Optimal Monetary Policy and Terms of Trade Shocks*

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Abstract

This paper derives a micro-founded utility based welfare loss function for the multi-sector closed economy NK-DSGE model in Ghate et al. (2016) for the Indian economy. The model consists of three sectors, namely grain, vegetables and manufacturing sector. The grain and vegetable sectors are flexible price sectors and the manufacturing sector is a standard sticky price sector. The presence of procurement in the grain sector makes the model a non-standard one. In general, for a multi sector set-up, the welfare loss function has quadratic terms of the inflation gap, the output gap and the terms of trade gap (from their respective efficient levels). The presence of procurement alters the coefficient in front of the gap terms mentioned above which makes the welfare loss function also a non-standard one. Due to this, the trade-off between inflation and the output gap changes. We show in this paper how an increase in procurement levels increases the trade-offs between core inflation and the output gap, and thus also increases the welfare losses. We derive two optimal rules by minimizing the welfare loss function under discretion and compare these two rules with each other and also with a simple Taylor rule. We compare results with alternate Taylor rules and show that some Taylor rules perform better then simple Taylor rule.

Keywords : Optimal Monetary Policy, Multi-sector New Keynesian DSGE Models, Terms of Trade Shocks, Indian Economy, Agricultural Procurement

JEL Codes: E31; E43; E52; E58; Q18

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1 Introduction

Monetary policy makers in emerging markets and developing countries (EMDEs) often struggle with stabilizing the rate of inflation and economic activity at the same time. Central bankers in many countries now rely on welfare based optimal monetary policy which provides a good benchmark to the normative analysis of policy. There are however additional challenges faced by central banks in EMDEs (see Hammond et al. (2009)) because of which it becomes difficult to derive such an optimal monetary policy. One such challenge is the inefficiencies present in the agricultural sector which spread to other sectors of the economy and affect macroeconomic aggregates. A classic example of such an inefficiency is the procurement distortion present in the grain sector of the Indian economy. The Indian government procures a certain proportion of produce in the grain sector at minimum support prices and distributes part of the procured good to the poor at a subsidized price. This policy of procurement distorts the market price, creates a shortage in the open market grain output and is inflationary (see Basu (2011) , Ramaswamy (2014) and Ghate et al. (2016)). Moreover, the mismanagement of the policy leads to wastage of grain products.

This paper attempts to derive optimal monetary policy and analyze some alternate monetary policy rules given procurement distortions present in the grain sector of the Indian economy. We build on the multi-sector NK-DSGE closed economy model in Ghate et al. (2016) . Our main contribution to the paper is deriving a micro-founded utility based welfare loss function and characterizing optimal discretionary monetary policy following Rotemberg and Woodford (1997, 1999) and Woodford (1999, 2003). We show that the welfare loss function has quadratic terms in the inflation gap, output gap, and terms of trade gap (from their respective efficient levels) as expected in the multi-sector model (see Benigno (2004), Aoki (2001)). The presence of procurement alters the coefficient in front of the gap terms mentioned above which makes the welfare loss function a non-standard one. This happens because the structure of the economy we assume here has a non-standard feature of procurement of grain by the government in the agriculture sector.

In Ghate et al. (2016) the focus is on explaining the transmission mechanism of a grain procurement shock and a productivity shock to sectoral and economy wide variables through a multi-sector NK- DSGE model. It is shown that the presence of procurement modifies the standard NKPC (new Keynesian Phillips curve) and DIS (dynamic IS) curves of the aggregate economy. They also show that procurement weakens monetary policy transmission, because of which the monetary policy response needs to be more aggressive to achieve a given inflation target. Our paper is an extension to this paper.

The presence of an additional inefficiency due to procurement makes this paper different from other multi-sector NK-DSGE models discussing optimal monetary policy either in a closed-economy framework (see Aoki (2001) and Huang and Liu (2005)) or in a open economy framework (see Benigno (2004)). In the Indian context, Ramaswamy et al. (2014) have estimated the welfare losses (monetary terms) generated from a rising MSP (Minimum Support Prices) between 1998-2011. They find that the welfare losses amount to 1.5 billion dollars to the Indian economy between 1998-2011.[1] We also show in this paper that increasing procurement levels increases welfare losses, but the welfare losses in our paper are deviations from the efficient allocation. Our paper contributes to a growing monetary literature in emerging and developing countries (EMDEs).

1.1 Main Results

Our main contribution is to derive a welfare loss function and characterize optimal monetary policy under discretion. We show that the welfare loss function depends on the square of core-inflation, consumption gap, and terms of trade gap, all measured as deviations from their respective efficient levels. We show that the presence of procurement alters the coefficient in front of the gap terms which makes the welfare loss function a non-standard one.

More specifically, we show that the welfare losses increase with the level of the steady state share of procured grain, c_p. We consider three rules, namely, two optimal rules and a simple Taylor rule. All three rules show the same pattern of increasing welfare losses as c_p increases. The increase in welfare losses happens due to higher inflation, a higher output gap and a terms of trade gap created due to higher c_p. Another reason for higher welfare losses is the higher trade-off between core-inflation and the output gap associated with a higher level of c_p. The trade-off between core-inflation and the output gap means that the monetary authority cannot stabilize both core-inflation and output gap at the same time. As discussed in Woodford (2003), this kind of trade-off does exists if the economy is characterized by inefficient shocks such that the flexible level of output deviates from the efficient level of output. The output gap now has two components, one that corresponds with the rise in inflation (as seen in the NKPC) and a second source which is the gap between the flexible level and the efficient level of output. In such cases, core-inflation targeting only stabilizes the output gap partially and in order to completely stabilize the output gap, the monetary authority needs to compromise on core-inflation deviating from its target. In the model considered here procurement acts as that inefficient shock. Any increase in its level, as captured by c_p, thus increases the trade-off leading to higher welfare losses.

Next our paper compares a simple Taylor rule and the optimal rules in terms of their

[1] Ghate et al. (2016) show how a rising MSP is associated with higher procurement levels.

implications for welfare losses. It is observed that the simple Taylor rule as considered in Ghate et al. (2016) performs poorly on welfare criterion. We also show that there exist alternate Taylor rules which generate results closer to optimal rules and thus can be considered as second best alternatives to optimal rules, since optimal rules are difficult to implement. The alternate rules with terms of trade gaps in the Taylor rule performs the best among the class of Taylor rules considered.

2 Overview of Ghate et al. (2016)

We briefly describe the model of Ghate et al. (2016).[2] The model is a three sector closed economy NK-DSGE model. The three sectors are namely, grain (G), vegetables (V) and manufacturing (M). A monopolistically competitive market structure is assumed for all the sectors, such that each sector has continuum of firms, each producing a differentiated good. Both the grain and vegetable sectors are part of a broader agriculture sector and have flexible prices. The manufacturing sector is a standard sticky price sector. The additional non-standard distortion considered in the model is the procurement of grains by the Indian government.[3] It is assumed that out of the total grain produced, $\widehat{Y}_{G,t}$, procured grain, $\widehat{Y}_{PG,t}$, gets wasted and it is only non-procured grain, $\widehat{Y}_{OG,t}$, which goes to the open market and is later consumed.[4] The following equation sums up this relation,

$$\widehat{Y}_{G,t} = (1 - c_p)\,\widehat{Y}_{OG,t} + c_p\widehat{Y}_{PG,t}$$

where $c_p = \frac{Y_{PG}}{Y_G}$, steady state share of the procured grain in total grain output. The parameter c_p is the distortionary parameter here which takes value between $\left[0, \frac{\theta-1}{\theta}\right]$, where $\theta > 1$ is the elasticity of substitution between the varieties within each sector and is assumed to be the same in all sectors. The household maximize the following expected lifetime utility at time 0,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_t) - \int_0^1 v(N_t(i))\ di \right] \tag{1}$$

where $U(C_t)$ is the utility from the aggregate consumption bundle C_t and $v(N_t(i))$ is the disutility of supplying labor, $N_t(i)$, by the i^{th} household. The demand functions for each

[2]The notations of the variables are kept same as the model discussed in Ghate et al. (2016) to avoid confusion.

[3]For details see Basu (2001), Ramaswamy (2014) and Ghate et al. (2016).

[4]For any variable X_t,

$$\widehat{X}_t = \ln(X_t) - \ln(X)$$

where X is the steady state value of X_t. $\widehat{Y}_{PG,t}$ follows an AR(1) process here.

sectoral good is as follows,

$$\widehat{Y}_{M,t} = \widehat{C}_t + \delta\widehat{T}_{AM,t} \tag{2a}$$

$$\widehat{Y}_{V,t} = \widehat{C}_t - (1-\delta)\,\widehat{T}_{AM,t} + (1-\mu)\,\widehat{T}_{OGV,t} \tag{2b}$$

$$\widehat{Y}_{OG,t} = \widehat{C}_t - (1-\delta)\,\widehat{T}_{AM,t} - \mu\widehat{T}_{OGV,t} \tag{2c}$$

where $\widehat{Y}_{M,t}$ and $\widehat{Y}_{V,t}$ are the output produced in the manufacturing and vegetable sector respectively, $\widehat{T}_{AM,t}$ and $\widehat{T}_{OGV,t}$ are the terms of trade between the agriculture and manufacturing sectors and between open grain and vegetable sector, respectively. The NKPC for the manufacturing sector is given by,

$$\pi_{M,t} = \beta E_t\{\pi_{M,t+1}\} + \lambda_M\,(\psi\Theta_1+\sigma)\,\widetilde{Y}_{M,t} - \lambda_M\delta\,(\psi\Theta_1+\sigma-1)\,\widetilde{T}_{AM,t}$$

or alternatively, using equation $(2a)$ we get

$$\pi_{M,t} = \beta E_t\{\pi_{M,t+1}\} + \lambda_M\,(\psi\Theta_1+\sigma)\,\widetilde{C}_t + \lambda_M\delta\widetilde{T}_{AM,t}, \tag{4}$$

where $\pi_{M,t}$ is inflation in the manufacturing sector. The dynamic-IS equation is given by,

$$\widetilde{Y}_t = E_t\{\widetilde{Y}_{t+1}\} - \frac{(1-\lambda_c)}{\sigma}[(\widehat{R}_t - E_t\{\pi_{t+1}\}) - \widehat{r}^n_t] - \lambda_c(1-\delta)E_t\left\{\Delta\widetilde{T}_{AM,t+1}\right\}, \tag{5}$$

where

$$\widetilde{Y}_t = (1-\lambda_c)\,\widetilde{C}_t + \lambda_c(1-\delta)\widetilde{T}_{AM,t} \tag{6}$$

and $\widehat{r}^n_t = \sigma E_t\{\Delta\widehat{C}^n_{t+1}\} - (1-\sigma)E_t\{\Delta\widehat{\Gamma}_{t+1}\}$, is the natural rate of interest.[5] We shall derive the welfare loss function in this paper on the model just briefed.[6]

3 Welfare loss function

We use the seminal work of Rotemberg and Woodford (1997, 1999) and Woodford (1999, 2003) to derive the welfare loss function. Their approach involves a second order approximation of the discounted sum of utility flows incurred by a representative consumer in a rational expectations equilibrium. The approximation to the utility is taken as its deviation

[5] Note that for a variable X_t,

$$\widetilde{X}_t = \widehat{X}_t - \widehat{X}^n_t.$$

The parameters Θ_1 and λ_c captures the distortions due to procurement and are 1 and 0 respectively when $c_p = 0$.

[6] For details please refer Ghate et al. (2016).

from the efficient allocation.[7] This approach to welfare criterion is very popular because it leads to a welfare loss function which is a function of squares of inflation and the output gap.[8] A few possible deviation from the standard welfare function as described comes with multi-sector models. A multi-sector model can either be a closed economy model with more than one sector (see Aoki (2001), Huang and Liu (2005)) or an open-economy model with two countries producing a similar good (see Benigno (2004)). With a multi-sector model in place an additional term namely, terms of trade gap from its efficient level also appears in the welfare loss function.

The welfare loss function is given by,[9]

$$\begin{aligned} W &= -E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{w_t}{U_C \ C} \right) \approx \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda_{\pi M} \left(\pi_{M,t} \right)^2 + \lambda_{\widetilde{C}} \left(\widetilde{C}_t \right)^2 + \right. \\ & \left. + \lambda_{\widetilde{TAM}} \left(\widetilde{T}_{AM,t} \right)^2 \right] + \|O\|^3 + t.i.p. \end{aligned} \tag{7}$$

where

$$\widetilde{C}_t = \frac{\widetilde{Y}_t}{(1-\lambda_C)} - \frac{\lambda_C (1-\delta)}{(1-\lambda_C)} \widetilde{T}_{AM,t}. \tag{8}$$

It can be seen that welfare is a function of squares of the consumption gap $\left(\widetilde{C}_t\right)$, core inflation $(\pi_{M,t})$ and the terms of trade gap $\left(\widetilde{T}_{AM,t}\right)$.[10] Also note that, here square of the consumption gap, $\widetilde{C}_t$, is not same as the square of the output gap $\left(\widetilde{Y}_t\right)$ due to the presence of procurement, but consumption gap $\left(\widetilde{C}_t\right)$ can be written as a function of the output gap $\left(\widetilde{Y}_t\right)$ and terms of trade gap $\left(\widetilde{T}_{AM,t}\right)$ as in equation (8) (for details see Ghate et al. (2016)).

3.1 Special case with $\mathbf{c}_p = 0$

The most important parameter which captures the procurement distortion is c_p i.e. the steady state level of the proportion of procured grain in total grain output. Welfare in equation (7) is derived for the general case where $c_p \in [0, \frac{\theta-1}{\theta}]$. The welfare loss function when $c_p = 0$ converges to its standard formulation where there is no procurement distortion:

[7] In our model, it is important to note that the efficient allocation of resources is the one where procurement is absent.

[8] Inflation and the output gap here are deviations from there efficient levels.

[9] For detailed derivations please refer to the technical appendix.

[10] Core inflation is defined as the inflation in the non-volatile sectors of the economy. In the context of the present model, manufacturing sector inflation represents core inflation.

it reduces to,

$$
\begin{aligned}
W &= -E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{w_t - w}{U_C\ C} \right) \approx \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda_{\pi M} \left(\pi_{M,t} \right)^2 + (\sigma + \psi) \left(\widetilde{Y}_t \right)^2 \right. \qquad (9) \\
&\quad \left. + (\psi + 1)(1 - \delta)\, \delta \left(\widetilde{T}_{AM,t} \right)^2 \right] + \| O \|^3 + t.i.p.
\end{aligned}
$$

The above welfare loss function, equation (9), is quite standard and is comparable to any loss function obtained in a multi-sector model setup with sticky prices. It will be shown later in the calibration section, how the increasing value of c_p increases welfare losses.

4 Optimal Monetary Policy

The loss functions (7) & (9) show how a high variance in the output gap/ consumption gap, inflation, and the terms of trade gap from their respective efficient level leads to higher welfare losses. Any policy which can minimize such losses i.e. in the variance in the output gap/ consumption gap, inflation and terms of trade gap, is termed as optimal monetary policy. We consider monetary policy under discretion for the present analysis. Under discretionary monetary policy, the monetary authorities optimize in each time period t given future expected terms.[11]

We thus minimize the welfare loss function as described in equation (7) at period t subject to the constraint that the NKPC (equation (4)) in the manufacturing sector holds (see Gali (2008) and Woodford (2003)). The problem can formally be written as,

$$
W = \min_{\{\pi_{M,t},\, \widetilde{C}_t,\, \widetilde{T}_{AM,t}\}} \frac{1}{2} \left[\lambda_{\pi M} \left(\pi_{M,t} \right)^2 + \lambda_{\widetilde{C}} \left(\widetilde{C}_t \right)^2 + \lambda_{\widetilde{TAM}} \left(\widetilde{T}_{AM,t} \right)^2 \right]
$$

subject to

$$
\pi_{M,t} = \lambda_M \left(\sigma + \psi \Theta_1 \right) \widetilde{C}_t - \lambda_M \delta \widetilde{T}_{AM,t}.
$$

The first order conditions are,

$$
\pi_{M,t} = -\frac{\lambda_{\widetilde{TAM}}}{\delta \lambda_M \lambda_{\pi M}} \widetilde{T}_{AM,t} \qquad (10a)
$$

$$
\widetilde{C}_t = -\frac{\lambda_M \left(\sigma + \psi \Theta_1 \right) \lambda_{\pi M}}{\lambda_{\widetilde{C}}} \pi_{M,t} \qquad (10b)
$$

Combining the above optimality conditions with the relation between the consumption gap,

[11] Its is assumed here that $E\left\{ \widehat{X}_{t+1} \right\}$ is given and $\widehat{X}_{t-1} = 0$.

$\widetilde{C}_t$, and the output gap, $\widetilde{Y}_t$, as described in equation (8), we can express the output gap in terms of core inflation as

$$\widetilde{Y}_t = -\left[(1-\lambda_C)\frac{\lambda_M(\sigma+\psi\Theta_1)\lambda_{\pi M}}{\lambda_{\widetilde{C}}} + \lambda_C(1-\delta)\frac{\delta\lambda_M\lambda_{\pi M}}{\lambda_{\widetilde{TAM}}}\right]\pi_{M,t}, \tag{11}$$

or alternatively, in terms of aggregate inflation as,

$$\widetilde{Y}_t = -\left((1-\lambda_C)\frac{\lambda_M(\sigma+\psi\Theta_1)\lambda_{\pi M}}{\lambda_{\widetilde{C}}} + \lambda_C(1-\delta)\frac{\delta\lambda_M\lambda_{\pi M}}{\lambda_{\widetilde{TAM}}}\right)\left(\frac{\lambda_{\widetilde{TAM}}}{\lambda_{\widetilde{TAM}} - \delta^2\lambda_M\lambda_{\pi M}}\right)\pi_{,t}. \tag{12}$$

Equations (11) and (12) can be re-written as,

$$\pi_{M,t} = -\kappa_1\widetilde{Y}_t \tag{13}$$

$$\pi_t = -\kappa_2\widetilde{Y}_t \tag{14}$$

respectively, where $\kappa_1 = \frac{1}{\left[(1-\lambda_C)\frac{\lambda_M(\sigma+\psi\Theta_1)\lambda_{\pi M}}{\lambda_{\widetilde{C}}} + \lambda_C(1-\delta)\frac{\delta\lambda_M\lambda_{\pi M}}{\lambda_{\widetilde{TAM}}}\right]}$ and $\kappa_2 = \frac{1}{\kappa_1\left(\frac{\lambda_{\widetilde{TAM}}}{\lambda_{\widetilde{TAM}} - \delta^2\lambda_M\lambda_{\pi M}}\right)}$. Equations (13) and (14) clearly show the possibility of a trade-off between core-inflation and the output gap and between general inflation and the output gap. Substituting equations (11) and (12) in to the DIS equation, (5) , separately, we get the nominal interest rate, $\widehat{R}_t$, as a function of core-inflation, $\pi_{M,t}$, and the natural rate of interest, $\widehat{r}_t^n$,

$$\widehat{R}_t = \widehat{r}_t^n + \frac{\sigma\lambda_M(\sigma+\psi\Theta_1)\lambda_{\pi M}}{\lambda_{\widetilde{C}}}\pi_{M,t} \tag{15}$$

and as a function of general-inflation, π_t, and the natural rate of interest, $\widehat{r}_t^n$,

$$\widehat{R}_t = \widehat{r}_t^n + \left(\frac{\sigma\lambda_M(\sigma+\psi\Theta_1)\lambda_{\pi M}}{\lambda_{\widetilde{C}}}\right)\left(\frac{\lambda_{\widetilde{TAM}}}{\lambda_{\widetilde{TAM}} - \delta^2\lambda_M\lambda_{\pi M}}\right)\pi_{,t}, \tag{16}$$

respectively. Note that the above nominal rate of interest is optimal as the optimizing conditions in equations (10) are used to derive these rules. We describe the first rule in equation (15) as the *optimal rule with a core inflation index* and the second rule in equation (16) as the *optimal rule with a general inflation index.* Later we compare the two optimal rules using the welfare-criterion in equation (7).[12]

[12]Note that both the optimal rules considered here satisfy Taylor Principle for the calibrated value of parameters.

4.1 $\mathbf{c}_p = 0$

When the procurement distortion is absent, $c_p = 0$, to derive the optimal rule under discretion we minimize welfare losses in equation (9) in time period t, subject to the NKPC in the manufacturing sector.[13] The formal problem can be expressed as,

$$\begin{aligned} L_t &= \min \frac{1}{2}\left[\lambda_{\pi M}\left(\pi_{M,t}\right)^2 + (\sigma+\psi)\left(\widetilde{Y}_t\right)^2 + (\psi+1)(1-\delta)\,\delta\left(\widetilde{T}_{AM,t}\right)^2\right] \\ &\quad -\phi_1\left[\pi_{M,t} - \lambda_M(\sigma+\psi)\,\widetilde{Y}_t - \lambda_M\delta\widetilde{T}_{AM,t}\right] \end{aligned}$$

The first order conditions imply,

$$\pi_{M,t} = -\frac{(\psi+1)(1-\delta)\,\delta}{\delta\lambda_M\lambda_{\pi M}}\widetilde{T}_{AM,t} \qquad (17a)$$

$$\widetilde{Y}_t = -\lambda_M\lambda_{\pi M}\pi_{M,t} \qquad (17b)$$

The second optimizing equation $(17b)$ can be re-written in general inflation terms as,

$$\widetilde{Y}_t = -\frac{\lambda_M\lambda_{\pi M}(\psi+1)(1-\delta)\,\delta}{(\psi+1)(1-\delta)\,\delta - \delta^2\lambda_M\lambda_{\pi M}}\pi_{,t} \qquad (18)$$

Equations $(17b)$ and (18) again show the possibility of a trade off between core-inflation and the output gap and between general inflation and the output gap respectively. We show later in the calibration exercise that the trade-off between between core-inflation and the output gap is an increasing function of c_p.

5 Calibration

We use the same calibrated parameter values as in Ghate (2016). Table 1 below summarizes the value of the parameters used here,

[13] Note that here the NKPC with $c_p = 0$ is considered.

Parameter	Notation	Value	Source
Discount factor	β	.9823	Levine, et al. (2012)
Inverse of Frisch elasticity of labor supply	ψ	3	Anand and Prasad (2010)
Inverse of inter-temporal elasticity of substitution	σ	1.99	Levine, et al. (2012)
Share of total consumption expenditure allocated to agriculture sector goods	δ	0.52	Ghate et al. (2016)
Share of total food consumption expenditure allocated to vegetable sector goods	μ	0.44	Ghate et al. (2016)
Elasticity of substitution between the varieties of same sector goods	θ	7.02	Levine, et al. (2012)
Measure of stickiness (M)	α_M	0.75	Levine, et al. (2012)
AR(1) coefficients			
Procurement in grain sector (PG)	$\rho_{Y_{PG}}$	0.4	Ghate et al. (2016)
Standard error of AR(1) process			
Procurement in grain sector (PG)	$\sigma_{Y_{PG}}$	0.66	Ghate et al. (2016)
Taylor rule Parameters			
Interest rate smoothing	ϕ_R	0	
Weight on inflation gap	ϕ_π	1.5	Taylor (1993)
Weight on output gap	ϕ_y	0.5	Taylor (1993)

Table 1: Calibrated parameter values from Ghate et. al (2016)

5.1 Welfare losses, Trade-offs and Procurement

Using the above parameters we find that the welfare losses increases monotonically with increasing value of c_p, as shown in Figure 1. Note that although the value of welfare losses for both the optimal rules is small, the graphs show a clear rise in the losses with increasing c_p. To show this we consider two optimal rules described earlier in equation (15), (16) and the simple Taylor rule considered in Ghate at al. (2016) of the following form,

$$\widehat{R}_t = \phi_R \widehat{R}_{t-1} + \phi_\pi \pi_t + \phi_y \widetilde{Y}_t. \tag{19}$$

We then calibrate the welfare losses in equation (7) for values of $c_p \epsilon [0, 0.2]$ for the above three mentioned rules for a one period procurement shock.[14]

[14] We use MATLAB version 2013 and Dynare version 4.4.3 for calibration.

[INSERT FIGURE 1 & 2]

All three graphs in Figure 1 show that the welfare losses increase with increasing c_p. When $c_p = 0$, procurement shocks do not induce any welfare losses ,i.e., welfare losses are zero. A positive value of c_p, which is 0.08 for the present model induces inflation, an output gap, and terms of trade gap as shown in Ghate et al. (2016). Another possible reason for the increase in welfare losses in the optimal rules is the increasing trade-off between core-inflation and the output gap as c_p increases, as shown in Figure 2.[15] The trade-off between core-inflation and the output gap means that the monetary authority cannot stabilize both core-inflation and output gap at the same time. As discussed in Woodford (2003), this kind of trade-off exists if the economy is characterized by inefficient shocks so that the flexible level of output deviates from the efficient level of output. The output gap now has two components, one that corresponds with the rise in inflation (as seen in the NKPC) and a second gap between the flexible level and the efficient level. In such cases, core-inflation targeting only stabilizes the output gap partially and in order to completely stabilize the output gap, the monetary authority needs to compromise on core-inflation by deviating from its target. In the model considered here the procurement level acts as that inefficient shock. Any increase in its level, as captured by c_p, thus increases the trade-off leading to higher welfare losses.

5.2 Comparing simple Taylor rule and optimal rules

Figure 3 below compares the welfare losses generated by a simple Taylor rule and the two optimal rules in equation (15) and (16). As can be seen from Figure 3, the welfare losses are minimum with the first *optimal rule with core inflation index* (very close to zero). This rule as discussed described in equation (15) generates unambiguously lower welfare losses than the *optimal rule with general inflation index* as described in equation (16) and the simple Taylor rule as described in equation (19).[16]

[INSERT FIGURE 3]

Thus the optimal monetary policy rule involves *core-inflation targeting.* This result thus reinforces the results that targeting inflation of the sticky price sector is optimal (see Aoki (2001), Huang and Liu (2005) and Benigno (2004)). The simple Taylor rule on the other hand, performs extremely poorly compared to the optimal rule. The simple Taylor rule

[15] There does not exist any trade-of between general inflation and the output gap as value of κ_2 is negative for all values of c_p, for calibrated parameter values.

[16] The calibrated values of the coefficient in front of core-inflation in equation (15) and in front of general inflation in equation (16) is greater than 1. Thus, the Taylor principle is satisfied and a unique solution could be obtained.

generates 470 times more welfare losses than the optimal monetary policy at $c_p = 0.08$. In the next section we explore other modified Taylor rules which perform better than the simple Taylor rule.

5.3 Modified alternate Taylor rules as the second best

Taylor (1999) discusses the advantages of the class of simple rules as discussed in equation (19) over the class of optimal rules. One of the reasons is the dependence of optimal rules on current and future paths of the shocks which are not known to the monetary authorities precisely. These imprecisions can lead to large welfare losses. On the other hand, simple Taylor rules are easy to implement and generate results which are close to the optimal monetary policy. We will explore and discuss some modified simple Taylor rules which will generate minimum welfare losses. We will introduce terms of trade gaps in the simple Taylor rule in an ad hoc way and show that the welfare losses reduces to a great extend compared to a simple Taylor rule. We will also rank these rules using a welfare loss criterion.

Consider the following general modified form of the Taylor Rule:

$$\widehat{R}_t = \phi_R \widehat{R}_{t-1} + \phi_\pi \pi_{i,t} + \phi_y \widetilde{Y}_t + \phi_{tam} \widetilde{T}_{AM,t}$$

where, $\widetilde{T}_{AM,t}$ is the terms of trade gap and ϕ_{tam} is the response of the nominal interest rate changes to $\widetilde{T}_{AM,t}$.[17] We consider seven rules in addition to the simple Taylor rule discussed earlier for comparison. We assume that the optimal core inflation targeting rule in equation (15) is the benchmark. Table 2 below summarizes the different rules considered.

Rule	**Inflation**	**Terms of Trade gap ($\widetilde{T}_{AM,t}$)**	$\boldsymbol{\phi_\pi}$	$\boldsymbol{\phi_y}$	$\boldsymbol{\phi_{tam}}$	$\boldsymbol{\phi_R}$
1	$\pi_{M,t}$	Yes	1.5	0.5	0.5	0.1
2	π_t	Yes	1.5	0.5	0.5	0.1
3	$\pi_{M,t}$	Yes	1.5	0.5	0.5	0
4	π_t	Yes	1.5	0.5	0.5	0
5	$\pi_{M,t}$	Yes	1.5	0.5	0	0
6	$\pi_{M,t}$	No	1.5	0	0	0
7	π_t	No	1.5	0	0	0
Simple Taylor rule	π_t	Yes	1.5	0.5	0	0

Table 2: Summarized alternate Taylor rules considered.

[17] We have used $\phi_{tam} = 0.5$ for this analysis. This value is completely arbitary. It is observed that with any positive value of ϕ_{tam} the welfare losses are reduced.

Rule 1 and 2 have positive persistence in the nominal interest rate and active terms of trade gaps. Rule 3 and 4 do not have positive persistence as compared to Rule 1 and 2. Rule 5 and the simple Taylor rule are flexible inflation targeting rules with core inflation/ general inflation and output gap terms. Rule 6 and 7 are pure inflation targeting rules with core-inflation and general inflation being used as an inflation index, respectively. We calculate the welfare losses for each of these rule fixing the procurement parameter, c_p, at 0.08. The results are summarized in the Table 3 below.

Rule	**Welfare losses**($\times 10^{-4}$)
1	1.31466
2	1.51991
3	1.53396
4	1.8804
5	2.65524
6	3.68824
7	5.92757
Simple Taylor rule	3.76873
Benchmark rule (Optimal Rule with core inflation index)	8.04763×10^{-3}

Table 3: Welfare losses for alternate Taylor rules.

As can be seen, Rule 1 and 2 generate the lowest welfare losses among the modified Taylor rules considered for monetary policy. All the rules with terms of trade gaps performs better than any other modified Taylor rule. A diagrammatic representation of the same is shown in figure 4.

[INSERT FIGURE 4]

The welfare losses reduce by 65% if rule 1 is used instead of the simple Taylor rule. Also in figure 4 it is clear that rule 1 lies closest to the optimal rule. Note that between rule 5 and 6, the losses are lower in the flexible inflation targeting rule 5 than the pure inflation targeting rule 6. One possible explanation is the trade-off present between core-inflation and output gap discussed earlier. This trade-off is higher for higher values of c_p. Our basic result is that, in the presence of distortions such as procurement, monetary policy can do better by considering alternate Taylor rules and one such rule could be adding terms of trade gaps to the simple Taylor rule.

6 Conclusion

This paper derives a micro-founded utility based welfare loss function for the multi-sector closed economy NK-DSGE model in Ghate et al. (2016) . Using this welfare function we derive the trade-off between core-inflation and the output gap and characterize optimal discretionary monetary policy. We show how increasing procurement levels increases the trade-offs in core-inflation and the output gap, and also increases welfare losses. We then derive two optimal rules, one where the nominal rate of interest is a function of core-inflation and another where the nominal rate of interest is a function of general inflation and show that the rule which targets core-inflation is optimal. We also show that the simple Taylor rule performs poorly compared to any of the optimal rules. We then discuss some modified Taylor rules. Most of these rules perform better than the simple Taylor rule. We show that adding terms of trade gaps to the Taylor rule reduces the welfare losses by 65%. We are currently working on how varying the inflation coefficients, output gap coefficients and persistence parameter in Taylor rules affects welfare losses.

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7 Figures

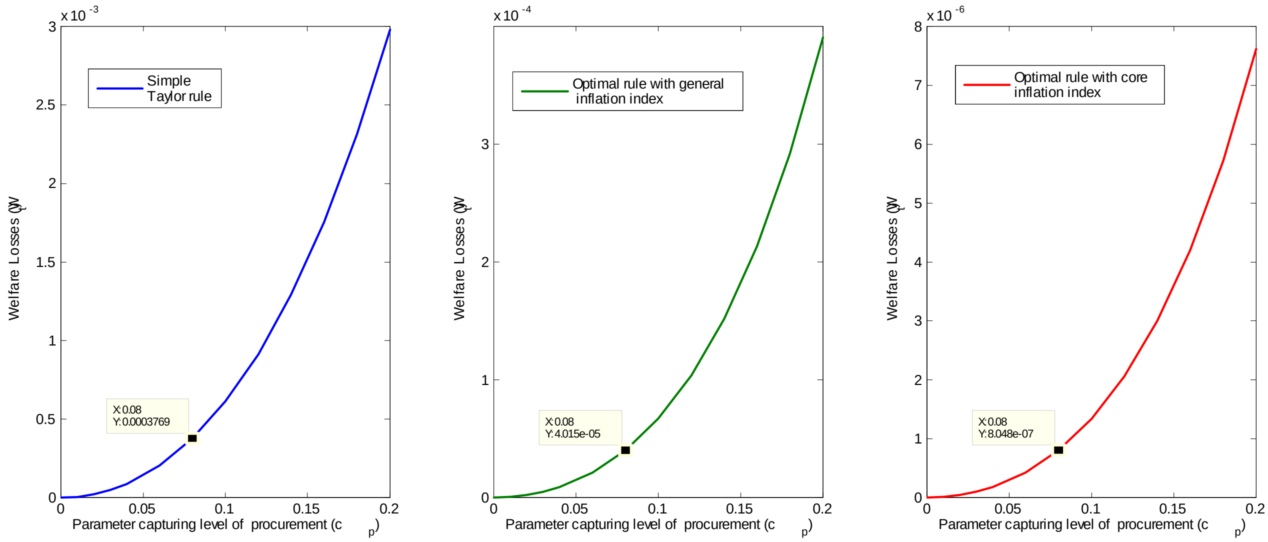

Figure 1: Welfare losses (W) and steady state share of procured grain (c_p).

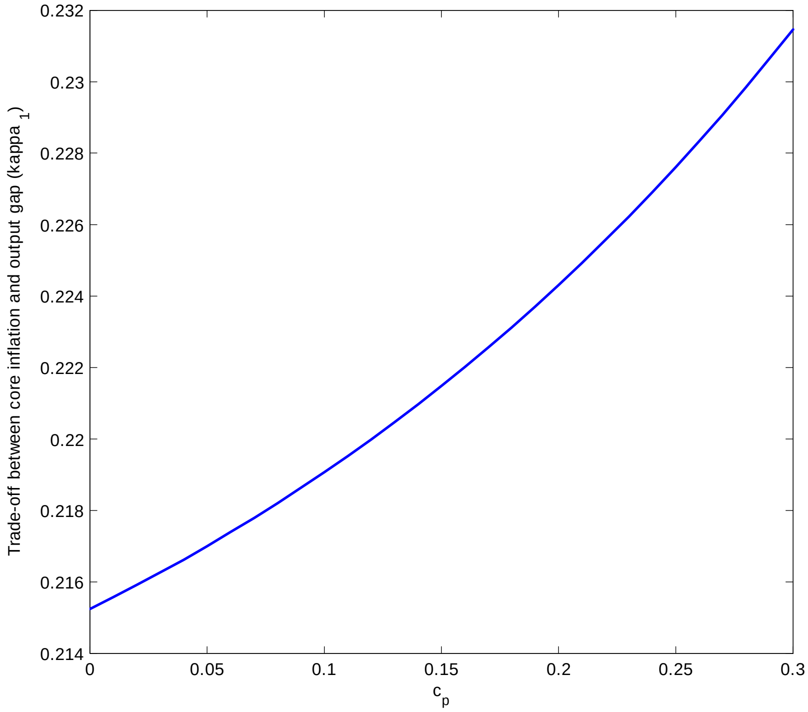

Figure 2: Trade-off between core-inflation and output gap (κ_1) and steady state share of procured grain (c_p).

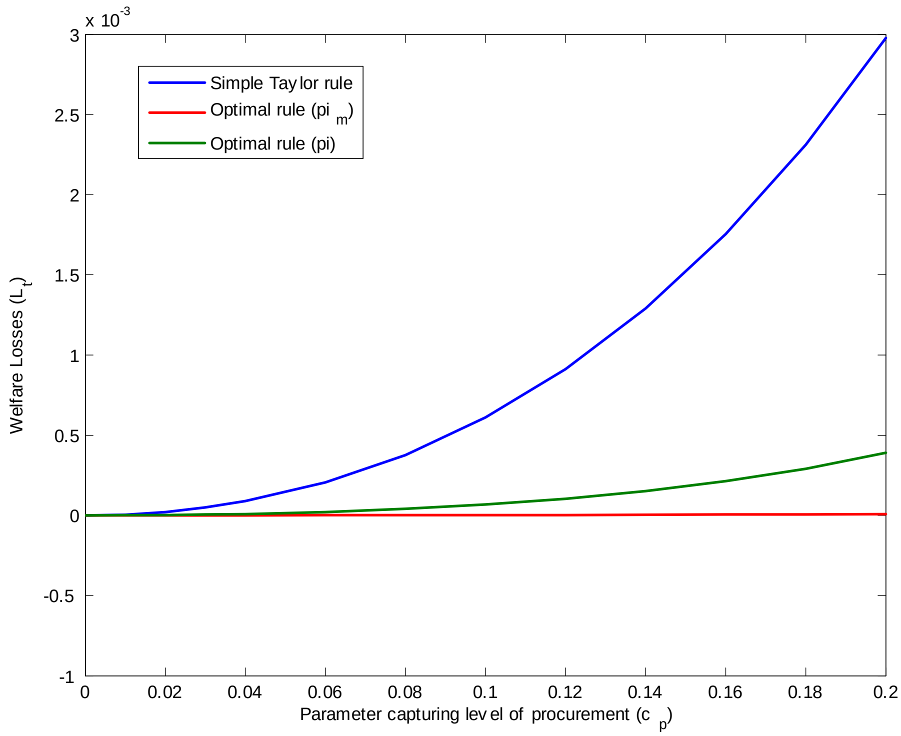

Figure 3: Comparing simple Taylor rule with the other two optimal rules.

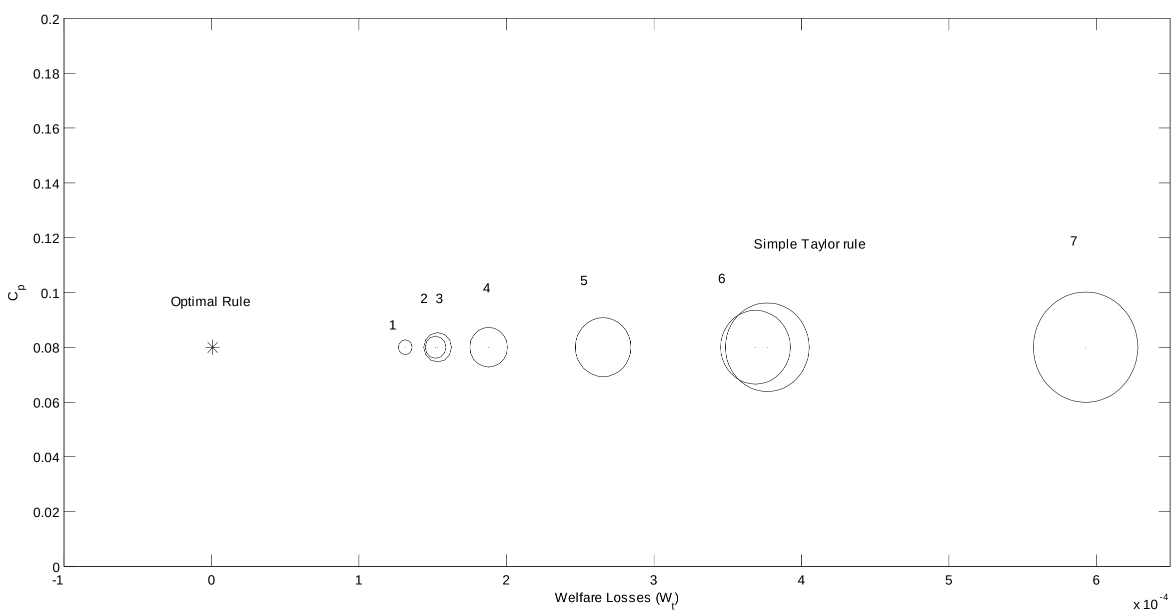

Figure 4: Welfare losses (W) for alternate Taylor rules.

8 Technical Appendix

The average utility flow at time t, is defined as

$$w_t = U\left(C_t\right) - \int_0^1 v(N_t\left(i\right))di \tag{20}$$

where $U\left(C_t\right)$ is the utility from the aggregate consumption bundle C_t and $v(N_t\left(i\right))$ is the disutility of supplying labor $N_t\left(i\right)$ by the i^{th} household. The welfare function would then become,

$$W = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{w_t - w}{U_C\ C}\right) \tag{21}$$

Alternatively, the welfare loss function would become

$$W = -E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{w_t - w}{U_C\ C}\right) \tag{22}$$

We take second order approximation to the $U\left(C_t,\right)$,

$$U\left(C_t,\right) \approx U_c\ C\left(\frac{C_t - C}{C}\right) + U_{cc}\ C^2\left(\frac{C_t - C}{C}\right)^2$$

using $\frac{Z_t - Z}{Z} \approx \widehat{Z}_t + \frac{1}{2}\widehat{Z}_t^2$ where $\widehat{Z}_t = \ln Z_t - \ln Z$

$$U\left(C_t,\right) \approx U_c\ C\left(\widehat{C}_t + \frac{1}{2}\widehat{C}_t^2\right) + \frac{1}{2}U_{cc}C^2\left(\widehat{C}_t + \frac{1}{2}\widehat{C}_t^2\right)^2$$

$$U\left(C_t,\right) \approx U_c\ C\left(\widehat{C}_t + \frac{1}{2}\widehat{C}_t^2\right) + \frac{1}{2}U_{cc}C^2\widehat{C}_t^2 + \left\|O\right\|^3$$

using $\sigma = -\frac{U_{cc}}{U_c}C$

$$U\left(C_t,\right) \approx U_c\ C\left(\widehat{C}_t + \frac{1}{2}\widehat{C}_t^2\right) + \frac{1}{2}\left(-\sigma U_c C\right)\widehat{C}_t^2 + \left\|O\right\|^3$$

$$U\left(C_t,\right) \approx U_c\ C\left[\widehat{C}_t + \frac{1}{2}\widehat{C}_t^2 - \frac{\sigma}{2}\widehat{C}_t^2\right] + \left\|O\right\|^3$$

$$U\left(C_t,\right) \approx U_c\ C\left[\widehat{C}_t + \frac{1}{2}\left(1-\sigma\right)\widehat{C}_t^2\right] + \left\|O\right\|^3 \tag{23}$$

Second order approximation to $V\left(N_t\left(i\right)\right)$,

An i^{th} household supplies labor to three sectors, i.e. grain $\left(G\right)$, vegetable $\left(V\right)$, manufacturing (M)

$$v\left(N_t\left(i\right)\right)=v\left(N_t^V\left(i\right)\right)+v\left(N_{G,t}\left(i\right)\right)+v\left(N_{M,t}\left(i\right)\right)$$

Now $v\left(N_{V,t}\left(i\right)\right)$ can be rewritten as $V\left(Y_{V,t}(i),A_{V,t}\right)$, since $Y_{V,t}(i)=A_{V,t}N_{V,t}\left(i\right)$. Similarly $v\left(N_{M,t}\left(i\right)\right)$ and $v\left(N_{G,t}\left(i\right)\right)$ can be rewritten as $V\left(Y_{M,t}(i),A_{M,t}\right)$ and $V\left(Y_t^{OG}(i),Y_{PG,t},A_{G,t}\right)$ respectively. Consider second order approximation to $v\left(N_t^V\left(i\right)\right)$, since $Y_{V,t}(i)=A_{V,t}N_t^V\left(i\right)$, $v\left(N_t^V\left(i\right)\right)=V\left(Y_{V,t}(i),A_{V,t}\right)$,

$$\begin{aligned}V\left(Y_{V,t}(i),A_{V,t}\right) &\approx V\left(Y_V,A_V\right)+V_Y\left(Y_{V,t}\left(i\right)-Y_V\right)+V_A\left(A_{V,t}-A_V\right)+V_{YA}\left(Y_{V,t}\left(i\right)-Y_V\right)\left(A_{V,t}-A_V\right)\\ &+\frac{V_{AA}}{2}\left(A_{V,t}-A_V\right)^2+\frac{V_{YY}}{2}\left(Y_{V,t}\left(i\right)-Y_V\right)^2+\left\|O\right\|^3\end{aligned}$$

$$\begin{aligned}V\left(Y_{V,t}(i),A_{V,t}\right) &\approx V\left(Y_V,A_V\right)+V_YY_V\left(\frac{Y_{V,t}\left(i\right)-Y_V}{Y_V}\right)+V_AA_V\left(\frac{A_{V,t}-A_V}{A_V}\right)\\ &+V_{YA}Y_VA_V\left(\frac{Y_{V,t}\left(i\right)-Y_V}{Y_V}\right)\left(\frac{A_{V,t}-A_V}{A_V}\right)+\frac{V_{AA}}{2}A_VA_V\left(\frac{A_{V,t}-A_V}{A_V}\right)^2\\ &+\frac{V_{YY}}{2}Y_VY_V\left(\frac{Y_{V,t}\left(i\right)-Y_V}{Y_V}\right)^2+\left\|O\right\|^3\end{aligned}$$

$$\begin{aligned}V\left(Y_{V,t}(i),A_{V,t}\right) &\approx V_YY_V\left(\widehat{Y}_{V,t}\left(i\right)+\frac{1}{2}\left(\widehat{Y}_{V,t}\left(i\right)\right)^2\right)+V_AA_V\left(\widehat{A}_{V,t}+\frac{1}{2}\left(\widehat{A}_{V,t}\right)^2\right)\\ &+V_{YA}Y_VA_V\left(\widehat{Y}_{V,t}\left(i\right)+\frac{1}{2}\left(\widehat{Y}_{V,t}\left(i\right)\right)^2\right)\left(\widehat{A}_{V,t}+\frac{1}{2}\left(\widehat{A}_{V,t}\right)^2\right)\\ &+\frac{V_{AA}}{2}A_VA_V\left(\widehat{A}_{V,t}+\frac{1}{2}\left(\widehat{A}_{V,t}\right)^2\right)^2+\frac{V_{YY}}{2}Y_VY_V\left(\widehat{Y}_{V,t}\left(i\right)+\frac{1}{2}\left(\widehat{Y}_{V,t}\left(i\right)\right)^2\right)^2\\ &+\left\|O\right\|^3+t.i.p.\end{aligned}$$

$$\begin{aligned}V\left(Y_{V,t}(i),A_{V,t}\right) &\approx V_YY_V\left(\widehat{Y}_{V,t}\left(i\right)+\frac{1}{2}\left(\widehat{Y}_{V,t}\left(i\right)\right)^2\right)+V_{YA}Y_VA_V\left(\widehat{Y}_{V,t}\left(i\right)\widehat{A}_{V,t}\right)\\ &+\frac{V_{YY}}{2}Y_VY_V\left(\widehat{Y}_{V,t}\left(i\right)\right)^2+\left\|O\right\|^3+t.i.p.\end{aligned}$$

Assuming the steady state to shocks is 1, i.e. $A_V=A_G=A_M=1$ and let $g_{V,t}=-\frac{V_{YA}\widehat{A}_{V,t}}{V_{YY}Y_V}$

$$\begin{aligned}V\left(Y_{V,t}(i),A_{V,t}\right) &\approx V_YY_V\left(\widehat{Y}_{V,t}\left(i\right)+\frac{1}{2}\left(\widehat{Y}_{V,t}\left(i\right)\right)^2\right)-g_{V,t}V_{YY}Y_VY_V\left(\widehat{Y}_{V,t}\left(i\right)\right)\\ &+\frac{V_{YY}}{2}Y_VY_V\left(\widehat{Y}_{V,t}\left(i\right)\right)^2+\left\|O\right\|^3+t.i.p.\end{aligned}$$

Using $V_{YY} = \psi \frac{V_Y}{Y_V}$

$$\begin{aligned} V\left(Y_{V,t}(i), A_{V,t}\right) &\approx V_Y Y_V \left(\widehat{Y}_{V,t}\left(i\right) + \frac{1}{2}\left(\widehat{Y}_{V,t}\left(i\right)\right)^2\right) - \psi g_{V,t} V_Y Y_V \left(\widehat{Y}_{V,t}\left(i\right)\right) \\ &\quad + \psi \frac{V_Y}{2} Y_V \left(\widehat{Y}_{V,t}\left(i\right)\right)^2 + \left\|O\right\|^3 + t.i.p. \end{aligned}$$

$$V\left(Y_{V,t}(i), A_{V,t}\right) \approx V_Y Y_V \left[\widehat{Y}_{V,t}\left(i\right) - \psi g_{V,t}\left(\widehat{Y}_{V,t}\left(i\right)\right) + \left(\frac{\psi+1}{2}\right)\left(\widehat{Y}_{V,t}\left(i\right)\right)^2\right] + \left\|O\right\|^3 + t.i.p. \tag{24}$$

Similarly for the manufacturing sector,

$$V\left(Y_{M,t}(i), A_{M,t}\right) \approx V_Y Y_M \left[\widehat{Y}_{M,t}\left(i\right) - \psi g_{M,t}\left(\widehat{Y}_{M,t}\left(i\right)\right) + \left(\frac{\psi+1}{2}\right)\left(\widehat{Y}_{M,t}\left(i\right)\right)^2\right] + \left\|O\right\|^3 + t.i.p. \tag{25}$$

where $g_{M,t} = -\frac{V_{YA}\widehat{A}_{M,t}}{V_{YY}Y_M}$.

For the grain sector,consider second order approximation to $v\left(N_{G,t}\left(i\right)\right)$, since $Y_{G,t}(i) = Y_{OG}(i) + Y_{PG,t} = A_{G,t}N_{G,t}\left(i\right)$,

$$\begin{aligned} V\left(Y_{OG,t}(i), Y_{PG,t}, A_{G,t}\right) &\approx V\left(Y_{OG}, Y_{PG,t}, A_G\right) + V_Y\left(Y_{OG,t}\left(i\right) - Y_{OG}\right) + V_Y\left(Y_{PG,t} - Y_{PG}\right) \\ &\quad + V_A\left(A_{G,t} - A_G\right) + V_{YA}\left(Y_{OG,t}\left(i\right) - Y_{OG}\right)\left(A_{G,t} - A_G\right) \\ &\quad + V_{YA}\left(Y_{PG,t} - Y_{PG,t}\right)\left(A_{G,t} - A_G\right) + \frac{V_{AA}}{2}\left(A_{G,t} - A_G\right)^2 \\ &\quad + V_{YY}\left(Y_{OG,t}\left(i\right) - Y_{OG}\right)\left(Y_{PG,t} - Y_{PG,t}\right) + \frac{V_{YY}}{2}\left(Y_{OG,t}\left(i\right) - Y_{OG}\right)^2 \\ &\quad + \frac{V_{YY}}{2}\left(Y_{PG,t} - Y_{PG,t}\right)^2 + \left\|O\right\|^3 \end{aligned}$$

$$\begin{aligned} V\left(Y_{OG,t}(i), Y_{PG,t}, A_{G,t}\right) &\approx V\left(Y_{OG}, Y_{PG,t}, A_G\right) + V_Y Y_{OG}\left(\frac{Y_{OG,t}\left(i\right) - Y_{OG}}{Y_{OG}}\right) + V_Y Y_{PG,t}\left(\frac{Y_{PG,t} - Y_{PG}}{Y_{PG}}\right) \\ &\quad + V_A A_G\left(\frac{A_{G,t} - A_G}{A_G}\right) + V_{YA} Y_{OG} A_G\left(\frac{Y_{OG,t}\left(i\right) - Y_{OG}}{Y_{OG}}\right)\left(\frac{A_{G,t} - A_G}{A_G}\right) \\ &\quad + V_{YA} Y_{PG} A_G\left(\frac{Y_{PG,t} - Y_{PG}}{Y_{PG}}\right)\left(\frac{A_{G,t} - A_G}{A_G}\right) + \frac{V_{AA}}{2} A_G A_G\left(\frac{A_{G,t} - A_G}{A_G}\right)^2 \\ &\quad + V_{YY} Y_{OG} Y_{PG}\left(\frac{Y_{OG,t}\left(i\right) - Y_{OG}}{Y_{OG}}\right)\left(\frac{Y_{PG,t} - Y_{PG}}{Y_{PG}}\right) \\ &\quad + \frac{V_{YY}}{2} Y_{OG} Y_{OG}\left(\frac{Y_{OG,t}\left(i\right) - Y_{OG}}{Y_{OG}}\right)^2 + \frac{V_{YY}}{2} Y_{PG} Y_{PG}\left(\frac{Y_{PG,t} - Y_{PG}}{Y_{PG}}\right)^2 + \left\|O\right\|^3 \end{aligned}$$

$$
\begin{aligned}
V\left(Y_{OG,t}(i), Y_{PG,t}, A_{G,t}\right) \approx\ & V_Y Y_{OG}\left(\widehat{Y}_{OG,t}(i)+\frac{1}{2}\left(\widehat{Y}_{OG,t}(i)\right)^2\right)+\frac{V_{YY}}{2} Y_{OG} Y_{OG} \\
& \left(\widehat{Y}_{OG,t}(i)+\frac{1}{2}\left(\widehat{Y}_{OG,t}(i)\right)^2\right)^2+V_{YA} Y_{OG} A_G\left(\widehat{Y}_{OG,t}(i)+\frac{1}{2}\left(\widehat{Y}_{OG,t}(i)\right)^2\right) \\
& \left(\widehat{A}_{G,t}+\frac{1}{2}\left(\widehat{A}_{G,t}\right)^2\right)+V_{YY} Y_{OG} Y_{PG}\left(\widehat{Y}_{OG,t}(i)+\frac{1}{2}\left(\widehat{Y}_{OG,t}(i)\right)^2\right) \\
& \left(\widehat{Y}_{PG,t}+\frac{1}{2}\left(\widehat{Y}_{PG,t}\right)^2\right)+\|O\|^3+t.i.p.
\end{aligned}
$$

$$
\begin{aligned}
V\left(Y_{OG,t}(i), Y_{PG,t}, A_{G,t}\right) \approx\ & V_Y Y_{OG}\left(\widehat{Y}_{OG,t}(i)+\frac{1}{2}\left(\widehat{Y}_{OG,t}(i)\right)^2\right)+\frac{V_{YY}}{2} Y_{OG} Y_{OG}\left(\widehat{Y}_{OG,t}(i)\right)^2 \\
& +V_{YA} Y_{OG} A_G\left(\widehat{Y}_{OG,t}(i)\,\widehat{A}_{G,t}\right)+V_{YY} Y_{OG} Y_{PG,t}\left(\widehat{Y}_{OG,t}(i)\,\widehat{Y}_{PG,t}\right)+\|O\|^3+t.i.p.
\end{aligned}
$$

Assuming the steady state to shocks is 1, i.e. $A_V = A_G = A_M = 1$ and let $g_{OG,t} = -\frac{V_{YA}\widehat{A}_{G,t}}{V_{YY}Y_{OG}}$ and $g_{PG,t} = -\widehat{Y}_{PG,t}$

$$
\begin{aligned}
V\left(Y_{OG,t}(i), Y_{PG,t}, A_{G,t}\right) \approx\ & V_Y Y_{OG}\left(\widehat{Y}_{OG,t}(i)+\frac{1}{2}\left(\widehat{Y}_{OG,t}(i)\right)^2\right)+\frac{V_{YY}}{2} Y_{OG} Y_{OG}\left(\widehat{Y}_{OG,t}(i)\right)^2 \\
& -g_{OG,t} V_{YY} Y_{OG} Y_{OG} \widehat{Y}_{OG,t}(i)-g_{PG,t} V_{YY} Y_{OG} Y_{PG} \widehat{Y}_{OG,t}(i)+\|O\|^3+t.i.p.
\end{aligned}
$$

Using $V_{YY} = \psi\frac{V_Y}{Y_G} = \psi\frac{V_Y}{Y_{OG}+Y_{PG,t}}$

$$
\begin{aligned}
V\left(Y_{OG,t}(i), Y_{PG,t}, A_{G,t}\right) \approx\ & V_Y Y_{OG}\left(\widehat{Y}_{OG,t}(i)+\frac{1}{2}\left(\widehat{Y}_{OG,t}(i)\right)^2\right)+\psi \frac{V_Y}{2\left(Y_{OG}+Y_{PG}\right)} Y_{OG} Y_{OG}\left(\widehat{Y}_{OG,t}(i)\right)^2 \\
& -g_{OG,t} \psi \frac{V_Y}{Y_{OG}+Y_{PG}} Y_{OG} Y_{OG} \widehat{Y}_{OG,t}(i)-g_{PG,t} \psi \frac{V_Y}{Y_{OG}+Y_{PG}} Y_{OG} Y_{PG} \widehat{Y}_{OG,t}(i) \\
& +\|O\|^3+t.i.p.
\end{aligned}
$$

$$
\begin{aligned}
V\left(Y_{OG,t}(i), Y_{PG,t}, A_{G,t}\right) \approx\ & V_Y Y_{OG}[\widehat{Y}_{OG,t}(i)+\frac{1}{2}\left(\widehat{Y}_{OG,t}(i)\right)^2+\psi \frac{Y_{OG}}{2\left(Y_{OG}+Y_{PG,t}\right)}\left(\widehat{Y}_{OG,t}(i)\right)^2 \\
& -g_{OG,t} \psi \frac{Y_{OG}}{Y_{OG}+Y^{PG}} \widehat{Y}_{OG,t}(i)-g_{PG,t} \psi \frac{Y^{PG}}{Y_{OG}+Y^{PG}} \widehat{Y}_{OG,t}(i)]+\|O\|^3+t.i.p.
\end{aligned}
$$

Since $c_p = \frac{Y^{PG}}{Y^{PG}+Y_{OG}}$,

$$
\begin{aligned}
V\left(Y_{OG,t}(i), Y_{PG,t}, A_{G,t}\right) \approx\ & V_Y Y_{OG}[\widehat{Y}_{OG,t}(i)+\frac{1}{2}\left(\widehat{Y}_{OG,t}(i)\right)^2+\frac{\psi\left(1-c_p\right)}{2}\left(\widehat{Y}_{OG,t}(i)\right)^2 \\
& -g_{OG,t} \psi\left(1-c_p\right) \widehat{Y}_{OG,t}(i)-g_{PG,t} \psi c_p \widehat{Y}_{OG,t}]+\|O\|^3+t.i.p.
\end{aligned}
$$

$$
\begin{aligned}
V\left(Y_{OG,t}(i), Y_{PG,t}, A_{G,t}\right) \;\approx\;& V_Y Y_{OG}[\widehat{Y}_{OG,t}\left(i\right) + \left(\frac{1+\psi\left(1-c_p\right)}{2}\right)\left(\widehat{Y}_{OG,t}\left(i\right)\right)^2 \\
& -\psi\left(g_{OG,t}\left(1-c_p\right)+g_{PG,t}c_p\right)\widehat{Y}_{OG,t}\left(i\right)] + \left\|O\right\|^3 + t.i.p.
\end{aligned} \tag{26}
$$

Therefore

$$
V\left(N_t\left(i\right)\right) = V\left(Y_{V,t}(i), A_{V,t}\right) + V\left(Y_{M,t}(i), A_{M,t}\right) + V\left(Y_{OG,t}(i), Y_{PG,t}, A_{G,t}\right)
$$

In the second order,

$$
v\left(N_t\left(i\right)\right) \approx (24) + (25) + (26)
$$

$$
\begin{aligned}
v\left(N_t\left(i\right)\right) \;\approx\;& V_Y Y_V\left[\widehat{Y}_{V,t}\left(i\right) - \psi g_{V,t}\left(\widehat{Y}_{V,t}\left(i\right)\right) + \left(\frac{\psi+1}{2}\right)\left(\widehat{Y}_{V,t}\left(i\right)\right)^2\right] \\
& +V_Y Y_M\left[\widehat{Y}_{M,t}\left(i\right) - \psi g_{M,t}\left(\widehat{Y}_{M,t}\left(i\right)\right) + \left(\frac{\psi+1}{2}\right)\left(\widehat{Y}_{M,t}\left(i\right)\right)^2\right] \\
& +V_Y Y_{OG}\left[\widehat{Y}_{OG,t}\left(i\right) + \left(\frac{1+\psi\left(1-c_p\right)}{2}\right)\left(\widehat{Y}_{OG,t}\left(i\right)\right)^2 - \psi\left(g_{OG,t}\left(1-c_p\right)+g_{PG,t}c_p\right)\widehat{Y}_{OG,t}\left(i\right)\right] \\
& +\left\|O\right\|^3 + t.i.p.
\end{aligned}
$$

Aggregating disutility over all households,

$$
\begin{aligned}
\int_0^1 v\left(N_t\left(i\right)\right)\; di \;\approx\;& V_Y Y_V\left[\int_0^1 \widehat{Y}_{V,t}\left(i\right) di - \psi g_{V,t}\int_0^1 \widehat{Y}_{V,t}\left(i\right) di + \left(\frac{\psi+1}{2}\right)\int_0^1 \widehat{Y}_{V,t}\left(i\right)^2 di\right] \\
& +V_Y Y_M\left[\int_0^1 \widehat{Y}_{M,t}\left(i\right) di - \psi g_{M,t}\int_0^1 \widehat{Y}_{M,t}\left(i\right) di + \left(\frac{\psi+1}{2}\right)\int_0^1 \widehat{Y}_{M,t}\left(i\right)^2 di\right] \\
& +V_Y Y_{OG}\left[\int_0^1 \widehat{Y}_{OG,t}\left(i\right) di + \left(\frac{1+\psi\left(1-c_p\right)}{2}\right)\int_0^1 \widehat{Y}_{OG,t}\left(i\right)^2 di\right. \\
& \left.-\psi\left(g_{OG,t}\left(1-c_p\right)+g_{PG,t}c_p\right)\int_0^1 \widehat{Y}_{OG,t}\left(i\right) di\right] \\
& +\left\|O\right\|^3 + t.i.p.
\end{aligned}
$$

$$
\begin{aligned}
\int_0^1 v\left(N_t(i)\right)\, di \;\approx\; & V_Y Y_V \left[E_i\left\{\widehat{Y}_{V,t}(i)\right\} - \psi g_{V,t} E_i\left\{\widehat{Y}_{V,t}(i)\right\} + \left(\frac{\psi+1}{2}\right) E_i\left\{\widehat{Y}_{V,t}(i)^2\right\}\right] \\
& + V_Y Y_M \left[E_i\left\{\widehat{Y}_{M,t}(i)\right\} - \psi g_{M,t} E_i\left\{\widehat{Y}_{M,t}(i)\right\} + \left(\frac{\psi+1}{2}\right) E_i\left\{\widehat{Y}_{M,t}(i)^2\right\}\right] \\
& + \left[V_Y Y_{OG} E_i\left\{\widehat{Y}_{OG,t}(i)\right\} + \left(\frac{1+\psi(1-c_p)}{2}\right) E_i\left\{\widehat{Y}_{OG,t}(i)^2\right\}\right. \\
& \left. - \psi\left(g_{OG,t}(1-c_p) + g_{PG,t} c_p\right) E_i\left\{\widehat{Y}_{OG,t}(i)\right\}\right] \\
& + \|O\|^3 + t.i.p.
\end{aligned}
$$

Since $Var(X) = E(X^2) - (E(X))^2$

$$
\begin{aligned}
\int_0^1 v\left(N_t(i)\right)\, di \;\approx\; & V_Y Y_V \left[E_i\left\{\widehat{Y}_{V,t}(i)\right\} - \psi g_{V,t} E_i\left\{\widehat{Y}_{V,t}(i)\right\} + \left(\frac{\psi+1}{2}\right)\left[Var\left\{\widehat{Y}_{V,t}(i)\right\}\right.\right. \\
& \left.\left. + \left[E_i\left\{\widehat{Y}_{V,t}(i)\right\}\right]^2\right]\right] + V_Y Y_M \left[E_i\left\{\widehat{Y}_{M,t}(i)\right\} - \psi g_{M,t} E_i\left\{\widehat{Y}_{M,t}(i)\right\}\right. \\
& \left. + \left(\frac{\psi+1}{2}\right)\left[Var\left\{\widehat{Y}_{M,t}(i)\right\} + \left[E_i\left\{\widehat{Y}_{M,t}(i)\right\}\right]^2\right]\right] \\
& + V_Y Y_{OG} \left[E_i\left\{\widehat{Y}_{OG,t}(i)\right\} + \left(\frac{1+\psi(1-c_p)}{2}\right)\left[Var\left\{\widehat{Y}_{OG,t}(i)\right\}\right.\right. \\
& \left.\left. + \left[E_i\left\{\widehat{Y}_{OG,t}(i)\right\}\right]^2\right] - \psi\left(g_{OG,t}(1-c_p) + g_{PG,t} c_p\right) E_i\left\{\widehat{Y}_{OG,t}(i)\right\}\right] \\
& + \|O\|^3 + t.i.p.
\end{aligned}
$$

$$
\begin{aligned}
\int_0^1 v\left(N_t(i)\right)\, di \;\approx\; & V_Y Y_V \left[\left(1-\psi g_{V,t}\right) E_i\left\{\widehat{Y}_{V,t}(i)\right\} + \left(\frac{\psi+1}{2}\right)\left[Var\left\{\widehat{Y}_{V,t}(i)\right\}\right.\right. \\
& \left.\left. + \left[E_i\left\{\widehat{Y}_{V,t}(i)\right\}\right]^2\right]\right] + V_Y Y_M \left[\left(1-\psi g_{M,t}\right) E_i\left\{\widehat{Y}_{M,t}(i)\right\}\right. \\
& \left. + \left(\frac{\psi+1}{2}\right)\left[Var\left\{\widehat{Y}_{M,t}(i)\right\} + \left[E_i\left\{\widehat{Y}_{M,t}(i)\right\}\right]^2\right]\right] \\
& + V_Y Y_{OG} \left[\left(1-\psi\left(g_{OG,t}(1-c_p) + g_{PG,t} c_p\right)\right) E_i\left\{\widehat{Y}_{OG,t}(i)\right\}\right. \\
& \left. + \left(\frac{1+\psi(1-c_p)}{2}\right)\left[Var\left\{\widehat{Y}_{OG,t}(i)\right\} + \left[E_i\left\{\widehat{Y}_{OG,t}(i)\right\}\right]^2\right]\right] \\
& + \|O\|^3 + t.i.p.
\end{aligned}
$$

It can be shown that (see Woodford (2003) and Gali and Monacelli (2005)),

$$\begin{aligned}
\widehat{Y}_{V,t} &= E_i\left\{\widehat{Y}_{V,t}(i)\right\} + \frac{1}{2}\left(\frac{\theta-1}{\theta}\right) Var\left\{\widehat{Y}_{V,t}(i)\right\} \\
\widehat{Y}_{M,t} &= E_i\left\{\widehat{Y}_{M,t}(i)\right\} + \frac{1}{2}\left(\frac{\theta-1}{\theta}\right) Var\left\{\widehat{Y}_{M,t}(i)\right\} \\
\widehat{Y}_{OG,t} &= E_i\left\{\widehat{Y}_{OG,t}(i)\right\} + \frac{1}{2}\left(\frac{\theta-1}{\theta}\right) Var\left\{\widehat{Y}_{OG,t}(i)\right\}
\end{aligned}$$

Therefore

$$\begin{aligned}
\int_0^1 v(N_t(i))\ di \approx\ & V_Y Y_V \left[(1-\psi g_{V,t})\left[\widehat{Y}_{V,t} - \frac{1}{2}\left(\frac{\theta-1}{\theta}\right) Var\left\{\widehat{Y}_{V,t}(i)\right\}\right] + \left(\frac{\psi+1}{2}\right)\left[Var\left\{\widehat{Y}_{V,t}(i)\right\}\right.\right. \\
& \left.\left. + \left[\widehat{Y}_{V,t} - \frac{1}{2}\left(\frac{\theta-1}{\theta}\right) Var\left\{\widehat{Y}_{V,t}(i)\right\}\right]^2\right]\right] \\
& + V_Y Y_M \left[(1-\psi g_{M,t})\left[\widehat{Y}_{M,t} - \frac{1}{2}\left(\frac{\theta-1}{\theta}\right) Var\left\{\widehat{Y}_{M,t}(i)\right\}\right]\right. \\
& \left. + \left(\frac{\psi+1}{2}\right)\left[Var\left\{\widehat{Y}_{M,t}(i)\right\} + \left[\widehat{Y}_{M,t} - \frac{1}{2}\left(\frac{\theta-1}{\theta}\right) Var\left\{\widehat{Y}_{M,t}(i)\right\}\right]^2\right]\right] \\
& + V_Y Y_{OG} \left[(1-\psi(g_{OG,t}(1-c_p) + g_{PG,t}c_p))\left[\widehat{Y}_{OG,t} - \frac{1}{2}\left(\frac{\theta-1}{\theta}\right) Var\left\{\widehat{Y}_{OG,t}(i)\right\}\right]\right. \\
& \left. + \left(\frac{1+\psi(1-c_p)}{2}\right)\left[Var\left\{\widehat{Y}_{OG,t}(i)\right\} + \left[\widehat{Y}_{OG,t} - \frac{1}{2}\left(\frac{\theta-1}{\theta}\right) Var\left\{\widehat{Y}_{OG,t}(i)\right\}\right]^2\right]\right] \\
& + \|O\|^3 + t.i.p.
\end{aligned}$$

$$\begin{aligned}
\int_0^1 v(N_t(i))\ di \approx\ & V_Y Y_V \left[(1-\psi g_{V,t})\left[\widehat{Y}_{V,t} - \frac{1}{2}\left(\frac{\theta-1}{\theta}\right) Var\left\{\widehat{Y}_{V,t}(i)\right\}\right] + \left(\frac{\psi+1}{2}\right)\left[Var\left\{\widehat{Y}_{V,t}(i)\right\}\right.\right. \\
& \left.\left. + \widehat{Y}_{V,t}^2\right]\right] + V_Y Y_M \left[(1-\psi g_{M,t})\left[\widehat{Y}_{M,t} - \frac{1}{2}\left(\frac{\theta-1}{\theta}\right) Var\left\{\widehat{Y}_{M,t}(i)\right\}\right]\right. \\
& \left. + \left(\frac{\psi+1}{2}\right)\left[Var\left\{\widehat{Y}_{M,t}(i)\right\} + \widehat{Y}_{M,t}^2\right]\right] \\
& + V_Y Y_{OG} \left[(1-\psi(g_{OG,t}(1-c_p) + g_{PG,t}c_p))\left[\widehat{Y}_{OG,t} - \frac{1}{2}\left(\frac{\theta-1}{\theta}\right) Var\left\{\widehat{Y}_{OG,t}(i)\right\}\right]\right. \\
& \left. + \left(\frac{1+\psi(1-c_p)}{2}\right)\left[Var\left\{\widehat{Y}_{OG,t}(i)\right\} + \widehat{Y}_{OG,t}^2\right]\right] + \|O\|^3 + t.i.p.
\end{aligned}$$

Using a result in Woodford (2003) , since the manufacturing sector has sticky prices in place,

$$Var\left\{\widehat{Y}_{M,t}(i)\right\} = \theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\}.$$

Similarly for the grain and vegetable sectors are flexible price sectors,

$$\begin{aligned} Var\left\{\widehat{Y}_{V,t}(i)\right\} &= \theta^2 Var\left\{\widehat{P}_{V,t}(i)\right\} = 0 \\ Var\left\{\widehat{Y}_{OG,t}(i)\right\} &= \theta^2 Var\left\{\widehat{P}_{OG,t}(i)\right\} = 0 \end{aligned}$$

Therefore,

$$\begin{aligned} \int_0^1 v\left(N_t(i)\right)\, di \approx\ & V_Y Y_V \left[\left(1 - \psi g_{V,t}\right)\widehat{Y}_{V,t} + \left(\frac{\psi+1}{2}\right)\widehat{Y}^2_{V,t}\right] \\ & + V_Y Y_M \left[\left(1 - \psi g_{M,t}\right)\left[\widehat{Y}_{M,t} - \frac{1}{2}\left(\frac{\theta-1}{\theta}\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\}\right]\right. \\ & \left. + \left(\frac{\psi+1}{2}\right)\left[\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\} + \widehat{Y}^2_{M,t}\right]\right] \\ & + V_Y Y_{OG}\left[\left(1 - \psi\left(g_{OG,t}\left(1-c_p\right) + g_{PG,t}c_p\right)\right)\left[\widehat{Y}_{OG,t}\right]\right. \\ & \left. + \left(\frac{1+\psi\left(1-c_p\right)}{2}\right)\left[\widehat{Y}^2_{OG,t}\right]\right] + \|O\|^3 + t.i.p. \end{aligned}$$

On simplifying we get,

$$\begin{aligned} \int_0^1 v\left(N_t(i)\right)\, di \approx\ & V_Y Y_V \left[\widehat{Y}_{V,t} - \psi g_{V,t}\widehat{Y}_{V,t} + \left(\frac{\psi+1}{2}\right)\widehat{Y}^2_{V,t}\right] \\ & + V_Y Y_M \left[\widehat{Y}_{M,t} - \psi g_{M,t}\widehat{Y}_{M,t} + \frac{1}{2}\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\}\right. \\ & \left. + \left(\frac{\psi+1}{2}\right)\widehat{Y}^2_{M,t}\right] + V_Y Y_{OG}\left[\widehat{Y}_{OG,t} - \psi\left(g_{OG,t}\left(1-c_p\right) + g_{PG,t}c_p\right)\widehat{Y}_{OG,t}\right. \\ & \left. + \left(\frac{1+\psi\left(1-c_p\right)}{2}\right)\widehat{Y}^2_{OG,t}\right] + \|O\|^3 + t.i.p. \end{aligned} \tag{27}$$

From the first order conditions,

$$\frac{V_{Y,t}}{U_{C,t}} = \frac{W_t}{P_t A_t}$$

at steady state $A = 1$, therefore

$$\frac{V_Y}{U_C} = \frac{W}{P}$$

where $P = P_A^{\delta} P_M^{1-\delta} = P_{OG}^{(1-\mu)\delta} P_V^{\mu\delta} P_M^{1-\delta}$. Using the technical appendix of Ghate et al. (2016),

$$
\begin{aligned}
P &= \left(\frac{\theta (1 - c_p)}{(\theta - 1)(1 - c_p) - c_p} W \right)^{(1-\mu)\delta} \left(\frac{\theta}{\theta - 1} W \right)^{\mu\delta} \left(\frac{\theta}{\theta - 1} W \right)^{(1-\delta)} \\
P &= \gamma^{-(1-\mu)\delta} \left(\frac{\theta - 1}{\theta} \right) W
\end{aligned}
$$

This implies,

$$
\frac{V_Y}{U_C} = \gamma^{(1-\mu)\delta}
$$

Again using the technical appendix of Ghate et al. (2016),

$$
\begin{aligned}
\frac{C_M}{C} &= (1 - \delta)\, \gamma^{-(1-\mu)\delta} \\
\frac{C_V}{C} &= \mu\delta\gamma^{-(1-\mu)\delta} \\
\frac{C_{OG}}{C} &= (1 - \mu)\, \delta\gamma^{-(1-\mu)\delta+1}
\end{aligned}
$$

Replacing Y_M, Y_V, Y_{OG} and V_Y in equation(27) with C_M, C_V, C_{OG} and $U_C\ \gamma^{(1-\mu)\delta}$ respectively we get,

$$
\begin{aligned}
\int_0^1 v(N_t(i))\ di \approx\ & U_C\ \gamma^{(1-\mu)\delta} C \left[\frac{C_V}{C} \left[\widehat{Y}_{V,t} - \psi g_{V,t} \widehat{Y}_{V,t} + \left(\frac{\psi + 1}{2} \right) \widehat{Y}_{V,t}^2 \right] \right. \\
& + \frac{C_M}{C} \left[\widehat{Y}_{M,t} - \psi g_{M,t} \widehat{Y}_{M,t} + \frac{1}{2} \left(\theta^{-1} + \psi \right) \theta^2 Var \left\{ \widehat{P}_{M,t}(i) \right\} \right. \\
& \left. + \left(\frac{\psi + 1}{2} \right) \widehat{Y}_{M,t}^2 \right] + \frac{C_{OG}}{C} \left[\widehat{Y}_{OG,t} - \psi \left(g_{OG,t} (1 - c_p) + g_{PG,t} c_p \right) \widehat{Y}_{OG,t} \right. \\
& \left. \left. + \left(\frac{1 + \psi (1 - c_p)}{2} \right) \widehat{Y}_{OG,t}^2 \right] \right] + \|O\|^3 + t.i.p.
\end{aligned}
$$

$$
\begin{aligned}
\int_0^1 v(N_t(i))\ di \approx\ & U_C\ C \left[\mu\delta \left[\widehat{Y}_{V,t} - \psi g_{V,t} \widehat{Y}_{V,t} + \left(\frac{\psi + 1}{2} \right) \widehat{Y}_{V,t}^2 \right] \right. \\
& + (1 - \delta) \left[\widehat{Y}_{M,t} - \psi g_{M,t} \widehat{Y}_{M,t} + \frac{1}{2} \left(\theta^{-1} + \psi \right) \theta^2 Var \left\{ \widehat{P}_{M,t}(i) \right\} \right. \\
& \left. + \left(\frac{\psi + 1}{2} \right) \widehat{Y}_{M,t}^2 \right] + (1 - \mu)\, \delta\gamma \left[\widehat{Y}_{OG,t} - \psi \left(g_{OG,t} (1 - c_p) + g_{PG,t} c_p \right) \widehat{Y}_{OG,t} \right. \\
& \left. \left. + \left(\frac{1 + \psi (1 - c_p)}{2} \right) \widehat{Y}_{OG,t}^2 \right] \right] + \|O\|^3 + t.i.p.
\end{aligned}
\tag{28}
$$

Now, we know that

$$w_t = U(C_t) - \int_0^1 v(N_t(i))\ di$$

Now, combining the second order approximation of utility from consumption (equation (23)) and the second order approximation of aggregated disutility from labour supply (equation (28)) in the average utility function (equation (20)), and using $\mu\delta\widehat{Y}_{V,t} + (1-\delta)\widehat{Y}_{M,t} + (1-\mu)\delta\widehat{Y}_{OG,t} = \widehat{C}_t$ we get,

$$\begin{aligned}
w_t \ &\approx\ U_C\ C\left[\widehat{C}_t + \frac{1}{2}(1-\sigma)\widehat{C}_t^2\right] - \left[U_C\ C\left[\mu\delta\left[\widehat{Y}_{V,t} - \psi g_{V,t}\widehat{Y}_{V,t} + \left(\frac{\psi+1}{2}\right)\widehat{Y}_{V,t}^2\right]\right.\right. \\
&\quad + (1-\delta)\left[\widehat{Y}_{M,t} - \psi g_{M,t}\widehat{Y}_{M,t} + \frac{1}{2}\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\}\right. \\
&\quad \left. + \left(\frac{\psi+1}{2}\right)\widehat{Y}_{M,t}^2\right] + (1-\mu)\delta\gamma\left[\widehat{Y}_{OG,t} - \psi\left(g_{OG,t}(1-c_p) + g_{PG,t}c_p\right)\widehat{Y}_{OG,t}\right. \\
&\quad \left.\left.\left. + \left(\frac{1+\psi(1-c_p)}{2}\right)\widehat{Y}_{OG,t}^2\right]\right]\right] + \|O\|^3 + t.i.p.
\end{aligned}$$

$$\begin{aligned}
w_t \ &\approx\ U_C\ C\left[\widehat{C}_t + \frac{1}{2}(1-\sigma)\widehat{C}_t^2 - \widehat{C}_t + (1-\mu)\delta(1-\gamma)\widehat{Y}_{OG,t} + \mu\delta\psi g_{V,t}\widehat{Y}_{V,t}\right. \\
&\quad + \mu\delta\left(\frac{\psi+1}{2}\right)\widehat{Y}_{V,t}^2 + (1-\delta)\psi g_{M,t}\widehat{Y}_{M,t} - \frac{1}{2}(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\} \\
&\quad - (1-\delta)\left(\frac{\psi+1}{2}\right)\widehat{Y}_{M,t}^2 + (1-\mu)\delta\gamma\psi\left(g_{OG,t}(1-c_p) - g_{PG,t}c_p\right)\widehat{Y}_{OG,t} \\
&\quad \left. - (1-\mu)\delta\gamma\left(\frac{1+\psi(1-c_p)}{2}\right)\widehat{Y}_{OG,t}^2\right] + \|O\|^3 + t.i.p.
\end{aligned} \tag{29}$$

Note that $\widehat{Y}_{OG,t} = \widehat{Y}_{OG,t}^n$, which is a function of shocks and thus t.i.p.

$$\begin{aligned}
w_t \ &\approx\ U_C\ C\left[\frac{1}{2}(1-\sigma)\widehat{C}_t^2 + \mu\delta\psi g_{V,t}\widehat{Y}_{V,t} + \mu\delta\left(\frac{\psi+1}{2}\right)\widehat{Y}_{V,t}^2\right. \\
&\quad + (1-\delta)\psi g_{M,t}\widehat{Y}_{M,t} - \frac{1}{2}(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\} - (1-\delta)\left(\frac{\psi+1}{2}\right)\widehat{Y}_{M,t}^2 \\
&\quad + (1-\mu)\delta\left(\gamma\psi\left(g_{OG,t}(1-c_p) - g_{PG,t}c_p\right) + (1-\gamma)\right)\widehat{Y}_{OG,t} \\
&\quad \left. - (1-\mu)\delta\gamma\left(\frac{1+\psi(1-c_p)}{2}\right)\widehat{Y}_{OG,t}^2\right] + \|O\|^3 + t.i.p.
\end{aligned}$$

Let $\mu\delta\psi g_{V,t} = \alpha_{1V}$ (coefficient of $\widehat{Y}_{V,t}$), $\mu\delta\left(\frac{\psi+1}{2}\right) = \alpha_{2V}$ (coefficient of $\widehat{Y}_{V,t}^2$),
$(1-\delta)\psi g_{M,t} = \alpha_{1M}$ (coefficient of $\widehat{Y}_{M,t}$), $(1-\delta)\left(\frac{\psi+1}{2}\right) = \alpha_{2M}$ (coefficient of $\widehat{Y}_{M,t}^2$),

$(1-\mu)\,\delta\left(\gamma\psi\left(g_{OG,t}\left(1-c_p\right)-g_{PG,t}c_p\right)+\left(1-\gamma\right)\right)=\alpha_{1G}$ (coefficient of $\widehat{Y}_{OG,t}$),
$(1-\mu)\,\delta\gamma\left(\frac{1+\psi(1-c_p)}{2}\right)=\alpha_{2G}$ (coefficient of $\widehat{Y}_{OG,t}^2$).

$$\begin{aligned} w_t \;\approx\; & U_C\; C\left[\frac{1}{2}\left(1-\sigma\right)\widehat{C}_t^2+\alpha_{1V}\widehat{Y}_{V,t}+\alpha_{2V}\widehat{Y}_{V,t}^2\right. \\ & +\alpha_{1M}\widehat{Y}_{M,t}-\frac{1}{2}\left(1-\delta\right)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}\left(i\right)\right\}-\alpha_{2M}\widehat{Y}_{M,t}^2 \\ & \left.+\alpha_{1G}\widehat{Y}_{OG,t}-\alpha_{2G}\widehat{Y}_{OG,t}^2\right]+\left\|O\right\|^3+t.i.p. \end{aligned}$$

Now substituting,

$$\begin{aligned} \widehat{Y}_{M,t} &= \widehat{C}_t+\delta\widehat{T}_{AM,t} \\ \widehat{Y}_{V,t} &= \widehat{C}_t-\left(1-\delta\right)\widehat{T}_{AM,t}+\left(1-\mu\right)\widehat{T}_{OGV,t} \\ \widehat{Y}_{OG,t} &= \widehat{C}_t-\left(1-\delta\right)\widehat{T}_{AM,t}-\mu\widehat{T}_{OGV,t} \end{aligned} \tag{30}$$

$$\begin{aligned} w_t \;\approx\; & U_C\; C\left[\frac{1}{2}\left(1-\sigma\right)\widehat{C}_t^2+\alpha_{1V}\left(\widehat{C}_t-\left(1-\delta\right)\widehat{T}_{AM,t}+\left(1-\mu\right)\widehat{T}_{OGV,t}\right)\right. \\ & +\alpha_{2V}\left(\widehat{C}_t-\left(1-\delta\right)\widehat{T}_{AM,t}+\left(1-\mu\right)\widehat{T}_{OGV,t}\right)^2+\alpha_{1M}\left(\widehat{C}_t+\delta\widehat{T}_{AM,t}\right) \\ & -\frac{1}{2}\left(1-\delta\right)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}\left(i\right)\right\}-\alpha_{2M}\left(\widehat{C}_t+\delta\widehat{T}_{AM,t}\right)^2 \\ & \left.+\alpha_{1G}\left(\widehat{C}_t-\left(1-\delta\right)\widehat{T}_{AM,t}-\mu\widehat{T}_{OGV,t}\right)-\alpha_{2G}\left(\widehat{C}_t-\left(1-\delta\right)\widehat{T}_{AM,t}-\mu\widehat{T}_{OGV,t}\right)^2\right]+\left\|O\right\|^3+t.i.p. \end{aligned}$$

$$\begin{aligned} w_t \;\approx\; & U_C\; C\left[\frac{1}{2}\left(1-\sigma\right)\widehat{C}_t^2+-\frac{1}{2}\left(1-\delta\right)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}\left(i\right)\right\}\right. \\ & +\left(\alpha_{1V}+\alpha_{1M}+\alpha_{1G}\right)\widehat{C}_t+\left(\alpha_{1M}\delta-\alpha_{1V}\left(1-\delta\right)-\alpha_{1G}\left(1-\delta\right)\right)\widehat{T}_{AM,t} \\ & +\left(\alpha_{1V}\left(1-\mu\right)-\alpha_{1G}\mu\right)\widehat{T}_{OGV,t}-\left(\alpha_{2V}+\alpha_{2M}+\alpha_{2G}\right)\widehat{C}_t^2-\alpha_{2M}\delta^2+\left[\alpha_{2V}\left(1-\delta\right)^2\right. \\ & \left.+\alpha_{2G}\left(1-\delta\right)^2\right]\widehat{T}_{AM,t}^2-\left(\alpha_{2V}\left(1-\mu\right)^2+\alpha_{2G}\mu^2\right)\widehat{T}_{OGV,t}^2 \\ & -\left(2\alpha_{2M}\delta-2\alpha_{2V}\left(1-\delta\right)-2\alpha_{2G}\left(1-\delta\right)\right)\widehat{C}_t\widehat{T}_{AM,t} \\ & \left.-\left(2\alpha_{2V}\left(1-\mu\right)-2\alpha_{2G}\mu\right)\widehat{C}_t\widehat{T}_{OGV,t}-\left(2\alpha_{2G}\left(1-\delta\right)\mu-2\alpha_{2V}\left(1-\delta\right)\left(1-\mu\right)\right)\widehat{T}_{AM,t}\widehat{T}_{OGV,t}\right] \\ & +\left\|O\right\|^3+t.i.p. \end{aligned}$$

Now we use the fact that $\widehat{Y}_{OG,t}$, $\widehat{T}_{OGV,t}$, $\widehat{Y}_{V,t}$, are $t.i.p.$ as they are natural levels.

$$\begin{aligned}\widehat{C}_t\widehat{T}_{AM,t} &= \left(\widehat{Y}_{V,t} + (1-\delta)\,\widehat{T}_{AM,t} - (1-\mu)\,\widehat{T}_{OGV,t}\right)\widehat{T}_{AM,t} \\ &= \widehat{Y}_{V,t}\widehat{T}_{AM,t} + (1-\delta)\,\widehat{T}^2_{AM,t} - (1-\mu)\,\widehat{T}_{OGV,t}\widehat{T}_{AM,t}\end{aligned}$$

$$\begin{aligned}w_t \approx\ & \Bigg[U_C\ C - \frac{1}{2}(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\} \\ & + \left[(\alpha_{1V}+\alpha_{1M}+\alpha_{1G}) - (2\alpha_{2V}(1-\mu) - 2\alpha_{2G}\mu)\,\widehat{T}_{OGV,t}\right]\widehat{C}_t + [(\alpha_{1M}\delta - \alpha_{1V}(1-\delta) \\ & -\alpha_{1G}(1-\delta)) - (2\alpha_{2M}\delta - 2\alpha_{2V}(1-\delta) - 2\alpha_{2G}(1-\delta))\left(\widehat{Y}_{V,t} - (1-\mu)\,\widehat{T}_{OGV,t}\right) \\ & - (2\alpha_{2G}(1-\delta)\,\mu - 2\alpha_{2V}(1-\delta)(1-\mu))\,\widehat{T}_{OGV,t}\Big]\,\widehat{T}_{AM,t} + (\alpha_{1V}(1-\mu) - \alpha_{1G}\mu)\,\widehat{T}_{OGV,t} - \\ & \left[-\frac{1}{2}(1-\sigma) + (\alpha_{2V}+\alpha_{2M}+\alpha_{2G})\right]\widehat{C}^2_t - \left[\left(\alpha_{2M}\delta^2 + \alpha_{2V}(1-\delta)^2 + \alpha_{2G}(1-\delta)^2\right)\right. \\ & \left.- (1-\delta)(2\alpha_{2M}\delta - 2\alpha_{2V}(1-\delta) - 2\alpha_{2G}(1-\delta))\right]\widehat{T}^2_{AM,t} - \left(\alpha_{2V}(1-\mu)^2 + \alpha_{2G}\mu^2\right)\widehat{T}^2_{OGV,t}\Bigg] \\ & + \|O\|^3 + t.i.p.\end{aligned}$$

Let $\left[(\alpha_{1V}+\alpha_{1M}+\alpha_{1G}) - (2\alpha_{2V}(1-\mu) - 2\alpha_{2G}\mu)\,\widehat{T}_{OGV,t}\right] = \beta_{1C}$ (coefficient of $\widehat{C}_t$),
$\left[-\frac{1}{2}(1-\sigma) + (\alpha_{2V}+\alpha_{2M}+\alpha_{2G})\right] = \beta_{2C}$ (coefficient of $\widehat{C}^2_t$),
$[(\alpha_{1M}\delta - \alpha_{1V}(1-\delta) - \alpha_{1G}(1-\delta)) - (2\alpha_{2M}\delta - 2\alpha_{2V}(1-\delta) - 2\alpha_{2G}(1-\delta))$
$\left(\widehat{Y}_{V,t} - (1-\mu)\,\widehat{T}_{OGV,t}\right) - (2\alpha_{2G}(1-\delta)\,\mu - 2\alpha_{2V}(1-\delta)(1-\mu))\,\widehat{T}_{OGV,t}\Big]$
$= \beta_{1TAM}$ (coefficient of $\widehat{T}_{AM,t}$),
$\left[\left(\alpha_{2M}\delta^2 + \alpha_{2V}(1-\delta)^2 + \alpha_{2G}(1-\delta)^2\right) - (1-\delta)(2\alpha_{2M}\delta - 2\alpha_{2V}(1-\delta) - 2\alpha_{2G}(1-\delta))\right]$
$= \beta_{2TAM}$ (coefficient of $\widehat{T}^2_{AM,t}$),
$(\alpha_{1V}(1-\mu) - \alpha_{1G}\mu) = \beta_{1TOG}$ (coefficient of $\widehat{T}_{OG,t}$),
$\left(\alpha_{2V}(1-\mu)^2 + \alpha_{2G}\mu^2\right) = \beta_{2TOG}$ (coefficient of $\widehat{T}^2_{OG,t}$).

$$\begin{aligned}w_t \approx\ & U_C\ C\left[-\frac{1}{2}(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\} + \beta_{1C}\widehat{C}_t\right. \\ & \left.+\beta_{ITAM}\widehat{T}_{AM,t} + \beta_{1TOGV}\widehat{T}_{OGV,t} - \beta_{2C}\widehat{C}^2_t - \beta_{2TAM}\widehat{T}^2_{AM,t} - \beta_{2TOGV}\widehat{T}^2_{OGV,t}\right] \\ & + \|O\|^3 + t.i.p.\end{aligned}$$

$$\begin{aligned} w_t &\approx -\frac{U_C\ C}{2}\Big[(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\} - 2\beta_{1C}\widehat{C}_t \\ &\quad -2\beta_{ITAM}\widehat{T}_{AM,t} - 2\beta_{2TOGV}\widehat{T}_{OGV,t} + 2\beta_{2C}\widehat{C}_t^2 + 2\beta_{2TAM}\widehat{T}_{AM,t}^2 + 2\beta_{2TOGV}\widehat{T}_{OGV,t}^2\Big] \\ &\quad + \|O\|^3 + t.i.p. \end{aligned}$$

$$\begin{aligned} w_t &\approx -\frac{U_C\ C}{2}\left[(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\} + 2\beta_{2C}\left(\widehat{C}_t^2 - \frac{\beta_{1C}}{\beta_{2C}}\widehat{C}_t\right) + \right. \\ &\quad \left. +2\beta_{2TAM}\left(\widehat{T}_{AM,t}^2 - \frac{\beta_{ITAM}}{\beta_{2TAM}}\widehat{T}_{AM,t}\right) + 2\beta_{2TOGV}\left(\widehat{T}_{OGV,t}^2 - \frac{\beta_{TOGV}}{\beta_{2TOGV}}\widehat{T}_{OGV,t}\right)\right] \\ &\quad + \|O\|^3 + t.i.p. \end{aligned}$$

Note here $\beta_{1C}, \beta_{1TAM}, \beta_{1TOGV}$ are functions of shocks and $\beta_{2C}, \beta_{2TAM}, \beta_{2TOGV}$ are constants.

$$\begin{aligned} w_t &\approx -\frac{U_C\ C}{2}\left[(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\} + 2\beta_{2C}\left(\widehat{C}_t - \widehat{C}_t^n\right)^2 + \right. \\ &\quad \left. +2\beta_{2TAM}\left(\widehat{T}_{AM,t} - \widehat{T}_{AMt}^n\right)^2 + 2\beta_{2TOGV}\left(\widehat{T}_{OGV,t} - \widehat{T}_{OGV,t}^n\right)^2\right] \\ &\quad + \|O\|^3 + t.i.p. \end{aligned}$$

where $\frac{\beta_{1C}}{\beta_{2C}} = \widehat{C}_t^n$, $\frac{\beta_{ITAM}}{\beta_{2TAM}} = \widehat{T}_{AMt}^n$, $\frac{\beta_{TOGV}}{\beta_{2TOGV}} = \widehat{T}_{OGV,t}^n$.Now since $\widehat{T}_{OGV,t} = \widehat{T}_{OGV,t}^n$, the welfare function reduces to,

$$\begin{aligned} w_t &\approx -\frac{U_C\ C}{2}\left[(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\} + 2\beta_{2C}\left(\widehat{C}_t - \widehat{C}_t^n\right)^2 + \right. \\ &\quad \left. +2\beta_{2TAM}\left(\widetilde{T}_{AM,t}\right)^2\right] + \|O\|^3 + t.i.p. \end{aligned}$$

$$\begin{aligned} w_t &\approx -\frac{U_C\ C}{2}\left[(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\} + 2\beta_{2C}\left(\widetilde{C}_t\right)^2 + \right. \\ &\quad \left. +2\beta_{2TAM}\left(\widetilde{T}_{AM,t}\right)^2\right] + \|O\|^3 + t.i.p. \end{aligned}$$

where $\widehat{C}_t - \widehat{C}_t^n = \widetilde{C}_t$, $\widehat{T}_{AM,t} - \widehat{T}_{AMt}^n = \widetilde{T}_{AM,t}$. Welfare loss function from equation (22) becomes,

$$\begin{aligned} W &= E_0\sum_{t=0}^{\infty}\beta^t\left(\frac{w_t - w}{U_C\ C}\right) \approx -\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left[(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\} + 2\beta_{2C}\left(\widetilde{C}_t\right)^2 + \right. \\ &\quad \left. +2\beta_{2TAM}\left(\widetilde{T}_{AM,t}\right)^2\right] + \|O\|^3 + t.i.p. \end{aligned}$$

Using the following result from Woodford (2003),[18]

$$E_0 \sum_{t=0}^{\infty} \beta^t Var\left\{\widehat{P}_{M,t}(i)\right\} = \frac{\alpha_M}{(1-\beta\alpha_M)(1-\alpha_M)} E_0 \sum_{t=0}^{\infty} \beta^t \pi_{M,t}^2$$

Therefore,

$$\begin{aligned} W &= -E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{w_t - w}{U_C\ C}\right) \approx -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\alpha_M (1-\delta)\left(\theta^{-1}+\psi\right)\theta^2}{(1-\beta\alpha_M)(1-\alpha_M)} \pi_{M,t}^2 + 2\beta_{2C}\left(\widetilde{C}_t\right)^2 + \right. \\ & \left. +2\beta_{2TAM}\left(\widetilde{T}_{AM,t}\right)^2\right] + \|O\|^3 + t.i.p. \end{aligned}$$

$$\begin{aligned} W_t &= -E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{w_t - w}{U_C\ C}\right) \approx -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda_{\pi M}(\pi_{M,t})^2 + \lambda_{\widetilde{C}}\left(\widetilde{C}_t\right)^2 + \right. \\ & \left. +\lambda_{\widetilde{TAM}}\left(\widetilde{T}_{AM,t}\right)^2\right] + \|O\|^3 + t.i.p. \end{aligned}$$

where $\lambda_{\pi M} = \frac{\alpha_M(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2}{(1-\beta\alpha_M)(1-\alpha_M)}$, $\lambda_{\widetilde{C}} = 2\beta_{2C}$, $\lambda_{\widetilde{TAM}} = 2\beta_{2TAM}$.

With special case when $c_p = 0$, continuing from equation (29) we get,

$$\begin{aligned} w_t &\approx U_C\ C\left[\widehat{C}_t + \frac{1}{2}(1-\sigma)\widehat{C}_t^2 - \widehat{C}_t + \mu\delta\psi g_{V,t}\widehat{Y}_{V,t} + \mu\delta\left(\frac{\psi+1}{2}\right)\widehat{Y}_{V,t}^2\right. \\ & + (1-\delta)\psi g_{M,t}\widehat{Y}_{M,t} - \frac{1}{2}(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\} - (1-\delta)\left(\frac{\psi+1}{2}\right)\widehat{Y}_{M,t}^2 \\ & \left. + (1-\mu)\delta\psi g_{OG,t}\widehat{Y}_{OG,t} - (1-\mu)\delta\left(\frac{1+\psi}{2}\right)\widehat{Y}_{OG,t}^2\right] + \|O\|^3 + t.i.p. \end{aligned}$$

, since $\gamma = 1$ when $c_p = 0$.

$$\begin{aligned} w_t &\approx U_C\ C\left[\frac{1}{2}(1-\sigma)\widehat{C}_t^2 + \mu\delta\psi g_{V,t}\widehat{Y}_{V,t} + \mu\delta\left(\frac{\psi+1}{2}\right)\widehat{Y}_{V,t}^2\right. \\ & + (1-\delta)\psi g_{M,t}\widehat{Y}_{M,t} - \frac{1}{2}(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\} - (1-\delta)\left(\frac{\psi+1}{2}\right)\widehat{Y}_{M,t}^2 \\ & \left. + (1-\mu)\delta\psi g_{OG,t}\widehat{Y}_{OG,t} - (1-\mu)\delta\left(\frac{1+\psi}{2}\right)\widehat{Y}_{OG,t}^2\right] + \|O\|^3 + t.i.p. \end{aligned}$$

[18] Please refer Chapter 6 of the book.

Using equation(30) in above,

$$
\begin{aligned}
w_t \;\approx\;& U_C\ C\left[\frac{1}{2}(1-\sigma)\widehat{C}_t^2+\mu\delta\psi g_{V,t}\left(\widehat{C}_t-(1-\delta)\widehat{T}_{AM,t}+(1-\mu)\widehat{T}_{OGV,t}\right)\right.\\
&+\left(\frac{\psi+1}{2}\right)\mu\delta\left(\widehat{C}_t-(1-\delta)\widehat{T}_{AM,t}+(1-\mu)\widehat{T}_{OGV,t}\right)^2+(1-\delta)\psi g_{M,t}\left(\widehat{C}_t+\delta\widehat{T}_{AM,t}\right)\\
&-\frac{1}{2}(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\}-(1-\delta)\left(\frac{\psi+1}{2}\right)\left(\widehat{C}_t+\delta\widehat{T}_{AM,t}\right)^2\\
&+(1-\mu)\delta\psi g_{OG,t}\left(\widehat{C}_t-(1-\delta)\widehat{T}_{AM,t}-\mu\widehat{T}_{OGV,t}\right)\\
&\left.-(1-\mu)\delta\left(\frac{1+\psi}{2}\right)\left(\widehat{C}_t-(1-\delta)\widehat{T}_{AM,t}-\mu\widehat{T}_{OGV,t}\right)^2\right]+\|O\|^3+t.i.p.
\end{aligned}
$$

$$
\begin{aligned}
w_t \;\approx\;& U_C\ C\left[\frac{1}{2}(1-\sigma)\widehat{C}_t^2-\frac{1}{2}(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\}+\mu\delta\psi g_{V,t}+((1-\delta)\psi g_{M,t}\right.\\
&+(1-\mu)\delta\psi g_{OG,t})\widehat{C}_t-\left(\frac{\psi+1}{2}\right)((1-\delta)+\mu\delta+(1-\mu)\delta)\widehat{C}_t^2\\
&+((1-\delta)\delta\psi g_{M,t}-\mu\delta(1-\delta)\psi g_{V,t}-(1-\delta)(1-\mu)\delta\psi g_{OG,t})\widehat{T}_{AM,t}\\
&+(\mu\delta(1-\mu)\psi g_{V,t}-\mu(1-\mu)\delta\psi g_{OG,t})\widehat{T}_{OGV,t}\\
&-\left(\frac{\psi+1}{2}\right)(2(1-\mu)\mu\delta-2(1-\mu)\mu\delta)\widehat{C}_t\widehat{T}_{OGV,t}\\
&-\left(\frac{\psi+1}{2}\right)\left((1-\delta)\delta^2+\mu\delta(1-\delta)^2+(1-\delta)^2(1-\mu)\delta\right)\widehat{T}_{AM,t}^2\\
&-\left(\frac{\psi+1}{2}\right)\left(\mu\delta(1-\mu)^2+(1-\mu)\mu^2\delta\right)\widehat{T}_{OGV,t}^2\\
&-\left(\frac{\psi+1}{2}\right)(2\delta(1-\delta)-2(1-\delta)\mu\delta-2(1-\delta)(1-\mu)\delta)\widehat{C}_t\widehat{T}_{AM,t}\\
&\left.+\left(\frac{\psi+1}{2}\right)(2(1-\mu)(1-\delta)\mu\delta-2(1-\mu)(1-\delta)\mu\delta)\widehat{T}_{AM,t}\widehat{T}_{OGV,t}\right]+\|O\|^3+t.i.p.
\end{aligned}
$$

Simplifying this expression, we get

$$
\begin{aligned}
w_t \;\approx\;& -\frac{1}{2}U_C\ C\left[(\sigma+\psi)\widehat{C}_t^2-2(\mu\delta\psi g_{V,t}+(1-\delta)\psi g_{M,t}+(1-\mu)\delta\psi g_{OG,t})\widehat{C}_t\right.\\
&-2(1-\delta)\delta\psi(g_{M,t}-\mu g_{V,t}-(1-\mu)g_{OG,t})\widehat{T}_{AM,t}+(\psi+1)(1-\delta)\delta\widehat{T}_{AM,t}^2\\
&-2\psi\mu\delta(1-\mu)(g_{V,t}-g_{OG,t})\widehat{T}_{OGV,t}+(\psi+1)\mu\delta(1-\mu)\widehat{T}_{OGV,t}^2\\
&\left.+(1-\delta)\left(\theta^{-1}+\psi\right)\theta^2 Var\left\{\widehat{P}_{M,t}(i)\right\}\right]+\|O\|^3+t.i.p.
\end{aligned}
$$

$$
\begin{aligned}
w_t &\approx -\frac{1}{2} U_C \; C \left[\left((\sigma + \psi) \right) \left(\widehat{C}_t - \widehat{C}_t^n \right)^2 + (\psi + 1)(1 - \delta)\delta \left(\widehat{T}_{AM,t} - \widehat{T}_{AM,t}^n \right)^2 \right. \\
&\quad + (\psi + 1)\mu\delta(1 - \mu) \left(\widehat{T}_{OGV,t} - \widehat{T}_{OGV,t}^n \right)^2 \\
&\quad \left. + (1 - \delta)\left(\theta^{-1} + \psi\right)\theta^2 Var\left\{ \widehat{P}_{M,t}(i) \right\} \right] + \|O\|^3 + t.i.p.
\end{aligned}
$$

where $\widehat{C}_t^n = \frac{\left(\mu\delta\psi g_{V,t} + (1-\delta)\psi g_{M,t} + (1-\mu)\delta\psi g_{OG,t}\right)}{(\sigma+\psi)}$, $\widehat{T}_{AM,t}^n = \frac{(1-\delta)\delta\psi\left(g_{M,t} - \mu g_{V,t} - (1-\mu) g_{OG,t}\right)}{(\psi+1)(1-\delta)\delta}$ and $\widehat{T}_{OGV,t}^n = \frac{\psi\mu\delta(1-\mu)\left(g_{V,t} - g_{OG,t}\right)}{(\psi+1)\mu\delta(1-\mu)}$.

$$
\begin{aligned}
w_t &\approx -\frac{1}{2} U_C \; C \left[(\sigma + \psi) \left(\widetilde{Y}_t^n \right)^2 + (\psi + 1)(1 - \delta)\delta \left(\widetilde{T}_{AM,t}^n \right)^2 \right. \\
&\quad \left. + (1 - \delta)\left(\theta^{-1} + \psi\right)\theta^2 Var\left\{ \widehat{P}_{M,t}(i) \right\} \right] + \|O\|^3 + t.i.p.
\end{aligned}
$$

where $\widetilde{Y}_t = \widehat{Y}_t - \widehat{Y}_t^n$, since $\widehat{C}_t = \widehat{Y}_t$ when $c_p = 0$, $\widetilde{T}_{AM,t} = \widehat{T}_{AM,t} - \widehat{T}_{AM,t}^n$, $\widetilde{T}_{OGV,t} = \widehat{T}_{OGV,t} - \widehat{T}_{OGV,t}^n$ and because $\widehat{T}_{OGV,t} = \widehat{T}_{OGV,t}^n$, $\widetilde{T}_{OGV,t} = 0$. Welfare loss function in equation (22) would thus reduce to,

$$
\begin{aligned}
W &= E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{w_t}{U_C \; C} \right) \approx -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[(\sigma + \psi) \left(\widetilde{Y}_t \right)^2 + (\psi + 1)(1 - \delta)\delta \left(\widetilde{T}_{AM,t} \right)^2 \right. \\
&\quad \left. + (1 - \delta)\left(\theta^{-1} + \psi\right)\theta^2 Var\left\{ \widehat{P}_{M,t}(i) \right\} \right] + \|O\|^3 + t.i.p.
\end{aligned}
$$

Note that,

$$
E_0 \sum_{t=0}^{\infty} \beta^t Var\left\{ \widehat{P}_{M,t}(i) \right\} = \frac{\alpha_M}{(1 - \beta\alpha_M)(1 - \alpha_M)} E_0 \sum_{t=0}^{\infty} \beta^t \pi_{M,t}^2
$$

$$
\begin{aligned}
W_t &= E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{w_t}{U_C \; C} \right) \approx -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda_{\pi M} \left(\pi_{M,t} \right)^2 + (\sigma + \psi) \left(\widetilde{Y}_t \right)^2 \right. \\
&\quad \left. + (\psi + 1)(1 - \delta)\delta \left(\widetilde{T}_{AM,t} \right)^2 \right] + \|O\|^3 + t.i.p.
\end{aligned}
$$

Discretionary monetary policy,

Minimize welfare loss function subject to the NKPC. The problem can formally be written as,

$$
W_t = \min_{\{\pi_{M,t},\, \widetilde{C}_t,\, \widetilde{T}_{AM,t}\}} \frac{1}{2} \left[\lambda_{\pi M} \left(\pi_{M,t} \right)^2 + \lambda_{\widetilde{C}} \left(\widetilde{C}_t \right)^2 + \lambda_{\widetilde{TAM}} \left(\widetilde{T}_{AM,t} \right)^2 \right]
$$

subject to

$$
\pi_{M,t} = \lambda_M \left(\sigma + \psi\Theta_1 \right) \widetilde{C}_t - \lambda_M \delta \widetilde{T}_{AM,t}
$$

Lagrangian,

$$L_t = \min \frac{1}{2}\left[\lambda_{\pi M}\left(\pi_{M,t}\right)^2 + \lambda_{\widetilde{C}}\left(\widetilde{C}_t\right)^2 + \lambda_{\widetilde{TAM}}\left(\widetilde{T}_{AM,t}\right)^2\right] - \phi_1\left[\pi_{M,t} - \lambda_M\left(\sigma + \psi\Theta_1\right)\widetilde{C}_t - \lambda_M \delta \widetilde{T}_{AM,t}\right]$$

First order conditions,

$$\begin{aligned}
\frac{\partial L_t}{\partial \pi_{M,t}} &= \lambda_{\pi M}\left(\pi_{M,t}\right) - \phi_1 = 0 \\
\frac{\partial L_t}{\partial \widetilde{C}_t} &= \lambda_{\widetilde{C}}\left(\widetilde{C}_t\right) + \phi_1 \lambda_M\left(\sigma + \psi\Theta_1\right) = 0 \\
\frac{\partial L_t}{\partial \pi_{M,t}} &= \lambda_{\widetilde{TAM}}\left(\widetilde{T}_{AM,t}\right) + \phi_1 \lambda_M \delta = 0
\end{aligned}$$

This implies,

$$\begin{aligned}
\pi_{M,t} &= -\frac{\lambda_{\widetilde{TAM}}}{\delta \lambda_M \lambda_{\pi M}} \widetilde{T}_{AM,t} \\
\widetilde{C}_t &= -\frac{\lambda_M\left(\sigma + \psi\Theta_1\right)\lambda_{\pi M}}{\lambda_{\widetilde{C}}} \pi_{M,t}
\end{aligned}$$

We know that,

$$\widetilde{Y}_t = \left(1 - \lambda_C\right)\widetilde{C}_t + \lambda_C\left(1 - \delta\right)\widetilde{T}_{AM,t}$$

Substituting $\widetilde{C}_t$ in above, we get

$$\begin{aligned}
\widetilde{Y}_t &= -\left(1 - \lambda_C\right)\frac{\lambda_M\left(\sigma + \psi\Theta_1\right)\lambda_{\pi M}}{\lambda_{\widetilde{C}}} \pi_{M,t} - \lambda_C\left(1 - \delta\right)\frac{\delta \lambda_M \lambda_{\pi M}}{\lambda_{\widetilde{TAM}}} \pi_{M,t} \\
\widetilde{Y}_t &= -\left[\left(1 - \lambda_C\right)\frac{\lambda_M\left(\sigma + \psi\Theta_1\right)\lambda_{\pi M}}{\lambda_{\widetilde{C}}} + \lambda_C\left(1 - \delta\right)\frac{\delta \lambda_M \lambda_{\pi M}}{\lambda_{\widetilde{TAM}}}\right]\pi_{M,t}
\end{aligned}$$

Also,

$$\begin{aligned}
\pi_{,t} &= \pi_{M,t} + \delta \widetilde{T}_{AM,t} \\
\pi_{,t} &= -\left[\frac{\lambda_{\widetilde{TAM}}}{\delta \lambda_M \lambda_{\pi M}} - \delta\right]\widetilde{T}_{AM,t} \\
\pi_{M,t} &= \pi_t + \delta \frac{1}{\left[\frac{\lambda_{\widetilde{TAM}}}{\delta \lambda_M \lambda_{\pi M}} - \delta\right]} \pi_{,t} \\
\pi_{M,t} &= \frac{\lambda_{\widetilde{TAM}}}{\lambda_{\widetilde{TAM}} - \delta^2 \lambda_M \lambda_{\pi M}} \pi_{,t}
\end{aligned}$$

$$\widetilde{Y}_t = -\left((1-\lambda_C) \frac{\lambda_M (\sigma + \psi\Theta_1) \lambda_{\pi M}}{\lambda_{\widetilde{C}}} + \lambda_C (1-\delta) \frac{\delta \lambda_M \lambda_{\pi M}}{\lambda_{\widetilde{TAM}}} \right) \left(\frac{\lambda_{\widetilde{TAM}}}{\lambda_{\widetilde{TAM}} - \delta^2 \lambda_M \lambda_{\pi M}} \right) \pi_{,t}$$

Substituting this in the DIS equation, we get

$$\widetilde{Y}_t = -\frac{(1-\lambda_C)}{\sigma} \left[\widehat{R}_t - \widehat{r}_t^n \right] + \lambda_C (1-\delta) \widetilde{T}_{AM,t}$$

$$\begin{aligned}
& -\left[(1-\lambda_C) \frac{\lambda_M (\sigma + \psi\Theta_1) \lambda_{\pi M}}{\lambda_{\widetilde{C}}} + \lambda_C (1-\delta) \frac{\delta \lambda_M \lambda_{\pi M}}{\lambda_{\widetilde{TAM}}} \right] \pi_{M,t} \\
&= -\frac{(1-\lambda_C)}{\sigma} \left[\widehat{R}_t - \widehat{r}_t^n \right] - \lambda_C (1-\delta) \frac{\delta \lambda_M \lambda_{\pi M}}{\lambda_{\widetilde{TAM}}} \pi_{M,t} \\
\widehat{R}_t &= \widehat{r}_t^n + \frac{\sigma \lambda_M (\sigma + \psi\Theta_1) \lambda_{\pi M}}{\lambda_{\widetilde{C}}} \pi_{M,t}
\end{aligned}$$

or alternatively

$$\widehat{R}_t = \widehat{r}_t^n + \left(\frac{\sigma \lambda_M (\sigma + \psi\Theta_1) \lambda_{\pi M}}{\lambda_{\widetilde{C}}} \right) \left(\frac{\lambda_{\widetilde{TAM}}}{\lambda_{\widetilde{TAM}} - \delta^2 \lambda_M \lambda_{\pi M}} \right) \pi_{,t}$$

Special case when $c_p = 0$, $\lambda_C = 0$, the welfare minimization problem under discretionary monetary policy would be,

$$\begin{aligned}
W_t &= E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{w_t}{U_C \ C} \right) \approx -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda_{\pi M} (\pi_{M,t})^2 + (\sigma + \psi) \left(\widetilde{Y}_t \right)^2 \right. \\
& \left. + (\psi + 1)(1-\delta) \delta \left(\widetilde{T}_{AM,t} \right)^2 \right] + \|O\|^3 + t.i.p.
\end{aligned}$$

$$\begin{aligned}
L_t &= \min -\frac{1}{2} \left[\lambda_{\pi M} (\pi_{M,t})^2 + (\sigma + \psi) \left(\widetilde{Y}_t \right)^2 + (\psi + 1)(1-\delta) \delta \left(\widetilde{T}_{AM,t} \right)^2 \right] \\
& -\phi_1 \left[\pi_{M,t} - \lambda_M (\sigma + \psi) \widetilde{Y}_t - \lambda_M \delta \widetilde{T}_{AM,t} \right]
\end{aligned}$$

First order conditions,

$$\begin{aligned}
\frac{\partial L_t}{\partial \pi_{M,t}} &= -\lambda_{\pi M}\left(\pi_{M,t}\right) - \phi_1 = 0 \\
\frac{\partial L_t}{\partial \widetilde{Y}_t} &= -\left(\sigma+\psi\right)\left(\widetilde{Y}_t\right) + \phi_1 \lambda_M \left(\sigma+\psi\right) = 0 \\
\frac{\partial L_t}{\partial \pi_{M,t}} &= -\left(\psi+1\right)\left(1-\delta\right)\delta \widetilde{T}_{AM,t} + \phi_1 \lambda_M \delta = 0
\end{aligned}$$

This implies,

$$\begin{aligned}
\pi_{M,t} &= -\frac{\left(\psi+1\right)\left(1-\delta\right)\delta}{\delta \lambda_M \lambda_{\pi M}} \widetilde{T}_{AM,t} \\
\widetilde{Y}_t &= -\lambda_M \lambda_{\pi M} \pi_{M,t}
\end{aligned}$$

Also,

$$\begin{aligned}
\pi_{,t} &= \pi_{M,t} + \delta \widetilde{T}_{AM,t} \\
\pi_{,t} &= -\left[\frac{\left(\psi+1\right)\left(1-\delta\right)\delta}{\delta \lambda_M \lambda_{\pi M}} - \delta\right] \widetilde{T}_{AM,t} \\
\pi_{M,t} &= \pi_t + \delta \frac{1}{\left[\frac{(\psi+1)(1-\delta)\delta}{\delta \lambda_M \lambda_{\pi M}} - \delta\right]} \pi_{,t} \\
\pi_{M,t} &= \frac{\left(\psi+1\right)\left(1-\delta\right)\delta}{\left(\psi+1\right)\left(1-\delta\right)\delta - \delta^2 \lambda_M \lambda_{\pi M}} \pi_{,t}
\end{aligned}$$

$$\widetilde{Y}_t = -\frac{\lambda_M \lambda_{\pi M}\left(\psi+1\right)\left(1-\delta\right)\delta}{\left(\psi+1\right)\left(1-\delta\right)\delta - \delta^2 \lambda_M \lambda_{\pi M}} \pi_{,t}$$