *}$ are well defined. In other words, it is shown that unless the players continue to play the strategies they used in the dominance solvable region of *Lemma 4* for even lower values of K, there must exist points (highest value of their respective observations) on the state space at which the players switch the strategies. The continuity result of *Lemma 2* is critical to establishing this result.

Let $B_i^\epsilon(z_1, z_2)$ denote the set of observations $k_i^\epsilon > -\epsilon$ such that $k_i^\epsilon \leq 1 - z_i + 3\epsilon$ and i plays *Stick* for such observations (i.e. $s_i(k_i^\epsilon) = 0$). Similarly let $B_j^\epsilon(z_1, z_2)$ denote the set of observations $k_j^\epsilon > -\epsilon$ such that $k_j^\epsilon \leq 1 - z_i + 3\epsilon$ and j plays *Accept* for such observations (i.e. $s_j(k_j^\epsilon) = 0$).

Lemma 5. *In any equilibrium of Γ^ϵ following a pair of incompatible demands (z_1, z_2) where $z_i - z_j > 4\epsilon$, either $B_i^\epsilon(z_1, z_2)$ is empty or $k_i^{\epsilon *} = \max\{x | x \in B_i^\epsilon(z_1, z_2)\}$ is well defined.*
Similarly, either $B_j^\epsilon(z_1, z_2)$ is empty or $k_j^{\epsilon *} = \max\{x | x \in B_j^\epsilon(z_1, z_2)\}$ is well defined.

The following lemma shows that if one player does not switch her second stage action at smaller values of observed cost from that used in the dominance solvable region of *Lemma 4*, then the other player would not make a switch either.

Lemma 6. *If $B_i^\epsilon(z_1, z_2)$ or $B_j^\epsilon(z_1, z_2)$ is empty then they are both empty.*

The next lemma establishes a relation between $k_i^{\epsilon *}$ and $k_j^{\epsilon *}$ when they are well defined. In particular it is shown that the switching points when they exist would be near each other.

Lemma 7. *In any equilibrium of Γ^ϵ following a pair of incompatible demands (z_1, z_2) where $z_i - z_j > 4\epsilon$ if the terms are well defined then, $k_i^{\epsilon *} < k_j^{\epsilon *} + 2\epsilon$.*

The next lemma contains the crucial argument that drives the result, since it shows that for incompatible demands with the higher demand sufficiently larger than the smaller one, the player with the higher demand always concedes whenever the observed cost is in the range that generated multiplicity in the complete information game. The symmetry of conditional beliefs guaranteed by *Lemma 1* plays a significant role here.

Lemma 8. *In any equilibrium of Γ^ϵ following a pair of incompatible demands (z_1, z_2) where $z_i - z_j \geq \max\{4\epsilon, \frac{(\kappa+2)\epsilon}{d}\}$, the sets $B_i^\epsilon(z_1, z_2)$ and $B_j^\epsilon(z_1, z_2)$ are empty.*

Lemma 8 makes it immediate that following an incompatible demand profile (z_1, z_2), where $z_i - z_j \geq \max\{4\epsilon, \frac{(\kappa+2)\epsilon}{d}\}$, player j plays *Stick* irrespective of the observation k_j^ϵ. On the other hand player i plays *Stick* for $k_i^\epsilon > 1-z_j+\epsilon$ while playing *Accept* for $k_i^\epsilon < 1-z_j-\epsilon$. This allows for a characterization of the expected payoffs in the first stage, from making such incompatible demands. Let $y_i(z_1, z_2)$ and $y_j(z_1, z_2)$ denote i and j's expected payoff in equilibrium from making demands z_i and z_j. The following lemma is delivered simply by calculating payoffs given the characterization of equilibrium behavior in the second stage discussed in *Lemmas 3, 4 and 8*.

Lemma 9. *In any equilibrium of* Γ^ϵ *following a pair of incompatible demands* (z_1, z_2) *where* $z_i - z_j \geq \max\{4\epsilon, \frac{(\kappa+2)\epsilon}{d}\}$, *it must be that*

$$z_j F_i^\epsilon(1 - z_j - \epsilon) \leq y_j \leq z_j F_i^\epsilon(1 - z_j + \epsilon) \tag{4}$$

$$y_i \leq \int_0^{1-z_j} (1 - z_j - w)h(w)dw \tag{5}$$

The analysis can now turn to the choice of first stage demands. Let the set of demand profiles that can be supported by equilibrium strategies in Γ^ϵ be denoted by Eq^ϵ. Further let $\phi(d) = \max\{4\epsilon, \frac{(\kappa+2)\epsilon}{d}\}$. The following lemma shows how equilibrium demands could never add up to less than 1. Also, it states the immediate implication of *Lemma 8* that incompatible demands with one player making a significantly higher demand than the other cannot be supported in equilibrium.

Lemma 10. *If* (z_1, z_2) *satisfies either of the following conditions,*

a. $z_1 + z_2 < 1$

b. $z_1 + z_2 > 1$ *and* $|z_1 - z_2| \geq \phi(d)$

then, $(z_1, z_2) \notin Eq^\epsilon$.

Let $\hat{k} = \int \min\{k, 1\}h(k)dk$. The following lemma shows that for an incompatible demand profile to be supported in equilibrium, the excess demand must be above a positive lower bound. If this were not to be the case then at least one of the players would have a strict incentive to deviate to making a compatible demand.

Lemma 11. *If* $z_1 + z_2 > 1$ *and* $d < \hat{k}/2$ *then* $(z_1, z_2) \notin Eq^\epsilon$.

Recall that $\phi(d) = \max\{4\epsilon, \frac{(\kappa+2)\epsilon}{d}\}$. Let $\phi^* = \phi(\hat{k}/8)$. The next lemma shows that incompatible demands that are close to each other but result in an excess demand that exceeds the bound from *Lemma 11* cannot be supported in equilibrium. Since the demand profile satisfies the lower bound, the result relies on the existence of some player i who can lower her demand enough to force the j to always do the conceding, thereby generating a higher expected payoff for i.

Lemma 12. *If $z_1 + z_2 > 1$, $d \geq \hat{k}/2$ and $|z_1 - z_2| < \phi(d)$ then $(z_1, z_2) \notin Eq^\epsilon$ for small enough ϵ.*

Proof of Proposition 3

Proof. Proposition 3 follows immediately from the observation that *Lemmas 10, 11 and 12* exhaust the entire set of incompatible demand profiles. □

4.2 Example of disagreement in the absence of noise

With revoking costs perfectly correlated and identical across players, Proposition 3 shows that with continuous density functions there is no disagreement while Proposition 2 shows that there can always be disagreement with binary distributions. The critical difference that gives rise to the contrasting results, however, is the presence of noisy signals in the continuous density case.[4] This can be seen by observing that without the noise there may be disagreement even in the continuous density case. An example of such a scenario follows.

Consider the game outlined earlier in this section with the additional assumption that $\epsilon = 0$. In other words, both players, following incompatible demands get to know the precise value of the revoking cost. Let the distribution function for the revoking cost be given by F with the interval $[0, \bar{k}]$ as its support where $\bar{k} > 1$. It is assumed that $F(1/4) = 9/10$ and $F(q) = 2/5$ where $q \in (0, 1/4)$. As outlined earlier, the players choose their demands in the first stage, with common knowledge regarding the distribution of the revoking cost, k. Following the demand stage, both players get to know the realized value of k and then decide simultaneously whether to stick to their demand or back down.

Consider the following subgame perfect strategy profile that leads to a positive probability of disagreement. The players demand identical amounts, namely $z_1 = z_2 = 3/4$. In the second stage, if $k \geq 1/4$, both players play *Stick*. If $k \in (q, 1/4)$ then player 1 plays *Accept* while player 2 plays *Stick*. If $k \leq q$ then player 1 plays *Stick* while player 2 plays *Accept*. In the subgame following player i making a demand, $\tilde{z}_i > z_i$, player $-i$ plays *Stick*

[4] Global game arguments require the state space to be a continuum and therefore has no analog in the discrete case.

irrespective of the realized value of k, while player i plays *Accept* if $k \leq 1/4$ and *Stick* otherwise. In the subgame following player i making a demand, $\tilde{z}_i < z_i$ but $\tilde{z}_i > 1 - z_i$, the following profile is played. If $k \geq 1 - \tilde{z}_i$ both players play *Stick*. If $k \in 1/4, (1 - \tilde{z}_i)$ then player i plays *Stick* and player $-i$ plays *Accept*. Finally, if $k \leq 1/4$ then player i plays *Accept* while player $-i$ plays *Stick*.

The expected payoff to player 1, y_1, from the above strategy profile is given by,

$$y_1 = \frac{3}{4} \cdot \frac{2}{5} + \frac{1}{4} \cdot \frac{1}{2} - \int_q^{1/4} k f(k)\, dk$$

The expected payoff to player 2 is given by,

$$y_2 = \frac{3}{4} \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{2}{5} - \int_0^{q} k f(k)\, dk$$

Clearly, $y_1 > \frac{3}{10}$ and $y_2 > \frac{3}{8}$. It can be easily checked that the second stage strategies are all Nash Equilibria of the subgames induced by the different values of k. To see that no player can do better by changing the first stage demand, notice first that by making a compatible demand a player would get, at best, $1/4$ which is lower than both y_1 and y_2. If a player deviates to making a higher demand than $3/4$ then her expected payoff would fall to strictly less than $1/4$, rendering it a loss making deviation. If either player makes a lower demand, then given the stated strategy profile her highest possible expected payoff must still be strictly less than $\frac{3}{4} \cdot \frac{1}{10}$, again less than both y_1 and y_2. The strategy profile outlined above is therefore subgame perfect and results in a positive probability of disagreement.

5 Independent revoking costs.

In this section I consider the opposite benchmark that involves the revoking costs being independently distributed. The first stage game is exactly as outlined in Section 4. Further the payoffs following incompatible demands are determined by the outcome of the game outlined in Table 1. In the first stage the players' common priors regarding the revoking costs k_1 and k_2 are given by the random vector K that takes values in $[0, \bar{k}_1] \times [0, \bar{k}_2]$ with $\bar{k}_i > 1$. Following incompatible demands both players observe the realized value of K before taking their second stage actions.

In the following analysis, whenever there is multiplicity in the second stage game, the risk dominant outcome will be selected. Instead of imposing it, this equilibrium selection criterion can indeed be derived by perturbing the model above to give it a global game information structure as was done in Section 4. The limit equilibrium prediction of such a perturbed model as the amount of noise is made arbitrarily small then delivers the equilibrium selection rule of the risk dominant outcome being played. The proof for this result, however, is largely of a technical nature and of marginal interest with respect to the results in CvD and Section 4.1 and is therefore omitted.[5] In particular, it requires the extension of the two dimensional version of the global games argument in CvD to the present game where the game itself is endogenously determined by the actions taken in the first stage.

One difference between the global games argument involved in Section 4.1 and the present section should be noted. In Section 4.1, when both players made equal demands that were incompatible, the global games argument could not resolve the subsequent second stage multiplicity. This is due to the lack of the required dominance solvable region. In this section, on the other hand, due to the independent distributions assumption, the required dominance solvable regions exist irrespective of the particular incompatible demand profile. This allows the expected payoff following any demand profile to be pinned down precisely.

Let $\Theta = [0, \bar{k}_1] \times [0, \bar{k}_2]$. While I do not explicitly solve the full global games model, the following assumption on the fundamentals of the model is required for the global game argument to work (with the addition of the noise parameters) and is therefore stated.

A1a. *K admits a density h that is strictly positive, continuously differentiable, and bounded and continuous on Θ.*

Suppose (z_1, z_2) is an incompatible demand profile. Let $D(z_1, z_2)$, $D_1(z_1, z_2)$ and $D_2(z_1, z_2)$ denote the part of the state space where the dominance solvable outcome of the second stage game are $(Stick, Stick)$, $(Stick, Accept)$ and $(Accept, Stick)$, respectively. Formally,

$$D(z_1, z_2) = \{k \in \Theta | k_1 > 1 - z_2 \text{ and } k_2 > 1 - z_1\}. \tag{6}$$

$$D_i(z_1, z_2) = \{k \in \Theta | k_i > 1 - z_j \text{ and } k_j < 1 - z_i\} \tag{7}$$

Figure 2 depicts the second stage equilibrium behavior over the entire state space, $0\bar{k}_1 W \bar{k}_2$, following an incompatible demand profile (z_1, z_2) where $z_1 > z_2$. The dominance solvable regions $D(z_1, z_2)$, $D_1(z_1, z_2)$ and $D_2(z_1, z_2)$ correspond to $MQWR$, $MP\bar{k}_1Q$ and $MN\bar{k}_2R$. $0NMP$ marks the region where both $(Stick, Accept)$ and $(Accept, Stick)$ are strict Nash equilibria.

[5]The proof for this result is available upon request.

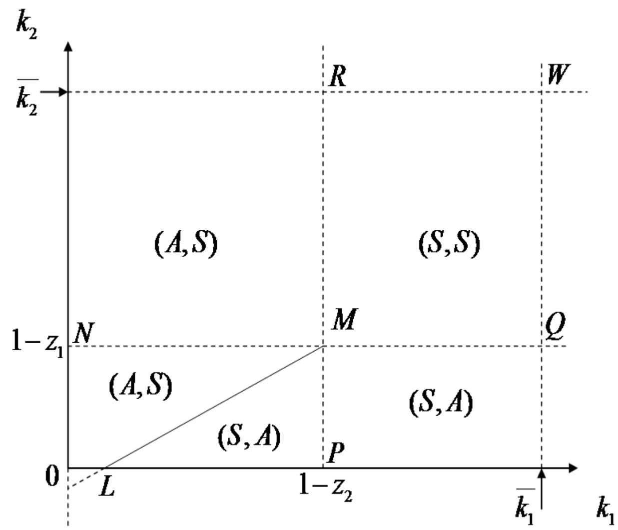

Figure 2: Second stage equilibrium behavior:Independent Costs

The equilibrium selection argument of risk dominance splits the state space (Θ) into three regions, following any incompatible demand profile, in terms of the action profile played in the second stage game. Let $R_i(z_1, z_2)$ denote the region of the state space where the risk dominant outcome in the second stage game following the incompatible demand profile (z_1, z_2) involves Player i playing *Stick* and Player j playing *Accept*. From Table 1 these regions can be completely characterized. In particular,

$$R_i(z_1, z_2) = \left\{ k \in \Theta | k_i < 1 - z_j \ \textit{and} \ k_j < 1 - z_i \ \textit{and} \ k_j < k_i \frac{d + 1 - z_i}{d + 1 - z_j} + \frac{d(z_j - z_i)}{d + 1 - z_j} \right\} \cup D_i(z_1, z_2) \tag{8}$$

In Figure 2, $R_1(z_1, z_2)$ and $R_2(z_1, z_2)$ correspond to $LMQ\bar{k}_1$ and $0LMR\bar{k}_2$. From (8) it can be seen that the line LM passes through the origin only if the two demands are the same.

Given this characterization it is possible to precisely pin down the payoffs following incompatible demands. In particular, following incompatible demands (z_1, z_2), if $k \in R_1(z_1, z_2)$, Player 1 gets z_1 while Player 2 gets $1 - z_1 - k_2$. Similarly if $k \in R_2(z_1, z_2)$, Player 1 gets $1 - z_2 - k_1$ while Player 2 gets z_2. Finally if $k \in D(z_1, z_2)$ then both players get 0. Notice that each player now faces a tradeoff between making a higher demand and increasing her risk dominant region where she actually receives her demand.

Figure 3 shows the changes in second stage behavior when Player 1 lowers her demand from z_1 to a still incompatible $\bar{z}_1$. Player 1, therefore, receives the lower share $\bar{z}_1$ whenever

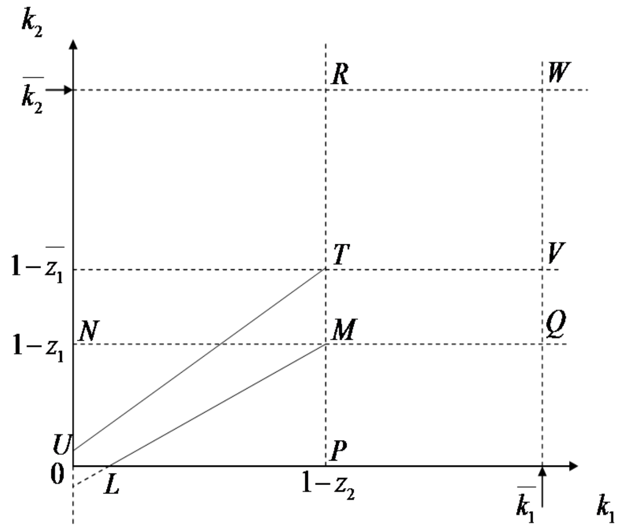

Figure 3: Lower demand and larger risk dominant region

k takes a value in her risk dominant region. However, her risk dominant region itself has now increased because of her lower demand, from $LMQ\bar{k}_1$ to $UTV\bar{k}_1 0$. Her greatest gain comes from converting the $TVQM$ region which earlier resulted in the full surplus being lost, to a region where she gets her exact demand. It is this tradeoff that prevents players from making arbitrarily high demands and results in the agreement results below.

Proposition 4. *If* ***A1a*** *is satisfied and K_1 and K_2 are independently distributed,with distribution functions F_1 and F_2, then the efficient demand profile* $(1/2, 1/2)$ *can be supported in equilibrium, for any pair of F_i that* First Order Stochastically Dominate *the uniform distribution.*

This efficiency argument is further strengthened by the non existence of equilibria supporting disagreement for the same range of distribution functions. In particular, the following result holds.

Proposition 5. *If* ***A1a*** *is satisfied and K_1 and K_2 are independently distributed, with distribution functions F_1 and F_2, then disagreement can not be supported in equilibrium for any pair of F_i that* First Order Stochastically Dominate *the uniform distribution.*

Example of Disagreement when FOSD Relation Fails: The following example was numerically computed using a program that calculated expected payoffs following incompatible demands exactly as outlined above, on Mathematica. Let K_1 and K_2 be identically and independently distributed according to a Beta distribution, $F(\alpha, \beta)$, with

$\alpha = 2$ and $\beta = 15$. Observe that F does not FOSD the uniform distribution. Let the two players make equal demands of $z_1 = z_2 = 0.5985$. Following any incompatible demand profile (z_1, z_2) and observations of (k_1, k_2), the corresponding unique risk dominant profile is played in the second stage. It can be checked that such a strategy profile satisfies subgame perfection. Being incompatible, such a demand profile gives rise to disagreement with positive probability.

6 Conclusion

The ability to attempt commitment to aggressive demands *does not* necessarily lead to disagreement in bargaining between two rational agents, when the success of the commitment attempt is ex ante uncertain. Firstly, it is important to specify the cause of such commitment ability. If players have access to exogenous random commitment devices, then disagreement would necessarily follow, as shown in EM. If the ability to commit arises from the presence of uncertain revoking costs, then the possibility of disagreement depends on the finer details of the players beliefs about such uncertainty. If the players believe that revoking costs can only take values of 0 or some number greater than the surplus, then disagreement can always be supported in equilibrium, even if they know that their revoking costs are identical (though uncertain). However, if the players' believe that the revoking costs can take all possible intermediate values as well then the possibility of disagreement is significantly limited. If the revoking costs are identical (but uncertain) then disagreement cannot obtain, irrespective of the particular distribution chosen. Even when the revoking costs are independent across players there cannot be any disagreement if the distribution functions FOSD the uniform distribution. In a sense if the ex ante probability of facing a high revoking cost is high enough, disagreement cannot occur.

Secondly, the key factor influencing the different results is the dependence of concession behavior on first stage demands. Binary distributions for revoking costs or the use of exogenous commitment devices result in equilibria where the probability of a successful commitment attempt does not depend on the demands made in the first stage. Continuous densities with noisy signals force equilibrium behavior in the game to establish a systematic dependence of concession behavior on first stage demands. In particular a higher demand always increases the success probability of the opponents commitment attempt while reducing one's own. Equilibria are therefore determined by the tradeoff between making a larger demand and increasing the probability of actually getting one's own demand. Such incentives often rule out the possibility of disagreement.

The analysis in this paper also highlights a particular feature of modeling behavioral types. In particular, models of behavioral types tend to be discrete in the sense that players are either fully rational or a specified type, due to the use of binary distributions. Allowing for the density, instead, to be continuous in the cost that must be paid to deviate from the actions of some type, necessarily makes the model a continuous one. In the present analysis this distinction led to sharply contrasting results. Whether such contrast applies more widely remains to be ascertained.

A Appendix

Proposition 1(a)

Proof. Fix $q \in (0,1)$. Let $z = \frac{q+1}{2}$. Following an incompatible demand profile (z_1, z_2), in the second stage Bayesian game, player i must always play the strictly dominant action *Stick* when $k_i > 1$. Equilibrium behavior when $k_i = 0$ needs to be pinned down. In this regard notice that playing *Accept* when $k_i = 0$ for both i, would constitute a Bayesian Nash Equilibrium if the following two inequalities hold.

$$q(1 - z_2) + (1 - q)(1 - z_2 + d) \geq (1 - q)z_1 \tag{9}$$

$$q(1 - z_1) + (1 - q)(1 - z_1 + d) \geq (1 - q)z_2 \tag{10}$$

The left hand (right hand) side of the inequalities gives the expected payoff to the player with $k_i = 0$ from playing *Accept* (*Stick*) when her opponent's strategy involves playing *Accept* when the cost is zero and *Stick* when it is greater than 1. (9) and (10) hold with equality if $z_1 = z_2 = z = \frac{q+1}{2}$.[6] Clearly the demand profile (z, z) is incompatible.

Consider now the following strategies. Each player demands z. Following the demand profile (z, z) player i plays *Accept* when $k_i = 0$ and *Stick* when $k_i > 1$. Following a demand profile where $z_i = z$ but $z_j > z$, player i plays *Stick* irrespective of k_i while j plays *Accept* when $k_j = 0$ and *Stick* when $k_j > 1$. Following an *incompatible* demand profile where $z_i = z$ but $z_j < z$, both players play *Accept* when their cost is 0 and *Stick*, when it is high. The strategies also subscribe actions that constitute a BNE for any subgame not considered above. It will be shown that such a strategy profile constitutes a Perfect Bayesian Nash Equilibrium of the game.

Consider first, behavior in the second stage subgames. Only the behavior of the types facing $k_i = 0$ needs to be checked, since i must always play *Stick* when $k_i > 1$ as it is

[6] Note that $d(z, z) = \frac{q}{2}$

the strictly dominant action in that case. Following the profile (z, z) both players with 0 cost play *Accept*. It has been shown earlier that for this to be a BNE (9) and (10) must be satisfied. Given the derivation of z, this is in fact the case. For incompatible demand profiles where $z_i = z$ and $z_j > z$, the strategies suggest that the low type of player i should play *Stick* while player j with $k_j = 0$ should play *Accept*. Given j's strategy i's low type choice would be optimal if

$$q(1 - z_j) + (1 - q)(1 - z_j + d) < (1 - q)z \tag{11}$$

Given that this relation holds with equality when $z_j = z$ and that the left hand side is strictly decreasing in z_j, it must be that for $z_j > z$, (11) is indeed satisfied. Further given that player i plays *Stick* always, player j does strictly better by playing *Accept* when $k_j = 0$. Finally for incompatible demand profiles with $z_i = z$ and $z_j < z$, notice that the inequalities (9) and (10) continue to be satisfied. As a result the strategies involving low cost types playing *Accept* does induce a BNE in such subgames. As for the first stage decisions, consider player 1. The expected payoff to 1 from demanding z when 2 demands z is given by $q(1-q)z+(1-q)[q(1-z)+(1-q)(1-z+(2z-1)/2)]$. If 1 demands less than z, $(z_1 < z)$ her expected payoff is $q(1-q)z_1+(1-q)[q(1-z)+(1-q)(1-z+(z+z_1-1)/2)]$ which is clearly less than her payoff from not deviating. If 1 demands $z_1 > z$ then her expected payoff is merely $(1 - q)(1 - z)$, again strictly less than if she had not deviated. It remains to be shown that no player would want to deviate from the profile (z, z) to making the compatible demand $1 - z$. Suppose this is a profitable deviation. Then it must be that,

$$\begin{aligned}
&q(1 - q)z + (1 - q)[q(1 - z) + (1 - q)(1 - z + d)] < 1 - z \\
\Rightarrow &q(1 - q)z + (1 - q)(1 - z) + (1 - q)^2 d < 1 - z \\
\Rightarrow &q(1 - q)z - q(1 - z) + (1 - q)^2 \frac{q}{2} < 0 \\
\Rightarrow &z - zq - 1 + z + \frac{(1 - q)^2}{2} < 0 \\
\Rightarrow &2z - 1 - zq + \frac{(1 - q)^2}{2} < 0 \\
\Rightarrow &q - \frac{q + 1}{2} q + \frac{(1 - q)^2}{2} < 0 \\
\Rightarrow &2q - q^2 - q + 1 - 2q + q^2 < 0 \\
\Rightarrow &q > 1
\end{aligned} \tag{12}$$

(12) contradicts the initial assumption of $q \in (0, 1)$. As a result no player would want to deviate to making a compatible offer, from the incompatible profile (z, z). □

Proposition 1(b)

Proof. Suppose not. Let the compatible demand profile supported in equilibrium be (z_1, z_2) where $z_1 + z_2 = 1$. WLOG let $z_1 \leq z_2$. Notice that substituting z_1 and z_2 into the inequalities (9) and (10) makes the inequalities strict. Further $d(z_1, z_2) = 0$. In particular, $q(1 - z_2) + (1 - q)(1 - z_2) > (1 - q)z_1$. Consequently if player 1 makes a higher demand, $z_1 + \delta$, the inequality will still be satisfied for small enough values of δ. Indeed, to satisfy the inequality (9), δ should satisfy, $q(1 - z_2) + (1 - q)(1 - z_2 + (\delta/2)) \geq (1 - q)(z_1 + \delta)$, which in turn implies that,

$$\delta \leq \frac{2qz_1}{1 - q} \tag{13}$$

To ensure that such a deviation maintains the second inequality it must be that, $q(1 - z_1 - \delta) + (1 - q)(1 - z_1 - \delta + (\delta/2)) \geq (1 - q)z_2$. This in turn, simplifies to,

$$\delta \leq \frac{2qz_2}{1 + q} \tag{14}$$

So if δ satisfies both (13) and (14), then following such a deviation, the subgame involving the incompatible demand profile, $(z_1 + \delta, z_2)$, would involve both players playing *Stick* when the cost is high and *Accept* when it is 0. To see that no other BNE exists in the second stage game, note that both low types playing *Stick* cannot occur in equilibrium. Further given that the inequalities (13) and (14) are satisfied, if one of the low types plays *Accept* then the low type of the other player must also play *Accept*. The expected payoff to player 1 from such a profile would therefore be, $q^2(0) + q(1 - q)(z_1 + \delta) + (1 - q)[q(1 - z_2) + (1 - q)(1 - z_2 + (\delta/2))]$. For this deviation to be profitable it must be that,

$$\begin{aligned}
&[q(1 - q) + (1 - q)]z_1 + q(1 - q)\delta + (1 - q)^2(\delta/2) > z_1 \\
\Rightarrow &q(1 - q)\delta + (1 - q)^2(\delta/2) > z_1 q^2 \\
\Rightarrow &(1 - q^2)\delta > 2z_1 q^2 \\
\Rightarrow &\delta > \frac{2z_1 q^2}{1 - q^2}
\end{aligned} \tag{15}$$

Let $z_1 > 0$. Then for such a deviation to exist, it simply needs to be shown that there exists $\delta > 0$ that simultaneously satisfies (13), (14) and (15). Notice that $\frac{2z_1q^2}{1-q^2} < \frac{2qz_1}{1-q} \Leftrightarrow \frac{q}{1+q} < 1$, and is satisfied for all $q > 0$. Further $\frac{2z_1q^2}{1-q^2} < \frac{2qz_2}{1+q} \Leftrightarrow \frac{z_1q}{1-q} < z_2$. Given that $z_1 \leq z_2$, this is satisfied for all $q < 1/2$. Consequently, if $z_1 > 0$ and $0 < q < 1/2$, there always exists a profitable deviation for player 1.

For the case where $z_1 = 0$ and $z_2 = 1$. If 1 deviates by demanding $\delta > 0$ that satisfies $\delta < \frac{2q}{1+q}$, the inequality (9) would be reversed and hold strictly. In other words following the demand profile $(\delta, 1)$, if player 2 plays *Accept* when $k_2 = 0$ and *Stick* otherwise, then player 1 would play *Stick* always. Also, given that 1 plays *Stick* always, 2's optimal action when $k_2 = 0$ is indeed to play *Accept* since it gives a payoff of $1 - \delta$ as opposed to the payoff of 0 if *Stick* is played. So these strategies constitute a BNE of the subgame following $(\delta, 1)$. Both players playing *Stick* always is not a BNE of this subgame since the low type of player 2 would strictly prefer to play *Accept*, as just described. The low types of both players playing *Accept* cannot happen due to the strict reversal of the inequality (9). So the only other potential BNE of this subgame involves player 2 playing *Stick* always while the low type of player 1 plays *Accept*. This would require the low type of player 2 to choose *Stick*, requiring, $q(1-\delta) + (1-q)(1-\delta+(\delta/2)) \le (1-q)(1)$. But, this inequality is violated if $\delta < \frac{2q}{1+q}$. The only BNE following a deviation to δ, therefore involves player 1 always playing *Stick* with the low type for player 2 playing *Accept*. Since this deviation gives a strictly positive payoff to player 1 it is a profitable deviation.

So it has been shown that given any compatible demand profile (z_1, z_2) with $z_1 \le z_2$ as long as $0 < q < 1/2$, there always exists a profitable deviation for player 1. Clearly, a symmetric argument applies for $z_2 \le z_1$. Consequently with $0 < q < 1/2$ there cannot be any equilibrium involving compatible demands. □

Proposition 2(a)

Proof. Consider the following strategies. Both players demand 1 in the first stage. Following any incompatible demand profile (z_1, z_2), player i plays *Stick* when $k_i > 1$. If $k_i = 0$ and $k_j > 1$, then player i plays *Accept*. If $k_1 = k_2 = 0$, then player 1 plays *Stick* while player 2 plays *Accept*.

Table 1 makes it clear that the strategies outlined above induce a Nash Equilibrium in every subgame following incompatible demand profiles. Notice that these subgames are dominance solvable except for the case where $k_1 = k_2 = 0$. In the latter case both (*Accept*, *Stick*) and (*Stick*, *Accept*) are Nash Equilibria. The particular selection made in this case is entirely arbitrary, but sufficient to support the incompatible profile as an equilibrium outcome.

The expected payoff to player 1 from the strategies above is $q(1-q)(1)+(1-q)(1-q)(1)$. Deviating to any lower incompatible demand z_1 gives an expected payoff, $q(1-q)(z_1) + (1-q)(1-q)(z_1)$, while making a compatible demand gives a payoff of 0. So player 1 has no incentive to deviate. Player 2's expected payoff from the stated strategies is $q(1-q)(1)$. Deviating to a lower but still incompatible demand, z_2, gives her $q(1-q)z_2$.

Finally deviating to a compatible demand gives her 0. As a result player 2 also has no incentive to deviate. □

Proposition 2(b)

Proof. Suppose not. Let (z_1, z_2) be supported in equilibrium, where $z_1 + z_2 = 1$. Suppose player i deviates to demanding $\tilde{z}_i = 1$. Player i's expected payoff from such a deviation must be no less than $q^2(0)+q(1-q)(1)+(1-q)q(1-z_j)+(1-q)^2(1-z_j) = q(1-q)+(1-q)z_i$. For such a deviation to not be profitable it must be that $z_i \geq q(1-q) + (1-q)z_i$. This implies, $z_i \geq 1-q$. Given that $q < 1/2$ and $z_1+z_2 = 1$, it must be that for some $i \in \{1, 2\}$, $z_i < 1-q$ holds. Such a player i would then do strictly better by deviating to a demand of 1. □

Proposition 2(c)

Proof. Let $k_1 = k_2 = k$. When $k > 1$, the unique Nash Equilibrium in the second stage game involves both players playing *Stick*. $k = 0$, on the other hand, results in two pure strategy NE, namely $(Accept, Stick)$ and $(Stick, Accept)$. Consider the following strategies. Both players demand 1. Following any incompatible demand profile (z_1, z_2), if $k = 0$, player 1 plays *Stick* while 2 plays *Accept*. Facing $k > 1$, both players play *Stick*. As mentioned earlier, the subgame strategies constitute Nash Equilibria. Player 1 gets an expected payoff of $1 - q$. By deviating to making any other demand z_1, the expected payoff would become strictly less, $(1-q)z_1$. Player 2, on the other hand, would always get 0 irrespective of her first stage demand and therefore has no incentive to deviate. Consequently the strategies support the demand $(1, 1)$ in equilibrium. The subsequent probability of disagreement is therefore q^2. □

Lemma 1(CvD)

Proof. Let $l = \max_{k\in[0,\bar{k}]} |h'(k)|$, where $h'(k)$ is the derivative of the function h at k for $k \in (0, \bar{k})$ with $h'(0)$ and $h'(\bar{k})$ defined as $\lim_{k\to 0} h'(k)$ and $\lim_{k\to\bar{k}} h'(k)$, respectively. Given **A1**, l is well defined with $l \geq 0$. Let $\nu = \min_{k\in[0,\bar{k}]} h(k)$. Given that h is continuous and strictly positive on $[0, \bar{k}]$, ν is well defined with $\nu > 0$. Let ϵ be such that $l\epsilon < \nu/2$. Then (1) leads to the following inequality for all $k_i, k_j \in [0, \bar{k}]$,

$$f_i^{\epsilon}(k_j|k_i) \leq \frac{(h(k_i) + l\epsilon)\int \varphi^{\epsilon}(k_1 - k, k_2 - k)dk}{(h(k_i) - l\epsilon)\int\int \varphi^{\epsilon}(k_1 - k, k_2 - k)dk_j dk} = \frac{(h(k_i) + l\epsilon)\psi^{\epsilon}(k_1 - k_2)}{h(k_i) - l\epsilon}$$

ψ^ϵ is the density function for $\epsilon E_1 - \epsilon E_2$ and is equal to the integral in the numerator of the second term for given values of k_1 and k_2 . Note that the double integral in the denominator of the second term above is equal to 1. Similarly, $\frac{(h(k_i)-l\epsilon)\psi^\epsilon(k_1-k_2)}{h(k_i)+l\epsilon} \leq f_i^\epsilon(k_j|k_i)$. For $k_i \in [-\epsilon, 0]$ the relevant inequality is $\frac{(h(0)-l\epsilon)\psi^\epsilon(k_1-k_2)}{h(0)+l\epsilon} \leq f_i^\epsilon(k_j|k_i) \leq \frac{(h(0)+l\epsilon)\psi^\epsilon(k_1-k_2)}{h(0)-l\epsilon}$. If $k_i \in [\bar{k}, \bar{k}+\epsilon]$ then the inequality is $\frac{(h(\bar{k})-l\epsilon)\psi^\epsilon(k_1-k_2)}{h(\bar{k})+l\epsilon} \leq f_i^\epsilon(k_j|k_i) \leq \frac{(h(\bar{k})+l\epsilon)\psi^\epsilon(k_1-k_2)}{h(\bar{k})-l\epsilon}$. Therefore,

$$(1-\frac{2l\epsilon}{h(k_i)+l\epsilon})\psi^\epsilon(k_1-k_2) \leq f_i^\epsilon(k_j|k_i) \leq (1+\frac{2l\epsilon}{h(k_i)-l\epsilon})\psi^\epsilon(k_1-k_2)^7$$

Further let $\kappa = \frac{8l}{\nu}$. Now,

$$\begin{aligned} 1+\frac{2l\epsilon}{h(k_i)-l\epsilon} &\leq 1+\frac{2l\epsilon}{\nu-l\epsilon} \\ &\leq 1+\frac{2l\epsilon}{\nu/2} \end{aligned}$$

Also,

$$\begin{aligned} 1-\frac{2l\epsilon}{h(k_i)+l\epsilon} &\geq 1-\frac{2l\epsilon}{h(k_i)-l\epsilon} \\ &\geq 1-\frac{2l\epsilon}{\nu-l\epsilon} \\ &\geq 1-\frac{2l\epsilon}{\nu/2} \end{aligned}$$

Then,

$$\psi^\epsilon(k_1-k_2)(1-(\kappa\epsilon)/2) \leq f_i^\epsilon(k_j|k_i) \leq \psi^\epsilon(k_1-k_2)(1+(\kappa\epsilon)/2) \tag{16}$$

$$\Rightarrow \int_{y\leq k_2} \psi^\epsilon(k_1-y)dy - (\kappa\epsilon)/2 \leq F_1^\epsilon(k_2|k_1) \leq \int_{y\leq k_2} \psi^\epsilon(k_1-y)dy + (\kappa\epsilon)/2 \tag{17}$$

(16) also implies,

$$\begin{aligned} &\int_{z\leq k_1} \psi^\epsilon(z-k_2)dz - (\kappa\epsilon)/2 \leq F_2^\epsilon(k_1|k_2) \leq \int_{z\leq k_1} \psi^\epsilon(z-k_2)dz + (\kappa\epsilon)/2 \\ &\Rightarrow \int_{z\geq k_1} \psi^\epsilon(z-k_2)dz + (\kappa\epsilon)/2 \geq 1-F_2^\epsilon(k_1|k_2) \geq \int_{z\geq k_1} \psi^\epsilon(z-k_2)dz - (\kappa\epsilon)/2 \\ &\Rightarrow \int_{y\leq k_2} \psi^\epsilon(k_1-y)dy + (\kappa\epsilon)/2 \geq 1-F_2^\epsilon(k_1|k_2) \geq \int_{y\leq k_2} \psi^\epsilon(k_1-y)dy - (\kappa\epsilon)/2 \end{aligned} \tag{18}$$

Subtracting (18) from (17) gives the required inequality.

□

[7] For values of k_i in $[-\epsilon, 0]$ and $[\bar{k}, \bar{k}+\epsilon]$ replace $h(k_i)$ by $h(0)$ and $h(\bar{k})$, respectively.

Lemma 2

Proof. The continuity of φ^ϵ is implied by the continuity of φ assumed in *A2.* Consider the numerator in the expression for $f_i^\epsilon(k_j|k_i)$ as expressed in (1). WLOG take a sequence k_1^n that converges to k_1, such that $k_1^n \in [-\epsilon, \bar{k}+\epsilon]$ for all n. Given the continuity of φ^ϵ it is immediate that holding k_2 fixed, $h(k)\varphi^\epsilon(k_1^n - k, k_2 - k) \to h(k)\varphi^\epsilon(k_1 - k, k_2 - k)$, *almost everywhere* in $[0, \bar{k}]$. Further $h(k)\varphi^\epsilon(k_1^n - k, k_2 - k) \leq h(k)\bar{\varphi}^\epsilon$ for all n and k, where $\bar{\varphi}^\epsilon$ is the maximum value taken by the function φ on $[-1, 1] \times [-1, 1]$. Consequently by the *Dominated Convergence Theorem*, $\int h(k)\varphi^\epsilon(k_1 - k, k_2 - k)dk = \lim_{n\to\infty} \int h(k)\varphi^\epsilon(k_1^n - k, k_2 - k)dk$. In other words, $\int h(k)\varphi^\epsilon(k_1 - k, k_2 - k)dk$ is continuous in k_i. For the denominator in (1), consider first the marginal density. Fix k. Let $k_1 \notin \{k-\epsilon, k+\epsilon\}$. Then for any sequence k_1^n that converges to k_1 it must be the case that $\varphi^\epsilon(k_1^n - k, k_2 - k) \to \varphi^\epsilon(k_1^n - k, k_2 - k)$ for all values of k_2, by **A3**. Again by the *Bounded Convergence Theorem*, the marginal $\int \varphi^\epsilon(k_1^n - k, k_2 - k)dk_2$ for a given value of k is found to be continuous at all k_1 other than potentially two points, $k - \epsilon$ and $k + \epsilon$. Consequently for any sequence k_1^n that converges to k_1, it is true that $h(k)\int \varphi^\epsilon(k_1^n - k, k_2 - k)dk_2 \to h(k)\int \varphi^\epsilon(k_1 - k, k_2 - k)dk_2$ for all values of k other than possibly $k_1 - \epsilon$ and $k_1 + \epsilon$. Further, $h(k)\int \varphi^\epsilon(k_1^n - k, k_2 - k)dk_2 \leq h(k)\bar{\varphi}^\epsilon$ for all k, n. By the *Dominated Convergence Theorem*, it must be that $\int h(k)\int \varphi^\epsilon(k_1^n - k, k_2 - k)dk_2dk$, the denominator in (1), is continuous in k_1. Given **A1** and the additive structure of the noise, the denominator is also strictly positive for all $k_1 \in (-\epsilon, \bar{k}+\epsilon)$. Therefore for all $k_1, k_2 \in [-\epsilon, \bar{k}+\epsilon]$, $f_i^\epsilon(k_j|k_i)$ is continuous in k_i. $f_i^\epsilon(k_j|k_i)$ is also continuous in k_j, since k_j does not affect the denominator of (1), while its influence on the numerator is symmetric to that of k_i. So let $\bar{f}_\epsilon$ be the maximum value taken by $f_i^\epsilon(k_j|k_i)$ for $k_1, k_2 \in [-\epsilon, \bar{k}+\epsilon]$. Then for any measurable function s_j, it must be that $s_j(k_j)f_i^\epsilon(k_j|k_i^n) \to s_j(k_j)f_i^\epsilon(k_j|k_i)$ if $k_i^n \to k_i$ and $s_j(k_j)f_i^\epsilon(k_j|k_i^n) \leq s_j k_j \bar{f}_\epsilon$, for all values of k_j. Therefore by the *Dominated Convergence Theorem*, $Pr(A_j|k_i^\epsilon, s_j) = \int s_j(k_j)f_i^\epsilon(k_j|k_i^\epsilon)dk_j$ is continuous in k_i^ϵ.

To show that $E^\epsilon(k|k_i^\epsilon)$ is continuous in k_i^ϵ consider first the conditional density of the true k given an observation k_i.

$$f_i^\epsilon(k|k_i) = \frac{\int h(k)\varphi^\epsilon(k_1 - k, k_2 - k)dk_j}{\int\int h(k)\varphi^\epsilon(k_1 - k, k_2 - k)dk_jdk} \tag{19}$$

Continuity of the denominator of (19) in k_i has already been established before. The numerator for a given k is the product of the strictly positive $h(k)$ and the marginal density of k_i. It has been shown earlier that for a given k the marginal density of k_i is continuous at all k_i other than possibly when $k_i \in \{k-\epsilon, k+\epsilon\}$, the boundary points.

As a result, for a given k, $f_i^\epsilon(k|k_i)$ is continuous for all k_i other than the two boundary points. Therefore for a sequence k_i^n that converges to k_i, $kf_i^\epsilon(k|k_i^n) \to kf_i^\epsilon(k|k_i)$ for all k other than possibly when $k \in \{k_i - \epsilon, k_i + \epsilon\}$. Further since the denominator in (19) is bounded below and the numerator bounded above, the *Dominated Convergence Theorem* delivers the continuity of $E^\epsilon(k|k_i^\epsilon) = \int kf_i^\epsilon(k|k_i^\epsilon)dk$ in k_i^ϵ. □

Lemma 3

Proof. Given the payoffs in Table 2, it is clear that whenever j chooses *Accept*, i always does strictly better by choosing *Stick*. Upon observing $k_i^\epsilon > 1 - z_j + \epsilon$ player i knows that for all the possible values that k can take she would get a strictly negative payoff by playing *Accept* if j plays *Stick*. As a result i would still strictly prefer to play *Stick* since it guarantees a payoff of 0 as opposed to the negative expected payoff from playing *Accept*, when j plays *Stick*. Consequently, upon observing $k_i^\epsilon > 1 - z_j + \epsilon$, *Stick* is the strictly dominant action for player i. □

Lemma 4

Proof. From lemma 3 it is already known that j plays *Stick* for every observation $k_j^\epsilon > 1 - z_i + \epsilon$. Player i making an observation $k_i^\epsilon \in (1 - z_i + 3\epsilon, 1 - z_j - \epsilon)$ learns two things. Firstly, she knows that j must have observed $k_j^\epsilon > 1 - z_i + \epsilon$ and must therefore be playing the strictly dominant *Stick*. Secondly, she knows that the true state k must lie in the interval $(1 - z_i + 2\epsilon, 1 - z_j)$. Conditional on j playing *Stick* for any such value of k, playing *Accept* strictly dominates playing *Stick* for i. The dominance solvable outcome following such an observation, therefore, involves i playing *Accept* while j plays *Stick*. □

Lemma 5

Proof. Suppose the statement is false for player i, who makes the higher demand. This means that $B_i^\epsilon(z_1, z_2)$ is non empty but $y = \sup\{x|x \in B_i^\epsilon(z_1, z_2)\} \notin B_i^\epsilon(z_1, z_2)$. So there exists a sequence of observations k_i^n that converge to y, with i playing *Stick* for all n but she plays *Accept* upon observing y. i's expected payoff from playing *Accept* following an observation k_i is given by $1 - z_j - E^\epsilon(k|k_i) + dPr(A_j|k_i)$ while it is $z_i Pr(A_j|k_i)$ from playing *Stick*. Given that i plays *Stick* for all observations in the sequence k_i^n it must be that $z_i Pr(A_j|k_i^n) \geq 1 - z_j - E(k|k_i^n) + dPr(A_j|k_i^n)$. By *Lemma 2*, $E^\epsilon(k|k_i)$ and $Pr(A_j|k_i)$ are continuous in k_i for all measurable strategies, s_j. So if $k_i^n \to y$ it must be that $z_i Pr(A_j|y) \geq 1 - z_j - E(k|y) + dPr(A_j|y)$. Given the tie break rule mentioned earlier this implies that i would play *Stick* upon observing y. This contradicts the earlier claim and proves the lemma for i. A symmetric argument proves the lemma for player j. $\square$

Lemma 6

Proof. Let $B_i^\epsilon(z_1, z_2)$ be empty. Then for all observations $k_i^\epsilon \leq 1 - z_i + 3\epsilon$ player i chooses to play *Accept*. In that case whenever player j receives a signal $k_j^\epsilon \leq 1 - z_i + 3\epsilon$ it is conditionally dominant for him to play *Stick*. This would imply that $B_j^\epsilon(z_1, z_2)$ is empty.

Now if $B_j^\epsilon(z_1, z_2)$ is empty then for all observations $k_j^\epsilon \leq 1 - z_i + 3\epsilon$ player j chooses to play *Stick*. Player i following an observation $k_i^\epsilon \leq 1 - z_i + 3\epsilon$ knows that the true value of k is such that $1 - z_j - k > 0$. Consequently conditional on j playing *Stick*, she is strictly better off playing *Accept*. As a result $B_i^\epsilon(z_1, z_2)$ is empty. $\square$

Lemma 7

Proof. Let $k_j^{\epsilon*} + 2\epsilon \leq k_i^\epsilon \leq 1 - z_i + 3\epsilon$. Conditional on such an observation player i knows that for all the possible values of k, $1 - z_j - k > 0$ and hence she would strictly prefer to play *Accept* if j plays *Stick*. Further such an observation implies that j has observed $k_j^\epsilon > k_j^{\epsilon*}$ implying that j would certainly play *Stick*. Consequently i's conditionally dominant action is to play *Accept*. $\square$

Lemma 8

Proof. Suppose not. Then, by *Lemmas 5 and 6*, $k_i^{\epsilon*}, k_j^{\epsilon*} > -\epsilon$ are well defined. Let player i's payoff from playing *Accept* and *Stick* upon observing $k_i^{\epsilon*}$ be denoted as $u_i(A_i|k_i^{\epsilon*})$ and $u_i(S_i|k_i^{\epsilon*})$ respectively. Given the payoffs in Table 2, $u_i(A_i|k_i^{\epsilon*}) = 1 - z_j - E^\epsilon(k|k_i^{\epsilon*}) + dPr(A_j|k_i^{\epsilon*})$. Also $u_i(S_i|k_i^{\epsilon*}) = z_i Pr(A_j|k_i^{\epsilon*})$. Given that i chooses *Stick* after such an observation, it must be that $u_i(S_i|k_i^{\epsilon*}) \geq u_i(A_i|k_i^{\epsilon*})$. This in turn implies the following inequality,

$$Pr(A_j|k_i^{\epsilon*}) \geq \frac{1 - z_j - E^\epsilon(k|k_i^{\epsilon*})}{z_i - d} \tag{20}$$

Similarly, player j choosing *Accept* upon observing $k_j^{\epsilon*}$ implies that $u_j(A_j|k_j^{\epsilon*}) \geq u_j(S_j|k_j^{\epsilon*})$. Writing out the payoffs, $1 - z_i - E^\epsilon(k|k_j^{\epsilon*}) + dPr(A_i|k_j^{\epsilon*}) \geq z_j Pr(A_i|k_j^{\epsilon*})$. This gives rise to the following inequality,

$$Pr(A_i|k_j^{\epsilon*}) \leq \frac{1 - z_i - E^\epsilon(k|k_j^{\epsilon*})}{z_j - d} \tag{21}$$

Now, player j plays *Stick* following any observation $k_j^\epsilon > k_j^{\epsilon*}$. Therefore, it must be that,

$$Pr(A_j|k_i^{\epsilon*}) \leq F_i^\epsilon(k_j^{\epsilon*}|k_i^{\epsilon*}) \tag{22}$$

On the other hand, player i plays *Accept* for observations $k_i^\epsilon > k_i^{\epsilon*}$ as long as $k_i^\epsilon < 1-z_j-\epsilon$. For values of k_i^ϵ that are within 2ϵ of $k_j^{\epsilon*}$ it must be that $k_i^\epsilon < 1-z_j-\epsilon$ since $k_j^{\epsilon*} \leq 1-z_i+\epsilon$ by *Lemma 3* and $1-z_i+\epsilon < 1-z_j-2\epsilon$ by assumption. As a result the following inequality holds.

$$Pr(A_i|k_j^{\epsilon*}) \geq 1 - F_j^\epsilon(k_i^{\epsilon*}|k_j^{\epsilon*}) \tag{23}$$

Subtracting (23) from (22) and using (2) from *Lemma 1* gives the inequality,

$$Pr(A_j|k_i^{\epsilon*}) - Pr(A_i|k_j^{\epsilon*}) \leq \kappa\epsilon \tag{24}$$

Finally combining (20), (21) and (24) gives,

$$\begin{aligned}
\kappa\epsilon &\geq \frac{1-z_j-E^\epsilon(k|k_i^{\epsilon*})}{z_i-d} - \frac{1-z_i-E^\epsilon(k|k_j^{\epsilon*})}{z_j-d} && (25)\\
&\geq \frac{1-z_j-k_i^{\epsilon*}-\epsilon}{z_i-d} - \frac{1-z_i-k_j^{\epsilon*}+\epsilon}{z_j-d} && (26)\\
&> \frac{1-z_j-k_j^{\epsilon*}-3\epsilon}{z_i-d} - \frac{1-z_i-k_j^{\epsilon*}+\epsilon}{z_j-d} && (27)
\end{aligned}$$

(25) $\Rightarrow$ (26) by the fact that $E^\epsilon(k|k_i^{\epsilon*}) \leq k_i^{\epsilon*}+\epsilon$ and $E^\epsilon(k|k_j^{\epsilon*}) \geq k_j^{\epsilon*}-\epsilon$. While the inequality from *Lemma 7*, namely $k_i^{\epsilon*} < k_j^{\epsilon*}+2\epsilon$, makes (26) $\Rightarrow$ (27).
(27) $\Rightarrow$

$$\begin{aligned}
&\kappa\epsilon(z_i-d)(z_j-d) > (z_j-z_i)(1-k_j^{\epsilon*}+d) - (z_j^2-z_i^2) - 3\epsilon z_j - \epsilon z_i + 4\epsilon d\\
\Rightarrow&\kappa\epsilon(1-(z_i-z_j)^2) > (z_j-z_i)(1-k_j^{\epsilon*}-(z_i+z_j)+d) + \epsilon(z_i-z_j) - 2\epsilon\\
\Rightarrow&\kappa\epsilon(1-(z_i-z_j)^2) > (z_i-z_j)(k_j^{\epsilon*}+d+\epsilon) - 2\epsilon\\
\Rightarrow&k_j^{\epsilon*}+d+\epsilon < \frac{\kappa\epsilon(1-(z_i-z_j)^2)}{z_i-z_j} + \frac{2\epsilon}{z_i-z_j}\\
\Rightarrow&k_j^{\epsilon*} < -\epsilon + \frac{\kappa\epsilon}{z_i-z_j} + \frac{2\epsilon}{z_i-z_j} - d - \kappa\epsilon(z_i+z_j)\\
\Rightarrow&k_j^{\epsilon*} < -\epsilon + \frac{(\kappa+2)\epsilon}{z_i-z_j} - d && (28)
\end{aligned}$$

Given that $k_j^{\epsilon*}$ must be a value strictly greater than $-\epsilon$, (28) delivers a contradiction to the initial claim if,

$$z_i - z_j \geq \frac{(\kappa+2)\epsilon}{d} \tag{29}$$

The premise in the lemma satisfies (29) and therefore it must be that $B_j^\epsilon(z_1,z_2)$ is empty. *Lemma 6* then guarantees that $B_i^\epsilon(z_1,z_2)$ is empty too. □

Lemma 10

Proof. (a) is immediate, since player i has an incentive to demand $1 - z_j$ and strictly increase her payoff by $1 - z_j - z_i > 0$. *Lemma 9* shows that following an incompatible demand profile such as (b), the player with the higher demand, say i, has an expected payoff $y_i \leq \int_0^{1-z_j}(1 - z_j - w)h(w)dw < 1 - z_j$ and could do strictly better by simply making the compatible demand $1 - z_j$. □

Lemma 11

Proof. Following an incompatible demand profile, the payoffs are determined by outcomes in the second stage game described in Table 2. Notice that following any possible realization, k, the maximum total payoff would be $\max\{1 - k, 0\}$. As a result the expected payoffs from making incompatible demands must satisfy, $y_1 + y_2 \leq 1 - \hat{k}$. Now for the incompatible profile (z_1, z_2) to be supported as an equilibrium in Γ^ϵ, it must be that neither player gains by making a compatible demand instead. This means, $y_i \geq 1 - z_j$. Summing across the two players gives, $y_1 + y_2 \geq 2 - z_1 - z_2$, which in turn implies, $2 - z_1 - z_2 \leq 1 - \hat{k}$. Given that $d = (z_1 + z_2 - 1)/2$ it must be that $d \geq \hat{k}/2$. □

Lemma 12

Proof. Equilibrium behavior in the second stage game involves a total payoff of 0 if both parties play *Stick* or $1 - k$ if (*Accept*, *Stick*) or (*Stick*, *Accept*) is the outcome. Players using mixed strategies results in the total payoff lying in the interval $[0, \max\{0, 1 - k\}]$. *Lemma 2* makes it clear that if $k > 1 - \min\{z_1, z_2\} + 2\epsilon$ then the players would always play (*Stick*, *Stick*). So it can be said for certain that following an incompatible demand profile, the total expected payoff in equilibrium must be no more than $(1 - \int kh(k|k \leq 1 - min\{z_1, z_2\} + 2\epsilon)dk)H(1 - \min\{z_1, z_2\} + 2\epsilon)$. This in turn implies that following incompatible demands there exists i with an expected payoff,

$$y_i \leq \frac{1}{2}(1 - \int kh(k|k \leq 1 - min\{z_1, z_2\} + 2\epsilon)dk)H(1 - \min\{z_1, z_2\} + 2\epsilon) \tag{30}$$

$d \geq \hat{k}/2$ implies $z_i + z_j - 1 \geq \hat{k}$. Also by the definition of ϕ, it must be that $\phi(d) \leq \phi^*$ since $d \geq \hat{k}/2$. So,

$$\begin{aligned} &|z_i - z_j| < \phi(d) \leq \phi^* \\ \Rightarrow &2\min\{z_1, z_2\} + \phi^* - 1 \geq \hat{k} \\ \Rightarrow &\min\{z_1, z_2\} \geq \frac{1}{2} + \frac{\hat{k}}{2} - \frac{\phi^*}{2} \end{aligned} \tag{31}$$

Let ϵ be small enough such that $\phi^* < \frac{\hat{k}}{8}$.

Then,

$$(31) \Rightarrow \min\{z_1, z_2\} > \frac{1}{2} + \frac{7}{16}\hat{k} \tag{32}$$

Now consider what happens if player i, who receives the payoff mentioned in (30), deviates to making a *still incompatible* demand of $\tilde{z}_i = 1/2$. Note that $z_j - \tilde{z}_i > \frac{7}{16}\hat{k} > \phi^*$. Further $d(\tilde{z}_i, z_j) > \frac{7}{32}\hat{k}$ which implies that $\phi(d) \leq \phi^*$. Therefore $z_j - \tilde{z}_i > \phi(d(\tilde{z}_i, z_j))$. As a result, the new demand profile satisfies the condition of *Lemma 9*, which implies that player i following such a deviation must expect a payoff $\tilde{y}_i$,

$$\tilde{y}_i \geq \frac{1}{2}F_i^{\epsilon}(\frac{1}{2} - \epsilon) \geq \frac{1}{2}H(\frac{1}{2} - 2\epsilon) \tag{33}$$

Player i's initial payoff inequality described in (30) along with (32) implies,

$$y_i < \frac{1}{2}H(\frac{1}{2} - \frac{7}{16}\hat{k} + 2\epsilon) \tag{34}$$

For small enough values of ϵ, it is clear that $y_i < \tilde{y}_i$. Given that such a profitable deviation exists, $(z_1, z_2) \notin Eq^{\epsilon}$. □

Proposition 4

Proof. Consider the following strategies. Both players demand 1/2 in the first stage. If player j makes a demand higher than 1/2 then in the second stage, in the event of multiplicity, both players play actions in accordance to the risk dominant outcome of the second stage game. Further if the state of the world (k_1, k_2) lies in the region $D(1/2, z_j)$ then both players play *Stick*. Given these strategies it is easy to see that second stage behavior satisfies equilibrium behavior since it either involves playing the unique dominance solvable action profile or playing one of the Nash Equilibria; in particular, the risk dominant action profile. However, it must be checked if any player has an incentive to deviate in the first stage. Deviating to a smaller demand is obviously less profitable to the deviator and hence ruled out.

Suppose Player 1 deviates to making a higher demand $z_1 > 1/2$. By doing so, Player 1 would gain a higher payoff for every k that lies in her new risk dominant region, $R_1(z_1, 1/2)$. Denote this gain by G. From (8), it must be that,

$$G \leq (z_1 - 1/2)(1 - F_1(1/2))F_2(1 - z_1) + 1/2(z_1 - 1/2)F_1(1/2)F_2(1/2). \tag{35}$$

On the other hand Player 1 ends up losing her share of 1/2 in the new disagreement region, $D(z_1, 1/2)$, while also paying her revoking cost in her opponents risk dominant region $R_2(z_1, 1/2)$. Denote this loss by L. From (8) and (6), it must be that,

$$L > \frac{1}{2}(1 - F_1(1/2))(1 - F_2(1 - z_1)) \tag{36}$$

The proposition will be first proven for F_1 and F_2 being uniform. If F_1 and F_2 are uniform then (35) can be rewritten as,

$$G \leq (z_1 - 1/2)\left(\frac{\bar{k}_1 - 1/2}{\bar{k}_1}\right)\left(\frac{1 - z_1}{\bar{k}_2}\right) + \frac{1}{2}(z_1 - 1/2)\left(\frac{(1/2)^2}{\bar{k}_1\bar{k}_2}\right) \tag{37}$$

Similarly (36) implies,

$$L > \frac{1}{2}\left(\frac{\bar{k}_1 - 1/2}{\bar{k}_1}\right)\left(\frac{\bar{k}_2 - 1/2}{\bar{k}_2}\right) + \frac{1}{2}\left(\frac{\bar{k}_1 - 1/2}{\bar{k}_1}\right)\left(\frac{z_1 - 1/2}{\bar{k}_2}\right) \tag{38}$$

Note that $z_1 > 1/2$ and $\bar{k}_i > 1$. Consequently, for such a deviation to be profitable it must be that $G > L$. This in turn, from (37) and (38), implies that a profitable deviation must involve,

$$\begin{aligned} &\frac{1}{2}\left(\frac{\bar{k}_1 - 1/2}{\bar{k}_1}\right)\left(\frac{\bar{k}_2 - 1/2}{\bar{k}_2}\right) < \frac{1}{2}(z_1 - 1/2)\left(\frac{(1/2)^2}{\bar{k}_1\bar{k}_2}\right) \\ \Rightarrow &1 < z_1 - 1/2 \end{aligned} \tag{39}$$

The impossibility of (39) rules out any profitable deviation for Player 1. A symmetric argument rules out any profitable deviation for Player 2. Consequently the strategies outlined above constitute a Subgame Perfect Equilibrium when the F_i are uniform distributions. To see how the argument then extends to any pair of distributions that *FOSD* the uniform distribution, notice that to arrive at the contradiction above, it was shown that,

$$\left(z_1 - \frac{1}{2}\right)\left(1 - F_1\left(\frac{1}{2}\right)\right)F_2(1 - z_1) + \left(z_1 - \frac{1}{2}\right)F_1\left(\frac{1}{2}\right)F_2\left(\frac{1}{2}\right) < \frac{1}{2}(1 - F_1(1/2))(1 - F_2(1 - z_1)) \tag{40}$$

when the F_i are uniform. It is easy to see that if the F_i *FOSD* the uniform distribution, the right hand side of (40) would be even higher, while the left hand side even lower than in the uniform case. Consequently the relationship $L > G$ would hold for all such distributions. The result follows. □

Proposition 5

Proof. First, note that an incompatible demand profile with at least one player making a demand of $z_i = 1$ cannot be supported in equilibrium. Following such an incompatible profile, player i either backs down in the second stage or the entire surplus is lost, since player j will never back down. In other words, $R_i(z_i = 1, z_j) = \emptyset$. Therefore player i's expected payoff must be strictly less than $1 - z_j$, which in turn makes the first stage deviation to a compatible demand a profitable one if $z_j < 1$. If, however, $z_1 = z_2 = 1$, then each player is better off making a demand of $1/2$ instead. The demand profile $(1, 1)$ yields a payoff of 0 to both players. If player 1 makes a demand of $1/2$ instead her expected payoff becomes, $(1/2)F_2(1/2)$ which is clearly payoff improving.

Having eliminated the possibility of a demand of 1 in equilibrium, the result shall first be proven for the F_i being uniform distributions. It will then be shown that the arguments generalize easily to any pair of distributions that each First Order Stochastically Dominates the uniform distribution.

The statement is proved by contradiction. Suppose (z_1, z_2) is an incompatible demand profile that is supported in equilibrium with F_i being a uniform distribution. It must be true then that neither player can have her payoff strictly increases by making a compatible demand in the first stage. Consider the options for Player 2. If she deviates to making a compatible demand she gains $1 - z_1$ in the region $D(z_1, z_2)$. She also gains the revoking cost she would have had to pay following incompatible demands in the region $R_1(z_1, z_2)$. The total expected gain from such a deviation is denoted by G where $G \geq (1 - z_1)[1 - F_2(1 - z_1)][1 - F_1(1 - z_2)] + E(k_2 | k_2 \leq 1 - z_1)[1 - F_1(1 - z_2)]F_2(1 - z_1)$. For the purpose of this proof the inequality, $G \geq (1 - z_1)[1 - F_2(1 - z_1)][1 - F_1(1 - z_2)]$ will suffice. Such a deviation, however, results in a loss of $z_2 - (1 - z_1)$ in the region $R_2(z_1, z_2)$. Denote the expected loss by L where $L \leq [z_2 - (1 - z_1)]F_1(1 - z_2)$.

Since the F_i are uniform distributions the relevant inequalities become,

$$G \geq (1 - z_1)\left(\frac{\bar{k}_2 - (1 - z_1)}{\bar{k}_2}\right)\left(\frac{\bar{k}_1 - (1 - z_2)}{\bar{k}_1}\right) \tag{41}$$

$$L \leq (z_2 - (1 - z_1))\frac{1 - z_2}{\bar{k}_1} \tag{42}$$

Given that such a deviation is not profitable by assumption it must be that $L \geq G$. Since $k_i > 1$, $L \geq G$ implies the following inequality,

$$(z_1 + z_2 - 1)(1 - z_2) \geq (1 - z_1)z_2 z_1 \tag{43}$$

A symmetric argument shows that for Player 1 to not be strictly better of from deviating to a compatible demand, the following inequality must hold.

$$(z_1 + z_2 - 1)(1 - z_1) \geq (1 - z_2)z_1 z_2 \tag{44}$$

Now suppose $z_1 \geq z_2$. Then to satisfy (44) it must be that $z_1 + z_2 - 1 \geq z_1 z_2$, which in turn implies $z_i \geq 1$. On the other hand if $z_2 \geq z_1$ then satisfying (43) would require $z_i \geq 1$. Since the possibility of $z_i = 1$ in equilibrium was ruled out earlier this delivers the contradiction.

The uniform distribution case was proved by essentially showing that at least one of the inequalities,

$$(1 - z_i)[1 - F_j(1 - z_i)][1 - F_i(1 - z_j)] > [z_j - (1 - z_i)]F_i(1 - z_j) \tag{45}$$

for $i \in \{1, 2\}$, holds if both players are not demanding 1 each. This made the deviation to a compatible demand a profitable one for Player j. Notice that for any other pair of distribution functions, F_1 and F_2 that FOSD the uniform distribution, this inequality would continue to hold since it would simply decrease the right hand side of (45) while increasing the left hand side. This would make the required inequality, $G > L$, hold for all such distributions. □

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