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Identifying destitution through linked subsets of multidimensionally poor

An ordinal approach

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Abstract: In order to understand whether a reduction in overall poverty has improved the situations of the poorest, it is crucial to distinguish them from the moderately poor population. In this paper, we explore the mechanisms to distinguish subsets of the poor in a multidimensional counting framework. We examine two approaches that capture two distinct forms of stringent multidimensional poverty: one uses a more stringent vector of deprivation cutoffs, and the other, a more stringent cross-dimensional poverty cutoff. To explore the distinction between these two approaches empirically, we examine the evolution of multidimensional poverty in Nepal. Our findings show crucial differences between these approaches.

Keywords: extreme poverty, ultra-poverty, multidimensional poverty, poverty characteristics, poverty profile, Nepal
JEL classification: I3, I32, D63, O1

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The World Institute for Development Economics Research (WIDER) was established by the United Nations University (UNU) as its first research and training centre and started work in Helsinki, Finland in 1985. The Institute undertakes applied research and policy analysis on structural changes affecting the developing and transitional economies, provides a forum for the advocacy of policies leading to robust, equitable and environmentally sustainable growth, and promotes capacity strengthening and training in the field of economic and social policy-making. Work is carried out by staff researchers and visiting scholars in Helsinki and through networks of collaborating scholars and institutions around the world.

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1 Introduction

The eradication of extreme forms of poverty has been at the top of the global as well as national development agendas. To accomplish this objective, it is necessary to identify the people who are the poorest of the poor. Being unable to distinguish the poorest from the moderately poor does not provide additional incentives for addressing the conditions of those at the very bottom of the distribution, who may be characteristically very different (Devereux 2003; Harriss-White 2005) and may require different types of resources and assistance (Lipton 1983) than those who are moderately poor. Deprivations among the poorest may also reflect more chronic forms of deprivations (Aliber 2003; McKay and Lawson 2003). This type of gradation is equivalent to setting different thresholds for 'identification', which is one of the crucial steps in the measurement of poverty (Sen 1976).

Discussions on poverty gradients are common with respect to monetary poverty. Since 1990, the World Bank has used different income poverty thresholds such as USD1.25/day and USD2/day to identify different levels of poverty (see World Bank 1990; Ravallion et al. 1991; Chen and Ravallion 2010). Poor people who suffer more stringent forms of poverty have been variously referred to as ultra-poor, destitute, extreme poor, and severely poor. The term 'ultra-poverty', pioneered by Lipton (1983, 1988), was coined for identifying the poorest of the poor by setting a more stringent monetary threshold.[1] In practice, however, organizations have been known to use multi-criteria approaches, which include both monetary and non-monetary criteria, to target the ultra-poor.[2]

The World Bank refers to those living below USD1.25/day as the 'extreme poor'. Indeed monetary deprivation is one form of many deprivations that may impair the freedom a person enjoys, but it may neither reflect nor capture deprivations in other dimensions. The term 'destitution' has been coined for identifying the poorest from a multidimensional perspective (see Devereux 2003; Harriss-White 2005; Alkire et al. 2014a).[3] However the poorest of the poor are referred to, the primary objective has been to identify a subset of the poor population who experience a more stringent form of poverty. We will use the word 'stringent' to refer to a subset of the multidimensionally poor who are strictly poorer than the subset we denote as 'moderately' poor.

Although various monetary approaches have been used to distinguish the poorer from the moderately poor, efforts within a multidimensional framework are scarce. Given the emerging interest and application of the multidimensional approach to the measurement of poverty as well as the identification of the poor, in this working paper, we explore the multidimensional identification technique proposed by Alkire and Foster (2011) within the counting framework that respects the

1 The approach of Lipton has been used by Kakwani (1993). For other monetary approaches, see Cornia (1994); Klasen (2000); Roberts (2001); Aliber (2003); IFPRI (2007); Bird and Manning (2008); Harrigan (2008); Ellis (2012).

2 Examples include but are not limited to BRAC in Bangladesh (Halder and Mosley 2004) and Bandhan in the state of West Bengal in India (Banerjee et al. 2011). See Chapter 4 of Alkire et al. (2015a) for further examples.

3 Devereux (2003) proposed identifying the destitute in terms of an inability to meet subsistence needs, assetlessness, and dependence on transfers; whereas, Harriss-White (2005) asserted that destitution had monetary as well as non-monetary (social and political) aspects. The Oxford Poverty and Human Development Initiative refer to those suffering a more stringent set of deprivations in the global Multidimensional Poverty Index (MPI) as 'destitute' (Alkire at al. 2014a).

ordinal nature of dimensions that are most commonly used in practice.[4] In the counting framework, there are two different forms of cutoffs. One is a set of deprivation cutoffs to identify deprivations in each dimension. The other is a poverty cutoff that identifies the poor population using the sum of their (weighted) deprivation profiles. Unlike monetary approaches, there is more than one way to identify a poorer subset of the poor population.

One straightforward way to identify a subset of poor people is by setting a more stringent poverty cutoff, similar to what is followed in monetary approaches. A more stringent poverty cutoff reflects a higher intensity of simultaneous deprivations in multiple dimensions. For example, a person may be identified as poor if the person is deprived in three or more of ten dimensions, but a stringent poverty cutoff may require a person to be deprived in, say, any five or more of ten dimensions. We refer to this approach for identifying the subset of the poor as the *intensity approach*.

A second approach for identifying a subset of the poor is to apply a set of more stringent (or ultra) deprivation cutoffs. For example, rather than defining deprivation in child undernutrition as two standard deviations below the median, one may define a more stringent deprivation cutoff for undernutrition, such as using three standard deviations below the median to identify a more extreme form of deprivation in that dimension: severe undernutrition. A subset of poor people may be identified as those who suffer at least the poverty cutoff level of ultra-deprivations. We refer to this approach as the *depth approach*.

If both of these measures are feasible, then which should be used and why? Addressing this question requires empirical as well as conceptual analysis. For example, it might be that both the intensity and depth approaches identify the same people as stringently multidimensionally poor and, furthermore, that their trends move together. In that case, it does not matter which measure is used. However, it also might be that the different approaches identify completely or partially distinct subsets of the poor and that the reduction of these subsets does not move in tandem. In this case, either a normative choice must be made to decide which subset of stringent poverty is more appropriate for the purpose at hand, or both may be reported. Furthermore, one may wish to deepen our understanding by analysing characteristics that are more associated with one form of stringent poverty than the other is.

To explore these empirical issues, we study the evolution of multidimensional poverty in Nepal. The country registered a very strong reduction in both monetary and multidimensional poverty in the pre-earthquake period. The reduction in USD1.25/day poverty rate was much faster than the reduction in USD2/day poverty, which suggests a pro-poor trend. The country also swiftly and significantly reduced its multidimensional poverty rate as measured by the global Multidimensional Poverty Index (MPI). Has this noteworthy reduction in the global MPI poverty rate been accompanied by a larger reduction in more stringent forms of multidimensional poverty? In order to answer this question, we define two identification criteria based on the intensity and the depth approaches, and apply them to identify two subsets of the MPI poor population. The measures are distinct, in that many people in intensity poverty are not deeply poor and vice versa. Nationally, the relative reduction in the intensity poverty rate was faster than the relative reduction in the MPI

[4] For a number of historical applications of counting approaches to identification, see Chapter 4, and for a list of international and national adaptations of the approach to measurement proposed by Alkire and Foster (2011), see Chapter 5 of Alkire et al. (2015a).

poverty rate, which is again positive, but the relative reduction in the depth poverty rate was slower than the MPI poverty reduction. Probing this with two logistic regression models, we find that different characteristics are associated with intense poverty than those that are associated with depth poverty.

This paper is structured as follows. Section 2 presents the identification methodology in the counting approach framework. We outline how linked subsets of the poor population may be identified in Section 3. Section 4 presents the level and evolution of multidimensional poverty in Nepal between 2006 and 2011. In Section 5, we examine how different characteristics are associated with different types of multidimensional poverty. Section 6 concludes.

2 Identification of the poor in the counting approach framework

We present the theoretical framework using a hypothetical society of population size n. The set of population in the society is denoted by N. The well-being of the society is assessed by $d \geq 2$ dimensions, which may include but are not limited to standard of living, health, education, access to basic services, etc. We denote the achievement or performance of any person i in dimension j by $x_{ij} \in \mathbb{R}_{+}$, where a higher value implies higher achievement. Achievements of the population in d dimensions are summarized by a matrix $X \in \mathbb{R}_{+}^{n \times d}$, where vector $x_{i\cdot}$ denotes its i^{th} row (contains d achievements of person i) and $x_{\cdot j}$ denotes its j^{th} column (contains n achievements in dimension j). Of the n persons, we denote the number of people identified as poor by q. The set of poor people and the set of non-poor people are denoted by Z and $N \backslash Z$, respectively. An identification function ρ is used to identify the poor population, such that $\rho(x_{i\cdot}; \theta) = 1$ whenever $i \in Z$ and $\rho(x_{i\cdot}, \theta) = 0$ whenever $i \notin Z$ or $i \in N \backslash Z$, where θ is the set of parameters. Unlike in the unidimensional framework, the identification function depends on the *identification approach*.

A frequently used approach for identification in a multidimensional context is the *censored achievement approach*, where a deprivation cutoff $z_j \in \mathbb{R}_{++}$ is defined for each dimension j, which determines whether any person i is deprived (whenever $x_{ij} < z_j$) or not deprived (whenever $x_{ij} \geq z_j$) in each dimension j.[5] All d deprivation cutoffs are summarized by vector $\boldsymbol{z} \in \mathbb{R}_{++}^{d}$. Note that we use a bold font to denote a vector throughout this paper. In the censored achievement approach, the achievement matrix X is censored by the deprivation cutoff vector $\boldsymbol{z}$ to obtain the achievement matrix $\tilde{X}$, such that $\tilde{x}_{ij} = x_{ij}$ if $x_{ij} < z_j$ and $\tilde{x}_{ij} = z_j$, otherwise. The censoring process ignores the achievements that are in excess of the deprivation cutoff of each dimension to prevent the substitution of achievements below the deprivation cutoff in one dimension by the achievement above the deprivation cutoff in another dimension. The justification behind such censoring is that permitting such substitutions may have misleading policy implications. For example, if a poor person has very inadequate access to basic health services but the person is not deprived in standard of living, then it is hard to justify that the person can be lifted out of poverty by further improving

[5] For a range of studies that use the censored achievement approach to identification, see Alkire et al. (2015a).

the living standard—either by providing a cash transfer or additional assets—without improving their ability to enjoy better health services.[6]

Some studies that primarily focus on the measurement of poverty rather than only the identification of the poor assume the underlying dimensions to be cardinally meaningful and use a union criterion to identify the poor. A union criterion identifies a person as poor if the person is deprived in any dimension, and its use is justified when it can be assumed that data are accurate, that each person's preferences are not to be deprived in any dimension, and that every deprivation should count. The assumption about cardinality is problematic as the majority of indicators used in practice are ordinal. In this paper, we explore the frequently used *dual-cutoff counting approach* to identification within the censored achievement approach put forward by Alkire and Foster (2011).

In the dual-cutoff counting approach, a poor person is identified in two stages. The first stage identifies the deprivations by assigning a deprivation status g_{ij} to each person in each dimension, such that $g_{ij} = 1$ if $\tilde{x}_{ij} < z_j$; and $g_{ij} = 0$, otherwise. We denote the relative value or weight assigned to dimension j by w_j such that $w_j > 0$ and $\sum_{j=1}^{d} w_j = 1$. These relative weights are summarized by the relative-weight vector w. The deprivation status and the relative weights are used to obtain the weighted deprivation score (c_i) for each person i: $c_i = \sum_{j=1}^{d} w_j g_{ij}$. The deprivation score indicates the share of weighted deprivations each person experiences. *Intensity* is the average share of deprivations experienced by those persons who are identified as poor.

In the second stage, the poor are identified by selecting a cross-dimensional poverty cutoff k. A person is identified as poor if the person's deprivation score is equal to or larger than k, where $k \in (0,1]$. Thus, the identification criterion involves two different types of cutoffs—a set of deprivation cutoffs z and a poverty cutoff k. The set of parameters in this framework is represented by $\theta = \{\boldsymbol{w}, \boldsymbol{z}, k\}$. Formally, any person i is identified as poor, i.e., $\rho(x_{i\cdot}; \theta) = 1$, if $c_i \geq k$ and non-poor, i.e., $\rho(x_{i\cdot}; \theta) = 0$, if $c_i < k$. Note two special cases: $k \in (0, \min\{w_1, \dots, w_d\}]$ denotes the *union* criterion to identification, whereas $k = 1$ denotes the *intersection* criterion to identification.

We denote the *multidimensional headcount ratio* or the proportion of the population identified as poor by $H = q/n$. The *dimensional uncensored headcount ratio* or the proportion of population deprived in a particular dimension j, irrespective of deprivations in any other dimension, is denoted by $h_j = \left[\sum_{i=1}^{n} g_{ij}\right]/n$. Any non-union criterion for identification; however, censors any deprivations of the non-poor. In that case, we denote the *dimensional censored headcount ratio* or the proportion of the population who are identified as poor, as well as deprived, in a given dimension j by $h_j(k) = \left[\sum_{i=1}^{n} \rho(x_{i\cdot}; \theta) g_{ij}\right]/n$.

[6] Another approach analogous to the consumption expenditure or income approach to identification is the aggregate achievement approach. Within this approach, the achievements of each person in all dimensions are aggregated using an aggregation function $f(x_{i\cdot})$ and then a poverty cutoff $\bar{f}$ is used to identify the poor, such that $\rho(x_{i\cdot}) = 1$ if $f(x_{i\cdot}) \leq \bar{f}$ and $\rho(x_{i\cdot}) = 0$, otherwise. This approach allows the substitution of achievements between any two dimensions at any level, permitting a poor person to become non-poor by improving her achievement in a non-deprived dimension even when her achievements are unaltered in deprived dimensions.

3 Identification of a linked subset of poor

We next explore ways by which a subset of the poor population can be identified in the dual-cutoff counting framework, such that each member of that subset is poorer than poor persons who do not belong to that subset. After obtaining the general result, we present two practical ways of identifying such a subset, where one way focuses purely on intensity and the other purely on depth. The task is quite straightforward in the unidimensional framework, where a more stringent poverty cutoff is used to identify the poorer population, who may be referred to as the *ultra-poor*. Lipton (1983, 1988), for example, used a more stringent income threshold that only reflected daily calorie requirements. In the context of Eastern European countries, Cornia (1994) classified those with incomes below a social minimum as poor but identified the ultra-poor population using an income threshold below the subsistence minimum. In the international context, the International Food Policy Research Institute (IFPRI) (2007) identified a subset of the USD1.25/day poor population as ultra-poor if they were living on less than USD0.50 a day. In South Africa, Klasen (2000) identified those in the poorest quintile ranked by their consumption expenditures as ultra-poor; whereas Roberts (2001) and Aliber (2003) identified ultra-poor as those whose income was less than half of the original poverty line.

Unlike in the unidimensional framework, the dual-cutoff counting approach framework uses two distinct types of thresholds for identification. In this paper, we use the term, *linked subset,* because each person in the poorer subset is *also* poor by the original identification criterion. Let us denote the subset of poorer people to be identified by $\bar{Z}$. A different set of parameter values are used to identify members of the subset, which we denote by $\bar{\theta} = \{\bar{w}, \bar{z}, \bar{k}\}$. Now we ask: What is the relationship between the set of parameters θ that is used for identifying the set of poor Z and the set of parameters $\bar{\theta}$ that is used for identifying the poorer subset $\bar{Z}$?

The following theorem sets out the general relationship between these parameters and sets of the poor.

Theorem: For any deprivation matrix X and for all deprivation cutoff vectors $\boldsymbol{z}, \bar{\boldsymbol{z}}$, for all relative-weight vectors $\boldsymbol{w}, \bar{\boldsymbol{w}}$ and for all $k, \bar{k} \in (0,1]$, (1) $\bar{Z} \subseteq Z$ if and only if $\bar{\boldsymbol{z}} \le \boldsymbol{z}$ whenever $k \in (0, \min\{w\}]$, (2) $\bar{Z} \subseteq Z$ if and only if $\bar{\boldsymbol{z}} \le \boldsymbol{z}$ whenever $k = \bar{k} = 1$, and (3) $\bar{Z} \subseteq Z$ if and only if $\bar{\boldsymbol{z}} \le \boldsymbol{z}$, $\bar{k} \ge k$ and $\bar{\boldsymbol{w}} = \boldsymbol{w}$ whenever $k \in (\min\{\boldsymbol{w}\}, 1)$.

Proof. See Appendix A.

For reasons of generality, the theorem identifies a weak subset of poor people ($\bar{Z} \subseteq Z$) and imposes weak restrictions on the deprivation cutoffs and the poverty cutoff. The theorem covers, for example, the particular possibility that all d dimensions may be binary or dichotomous so that it is only possible to have $\bar{\boldsymbol{z}} = \boldsymbol{z}$. The expression $\bar{\boldsymbol{z}} \le \boldsymbol{z}$ permits all deprivation cutoffs to be identical in both vectors, or to be strictly more stringent. Similarly, the expression $\bar{k} \ge k$ permits equality in the poverty cutoffs as well as a situation in which $\bar{k}$ is greater than k such that some person could be defined as poor by k who would not be identified as poor by $\bar{k}$.

The main motivation of this paper however is to define situations in which strict subsets of the poor could possibly be identified. We define the expression $\bar{\boldsymbol{z}} < \boldsymbol{z}$ to imply $\bar{z}_j \le z_j$ for all j and $\bar{z}_j < z_j$

for at least one j and expression $\bar{k} > k$ to indicate that the poverty cutoff $\bar{k}$ is strictly higher than (more stringent than) k in the non-intersection case. Note that even when dimensions have more than one category and strict restrictions $\bar{\boldsymbol{z}} < \boldsymbol{z}$ and $\bar{k} > k$ are applied singly or together, these are not sufficient for ensuring the identification of a strict subset of poor people. For example, it might be the case that no person is deprived according to the more stringent deprivation or poverty cutoff(s)—indeed, that would be a desirable situation, but it is already included because the empty set is a subset of all sets. It also might be that all poor persons in Z were identified as stringently poor and thus are members of $\bar{Z}$. Even if the sets are identical or if one is the empty set, this result still adds information regarding the structure of poverty. Although restrictions $\bar{\boldsymbol{z}} < \boldsymbol{z}$ and $\bar{k} > k$ are not sufficient for ensuring $\bar{Z} \subset Z$, these restrictions are necessary in order to have $\bar{Z} \subset Z$. This is shown in the following corollary.

Corollary: For any deprivation matrix X and for all deprivation cutoff vectors $\boldsymbol{z}, \bar{\boldsymbol{z}}$, for all relative-weight vectors $\boldsymbol{w}, \bar{\boldsymbol{w}}$ and for all $k, \bar{k} \in (0,1]$ and for any non-empty set of poor Z, (1) $\bar{Z} \subset Z$ only if $\bar{\boldsymbol{z}} < \boldsymbol{z}$ whenever $k \in (0, \min\{w\}]$, (2) $\bar{Z} \subset Z$ only if $\bar{\boldsymbol{z}} < \boldsymbol{z}$ whenever $k = \bar{k} = 1$, and $\bar{Z} \subset Z$ only if either (3a) $\bar{\boldsymbol{z}} < \boldsymbol{z}$, $\bar{k} \geq k$ and $\bar{\boldsymbol{w}} = \boldsymbol{w}$ or (3b) $\bar{\boldsymbol{z}} \leq \boldsymbol{z}$, $\bar{k} > k$ and $\bar{\boldsymbol{w}} = \boldsymbol{w}$ whenever $k \in (\min\{\boldsymbol{w}\}, 1)$.

Proof: The proof directly follows from the theorem above and additionally requires eliminating the cases where $\bar{\boldsymbol{z}} = \boldsymbol{z}$ and/or $\bar{k} = k$.

What are the interpretations of the results in the theorem and in the corollary? The overall results have three parts, all of which require that $\bar{\boldsymbol{z}} \leq \boldsymbol{z}$ or $\bar{\boldsymbol{z}} < \boldsymbol{z}$, respectively. Intuitively from the theorem, the ultra-deprivation cutoffs for identifying the poorer subset must not be higher (less *stringent*) than the corresponding deprivation cutoffs used for identifying the poor population. In other words, a person in $\bar{Z}$ should not be identified as deprived in a dimension with respect to the deprivation cutoff $\bar{z}_j$ if the person is not identified as deprived with respect to the deprivation cutoff z_j. Why is this requirement necessary? Consider any non-poor person i, but suppose the ultra-deprivation cutoff is such that $\bar{z}_j > x_{ij} \geq z_j$. Clearly, person i is ultra-deprived in dimension j. Now if we assign enough weight to dimension j so that the weight is equal to or higher than the poverty cutoff, then person i belongs to $\bar{Z}$. Hence, $\bar{Z}$ cannot be a subset of Z. Note that if $\bar{z}_j > z_j$, then there always exists some combination of achievements, weights, and poverty cutoffs for which $i \in \bar{Z}$ but $i \notin Z$. Based on the corollary, $\bar{z}$ should be a vector containing a set of d ultra-deprivation cutoffs such that $\bar{\boldsymbol{z}} < \boldsymbol{z}$.

Unlike the relationship between the two sets of deprivation cutoffs $\boldsymbol{z}$ and $\bar{\boldsymbol{z}}$, relationships between the two poverty cutoffs k and $\bar{k}$ and between the two sets of relative weights $\boldsymbol{w}$ and $\bar{\boldsymbol{w}}$ are not universal. Whenever, $k \in (0, \min\{\boldsymbol{w}\}]$, which is equivalent to the union criterion of identification, it is both necessary and sufficient to have $\bar{\boldsymbol{z}} \leq \boldsymbol{z}$ in order to have $\bar{Z} \subseteq Z$ and it is necessary to have $\bar{\boldsymbol{z}} < \boldsymbol{z}$ in order to create the potential to identify a poorer subset.

Intuitively, a union criterion identifies anyone having a deprivation in any dimension as poor and, therefore, as long as a set of ultra-deprivation cutoffs is used, the identification of a poorer subset is possible. Similarly, whenever $k = 1$, which is equivalent to the intersection criterion of identification, requiring a person to be deprived in all dimensions simultaneously, it is both necessary and sufficient to have $\bar{\boldsymbol{z}} \leq \boldsymbol{z}$ and $\bar{k} = k = 1$ in order to ensure $\bar{Z} \subseteq Z$ and it is necessary to have $\bar{z} < z$ and $\bar{k} = k = 1$ in order to ensure $\bar{Z} \subset Z$.[7] Finally, whenever $k \in (\min\{\boldsymbol{w}\}, 1)$, which is equivalent to an intermediate criterion for identification, it is necessary and sufficient to have $\bar{\boldsymbol{z}} \leq \boldsymbol{z}$, $\bar{k} \geq k$, and $\bar{\boldsymbol{w}} = \boldsymbol{w}$ in order to have $\bar{Z} \subseteq Z$ and necessary to have either $\bar{\boldsymbol{z}} < \boldsymbol{z}$, $\bar{k} \geq k$, and $\bar{\boldsymbol{w}} = \boldsymbol{w}$ or $\bar{\boldsymbol{z}} \leq \boldsymbol{z}$, $\bar{k} > k$, and $\bar{\boldsymbol{w}} = \boldsymbol{w}$ in order to have $\bar{Z} \subset Z$. Intuitively, to ensure the identification of the poorer subset in this case, we must ensure that either a set of ultra-deprivation cutoffs, a more stringent poverty cutoff, or both is used, but, importantly, the set of relative weights should be the same.

It is clear from the above discussion that there is more than one way to draw distinctions between the moderately poor population and the ultra-poor population. Let us look at the cases more closely. The use of a union approach for identification must be carefully understood. Some regard it to be an advantage that the union criterion does not require imposing any restriction on the set of weights and thus avoids normative judgements. This is a misunderstanding. The size of a union headcount ratio can be reduced by dropping dimensions that have higher deprivation headcount ratios than other dimensions. Thus, the union approach does not avoid normative decisions; instead, the normative decisions are concentrated in the selection of dimensions to include or exclude, and these decisions may greatly affect identification.

Still, if a rights-based approach is used where every dimension is accurately measured and deprivations are of universal, equal, and inalienable importance, then a union approach may appear to be appropriate. Note, however, that the union approach often identifies an unreasonably large fraction of the population as poor. In addition, while relative-weights are not required for identification using the union approach, they are still required for exploring intensity (the average share of deprivations poor people experience). That is, relative weights are still required to assess who is suffering a higher share of multiple deprivations within the poor subset of population.

Similar restrictions on parameters are also required for the intersection criterion, which requires that a person should be identified as poor only when the person is simultaneously deprived in all dimensions. Like the union criterion, the intersection criterion also is often claimed to have an advantage in that it avoids normative judgements because it does not require imposing any restriction on the set of weights as the choice of weights does not matter in this case. However, again this is a misunderstanding. The identification of who is poor is highly sensitive to the choice of dimensions, and so in the intersection approach, like union, the selection of dimensions requires very demanding normative judgements. Moreover, this approach often identifies a strikingly low fraction of the population as poor. In order to prevent this, the dimensions must be carefully restricted in terms of size and joint distribution. Thus again in the intersection approach, the measure can be highly sensitive to the choice of dimensions; hence their selection must be normatively justified.

The intermediate criterion may appear to be demanding because it requires a larger number of restrictions on parameters, but this criterion allows more flexibility in practice.

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[7] It is straightforward to verify that the restrictions $\bar{k} \in (0,1]$, $\bar{k} \geq k$ and $k = 1$ together imply $\bar{k} = 1$.

Based on the restrictions on parameters in the theorem, we now define two restricted approaches for identifying the poor: the *depth approach* and the *intensity approach*. For a given set of dimensions and a fixed poverty cutoff k and weight vector $\boldsymbol{w}$, if the poorer subset of the population is identified using a set of ultra-deprivation cutoffs $\bar{\boldsymbol{z}} < \boldsymbol{z}$, we refer to this approach as the *depth approach*. Let us denote the ultra-deprivation status of depth-poor person i in dimension j by $\bar{g}_{ij}$ such that $\bar{g}_{ij} = 1$ if $x_{ij} < \bar{z}_j$ and $\bar{g}_{ij} = 0$ otherwise, and, given that $\bar{\boldsymbol{w}} = \boldsymbol{w}$, the weighted ultra-deprivation score for each person i is denoted by $\bar{c}_i = \sum_{j=1}^{d} w_j \bar{g}_{ij}$. If the set of parameters for identification is denoted by $\bar{\theta}_D = \{\boldsymbol{w}, \bar{\boldsymbol{z}}, k\}$, then $\rho(x_{i\cdot}; \boldsymbol{w}, \bar{\boldsymbol{z}}, k) = 1$ if $\bar{c}_i \geq k$ and $\rho(x_{i\cdot}; \boldsymbol{w}, \bar{\boldsymbol{z}}, k) = 0$ if $\bar{c}_i < k$. We denote the subset of poor people who are *depth poor* by $\bar{Z}_D$, their number by $\bar{q}_D$, and the *depth poverty rate* by $H_D = \bar{q}_D/n$. The depth approach is compatible with all three identification criteria—union, intersection, and intermediate—subject to different restrictions on parameters as presented in the theorem earlier.

Unlike the depth approach, the intensity approach identifies the subset of poor people by choosing a more stringent poverty cutoff $\bar{k}_I > k$, while keeping fixed the set of dimensions, the weight vector, and the set of deprivation cutoffs used for identifying the set of poor in Z. The intensity approach is analogous to the identification of the poorer subset in the unidimensional context. We denote the set of parameters to identify the intensity poor by $\bar{\theta}_I = \{\boldsymbol{w}, \boldsymbol{z}, \bar{k}_I\}$ such that $\rho\left(x_{i\cdot}; \boldsymbol{w}, \boldsymbol{z}, \bar{k}_I\right) = 1$ if $c_i \geq \bar{k}_{I,}$ and $\rho\left(x_{i\cdot}; \boldsymbol{w}, \boldsymbol{z}, \bar{k}_I\right) = 0$ if $c_i < \bar{k}_I$. Note that the intensity approach is not compatible with all three identification criteria. For the intersection criterion, the poverty cutoff is already equal to its maximum possible value and so setting a more stringent poverty cutoff is not feasible. The union criterion in the intensity approach can be used as the original identification criterion, in which case the stringent poverty cutoff requires a non-union criterion such that $\bar{k}_I > k$. The intermediate criterion requires both $\bar{k}_I > k$ and $\bar{\boldsymbol{w}} = \boldsymbol{w}$. We denote the subset of poor who are *intensity poor* by $\bar{Z}_I$, their number by $\bar{q}_I$, and the intensity poverty rate by $H_I = \bar{q}_I/n$.

The intensity approach has been used by the United Nations Development Programme (UNDP 2010) and the Oxford Poverty and Human Development Initiative (OPHI) to identify the severely poor population (see methodological details in Alkire et al. 2014a), which is a subset of the MPI poor population. The MPI poor are those who suffer one-third or more of the weighted deprivations in ten indicators (Alkire and Santos 2010, 2014); whereas the severely poor are those who suffer half or more of the weighted deprivations in the same set of ten indicators and with respect to the same set of deprivation cutoffs. The intensity approach has also been used by Alkire et al. (2015b) to identify the poorest billion people living across 109 developing countries. Alkire and Seth (2015) use both the depth and intensity approaches, and their intersection, to divide the overall MPI poor population into different subsets of the poor to study the evolution of multidimensional poverty according to these subsets in India between 1999 and 2006. The depth approach has recently been used by Alkire et al. (2014a) and Alkire and Robles (2015) to study both depth and intensity approaches in 49 and 100 developing countries, respectively. They measure destitution using the same set of ten indicators but apply more stringent deprivation cutoffs for eight of these ten indicators. A person is destitute if the person is deprived in one-third or more of the weighted deprivations in these ten indicators subject to the stringent deprivation cutoffs; a cutoff of one-half is also applied to identify the severely poor according to the intensity approach.

The depth and intensity approaches capture different forms of multidimensional poverty and need not identify the same poorer subset of the poor. The intensity approach identifies the poorer subset by capturing a multiplicity of deprivations in the same set of indicators but ignores information on the depth of deprivations even when the information can be feasibly captured. The depth approach on the other hand captures a multiplicity of deprivations but in terms of ultra-deprivations whenever the information on depth is available. When all dimensions are cardinal (ratio scale), the information on depth can be reflected by computing normalized shortfalls from the deprivation cutoff. However, when dimensions are ordinal, such computations are not feasible, but the depth approach can still be implemented and proves to be a useful tool.

4 Evolution of multidimensional poverty in Nepal between 2006-2011

We now provide an inter-temporal illustration of the methodology using the evolution of multidimensional poverty in Nepal, a land-locked country bordered by India and China. In the first decade of the new millennium, before their tragic earthquake of 2015, Nepal had shown dramatic improvements both in terms of reducing monetary poverty as well as multidimensional poverty. The World Bank data show that between 2003 and 2010, the proportion of the population living below USD1.25/day fell from 53.1 per cent to 23.7 per cent, by 10.9 per cent per annum in relative terms. The proportion of the population living below USD2/day, however, fell only from 77.3 per cent to 56 per cent, by 4.5 per cent per annum in relative terms. The pattern of reduction in monetary poverty clearly shows that the relative reduction in the USD2/day poverty rate has been much slower than the relative reduction in the USD1.25/day poverty rate, which implies that the proportion of population suffering from the extreme form of monetary poverty fell faster.

Data from Alkire et al. (2014b) show that between 2006 and 2011, the proportion of the MPI poor population or the MPI poverty rate fell from 64.7 per cent to 44.2 per cent, by 7.4 per cent per annum in relative terms. Thus, the annual relative reduction in the proportion of MPI poor has been slower than that in the USD1.25/day poverty rate but faster than the USD2/day poverty rate. What happened to the more stringent forms of multidimensional poverty? Like monetary poverty, did these more stringent forms of multidimensional poverty rate fall faster than moderate poverty?[8] We explore this question by applying the depth approach and the intensity approach defined above to identify two more stringent forms of multidimensional poverty and explore their interrelationships.[9]

[8] Exploring whether the stringently poorer subset of the poor are 'catching up' or 'falling behind' is not a straightforward comparison. It entails comparing both the absolute rates of change and relative rates of change (see Chapter 9 of Alkire et al. 2015a). If the poorer subset has faster poverty reduction in both relative and absolute terms, the poorer subset is 'catching up'. When poverty rates go down for both the moderately poor and the poorer subset, however, it is unlikely that the absolute reduction among the poorer would be larger due to a much lower initial poverty rate. In this case, one may need to rely on comparing the relative rates of change in order to examine whether, given their starting levels, the poorer group has a proportionately larger reduction. On the other hand, if the poorer subset had slower poverty reduction in both relative and absolute terms, they were 'falling behind'. There may be other intermediary possibilities. Note that at present, we could not assess the statistical significance of differences in relative poverty rates.

[9] Our main purpose in this paper is to explore the counting approach identification technique and not poverty measurement. In order to assess a robust reduction in multidimensional poverty, one may apply a dominance approach (for such a tool, see Yalonetzky 2014) or assess the statistical significance of change (as was done for Nepal in Alkire et al. 2014b).

Note that from this point, we are using modified terms and refer to the d columns of the deprivation matrix—introduced as 'dimensions'—as **indicators**, following the language of the global MPI. The conceptual categories of indicators we henceforth refer to as **dimensions**.

The subsequent analysis uses the global MPI framework developed by Alkire and Santos (2010, 2014), i.e., we use the same set of ten indicators and, because the global MPI identifies a poor person using an intermediate criterion, we use the nested set of MPI relative weights (Table 1, column 3) for both intensity and depth approaches. The global MPI identifies a person as poor if the person is deprived in one-third or more of weighted indicators ($k = 1/3$). The MPI figures for Nepal for both 2006 and 2011 are computed using Demographic Health Survey (DHS) datasets (Alkire et al. 2014a). We use the same datasets, where both datasets have been harmonized following Alkire et al. (2014b) to preserve strict comparability over time. The harmonized dataset for 2006 contains information on 8,624 households or 41,937 persons; whereas the 2011 dataset contains information on 5,208 households or 23,864 persons.[10] Both datasets are nationally representative as well as representative across rural/urban areas and development regions. Households in both 2006 and 2011 were selected through two-stage stratified sampling. All our statistical inferences respect this complex survey design.

In order to identify the *intensity poor*, we use the MPI deprivation cutoffs but increase the poverty cutoff to define a person to be *intensity poor* if the person is deprived in half or more of weighted indicators. In order to identify the *depth poor*, we use a set of more stringent deprivation cutoffs. In this example, a person is identified as *depth poor* if the person suffers one-third or more of weighted ultra-deprivations.

The set of indicators, their relative weights, and deprivation cutoffs for identifying the MPI poor are outlined in the first four columns of Table 1. The next three columns of the table report the uncensored headcount ratios in 2006 and 2011 subject to the MPI deprivation cutoffs and their relative changes over time. All uncensored headcount ratios have improved statistically significantly. The final four columns report the more stringent deprivation cutoffs, which we will refer to as 'ultra' cutoffs. They also provide the corresponding uncensored headcount ratios according to the ultra-deprivation cutoffs and their relative changes between 2006 and 2011.

[10] Anthropometric information in 2011 was not collected for women and children from all households. Rather around half of the sample households were randomly selected for this purpose. Therefore, we were only able to use half of the samples in 2011 but they were still nationally representative.

Table 1: Dimensions, indicators, relative weights, and uncensored headcount ratios

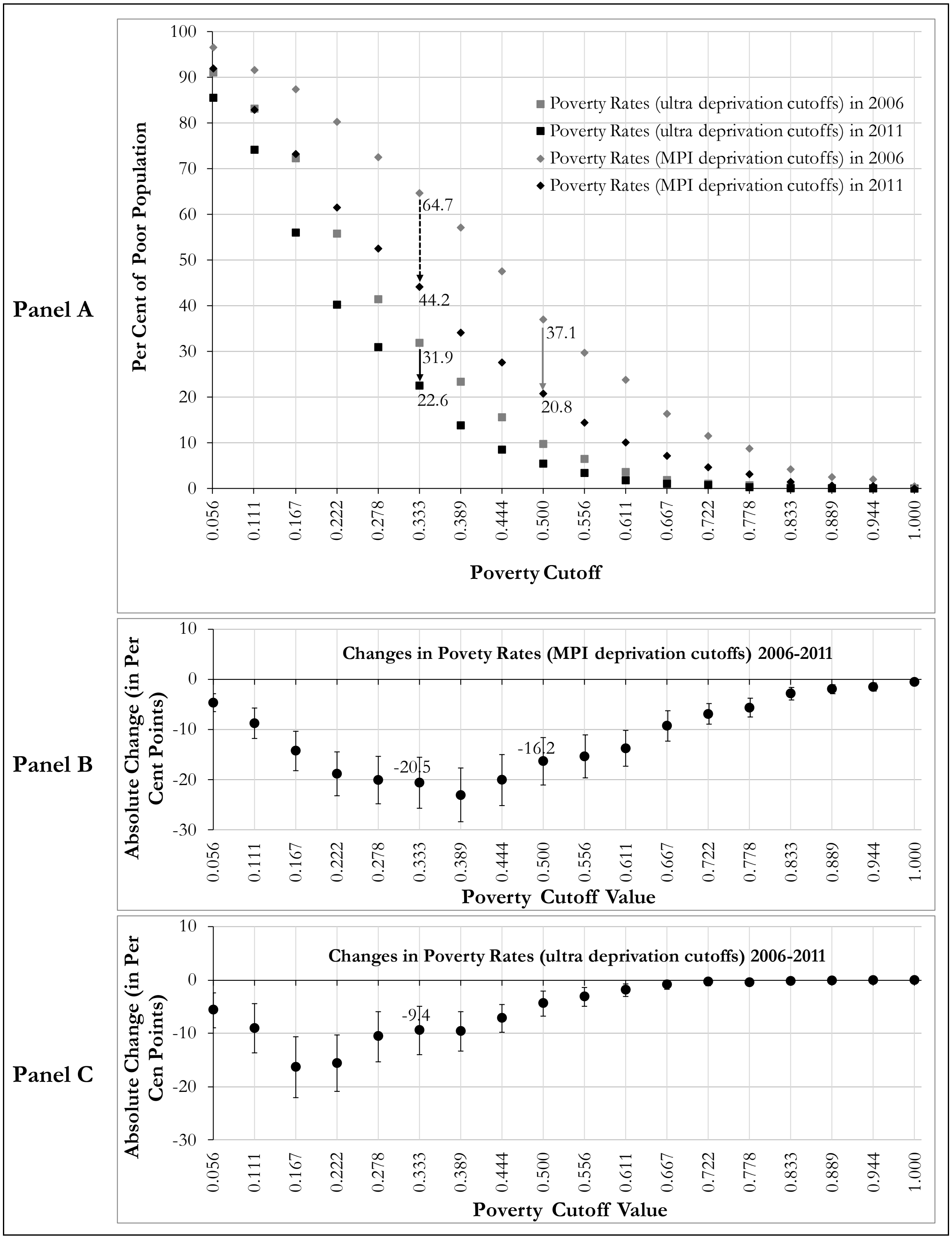
Dimension	Indicator	Relative Weight	Deprivation Cutoff (MPI)	Uncensored Headcount Ratio in Per Cent				Depth Deprivation Cutoff	Uncensored Headcount Ratio for Depth Cutoff in Per Cent			
				2006	2011	Change			2006	2011	Change	
Health	Nutrition	1/6	1 if any adult or child in the household with nutritional information is undernourished (Adult: BMI<18.5 kg/m^2 or Child: -2 standard deviations from the median weight-for-age z-score); 0 otherwise	44.0	32.1	6.1	***	1 if any adult or child in the household with nutritional information is severely undernourished (Adult: BMI<17 kg/m^2 or Child: -3 standard deviations from the median weight-for-age z-score); 0 otherwise	17.4	12.3	6.6	***
	Mortality	1/6	1 if any child has died in the household; 0 otherwise	32.6	22.6	7.1	***	1 if two or more children have died in the household; 0 otherwise	13.4	13.8	-0.5	
Education	Schooling	1/6	1 if no household member has completed five years of schooling; 0 otherwise	30.3	22.2	6.0	***	1 if no household member has completed at least one year of schooling; 0 otherwise	10.0	8.1	4.3	**
	Attendance	1/6	1 if any school-aged child in the household is not attending school up to class 8; 0 otherwise	16.1	8.4	12.1	***	1 if no children are attending school up to the age at which they should finish class 6; 0 otherwise	4.7	2.5	11.9	***
Standard of Living	Electricity±	1/18	1 if the household has no electricity; 0 otherwise	50.7	24.4	13.6	***	1 if the household has no electricity (no change); 0 otherwise	50.7	24.4	13.6	***
	Sanitation	1/18	1 if the household's sanitation facility is not improved or it is shared with other households; 0 otherwise	75.6	60.3	4.4	***	1 if there is no sanitation facility (open defecation); 0 otherwise	52.0	38.3	5.9	***
	Water	1/18	1 if the household does not have access to safe drinking water or safe water is more than a 30-minute walk, round trip; 0 otherwise	17.1	12.9	5.5	**	1 if the household does not have access to safe drinking water or safe water is more than a 45-minute walk, round trip; 0 otherwise	16.5	12.6	5.2	**
	Floor±	1/18	1 if the household has a dirt, sand, or dung floor; 0 otherwise	76.7	70.0	1.8	***	1 if the household has a dirt, sand, or dung floor; 0 otherwise (no change)	76.7	70.0	1.8	***
	Cooking Fuel	1/18	1 if the household cooks with dung, wood, or charcoal; 0 otherwise	86.8	79.3	1.8	***	1 if the household cooks with dung or wood (coal/lignite/charcoal are now non-deprived); 0 otherwise	86.7	79.0	1.8	***
	Assets	1/18	1 if the household does not own more than one of the following: radio, TV, telephone, bike, motorbike, or refrigerator, and does not own a car or truck; 0 otherwise	59.2	28.5	13.6	***	1 if the household has no assets (radio, mobile phone, refrigerator, etc.) and no car; 0 otherwise	20.0	11.0	11.3	***

Notes: The statistical tests of differences are one-tailed tests. ***-Statistically significant at α = 1 per cent, **-Statistically significant at α = 5 per cent, and *-Statistically significant at α = 10 per cent.

± The Deprivation Cutoff (MPI) and the Depth Deprivation Cutoff are the same for this indicator.

Source: Alkire et al. (2014a); Alkire et al. (2014b); plus authors' computations.

Figure 1: Robust multidimensional poverty reduction in Nepal between 2006 and 2011



Source: Authors' computations.

Although different indicators have changed at different rates, how did their joint distribution change? This is captured by identifying the share of people suffering from simultaneous multiple deprivations. Figure 1 presents the evolution of multidimensional poverty in Nepal between 2006 and 2011. The figure has three panels. Panel A presents the proportion of population that are multidimensionally poor for all combinations of the two sets of deprivation cutoffs and eighteen meaningful poverty cutoffs.[11] Multidimensional headcount ratios with respect to the set of MPI deprivation cutoffs have gone down statistically significantly for all poverty cutoffs ranging between 0.056 and 1. The amount of reduction, along with the 95 per cent confidence intervals, is reported in Panel B. The depth poverty rates with respect to the set of ultra-deprivation cutoffs have gone down as well but not statistically significantly for all poverty cutoffs. A statistically significant absolute reduction was evident for all poverty cutoffs ranging from 0.056 to 0.661. The absolute size of the reduction and the 95 per cent confidence intervals are presented in Panel C.

The MPI poverty rate ($k = 1/3$) has gone down from 64.7 per cent to 44.2 per cent (denoted by the dotted arrow)—20.5 percentage points between 2006 and 2011. This is equivalent to a reduction of 7.4 per cent per annum in relative terms. The proportion of intensity poor has gone down nationally from 37.1 per cent to 20.8 per cent (denoted by the solid grey arrow)—16.2 percentage points, which is equivalent to a reduction of 10.9 per cent per annum in relative terms. The reduction pattern in the MPI poverty rate and the intense poverty rate closely follows the reduction pattern in the USD1.25/day poverty rate and USD2/day poverty rate in the sense that rates of the more stringent form of poverty have gone down relatively faster. Similar conclusions however cannot be reached when we look at the reduction in the depth poverty rate across two years, which has gone down from 31.9 per cent to 22.6 per cent (denoted by the solid black arrow)—9.4 percentage points. This is equivalent to a relative reduction of only 6.7 per cent per annum, which is even slower than the reduction in MPI poverty rate. When we view the more stringent form of multidimensional poverty through the depth approach, the reduction has not been faster than the reduction in the MPI poverty rate.

5 Characteristics of multidimensional poverty

The illustration in the previous section clearly suggests that the depth approach and the intensity approach do not necessarily identify the same set of poor people. In fact, both in 2006 and 2011, nearly one-third (32.5 per cent and 32.2 per cent, respectively) of MPI poor people were either identified as depth poor or as intensity poor but not both. In 2006, out of the total MPI poor population, 57.2 per cent are intensity poor and 49.3 per cent are depth poor. Looking further, 20.2 per cent are only intensity poor, 12.3 per cent are only depth poor, 37 per cent are both intensity poor and depth poor, and 30.4 per cent are neither intensity poor nor depth poor—they are moderately poor. Thus, the overlap between the two subsets of the poor is moderate, with 65 per cent of the intensity poor being depth poor and 75 per cent of the depth poor being intensity poor. In 2011, of the total MPI poor population 14.1 per cent are only intensity poor, 18.1 per cent are only depth poor, 33 per cent are both intensity poor and depth poor, and 34.8 per cent are neither intensity poor nor depth poor—and the overlaps among the depth and intensity poor are 70 per cent and 69 per cent, respectively.

[11] Given the weighting structure, the meaningful poverty cutoffs are multiples of 1/18. The poverty cutoff equal to 0.056 is equivalent to the union criterion; whereas the poverty cutoff equal to one is equivalent to the intersection criterion.

The analysis in the previous section raises at least two interesting questions.[12] First, given that the depth approach and the intensity approach do not always coincide, are households with certain characteristics more likely to suffer one form of stringent multidimensional poverty than the other is? Second, given that the pace of changes in the MPI poverty rate and two different forms of stringent poverty rates have been different, are the households facing one or both stringent forms of poverty characteristically different from the households facing only a moderate form of poverty? In this section, we conduct two exercises aiming to explore these questions using a binary response logit model and a multinomial logit model. Because our identification is conducted at the household level and all members residing within a household have the same poverty status, we undertake the regression analysis at the household level.

In the first exercise, we divide the households into two groups: those that are identified as poor by a certain identification method and those that are not identified as poor by the same identification method. Let us denote the set of households identified as poor by $\mathcal{Z}^H$, which may represent, depending on the situation, the set of MPI poor households, the set of depth poor households, or the set of intensity poor households. We may assign a value of $y_h = 1$ for any household h if $h \in \mathcal{Z}^H$ and $y_h = 0$ if $h \notin \mathcal{Z}^H$. In this model, we are primarily interested in understanding how the set of L characteristics is related to the response probability $P(h \in \mathcal{Z}^H | \mathbf{x}_h)$, where the vector $\mathbf{x}_h = (x_h^1, \dots, x_h^L)$ summarizes the L characteristics of a given household h. In other words, we are interested in understanding which of the L characteristics are associated with increasing the probability of a household suffering a particular form of poverty. This can be accomplished by estimating the following binary response logit regression model:

$$P(h \in \mathcal{Z}^H | \mathbf{x}_h) = \frac{\exp(\beta_0 + \mathbf{x}_h \boldsymbol{\beta})}{1 + \exp(\beta_0 + \mathbf{x}_h \boldsymbol{\beta})} \tag{1}$$

where β_0 is the constant term and $\mathbf{x}_h \boldsymbol{\beta} = x_h^1 \beta_1 + \cdots + x_h^L \beta_L$ and $\exp(\cdot)$ is the exponential of its argument. The sign of the regression coefficient for characteristic l, which we denote by β_l, captures the direction of change in the probability of being in poverty associated with an increase in x^l. For example, a positive sign of β_l implies that an increase in x^l is associated with an increase in the probability of being poor.

In a binary response logit model, the change in the probability of being in poverty associated with a one unit increase in the characteristics is determined by computing the *marginal effect* (p_l) of characteristic l as

$$p_l = \frac{\exp(\beta_0 + \tilde{\mathbf{x}} \boldsymbol{\beta})}{[1 + \exp(\beta_0 + \tilde{\mathbf{x}} \boldsymbol{\beta})]^2} \beta_l \,. \tag{2}$$

Note that the marginal effect of a particular characteristic depends on the levels of all characteristics given by the L-dimensional vector $\tilde{\mathbf{x}}$. Thus, the marginal effect of a particular characteristic may be

[12] It would also be useful to explore the dimensional and subnational composition of depth poverty and its change; that analysis lays beyond the scope of this paper (See Alkire et al. 2014a; Alkire and Robles 2015).

different if evaluated at a different vector of values. If any characteristic l is a dummy variable, then its marginal effect is the difference between the probability values evaluated at 1 and at 0, while keeping the values of other characteristics unchanged. Typically, marginal effects are reported at the average of the characteristics, and we do the same in this paper.

The regression coefficients in the logit model have another interesting interpretation in terms of the *odds of* being in poverty. Using the response probability, the odds of household h being in poverty can be computed as

$$\frac{P(h\in Z^H|\mathbf{x}_h)}{P(h\notin Z^H|\mathbf{x}_h)} = \frac{P(h\in Z^H|\mathbf{x}_h)}{1-P(h\in Z^H|\mathbf{x}_h)} = \exp(\beta_0 + \mathbf{x}_h\boldsymbol{\beta}) = \exp(\beta_0) \times \prod_{l=1}^{L} \exp(x_h^l \beta_l). \quad (3)$$

In other words, the odds for a household being in poverty are the ratio of the probability of the household being in poverty to the probability of the household not being in poverty. If the probability of being in poverty is equal to that of not being in poverty, then the odds for being in poverty are *even*. If the probability of being in poverty is larger than the probability of not being in poverty, then the odds of being in poverty are less than one; whereas if the probability of being in poverty is less than the probability of not being in poverty, then the odds of being in poverty are larger than one. It is straightforward to check that whenever $\beta_l > 0$, a one-unit increase in x_l is associated with a $[\exp(\beta_l) - 1]\times 100$ per cent increase in the odds of being in poverty. Similarly, whenever $\beta_l < 0$, a one-unit increase in x_l is associated with a $[1 - \exp(\beta_l)]\times 100$ per cent decrease in the odds of being in poverty.[13]

In the second exercise, we restrict our sample to only MPI poor households and divide them into four groups: neither intensity nor depth poor (M), depth but not intensity poor (D), intensity but not depth poor (I), and both intensity and depth poor (B). We denote any of these four groups by m. We now explore which characteristics are associated with a household falling in one of these four groups of MPI poor households. We know clearly that group M consists of the least poor households and group B consists of the poorest households, but we cannot make any claim on which group is poorer among group D and group I. We thus use the multinomial logit model (MNL), which is a categorical variable model where categories of the dependent variable cannot be ordered.

We assign a value of $y_h^m = 1$ to any MPI poor household h if $h \in \bar{\mathcal{Z}}_m^H | h \in Z^H$ and $y_h^m = 0$ if $h \notin \bar{\mathcal{Z}}_m^H | h \in Z^H$, where Z^H is the set of MPI poor households and $\bar{\mathcal{Z}}_m^H$ is the set of MPI poor households falling in group m. Let us denote the probability of the MPI poor household h falling in group m by π_h^m, such that $\pi_h^M + \pi_h^D + \pi_h^I + \pi_h^B = 1$. The MNL model can be written as

$$\pi_h^m = P(h \in \bar{\mathcal{Z}}_m^H | h \in Z^H; \mathbf{x}_h) = \frac{\exp(\gamma_0^m + \mathbf{x}_h\boldsymbol{\gamma}^m)}{\sum_m \exp(\gamma_0^m + \mathbf{x}_h\boldsymbol{\gamma}^m)}, \quad (4)$$

[13] For a detailed discussion, see Chapter 10 of Alkire et al. (2015a).

where γ_0^m is the constant term and $\mathbf{x}_h\boldsymbol{\gamma}^m = x_h^1\gamma_1^m + \cdots + x_h^L\gamma_L^m$ for all $m = M, D, I, B$. Note that in this case, we need to estimate four constant terms and $4 \times L$ parameters. In order to obtain a unique solution of these parameters, it is usual practice to set (Theil normalization) $\exp(\gamma_0^m + \mathbf{x}_h\boldsymbol{\gamma}^m) = 1$ for any one category. We aim to obtain our result with respect to group M and so we set $\exp(\gamma_0^M + \mathbf{x}_h\boldsymbol{\gamma}^M) = 1$. The revised probabilities with this normalization may be written as

$$\pi_h^M = \frac{1}{1 + \exp(\gamma_0^D + \mathbf{x}_h\boldsymbol{\gamma}^D) + \exp(\gamma_0^I + \mathbf{x}_h\boldsymbol{\gamma}^I) + \exp(\gamma_0^B + \mathbf{x}_h\boldsymbol{\gamma}^B)},$$

$$\pi_h^D = \frac{\exp(\gamma_0^D + \mathbf{x}_h\boldsymbol{\gamma}^D)}{1 + \exp(\gamma_0^D + \mathbf{x}_h\boldsymbol{\gamma}^D) + \exp(\gamma_0^I + \mathbf{x}_h\boldsymbol{\gamma}^I) + \exp(\gamma_0^B + \mathbf{x}_h\boldsymbol{\gamma}^B)},$$

$$\pi_h^I = \frac{\exp(\gamma_0^I + \mathbf{x}_h\boldsymbol{\gamma}^I)}{1 + \exp(\gamma_0^D + \mathbf{x}_h\boldsymbol{\gamma}^D) + \exp(\gamma_0^I + \mathbf{x}_h\boldsymbol{\gamma}^I) + \exp(\gamma_0^B + \mathbf{x}_h\boldsymbol{\gamma}^B)},$$

$$\pi_h^B = \frac{\exp(\gamma_0^B + \mathbf{x}_h\boldsymbol{\gamma}^B)}{1 + \exp(\gamma_0^D + \mathbf{x}_h\boldsymbol{\gamma}^D) + \exp(\gamma_0^I + \mathbf{x}_h\boldsymbol{\gamma}^I) + \exp(\gamma_0^B + \mathbf{x}_h\boldsymbol{\gamma}^B)}.$$

Using these revised probabilities, the MNL model that we are required to estimate may be written as

$$\frac{\pi_h^D}{\pi_h^M} = \exp(\gamma_0^D) \times \prod_{l=1}^{L} \exp(x_h^l\gamma_l^D), \tag{5}$$

$$\frac{\pi_h^I}{\pi_h^M} = \exp(\gamma_0^I) \times \prod_{l=1}^{L} \exp(x_h^l\gamma_l^I), \tag{6}$$

$$\frac{\pi_h^B}{\pi_h^M} = \exp(\gamma_0^B) \times \prod_{l=1}^{L} \exp(x_h^l\gamma_l^B). \tag{7}$$

Note that these equations resemble the log-odds in equation (3) with similar interpretations. Consider any parameter, say, γ_l^D. If $\gamma_l^D > 0$, a one-unit increase in x_l is associated with a $[\exp(\gamma_l^D) - 1]\times 100$ per cent increase in the odds of being *only depth poor* compared to being *neither depth nor intensity poor*. Similarly, whenever $\gamma_l^D < 0$, a one-unit increase in x_l is associated with a $[1 - \exp(\beta_l)]\times 100$ per cent decrease in the odds of being *only depth poor* compared to being *neither depth nor intensity poor*. The marginal effect or the change in probability of being in any of these four groups due to the change in any of the exogenous variables may be similarly computed using the revised probabilities reported above.

For our application, we consider different household characteristics, such as the size of each household, the number of members per sleeping room, the share of working-age male members (15-60 years old), the dependency rate or the share of children (younger than 14 years) and elderly members (older than 60 years), and each household's land ownership information, as well as certain characteristics of head of each household, such as the head's gender, age, and years of education.

Unfortunately, the occupational information is not available for all households in the DHS, and so we instead use the share of working-age male members within each household as a proxy.[14] Table 2 presents summary statistics of various characteristics in 2006 and 2011 across the entire sample. The average values of different characteristics, except land ownership, show statistically significant changes between 2006 and 2011.

Table 2: Summary statistics of household characteristics in 2006 and 2011

Population Characteristics in Nepal	2006	2011	Change	
Average share of children and elderly members (per cent)	44.6	41.9	-2.7	***
Average share of working-age male members (per cent)	38.7	36.4	-2.3	***
Average household size	4.9	4.4	-0.5	***
Average number of household members per bedroom	2.9	2.4	-0.4	***
Average land holding (ha.)	0.41	0.42	0.01	
Share own land usable for agriculture (per cent)	68.1	67.5	-0.6	
Average land holding for usable land (ha.)	0.61	0.63	0.02	
Share of household with male head (per cent)	76.6	70.5	-6.1	***
Average age of household heads (years)	44.2	45.5	1.3	***
Average years of education of household heads	3.4	3.7	0.3	**

Notes: The statistical tests of differences are one-tailed tests. ***-Statistically significant at α = 1 per cent, **-Statistically significant at α = 5 per cent, and *-Statistically significant at α = 10 per cent.

Source: Authors' computations.

Although most characteristics changed between 2006 and 2011, are some of these characteristics associated with particular forms of multidimensional poverty, irrespective of the time period? We explore the relationship by pooling the household survey datasets for both years, which allows us to purge the time-specific effects. Table 3 reports three sets of regression results for MPI poverty, intensity poverty, and depth poverty. The dependent variable for MPI poverty regression is 1 if a household is MPI poor and 0 otherwise; the dependent variable for intensity poverty regression is 1 if a household is intensity poor and 0 otherwise; and the dependent variable for the depth poverty regression is 1 if a household is depth poor and 0 otherwise. The first column of the table reports different characteristics. Within each of the three sets of regression results, the first column reports the regression coefficients (β_0 and $\boldsymbol{\beta}$ as in Equation (1)) and their statistical significance, the second column reports the marginal effects of the characteristics (p_l as in equation (2)), and the third column reports the exponential coefficients ($\exp(\beta_l)$ as in Equation (3)) reflecting the changes in the odds of being in poverty.

Let us first look at the MPI poverty regression results. As expected, the estimated coefficients show that the probability of being in MPI poverty is higher for households with a higher dependency rate and a larger number of household members per bedroom. The probability is lower for households with a larger share of working-age male members and with heads completing more years of education. The probability of being in MPI poverty is lower in urban areas than in rural areas and in

[14] This is admittedly imperfect and is used merely for the purpose of illustration. The Nepal Labour Force Survey 2008 Statistical Report (Government of Nepal 2009) reveals that the labour force participation rate of adult males was 87.5 per cent, whereas that of adult females was 80.1 per cent. For those who were employed, a much larger fraction of males were employed outside the subsistence agricultural sector and on average males worked nearly 8.3 hours more per week than females. Therefore, on average, larger economic support may be expected from working males within households. Note that this should not be generalized for all households.

year 2011 than in year 2006. Interestingly, however, the probability of being in MPI poverty is larger for single-member households and households with male heads. Now the question is which of these characteristics are attributed to a larger change in the probability of being in MPI poverty. Surprisingly, the probability of an average household to be in multidimensional poverty increases by 0.307 units if the household consists of only one member.[15] In other words, the odds of being in MPI poverty increase by 376 per cent [(4.76−1]×100] or 3.76 times for single member households. The probability of an average household being in MPI poverty increases by 0.075 units if the number of household members per bedroom goes up by one unit from the average, and the probability strongly decreases by 0.351 units if the average household lives in an urban instead of rural area.

How are these characteristics related to the probability of being in intensity poverty or in depth poverty? Some characteristics that are consistently related to both intensity poverty and depth poverty in ways similar to how they are related to MPI poverty are the dependency rate, the share of working-age male members, male household head, household head's years of education, and whether the households reside in urban areas. This is not surprising; as described above, more than 65 per cent of the members of one group are also poor according to the other. However, there are some interesting differences. One noticeable difference is that, like MPI poverty, there is a large increase in the probability of being in depth poverty (0.434 units) for the single-member households; whereas no significant change in the probability of being in intensity poverty for these households is observed. Instead, the probability of being in intensity poverty increases with an increase in household size. Second, the land ownership variable is differently related to different forms of poverty. Whether a household owns any land usable for agriculture does not influence the probability of being in MPI poverty but does influence the probability of an average household suffering both forms of stringent multidimensional poverty. Note that the increase in the probability of being in depth poverty (0.53) is more than twice as high as the increased probability of being in intensity poverty (0.25).

This first exercise shows how characteristics may be associated in similar or different ways with different forms of poverty without shedding any light on whether the moderately poor are characteristically different from the poor population facing more stringent forms of multidimensional poverty. In the second exercise, we restrict our sample to only those who are identified as MPI poor. The intensity approach and the depth approach then divide this truncated sample into four groups as we discussed while introducing the MNL earlier. We present the second set of regression results in Table 4, exploring which characteristics are associated with a household's odds of falling in any of the three poorer groups compared to group M. The first column reports the same set of characteristics as in Table 3. Within each set of regression results presented in the next six columns, the first column reports the regression coefficients (γ_0^m and $\boldsymbol{\gamma}^m$) and the second column reports the exponential coefficients reflecting the changes in the odds of being in any poorer group compared to being neither depth nor intensity poor. The marginal effects or changes in probability of being in any group due to changes in different exogenous characteristics are reported in the final three columns. Figures in the last column are the most important for empirical analysis.

[15] The shares of single-member households were 5.1 per cent and 5.6 per cent in 2006 and 2011, respectively.

Table 3: Household characteristics and the probability of being in different forms of multidimensional poverty

Household Characteristics[16]	MPI Poverty			Intensity Poverty			Depth Poverty		
	Coefficient	Marginal Effect	Exp. Coef.	Coefficient	Marginal Effect	Exp. Coef.	Coefficient	Marginal Effect	Exp. Coef.
Dependency Rate (per cent)	0.011***	0.003***	1.01	0.012***	0.002***	1.01	0.008***	0.001***	1.01
Share of working-age male members (per cent)	-0.011***	-0.003***	0.99	-0.006***	-0.001***	0.99	-0.009***	-0.001***	0.99
Household size: six or more (Dummy)	0.050	0.013	1.05	0.302***	0.040***	1.35	0.002	0.000	1.00
Single-member household (Dummy)	1.565***	0.339***	4.78	-0.045	-0.006	0.96	1.981***	0.432***	7.25
Household members per bedroom	0.295***	0.074***	1.34	0.256***	0.032***	1.29	0.117***	0.018***	1.12
Own land usable for agric. (Dummy)	-0.083	-0.021	0.92	-0.172**	-0.022*	0.84	-0.325***	-0.052***	0.72
Male household head (Dummy)	0.970***	0.234***	2.64	0.721***	0.081***	2.06	0.941***	0.128***	2.56
Household head’s age	0.029**	0.007**	1.03	0.032*	0.004*	1.03	0.024*	0.004*	1.02
Household head’s age square	-0.001***	0.000***	1.00	-0.001***	0.000***	1.00	0.000***	0.000***	1.00
Head’s years of education	-0.262***	-0.065***	0.77	-0.260***	-0.033***	0.77	-0.341***	-0.053***	0.71
Head’s years of education square	0.000	0.000	1.00	-0.003	0.000	1.00	0.010***	0.002***	1.01
Urban areas (Dummy)	-1.587***	-0.351***	0.20	-1.332***	-0.122***	0.26	-1.526***	-0.170***	0.22
Eastern region (Dummy)	-0.704***	-0.172***	0.50	-0.603***	-0.068***	0.55	-0.508***	-0.073***	0.60
Central region (Dummy)	-0.532***	-0.132***	0.59	-0.263	-0.032*	0.77	-0.080	-0.012	0.92
Western region (Dummy)	-0.721***	-0.176***	0.49	-0.531***	-0.060***	0.59	-0.556***	-0.079***	0.57
Far-Western region (Dummy)	0.234	0.059	1.26	0.166	0.022	1.18	0.225	0.037	1.25
Year: 2011 (Dummy)	-0.848***	-0.209***	0.43	-0.748***	-0.097***	0.47	-0.422***	-0.066***	0.66
Constant	0.441			-1.102**			-0.832**		
Number of Observations	13,826			13,826			13,826		
F Statistic	87.5			80.7			56.3		

Notes: ***-Statistically significant at α = 1 per cent, **-Statistically significant at α = 5 per cent, and *-Statistically significant at α = 10 per cent.

Source: Authors’ computations.

[16] Population shares for each group are presented in Appendix B.

Table 4: Household characteristics and the probability of MPI poor households being in different poverty groups

Household Characteristics[17]	Only Depth Poor (D)		Only Intensity Poor (I)		Both Intensity & Depth Poor (B)		Marginal Effects		
	Coefficient	**Exp. Coef.**	**Coefficient**	**Exp. Coef.**	**Coefficient**	**Exp. Coef.**	**Only Depth Poor (D)**	**Only Intensity Poor (I)**	**Both Depth & Intensity Poor (B)**
Dependency Rate per cent	-0.001	1.00	0.009***	1.01	0.007***	1.01	-0.001**	0.001***	0.001***
Share of working-age male members (per cent)	-0.005**	0.99	-0.002	1.00	-0.005**	1.00	0.000	0.000	-0.001
Household size: 6 or more (Dummy)	-0.166	0.85	0.307***	1.36	0.266***	1.30	-0.047***	0.029**	0.054***
Single-member household (Dummy)	2.104***	8.20	-1.600***	0.20	0.899***	2.46	0.367***	-0.143***	-0.008
Household members per bedroom	-0.052	0.95	0.204***	1.23	0.138***	1.15	-0.021***	0.019***	0.024***
Own land usable for agric. (Dummy)	-0.117	0.89	0.194	1.21	-0.296***	0.74	-0.003	0.038***	-0.068***
Male household head (Dummy)	0.683***	1.98	0.288*	1.33	0.686***	1.99	0.052***	-0.011	0.098***
Household head's age	0.019	1.02	0.023	1.02	0.028	1.03	0.001	0.001	0.004
Household head's age square	0.000	1.00	0.000*	1.00	-0.001***	1.00	0.000	0.000	0.000**
Household head's years of education	-0.312***	0.73	-0.126**	0.88	-0.380***	0.68	-0.020***	0.010*	-0.060***
Household head's years of educ. Square	0.028***	1.03	-0.004	1.00	0.018***	1.02	0.003***	-0.002***	0.002***
Urban areas (Dummy)	-0.575***	0.56	-0.298**	0.74	-0.923***	0.40	-0.031*	0.013	-0.142***
Eastern region (Dummy)	-0.056	0.95	-0.274	0.76	-0.484**	0.62	0.026	-0.011	-0.088***
Central region (Dummy)	0.226	1.25	-0.197	0.82	0.085	1.09	0.034*	-0.032**	0.014
Western region (Dummy)	-0.393**	0.68	-0.350*	0.70	-0.413*	0.66	-0.025	-0.014	-0.053
Far-Western region (Dummy)	0.143	1.15	0.050	1.05	0.160	1.17	0.010	-0.005	0.025
Year: 2011 (Dummy)	0.146	1.16	-0.543***	0.58	-0.363***	0.70	0.057***	-0.051***	-0.064***
Constant	-1.280**	0.28	-1.678***	0.19	-0.264	0.77			
Number of Observations: 7464									
F Statistic = 20.95									

Notes: ***-Statistically significant at α = 1 per cent, **-Statistically significant at α = 5 per cent, and *-Statistically significant at α = 10 per cent.

Source: Authors' computations.

[17] Population shares for each group are presented in Appendix B.

Household characteristics whose effect on the odds of being in any of these three groups compared to being in group B is negligible are dependency rate, share of working-age male members, and household head's age. Marginal effects of these characteristics are quite different across groups D, I, and B. The probability of being in group D and in group B compared to being in group M increases statistically significantly for households with a male head, for households with heads who have fewer years of education, and for households residing in rural areas. The differences in probabilities are much higher for the poorest group B, which implies that these three characteristics are quite important in differentiating the poorest group B from the least poor group M. Marginal effects of these characteristics are either statistically insignificant or of lower magnitude for group I.

Being in a single-member household increases the probability of being in group D compared to being in group M by 0.367 probability points; whereas the same characteristic reduces the probability of being in group I compared to being in group M. For large households (6 or more members), the probability of being in group I and in group B compared to being in group M increase. Quite interestingly, the ownership of usable land for agriculture appears to affect the affiliation to the poorest group. This characteristic reduces the probability of a household being in group B compared to being in group M by 0.068 probability points.

6 Concluding remarks

The primary objective behind identifying the poorest is to ensure that they are not left behind and less likely to enjoy the benefits of poverty reduction. In this paper, we have explored how the poorest may be identified through the counting-based identification method proposed by Alkire and Foster (2011). We especially concentrate on two different approaches one may undertake to identify a subset containing the poorest of the poor population: the intensity approach and the depth approach. An obvious concern one may raise is 'How much difference does it make whether the poorest are identified through one approach over the other'.

We try to explore this concern by investigating the pattern of multidimensional poverty reduction in Nepal, where the overlap between intensity and depth poverty is only moderate. The country has reduced the stringent form of monetary poverty measured by the USD1.25/day poverty rate faster than the more moderate form of poverty measured by the USD2/day poverty rate. We however could not conclude the same in terms of multidimensional poverty reduction. Although the MPI poverty rate of the country has gone down sharply between 2006 and 2011, the pace of the more stringent form of poverty reduction has been ambiguous, depending on the approach we use for identifying the poorest. The intensity poverty rate has gone down much faster than the MPI poverty rate, but the reduction in the depth poverty rate has been slower.

It has been argued in the literature, as we have discussed in the introduction, that the poorest may be characteristically very different from the moderately poor. Our findings based on the two exercises that we perform in Section 0 of this paper support this point. The first exercise divides the population into two groups by three different criteria: MPI poor vs. non-MPI poor, intensity poor vs. non-intensity poor, and depth poor vs. non-depth poor. Applying a binary response logit regression model, we find that certain characteristics are common across different forms of multidimensional poverty, whether it is MPI poverty or any of the two types of stringent poverty, but certain characteristics are indeed associated with different forms of multidimensional poverty. For example, landlessness is more related to depth poverty than to intensity or MPI poverty.

In our second exercise, we restrict our attention to the sample of MPI poor only and then divide the sample of MPI poor into four groups, where the least poor group consists of those who are neither depth nor intensity poor and the most poor group consists of those who are depth as well as intensity poor. The other two groups of MPI poor are those that are only depth poor and only intensity poor. Applying a multinomial logit regression model, we found five characteristics—urban/rural residence, ownership of land usable for agriculture, household size, and household head's years of education and gender—played a crucial role in the Nepalese context in distinguishing the moderately poor from the poorest of the poor.

Both approaches to identifying the poorest among the poor add value because each of them captures a unique form of stringent poverty. Given that both approaches disagree somewhat in the identification of the poorer, it is crucial to apply both approaches and study their levels, trends, and the characteristics associated with them in a larger set of countries in order to understand the dynamics of stringent poverty better. Given the current global attention on extreme poverty alleviation, it is important that the poorest are identified properly.

References

Aliber, M. (2003). 'Chronic Poverty in South Africa: Incidence, Causes and Policies'. *World Development*, 31(3): 473-490.

Alkire, S., A. Conconi, and S. Seth (2014a). 'Measuring Destitution in Developing Countries: An Ordinal Approach for Identifying Linked Subsets of the Multidimensionally poor'. *OPHI Research in Progress 42a*. Oxford: University of Oxford.

Alkire, S., and J.E. Foster (2011). 'Counting and Multidimensional Poverty Measurement'. *Journal of Public Economics*, 95(7): 476-87.

Alkire, S., J.E. Foster, S. Seth, M.E. Santos, J.M. Roche, and P. Ballon (2015a). *Multidimensional Poverty Measurement and Analysis*. Oxford: Oxford University Press.

Alkire, S., and G. Robles (2015). 'Multidimensional Poverty Index – Winter 2015/16: Brief Methodological Note and Results'. *OPHI Briefing No. 36*. Oxford: University of Oxford.

Alkire, S., J.M. Roche, S. Seth, and A. Sumner (2015b). 'Identifying the Poorest People and Groups: Strategies Using the Global Multidimensional Poverty Index;. *Journal of International Development*, 27(3): 363-87.

Alkire, S., J.M. Roche, and A. Vaz (2014b). 'How Countries Reduce Multidimensional Poverty: A Disaggregated Analysis'. *OPHI Working Paper 76*. Oxford: University of Oxford.

Alkire, S., and M.E. Santos (2010). 'Acute Multidimensional Poverty: A New Index for Developing Countries'. *OPHI Working Paper No. 38*. Oxford: University of Oxford.

Alkire, S., and M.E. Santos (2014). 'Measuring Acute Poverty in the Developing World: Robustness and Scope of the Multidimensional Poverty Index'. *World Development*, 59: 251-74.

Alkire, S., and S. Seth (2015). 'Multidimensional Poverty Reduction in India between 1999 and 2006: Where and How?'. *World Development*, 72: 93-108.

Banerjee, A., E. Duflo, R. Chattopadhyay, and J. Shapiro (2011). 'Targeting the Hard-Core Poor: An Impact Assessment'. Draft report. Cambridge, MA: Massachusetts Institute of Technology, Poverty Action Lab. Available at: https://www.povertyactionlab.org/sites/default/files/publications/110-%20November%202011_0.pdf (accessed on 30 December 2015).

Bird, K., and C. Manning (2008). 'Minimum Wages and Poverty in a Developing Country: Simulations from Indonesia's Household Survey'. *World Development*, 36(5): 916-33.

Chen, S., and M. Ravallion (2010). 'The Developing World is Poorer than We Thought, but No Less Successful in the Fight against Poverty'. *The Quarterly Journal of Economics*, 125(4): 1577-625.

Cornia, G.A. (1994). 'Poverty, Food Consumption, and Nutrition during the Transition to the Market Economy in Eastern Europe'. *The American Economic Review*, 84: 297-302.

Devereux, S. (2003). 'Conceptualising Destitution'. *IDS Working Paper No. 216*. Brighton: Institute of Development Studies.

Ellis, F. (2012). 'We are All Poor Here: Economic Difference, Social Divisiveness and Targeting Cash Transfers in sub-Saharan Africa'. *Journal of Development Studies*, 48(2): 201-14.

Government of Nepal (2009). 'Report on the Nepal Labour Force Survey 2008'. Kathmandu, Nepal: Central Bureau of Statistics, National Planning Commission Secretariat.

Halder, S.R., and P. Mosley (2004). 'Working with the Ultra-Poor: Learning from BRAC Experiences'. *Journal of International Development*, 16(3): 387-406.

Harrigan, J. (2008). 'Food Insecurity, Poverty and the Malawian Starter Pack: Fresh Start or False Start?'. *Food Policy*, 33: 237-49.

Harriss-White, B. (2005). 'Destitution and the Poverty of its Politics—with Special Reference to South Asia;. *World Development*, 33(6): 881-91.

IFPRI (2007). 'The World's Most Deprived. Characteristics and Causes of Extreme Poverty and Hunger'. *2020 Discussion Paper No. 43*. Washington DC: International Food Policy Research Institute.

Kakwani, N. (1993). 'Poverty and Economic Growth with Application to Côte d'Ivoire'. *Review of Income and Wealth,* 39(2): 121-39.

Klasen, S. (2000). 'Measuring Poverty and Deprivation in South Africa'. *Review of Income and Wealth*, 46(1): 33-58.

Lipton, M. (1983). 'Poverty, Undernutrition and Hunger'. *World Bank Staff Working Papers No. 597*. Washington DC: The World Bank.

Lipton, M. (1988). 'The Poor and the Poorest. Some Interim Findings'. *World Bank Discussion Paper No. 25*. Washington DC: The World Bank.

McKay, A., and D. Lawson (2003). 'Assessing the Extent and Nature of Chronic Poverty in Low Income Countries: Issues and Evidence'. *World Development*, 31(3): 425-39.

Ravallion, M., G. Datt, and D. Walle (1991). 'Quantifying Absolute Poverty in the Developing World'. *Review of Income and Wealth*, 37(4): 345-61.

Roberts, B.J. (2001). 'Chronic and Transitory Poverty in Post-Apartheid South Africa: Evidence from KwaZulu-Natal'. *Journal of Poverty*, 5(4): 1-28.

Sen, A.K. (1976). 'Poverty: An Ordinal Approach to Measurement'. *Econometrica*, 44: 219-31.

UNDP (2010). *Human Development Report 2010*: *The Real Wealth of Nations: Pathways to Human Development*. New York: United Nations Development Programme.

World Bank (1990). *World Development Report: Poverty*. Oxford: Oxford University Press.

Appendix A

Proof of the theorem

Suppose the deprivation matrix corresponding to any $X \in \mathbb{R}_+^{n\times d}$ and $\boldsymbol{z} \in \mathbb{R}_+^d$ is g such that $g_{ij} = 1$ if $x_{ij} < z_j$ and $g_{ij} = 0$ if $x_{ij} \geq z_j$. The deprivation score of any person i is computed as $c_i = \sum_{j=1}^{d} w_j g_{ij}$. Any person i is poor, i.e., $i \in Z,$ if $c_i \geq k$ and person i is not poor, i.e., $i \in N\backslash Z$, if $c_i < k$.

(1) Let us prove the necessary and sufficient condition whenever $k \in (0, \min\{w_1, \dots, w_d\}]$. In this case, for all $i \in N\backslash Z$, $g_{ij} = 0$ for all $j \in D$, and for all $i \in Z$, $g_{ij} = 1$ for at least one $j \in D$. To prove the sufficiency condition, consider the deprivation cutoff vector $\bar{z}$ such that $\bar{z} \leq z$. We denote the corresponding deprivation matrix by $\bar{g}$ such that $\bar{g}_{ij} = 1$ if $x_{ij} < \bar{z}_j$ and $\bar{g}_{ij} = 0$ if $x_{ij} \geq \bar{z}_j$. Let us denote the set of poor persons by $\bar{Z}$. Following $\bar{z} \leq z$, for any (i, j), $g_{ij} = 0$ implies $\bar{g}_{ij} = 0$. Thus, $i \in N\backslash Z$ implies $i \in N\backslash \bar{Z}$ or $N\backslash Z \subseteq N\backslash \bar{Z}$, which in turn implies $\bar{Z} \subseteq Z$. In order to prove the necessary condition, suppose $i \in N\backslash Z$. Suppose further for any $j \in D$ that $\bar{z}_j > x_{ij} \geq z_j$ and $\bar{w}_j \geq \bar{k}$. Then, $i \in \bar{Z}$. Hence, $\bar{Z}$ is not a subset of Z.

(2) Next, let us consider the situation whenever $k = 1$. In this case, for all $i \in Z$, $g_{ij} = 1$ for all $j \in D$, and for all $i \in N\backslash Z$, $g_{ij} = 0$ for at least one $j \in D$. To prove the sufficiency condition, consider the deprivation cutoff vector $\bar{z} \leq z$ and $\bar{k} = 1$. We denote the corresponding deprivation matrix by $\bar{g}$ and the set of poor by $\bar{Z}$. As the proof of (1) above, for any (i, j), $g_{ij} = 0$ implies $\bar{g}_{ij} = 0$. Then $i \in N\backslash Z$ implies $i \in N\backslash \bar{Z}$, which in turn implies $\bar{Z} \subseteq Z$. In order to prove the necessary condition, we need to show that $\bar{Z}$ is not a subset of Z either if $\bar{z} \nleq z$ or if $\bar{k} < k = 1$. Consider any $i \in N\backslash Z$ such that $x_{ij} \geq z_j$ for all $j \in D' \subset D$ and $x_{ij} < z_j$ for all $j \in D\backslash D'$. First, suppose $\bar{k} = 1$ but $\bar{z}_j > x_{ij} \geq z_j$ for all $j \in D$. Then $i \in \bar{Z}$ and so $\bar{Z}$ is not a subset of Z. Second, suppose $\bar{z} \leq z$ but $0 < \bar{k} \leq \bar{c}_i < k = 1$. Then $i \in \bar{Z}$ despite $i \in N\backslash Z$. Hence, $\bar{Z}$ is not a subset of Z.

(3) Finally, let us consider the situation whenever $k \in (\min\{w_1, \dots, w_d\}, 1)$. In order to prove the sufficiency condition, suppose $\bar{z} \leq z$, $\bar{k} \geq k$ and $\bar{w} = w$. Consider any $i \in N\backslash Z$. Then, $c_i < k$. If $\bar{z} \leq z$, then for any $j \in D$, $g_{ij} = 0$ implies $\bar{g}_{ij} = 0$. In addition, if $\bar{w} = w$, then $\bar{c}_i = \sum_{j=1}^{d} w_j \bar{g}_{ij} \leq \sum_{j=1}^{d} w_j g_{ij} = c_i$. Finally, if $\bar{k} \geq k$, then definitely $\bar{c}_i < \bar{k}$ and so $i \in N\backslash \bar{Z}$. Hence, $\bar{Z} \subseteq Z$.

In order to prove the necessary condition, we need to prove that $\bar{Z}$ is not a subset of Z either if $\bar{z} \nleq z$ or if $\bar{k} < k$ or if $\bar{w} \neq w$. Consider any $i \in N\backslash Z$ so that $c_i < k$. First, consider a situation when $\bar{k} \geq k$ and $\bar{w} = w$, but $\bar{z}_j > x_{ij} \geq z_j$ for some $j \in D' \subset D$ such that $\sum_{j \in D'} w_j \geq \bar{k}$. Thus, $\bar{c}_i \geq \bar{k}$ and so $i \in \bar{Z}$. Hence, $\bar{Z}$ is not a subset of Z. Second, consider a situation when $\bar{z} \leq z$ and $\bar{w} = w$, but $\bar{k} \leq \bar{c}_i < k$. Then also $i \in \bar{Z}$ and hence $\bar{Z}$ is not a subset of Z. Finally, consider a situation when $\bar{z} \leq z$ and $\bar{k} \geq k$, but $\bar{w} \neq w$. Suppose, for some $j \in D' \subset D$, $\sum_{j \in D'} w_j < k$ such that $c_i < k$, but $\sum_{j \in D'} \bar{w}_j \geq \bar{k}$, which implies $\bar{c}_i \geq \bar{k}$. Then $i \in \bar{Z}$ and so $\bar{Z}$ is not a subset of Z. This completes the proof.

Appendix B

Per cent changes in poverty rates by different characteristics in Nepal between 2006 and 2011

	Household Share in Per Cent		Population Share in Per Cent		MPI Poverty Rate in Per Cent				Intense Poverty Rate in Per Cent				Depth Poverty Rate in Per Cent			
Variable	2006	2011	2006	2011	2006	2011	Relative Change		2006	2011	Relative Change		2006	2011	Relative Change	
Dependency rate																
No dependence	14.1	18.6	7.2	10.9	38.5	23.0	-9.8	***	11.9	5.7	-13.9	***	18.3	13.4	-6.1	***
0.01-0.40	21.9	22.8	23.2	25.4	56.6	34.2	-9.6	***	28.2	12.9	-14.5	***	26.7	16.8	-8.8	***
0.41-0.50	28.6	26.9	30.7	29.0	63.8	43.7	-7.3	***	34.9	20.0	-10.6	***	31.5	22.7	-6.4	***
0.51-1	35.5	31.7	39.0	34.7	75.2	58.6	4.8	**	48.6	32.1	-8.0	***	37.9	29.6	-4.8	**
Household size																
1 member	5.1	5.6	1.1	1.3	63.9	51.1	-4.4		15.3	6.0	-17.1	***	53.8	43.6	-4.1	
2-5 members	62.2	68.8	47.3	56.2	57.1	38.0	-7.8	***	29.2	15.9	-11.5	***	27.1	18.4	-7.4	***
6 or more members	32.7	25.7	51.7	42.6	71.7	52.1	-6.2	**	44.7	27.8	-9.1	***	35.9	27.4	-5.3	**
Bedroom(s) per person																
0.5 or more	44.0	58.3	32.0	46.9	49.5	29.5	-9.8	***	20.3	9.2	-14.6	***	23.2	14.4	-9.1	***
0.25-0.49	41.2	33.0	47.4	40.6	66.9	51.5	-5.1	**	38.0	25.7	-7.5	***	31.7	27.0	-3.2	*
0-0.24	14.8	8.7	20.6	12.5	83.5	75.4	-2.0		60.9	48.1	-4.6	*	46.1	38.9	-3.3	
Male members (age 15-60)																
No male member	24.3	28.8	16.3	21.0	68.7	51.6	-5.6	**	37.6	23.6	-8.9	***	36.8	25.5	-7.1	***
0–50 per cent male members	59.7	56.3	67.6	64.6	66.3	44.8	-7.5	***	39.1	22.3	-10.6	***	31.8	22.7	-6.6	***
More than 50per cent males	16.0	14.8	16.1	14.4	54.3	30.6	-10.9	***	28.0	10.2	-18.3	***	27.5	17.8	-8.3	**
Land Ownership																
No Land	31.9	32.6	28.1	30.9	59.8	48.4	-4.1	*	36.5	26.1	-6.5	**	31.8	28.4	-2.2	
Marginal (0.1 ha or less)	16.5	16.6	15.9	16.5	68.6	48.0	-6.9	**	40.5	22.1	-11.4	***	38.3	22.8	-9.9	***
Small (0.1-0.5 ha)	21.6	23.0	20.6	22.3	68.0	48.2	-6.6	**	37.0	22.9	-9.2	***	29.3	22.3	-5.3	**
Medium & Large (>0.5 ha)	30.1	27.9	35.3	30.4	65.0	34.9	-11.7	***	36.0	13.1	-18.2	***	30.7	16.7	-11.4	***
Gender of HH head																
Female	23.4	29.4	17.4	24.0	63.7	44.6	-6.8	**	34.0	22.1	-8.3	***	31.9	20.4	-8.6	***
Male	76.6	70.6	82.6	76.0	65.0	44.1	-7.5	***	37.7	20.4	-11.6	***	31.9	23.3	-6.1	***
Age of HH head																
30 years or less	21.0	18.3	16.4	14.5	60.9	48.1	-4.6	**	35.4	26.4	-5.7	**	30.7	23.7	-5.1	**
30 and 60 years	62.3	64.2	67.4	68.6	66.4	44.8	-7.6	***	39.4	21.1	-11.7	***	33.3	23.1	-7.0	***
Older than 60 years	16.7	17.5	16.2	17.0	61.8	38.6	-9.0	***	28.8	14.6	-12.7	***	27.6	19.4	-6.8	***
Education of HH head																

Variable	Household Share in Per Cent 2006	Household Share in Per Cent 2011	Population Share in Per Cent 2006	Population Share in Per Cent 2011	MPI Poverty Rate in Per Cent 2006	MPI Poverty Rate in Per Cent 2011	MPI Poverty Rate in Per Cent Relative Change	Intense Poverty Rate in Per Cent 2006	Intense Poverty Rate in Per Cent 2011	Intense Poverty Rate in Per Cent Relative Change	Depth Poverty Rate in Per Cent 2006	Depth Poverty Rate in Per Cent 2011	Depth Poverty Rate in Per Cent Relative Change
No education/preschool	47.5	45.1	46.7	46.0	77.5	60.2	-4.9 **	48.1	33.4	-7.0 ***	43.4	34.5	-4.5 **
Primary completed	23.4	21.1	25.3	21.9	71.2	46.9	-8.0 ***	39.8	18.5	-14.2 ***	29.6	18.2	-9.2 ***
Secondary completed	22.6	25.8	22.8	25.2	42.9	22.0	-12.5 ***	19.3	5.3	-22.8 ***	17.5	10.0	-10.6 ***
Higher education	6.5	8.0	5.2	6.8	14.4	9.4	-8.2 ***	2.0	0.4	-26.9 ***	4.0	2.6	-8.3 ***
Development regions													
Eastern	21.9	24.8	21.8	23.7	62.2	37.4	-9.7 **	31.9	15.9	-13.0 ***	27.7	16.7	-9.6 ***
Central	35.3	33.1	33.4	32.5	59.0	46.2	-4.8 *	36.6	23.5	-8.5 **	31.9	28.6	-2.2
Western	22.8	21.5	22.7	21.0	62.6	33.4	-11.8 ***	31.9	13.7	-15.5 ***	28.7	11.9	-16.1 ***
Mid-western	10.8	11.5	10.4	12.4	72.8	59.1	-4.1 *	42.5	29.2	-7.2 **	32.6	32.0	-0.3
Far-western	9.2	9.1	11.7	10.3	82.6	57.7	-6.9 *	53.0	27.8	-12.1 **	45.8	27.3	-9.8 **
Rural/urban													
Rural	83.2	86.3	85.0	87.2	71.3	48.4	-7.5 ***	41.4	23.2	-11.0 ***	35.8	25.1	-6.8 ***
Urban	16.8	13.8	15.0	12.8	27.4	15.4	-10.9 ***	12.5	4.7	-17.7 ***	10.2	5.2	-12.5 ***
Nepal	100	100	100	100	64.7	44.2	-7.4 ***	37.1	20.8	-10.9 ***	31.9	22.6	-6.7 ***

Notes: The statistical tests of differences are one-tailed tests. ***-Statistically significant at α = 1 per cent, **-Statistically significant at α = 5 per cent, and *-Statistically significant at α = 10 per cent.

Source: Authors' computations.