Age-Specific Effects of Mortality Shocks and Economic Development[*]

Shankha Chakraborty — University of Oregon

Fidel Pérez-Sebastián — U. Alicante and U. Hull

August 2016

Version 1

Abstract

This paper examines the effect of age-specific mortality on the pattern of investment and economic development. In the presence of mortality risk, altruistic parents have an incentive to invest more in tangible assets (physical capital) that are readily transferable to future generations compared to their own intangible human capital. They also invest in the human capital of their children. The model is calibrated to the US and South Africa. The effect a mortality shock is shown to have strong distributional effects across cohorts and aggregate effects. The loss of human capital from an HIV/AIDS shock sets back economic development; advanced economies, more reliant on human capital, suffer more.

KEYWORDS: Life Expectancy, Mortality, Health, Physical Capital, Human Capital, Parental Altruism, HIV epidemic

JEL CLASSIFICATION: I10, O10, O40

[*]We thank seminar participants at the 2nd Barcelona Workshop on Towards Sustained Economic Growth, and the 13th Viennese Workshop on Optimal Control and Dynamic Games. Perez-Sebastian acknowledges financial support from the Spanish Ministry of Science and Innovation (ECO2012-36719), and the Generalitat Valenciana (PROMETEO/2013/037).

1 Introduction

This paper studies the impact of adult mortality on the choice between different income generating assets and its consequences for intergenerational transfer and economic development. We differentiate between physical assets and human capital as alternative sources of future income, one of the key distinction being the latter's "inalienability" (Hart and Moore, 1994). Physical assets such as land and capital are readily transferable across people in a way that human capital is not. This difference becomes important when an investor faces lifetime uncertainty that can cut short her amortization period.

When altruistic parents derive pleasure from bequests, the utility of an asset depends on its transferability to the future generation. The possibility that an investor may die prematurely but leave some of her physical assets to her survivors enhances the internal return on physical assets vis-a-vis human capital. This creates an incentive for parents to invest in physical assets versus their own human capital. Altruistic parents also have the option of improving their children's lives by directly imparting human capital to them through time investment. This too suffers when parents die prematurely.

These effects of lifetime uncertainty on portfolio choice are embedded in a dynamic dynastic model. We use it to study the effect of mortality shocks. The HIV/AIDS epidemic in sub-Saharan Africa over the last three decades has caused untold misery. However, its economic impact is still debated. Authors such as Young (2005) and Acemoglu and Johnson (2009) argue that epidemics can actually generate a long-run per-capita income boom due to higher labor productivity effect from a diminished work force. Weil (2007) finds similar evidence. Other papers such as Bell *et al.* (2006), Chakraborty *et al.* (2010) and Aghion *et al.* (2010) find the opposite. One outcome of this debate is the recognition that we need better, micro-founded models of health behavior that can offer testable predications about the effect of health at a disaggregated level.

Much of the literature studying the effect of infectious disease on economic development has considered that disease affects the whole population in the same manner. This is, however, ignores the fact that diseases such as HIV/AIDS have affected different age groups very differently across societies. Therefore it is essential that we understand better the compositional effects of age-specific mortality on economic development.

An overlapping-generations model people with four distinct "ages" – childhood, youth, middle age and old age – each with age-specific mortality risk is calibrated to the US and South Africa. Agents are allowed to accumulate wealth through physical capital and human capital investment. They value child quality and the bequests they leave in the event of pre-mature death. The model economies are then subject to age-specific mortality shocks. A negative shock to middle-age survival lowers steady-state consumption across cohorts yet improves output per capita, albeit by a small margin, as the human capital scarcity prompts higher educational investment by the youth in themselves and in their children. Surprisingly a negative shock to child mortality has an even stronger positive effect on output per capita despite consumption suffering: the human capital response is stronger still. A counterpoint to these "gifts of the dying" (Young, 2005) is the effect of a mortality shock among the youth. By preventing investment in their children and in their own human capital, the shock adversely affects overall human capital. Consumption and output per capita drop sharply.

We conclude our analysis by producing an HIV/AIDS shock of the kind that South Africa suffered during 1990-2000 when age-specific survival rates fell among children and, especially, young and middle-aged workers. Quantitative results show that consumption falls over the lifetime, the surviving young accumulate more assets but the surviving middle-aged and old have less. Education suffers the most since the epidemic targeted prime-age adults. The overall effect is to lower South Africa's GDP per capita by 8%. Interestingly the same HIV/AIDS shock can have an even more adverse effect on an advanced economy such as the US: the mortality shock lowers GDP per capita by as much as 16%. Underlying this difference is the non-transferability of human capital: in a more physical-capital reliant economy, death among adult cohorts is partly offset by the higher bequests (transfers) received by surviving cohorts. In a human-capital reliant economy such as the US, the human capital of the deceased is lost and with it is lost productive capacity. AIDS orphans' human capital suffers too because of parental absence. While surviving adults do respond to the human capital scarcity by stepping up investment in themselves and their children, it is not enough to compensate for the loss.

The paper is related to several strands of work in the literature. On one side, the differential nature of human and physical capital investment, the compensatory effect of lifetime uncertainty on human capital and the role of bequests for wealth and consumption distributions are studied by Razin (1976), Krebs (2003) and De Nardi (2004) respectively. On the other side, the economic effects of health and epidemics is a theme running through several recent works such as Acemoglu and Johnson (2009), Aghion *et al.* (2010), Bell *et al.* (2006), Chakraborty *et al.* (2016) and Young (2005).

Our analysis specifically suggests the positive effect of the AIDS epidemic on income per capita in Young (2005) owes to the focus on child mortality and reduced fertility. We do not model the fertility channel, but subsequent works have questioned if the fertility effect was negative (Kalemli-Ozcan and Turan, 2011). What our work suggests instead is that it is important to be mindful of the age-distribution of mortality shocks. HIV/AIDS took its heaviest toll among working-age adults whose loss of lives extracted a much higher economic cost through the human capital channel.[1]

[1]Age-specific effects of the HIV epidemic on fertility behavior, are also studied in Durevall and Lindskog (2011) and Wang *et al.*'s (2013).

Related to our work, Galor and Moav (2004) differentiate between physical and human capital in terms of their technological characteristics. According to these authors, the fundamental asymmetry between human and physical capital accumulation is that the former is subject to quicker diminishing returns due to physiological constraints. Our focus on another asymmetry, in transferability, complements Galor and Moav.

The following section presents the overall structure of the model and household behavior. Household optimal choices and the aggregate equilibrium are constructed in section 3. The model is calibrated and the effect of mortality shocks are analyzed in section 4.

2 Structure of the Economy

In a discrete-time economy, overlapping generations of households invest in human and physical capital subject to lifetime uncertainty.

Specifically, each individual potentially lives for four periods, the first period being childhood. The three subsequent periods of adulthood are labeled as youth, middle-age and old-age. Every period, the individual faces mortality risk. Let s denote the survival probability in a particular period of life and suppose survival outcomes are realized at the end of each of the 4 periods. Fertility is exogenous – a young adult gives birth to $1+n$ offsprings at some point in her early youth if she survives from childhood to youth. $n > 0$ ensures an ever shrinking population since childhood survival rates can be less than 1. The size of the cohort born at date t is denoted by N_t.

Children are passive, only receiving parent-provided education. Individuals are retired in the last period of their lives when they draw down their savings. Successive generations are linked through parental educational investment and bequests. Under "warm glow" altruism, parents can derive pleasure from bequests in any period of their lives depending on their own survival. For example, in investing, a young person will take into account the added benefit from transferring financial (tangible) assets in the event she does not survives into middle-age.

For simplicity, suppose that all bequests are made through a fund at the beginning of each period, and the fund allocates them to all surviving individuals in the same amount. This could be the case, for example, if agents take out insurance claims so that they receive a share of the bequests left should their parents survive and pay a fraction of the bequest inherited if they die. In a developed society this deal could be carried out through an insurance company. In less developed societies, it could be implemented at the village level by some sort of "communal trust".

2.1 The Household's Recursive Problem

Consider the problem faced by an agent born in period $t-1$. For given prices, the state variables defining her decision problem at t are collected in the vector (a_t, h_t, b_t). These represent the agent's financial assets carried over from the previous period, her human capital from prior (net) investment and inheritances (if any) received.

A person born in $t-1$ is a child (c) at $t-1$, young (y) in t, middle-aged (m) in $t+1$ and old (o) in $t+2$. Individuals start their adult lives at t with no financial assets but human capital h_t^{t-1} that is realized through parental investment during their childhood. For survival rates we use the notation s_t^τ to denote the probability for an individual born in $\tau \le t$ of surviving period t. For example, $\mathbf{s}^t = (s_t^t, s_{t+1}^t, s_{t+2}^t, s_{t+3}^t)$ denotes the vector of survival rates for an individual born in period t over her lifetime. The first is an unconditional probability, the rest are conditional on living up to that date; $s_{t+3}^t = 0$.

It will be useful to specify the individual's optimization problem for each period of adulthood.

Decisions in Youth

In period t, the agent's problem is

$$V_t(a_t, h_t, b_t) = \max_{c_t, a_{t+1}, e_t, x_t} \left\{ \begin{array}{c} u(c_t) + \gamma(1+n)\psi(h_t^c) + \beta s_t^{t-1} V_{t+1}(a_{t+1}, h_{t+1}, 0) \\ +(1-s_t^{t-1}) s_t^c (1+n)\phi\left(\frac{a_{t+1}}{s_t^c(1+n)}\right) \end{array} \right\} \tag{1}$$

subject to

$$\begin{aligned} c_t &= [1-e_t - x_t(1+n)]\, w_t h_t + b_t - a_{t+1}, & (2)\\ h_{t+1} &= g(e_t, h_t), & (3)\\ h_t^c &= q(x_t, h_t). & (4) \end{aligned}$$

There is no cost to raising children, only in educating them at a time cost of x per child. In the problem, s_t^i refers to the (unconditional) probability that an agent is alive at the end of her i age given that she was age i in period t, with $i \in \{c, y, m, o\}$. In particular, $s_t^c = s_t^t$ in the above problem, and $s_t^o = 0$ for all t.[2] A parent invests in the human capital of all children (denoted by h_t^c) but, should he himself die, enjoys altruistic pleasure from (equitable) asset transfers to his surviving children by society. We are imposing certainty equivalence with respect to childhood survival, i.e., the parent behaves as if $s_t^c(1+n)$ children survive and weigh the bequest motive accordingly.

[2] Since not much educational investment takes place in the first five years of life, s_t^c should correspond to survival rates among 5 – 15 year olds.

We will consider the CES functional specifications

$$u(c_t) = \ln c_t, \tag{5}$$

$$\psi(h_t) = \ln h_t, \tag{6}$$

and

$$\phi(z_t) = \phi \ln(1 + z_t). \tag{7}$$

The production functions are

$$g(e_t, h_t) = \left(1 + \frac{e_t}{v_e}\right)^{\theta} h_t^{1-\theta},\ \theta \in (0,1) \tag{8}$$

and

$$q(x_t, h_t) = \left(1 + \frac{x_t}{v_x}\right)^{\eta} h_t^{1-\eta},\ \eta \in (0,1). \tag{9}$$

Specifications (8) and (9) for g and q respectively ensure that corner solutions (e_t or $x_t = 0$) are feasible. The q specification incorporates an intergenerational externality while for g, the complementarity between h_t and e_t follows from Heckman's (XXXX) emphasis on the importance of early childhood education for future outcomes. Again, the individual's h_t is the h_{t-1}^c that his parent chose in the previous period.

Decisions in Middle-Age

The decision problem in $t+1$ becomes

$$V_{t+1}(a_{t+1}, h_{t+1}, 0) = \max_{c_{t+1}, a_{t+2}} \left\{ \begin{array}{c} u(c_{t+1}) + \beta s_{t+1}^{t-1} V_{t+2}(a_{t+2}, h_{t+2}, 0) \\ +(1 - s_{t+1}^{t-1}) s_{t+1}^{y} (1+n) \phi\left(\frac{a_{t+2}}{s_{t+1}^{y}(1+n)}\right) \end{array} \right\} \tag{10}$$

subject to

$$c_{t+1} = (1 + r_{t+1}) a_{t+1} + w_{t+1} h_{t+1} - a_{t+2}. \tag{11}$$

The probability that the child is alive at the end of date $t+1$ is given by $s_{t+1}^{y} = s_t^t s_{t+1}^t$. The individual has no incentive to invest in skills since she will retire the following period.

Decisions in Old-Age

The decision problem becomes much simpler in $t+2$

$$V_{t+2}(a_{t+2}, h_{t+2}, 0) = \max_{c_{t+2}, a_{t+3}} \left\{ u(c_{t+2}) + s_{t+2}^{m} (1+n) \phi\left(\frac{a_{t+3}}{s_{t+2}^{m}(1+n)}\right) \right\} \tag{12}$$

subject to

$$c_{t+2} = (1 + r_{t+2}) a_{t+2} - a_{t+3} \tag{13}$$

where $s^m_{t+2} = s^t_t s^t_{t+1} s^t_{t+2}$.

2.2 Production

The unique final good is produced from physical capital and labor:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha},\ \alpha \in (0,1); \tag{14}$$

where L_t is effective labor supply by young and middle-aged workers at time t and A_t is aggregate productivity that grows at the exogenous rate $G \geq 0$.

Equilibrium wage and interest rate, w_t and r_t respectively, are determined by the usual competitive market conditions

$$w_t = (1-\alpha) A_t \left(\frac{K_t}{L_t}\right)^{\alpha}, \tag{15}$$

$$1 + r_t = \alpha A_t \left(\frac{L_t}{K_t}\right)^{1-\alpha}. \tag{16}$$

assuming that physical capital fully depreciates in one generation.

3 Equilibrium

3.1 Household Decisions

The solution to the generation-$t-1$ household's optimization problem is derived backwards starting with the last period of life.

Old Age

The Kuhn-Tucker necessary and sufficient first order condition associated with bequests is

$$u'(c_{t+2}) \geq \phi' \left[\frac{a_{t+3}}{s^m_{t+2}(1+n)}\right] \tag{17}$$

and $a_{t+3} \geq 0$.

1. **Case 1:** If the agent leaves no bequests ($a_{t+3} = 0$), then she consumes her entire accumulated savings:

$$c_{t+2} = (1 + r_{t+2}) a_{t+2}. \tag{18}$$

2. **Case 2:** If on the other hand, positive bequests are left, (17) holds with equality:

$$c_{t+2} = \frac{\phi s_{t+2}^m (1+n) + (1+r_{t+2}) a_{t+2}}{1 + \phi s_{t+2}^m (1+n)} \tag{19}$$

and

$$a_{t+3} = \left[\frac{\phi(1+r_{t+2}) a_{t+2} - 1}{1 + \phi s_{t+2}^m (1+n)} \right] s_{t+2}^m (1+n) \tag{20}$$

from the budget constraint (13).

Middle Age

The consumption Euler condition

$$u'(c_{t+1}) \geq \beta s_{t+1}^{t-1} \frac{\partial V_{t+2}}{\partial a_{t+2}} + (1 - s_{t+1}^{t-1})\, \phi' \left[\frac{a_{t+2}}{s_{t+1}^y (1+n)} \right] \tag{21}$$

can be rewritten as

$$u'(c_{t+1}) \geq \beta s_{t+1}^{t-1} (1+r_{t+2}) \phi' \left[\frac{a_{t+3}}{s_{t+2}^m (1+n)} \right] + (1 - s_{t+1}^{t-1}) \phi' \left[\frac{a_{t+2}}{s_{t+1}^y (1+n)} \right] \tag{22}$$

using optimal decisions in old age and the Envelope Theorem.

Under logarithmic preferences, consumption is always desirable and the optimality condition must hold with equality. Therefore,

$$\frac{1}{c_{t+1}} = \frac{\beta s_{t+1}^{t-1} \left[1 + s_{t+2}^m (1+n) \phi \right] (1+r_{t+2})}{s_{t+2}^m (1+n) + (1+r_{t+2}) a_{t+2}} + \frac{\left(1 - s_{t+1}^{t-1}\right) \phi s_{t+1}^y (1+n)}{s_{t+1}^y (1+n) + a_{t+2}}. \tag{23}$$

It can be shown that the optimal c_{t+1}^{t-1} that solves this expression is:[3]

$$c_{t+1}^{t-1} = \frac{-\tau_b - \sqrt{\tau_b^2 - 4\tau_a \tau_c}}{2\tau_a}; \tag{24}$$

[3]This is one of two solutions to the second-order polynomial $\tau_a c_{t+1}^2 + \tau_b c_{t+1} + \tau_c = 0$. It can be shown that the other solution always leads to negative values of a_{t+2}.

where

$$
\begin{aligned}
\tau_a &= (1+r_{t+2}) + z_{4t+1} \\
\tau_b &= -\{[(1+r_{t+1})a_{t+1} + w_{t+1}h_{t+1}]\,[2(1+r_{t+2}) + z_{4t+1}] + z_{2t+1} + z_{3t+1}\} \\
\tau_c &= z_{1t+1} + [(1+r_{t+1})a_{t+1} + w_{t+1}h_{t+1}]^2\,(1+r_{t+2}) \\
&\quad + z_{2t+1}\,[(1+r_{t+1})a_{t+1} + w_{t+1}h_{t+1}]
\end{aligned}
$$

and

$$
\begin{aligned}
z_{1t+1} &= \left[s^m_{t+2}(1+n)\right]\; s^y_{t+1}(1+n) \\
z_{2t+1} &= \left[s^m_{t+2}(1+n)\right] + (1+r_{t+2})s^y_{t+1}(1+n) \\
z_{3t+1} &= s^y_{t+1}(1+n)\;\beta s^{t-1}_{t+1}\left[1+s^m_{t+2}(1+n)\phi\right](1+r_{t+2}) + \left[s^m_{t+2}(1+n)\right]\left(1-s^{t-1}_{t+1}\right)\phi \\
z_{4t+1} &= (1+r_{t+2})\;\left\{\beta s^{t-1}_{t+1}\left[1+s^m_{t+2}(1+n)\phi\right] + \left(1-s^{t-1}_{t+1}\right)\phi s^y_{t+1}(1+n)\right\}
\end{aligned}
$$

Youth

Young agents allocate resources towards their own consumption and education and their children's education with the associated optimality conditions

$$
u'(c_t) \geq \beta s^{t-1}_t \frac{\partial V_{t+1}}{\partial a_{t+1}} + (1-s^{t-1}_t)\;\phi'\left[\frac{a_{t+1}}{s^c_t(1+n)}\right] \tag{25}
$$

$$
\beta s^{t-1}_t \frac{\partial V_{t+1}}{\partial h_{t+1}} g_e(e_t, h_t) \geq \left\{\beta s^{t-1}_t \frac{\partial V_{t+1}}{\partial a_{t+1}} + (1-s^{t-1}_t)\;\phi'\left[\frac{a_{t+1}}{s^c_t(1+n)}\right]\right\} w_t h_t \tag{26}
$$

$$
\gamma(1+n)\psi'(h^c_t)\;q_x(x_t, h_t) \geq \left\{\beta s^{t-1}_t \frac{\partial V_{t+1}}{\partial a_{t+1}} + (1-s^{t-1}_t)\;\phi'\left[\frac{a_{t+1}}{s^c_t(1+n)}\right]\right\} w_t h_t(1+n). \tag{27}
$$

Potentially (25) can hold with inequality since middle-age consumption can be financed exclusively from labor income (unlike in old age) and still leave income for saving. For computational convenience, we will however restrict the parameter space such that (25) holds with equality.

Applying the envelope theorem to (10), we have

$$
\frac{\partial V_{t+1}}{\partial a_{t+1}} = \beta s^{t-1}_{t+1}(1+r_{t+1})(1+r_{t+2})\phi'\left[\frac{a_{t+3}}{s^m_{t+2}(1+n)}\right] + (1-s^{t-1}_{t+1})\phi'\left[\frac{a_{t+2}}{s^y_{t+1}(1+n)}\right] \tag{28}
$$

$$
\frac{\partial V_{t+1}}{\partial h_{t+1}} = \frac{\partial V_{t+1}}{\partial a_{t+1}} \frac{w_{t+1}}{1+r_{t+1}}. \tag{29}
$$

Because the human capital production functions and bequest utility can deliver corner solutions, we have four cases:

1. **Case 1:** (26) and (27) hold with inequality when $x_t = e_t = 0$. Using expression (28), (25) becomes

$$\begin{aligned} u'(c_t) \quad \geq \quad & \beta^2 s_t^{t-1} s_{t+1}^{t-1}(1+r_{t+1}) \left\{ (1+r_{t+2})\phi' \left[\frac{a_{t+3}}{s_{t+2}^m(1+n)} \right] \right. \\ & \left. +(1-s_{t+1}^{t-1})\phi' \left[\frac{a_{t+2}}{s_{t+1}^y(1+n)} \right] \right\} + (1-s_t^{t-1})\phi' \left[\frac{a_{t+1}}{s_t^c(1+n)} \right]. \end{aligned} \tag{30}$$

2. **Case 2:** Only (26) holds with inequality, $e_t = 0$. FOCs (25) and (27) hold with equality. Combining them along with (28) and (29) give

$$c_t = \frac{h_t^c w_t}{\gamma \chi \eta} \, (v_x + x_t)^{1-\eta} \, h_t^\eta. \tag{31}$$

   This taken to (30) provides c_t. Alternatively, we can write x_t as a function of c_t as follows

$$x_t = \frac{c_t \gamma \eta}{w_t h_t} - v_x. \tag{32}$$

3. **Case 3:** Only (27) holds with inequality. Combining (25), (26), (28) and (29) obtains

$$e_t = 1 - x_t(1+n) + \frac{I_t b_t - c_t}{w_t h_t} + \frac{s_t^c(1+n)}{w_t h_t} \left[1 - \phi \frac{c_t(1-s_t^{t-1})}{1 - \frac{(1+r_{t+1}) w_t h_t^\theta}{\mu w_{t+1}}} \right]. \tag{33}$$

   In this case $x_t = 0$. Again, equation (33) together with (30) determines equilibrium c_t.

4. **Case 4:** All optimality conditions hold with equality. As long as $x_t > 0$ and $c_t > 0$, the consumption decision is given by (31). In the same way, as long as $e_t > 0$ and $c_t > 0$, the middle-age agent's schooling decision is given by (33). Combining them with (30) we end up with a system in which the only unknown is x_t.

3.2 Aggregation and Market Clearing

Let us change notation a bit and denote stock variables a and h corresponding to different cohorts by z_t^τ, the value of z in period t for an individual born in $\tau \leq t$. If we assume that those claims exist, bequest per individual is given by

$$b_t = \frac{s_{t-2}^m N_{t-4} \, a_t^{t-4} + s_{t-2}^y (1 - s_{t-1}^{t-3}) N_{t-3} \, a_t^{t-3} + s_{t-2}^c (1 - s_{t-1}^{t-2}) N_{t-2} \, a_t^{t-2}}{s_{t-1}^m N_{t-3} + s_{t-1}^y N_{t-2} + s_{t-1}^c N_{t-1}}. \tag{34}$$

The numerator collects the bequests left by individuals who died last period, and the denominator gives the population size alive in t.

In addition, given that physical capital depreciates completely, the labor and capital aggregates are given that the following equilibrium conditions:

$$L_t = s^c_{t-1} N_{t-1} h^{t-1}_t + s^y_{t-1} N_{t-2} h^{t-2}_t, \tag{35}$$

and

$$K_t = s^y_{t-1} N_{t-2}\ a^{t-2}_t + s^m_{t-1} N_{t-3}\ a^{t-3}_t. \tag{36}$$

the right hand side corresponding to labor and capital supplied by agents alive at t. Bequests are invested in capital, but only after being inherited for a period. Labor is supplied by the young and middle-aged, and capital by the middle-aged and old.

Finally, note that the interest rate is pinned down by the small open economy assumption. Specifically, due to capital mobility, the domestic interest factor is $R = R^*$ for all $t \geq 0$.

4 Quantitative Exercises

To what extent does the transferability of physical assets versus the inalienability of human capital affect welfare when the population is subject to mortality shocks? We analyze this and other implications of the model for two different countries: the U.S. and South Africa. The US represents the benchmark economy to which the deep parameters of the model are calibrated. South Africa, on the other hand, corresponds to an economy that has suffered a high burden of the HIV/AIDS epidemics, and has been the subject of many studies dealing with the disease impact. In our quantitative analysis, our focus is understanding how changing age-specific mortality rates from the HIV/AIDS epidemic has affected South Africa.

4.1 Calibration

Childhood is taken to last 15 years. Each subsequent period, and there are three of them, is 25 years long. This means a maximum lifetime of 90 years. The life expectancy of cohort born at t is

$$LE_t = 15 + 25 s^t_t \left(1 + s^t_{t+1} + s^t_{t+1} s^t_{t+2}\right). \tag{37}$$

Table 1 provides the parameter values. Benchmark values are the ones for the U.S. economy. Some of them are standard. The growth rate of income per capita is set to 2% per year, which implies a gross growth rate for a 25-year period of 1.02^{25}. The discount coefficient (β) equals 0.99 per quarter, which agrees with the business cycle literature. The share of capital in aggregate output (α) is, in turn, 0.34, close to one third.

Table 1: Parameters values assigned in the calibration

Parameters			Criteria
$s^c; s^y; s^m$			
	U.S.	0.9847; 0.9374; 0.7405	WHO Life Tables, males, 1990
	S.Africa	0.9199; 0.8387; 0.4891	WHO Life Tables, males, 1990
	S.Africa	0.9095; 0.7738; 0.4193	WHO Life Tables, males, 2000
$1+n$			
	U.S.	1.0094^{25}	World Bank, Average 1981-90
	S.Africa	1.024^{25}	World Bank, Average 1981-90
$\alpha;\ \beta;\ 1+g_y$		$0.34;\ 0.99^{4*25};\ 1.02^{25}$	Standard in literature
ϕ			
	U.S.	1.8	Consumption share 67% of GDP
	S.Africa	2.17	Consumption share 61% of GDP
$\gamma;\ \eta=\theta$		0.271; 0.635	de la Croix and Doepke (2003)
υ_x		0.041	$x = 0.082$
υ_e		0.0689	Average years of schooling of 12.3
$1+r$		1.043^{25}	Capital-ouput ratio of 3/25

The evolution of population size depends on fertility and mortality rates. Mortality rates for the years 1990 and 2000 are obtained from the Life Tables provided by the WHO in its Global Health Observatory Data repository. The HIV virus spread rapidly during 1990–2000. The benchmark population growth rate is chosen to be 0.94% per year, the U.S. mean for the period 1981-1990, that is, during the decade before the start of the HIV outbreak. Benchmark mortality rates, in turn, are given by the U.S. economy in 1990 through the l_x indicator, that is, by the number of people left alive at ages 15, 40 and 65. This gives values of 98.47%, 93.74% and 74.05% for the unconditional survival probability of children, young and middle-age agents, respectively. Table 1 shows alternative values too (from the same sources) that correspond to South Africa. These are later used in the computational experiments.

For the bequest function (7), we pick the benchmark value of ϕ so as to reproduce a household consumption share in aggregate output of 67%, the average in the U.S. economy during 1991–2010 (World Bank). Later, we also target the mean for South Africa's consumption share, 61%, during the same period. This implies a benchmark value of $\phi = 1.8$, and a value 2.17 specific to South Africa.

We follow de la Croix and Doepke (2003) in choosing children-related parameters. The authors assign a value of 0.271 to the weight γ placed by parents on child quality and a value of

0.635 to a parameter similar to η that influences the elasticity of human capital with respect to a child's education. In addition, they calculate that the opportunity cost of a child till the age of 15 is equivalent to about 7.5% of the parents' time endowment. In their model, however, each time interval is equivalent to 30 years, whereas in ours, periods last 25 years. We then pick υ_x so that x_t at steady state equals 8.2% in the benchmark case, slightly above the one chosen by de la Croix and Doepke. The value of υ_x obtained is 0.041.

Finally, we pick the parameters governing human capital accumulation in youth, (8). The elasticity of human capital with respect to education θ is set equal to η, the elasticity for human capital formation during childhood: $\theta = \eta = 0.635$. The value of υ_e is chosen so as to generate a total of 12.3 years of education in steady state. This corresponds to the average educational attainment in the U.S. at the beginning of the relevant time interval, 1990. To reach it, a young agent needs to generate 3.3 additional years to the 9 that were already acquired as a child, assuming that schooling starts at 6 years of age. This means that the variable e_t in steady state must equal 0.132, that is, young individuals allocate 13.2% of their time to education. The implied value of υ_e is 0.0689. Notice that because $\upsilon_e > \upsilon_x$, this means that investment in the child's education is more productive than investment in human capital in youth.

4.2 Quantitative Results

Restricting ourselves to the steady-state of the model, we now turn to quantifying the effects on consumption, saving, physical and human capital investments, and GDP per capita of several key parameters. More specifically, we conduct three experiments. First, we look for the response to a weaker preference for bequests. Second, we assess the effect of changes in age-specific survival rates. Third, and most importantly, we evaluate through the lenses of our model the impact of the mortality rate variations observed between 1990 and 2000.

Table 2 presents results from the first two exercises, and compares them to the benchmark case. The values are normalized except for time fractions e and x. This normalization divides each variable by $G^{1/(1-\alpha)}$, the long-run growth rate of $G^{1/(1-\alpha)}$ for per capita output because of exogenous total factor productivity in aggregate output (14).

4.2.1 Lower Preference for Bequests

Bequests are essential in the model to replicate the appropriate consumption share in GDP, else household consumption would be too high. In addition, the possibility of leaving bequests opens the door to different roles of human and physical capital. The reason is that physical capital is transferrable through bequests, whereas human capital is transferable only through education within the family. Furthermore, the importance of the bequest motive may vary with economic development. This is something that we have allowed in our calibration; recall that the value of ϕ in the case of South Africa has turned out larger than for the U.S.. development.

Comparing the first and second rows in Table 2, we observe the response to a weaker preference for bequests. The fall in ϕ from 1.8 to 1 leads to more consumption in youth and middle-age and, from further down the table, a larger consumption share (C_t/GDP_t). Assets $(a^y_{t+1}, a^m_{t+1}, a^o_{t+1})$ and education (e_t) investment both fall, the former by relatively more than the latter. Consequently the value of a young person's education investment relative to assets $(e_t w_t / a^y_{t+1})$ rises. Since parents still value child quality to the same extent as before, not surprisingly, investment in child's education (x_t), increases when they derive lower utility from financial bequests. Finally as expected, aggregate output per capita $(GDPpc, GNPpc)$ both fall.

Table 2: Normalized steady-state values, US benchmark, selected variables

	c^y_t	c^m_t	c^o_t	a^y_{t+1}	a^m_{t+1}	a^o_{t+1}
Benchmark	0.474	0.432	0.351	0.070	0.304	0.360
$\phi_1 = 1$	0.520	0.435	0.321	0.041	0.204	0.183
$s^t_t - 0.06$	0.300	0.697	0.570	-0.005	0.487	0.584
$s^{t-1}_t - 0.06$	0.441	0.383	0.288	0.113	0.249	0.295
$s^{t-2}_t - 0.06$	0.472	0.428	0.381	0.070	0.313	0.359
	C_t/GDP_t	x_t	e_t	$e_t w_t / a^y_{t+1}$	$GDP_t pc$	$GNP_t pc$
Benchmark	0.671	0.082	0.132	0.420	5.375	0.493
$\phi_1 = 1$	0.780	0.088	0.106	0.568	5.365	0.436
$s^t_t - 0.06$	0.620	0.057	0.661	-27.761	7.469	0.674
$s^{t-1}_t - 0.06$	0.663	0.078	0.029	0.057	4.369	0.434
$s^{t-2}_t - 0.06$	0.674	0.082	0.136	0.431	5.458	0.494

4.2.2 Shocks to Age-specific Mortality

In our second experiment, we decrease the age-specific survival rates, one at a time. The shock that we apply to the benchmark case is a 6% fall, similar to the one experienced by South Africa between 1990 and 2000. When the survival probability of middle-aged individuals decreases $(s^{t-2}_t - 0.06$, fourth row in Table 2), the effect is small, with a slightly increase in the standard of living, as measured by aggregate output, of those alive. In contrast, when the mortality rate of other age groups change, the effect is sizable. Furthermore, the effect of lower survival probabilities of children and young go in opposite directions.

As we can see in row three $(s^t_t - 0.06)$, higher mortality among children adversely affects a child's education and consumption and saving among young parents. However, this initial negative effect is offset later in life with substantial increases in middle-aged and old individuals' consumption, assets, and human capital. The net result is a 39% and 37% increase in per capita GDP and per capita GNP, respectively. The main reason is the higher productivity of the surviving labor. However, unlike in Young (2005) where it is from higher labor market participation by the survivors,[4] here it occurs due to higher labor quality in the survivors – own-education investment by the young rises five-fold. The young are responding to the lower chance of their children receiving their assets by reallocating towards human capital investment.

The decline in the survival rate of the young, on the other hand, is disastrous for the economy. As we see in row four of Table 2 ($s_t^{t-1} - 0.06$), the adverse consequences are especially felt by middle-age and old individuals whose consumption and assets decrease by as much as 18%. The main cause is the young's investment in education that falls from 13.2% to 2.9% of their available time. As a result GDP per capita falls by 18%.

Aggregate Effects

So far, the model has shown that mortality shocks have differential effects on various age groups. Our next task is assessing the impact of the specific mortality shocks experienced by South Africa between 1990 and 2000. It is well known that these shocks are a main consequence of the HIV/AIDS epidemic, e.g., South Africa during that decade saw its surviving children, youth and middle-age citizens fall by 1.04%, 6.49% and 6.98%, respectively (Table 1).

Table 3 shows the effects of these shocks in three different scenarios. Rows 1 and 2 give results before and after the shocks in the benchmark (or U.S.) case. Rows 3 and 4 provide predictions before and after the shocks starting from South Africa's mortality rates in 1990 and population growth. Rows 5 and 6 do the same exercise for South Africa as rows 3 and 4 but use the alternative bequest parameter value of $\phi = 2.17$.

Qualitatively, results are similar in the three scenarios: a fall in consumption all along the life cycle, an increase in asset accumulation when young but a decline in the last two periods of life, and a decrease in education investment especially for young agents. Importantly, unlike in Young (2005), the economies end up worse off in terms of aggregate per capita output after the disease shock. Quantitatively, the two South African cases provide almost the same predictions; differences arise only when we compare the South African and the U.S. scenarios. The main difference between the two is that the negative effect in the more developed nation is stronger: while disease shocks in the U.S. reduce GDP per capita to 84% of its 1990 level, GDP per capita in South Africa falls to 92%. This is due to the larger fall in schooling in the former country. In other words, the higher dependence on human capital in a developed country also means less capacity to respond to mortality shocks as human capital is inherently non-transferable.

[4] In our model, the wage rate per unit of effective unit of labor w_t pretty much remains the same.

Table 3: HIV/AIDS shock, normalized steady-state values, selected variables

	c_t^y	c_t^m	c_t^o	a_{t+1}^y	a_{t+1}^m	a_{t+1}^o
US 1990	0.474	0.432	0.351	0.070	0.304	0.360
US after	0.445	0.386	0.320	0.110	0.260	0.297
S.A. 1990*	0.377	0.281	0.328	0.102	0.284	0.318
S.A. after*	0.353	0.270	0.334	0.121	0.262	0.277
S.A. 1990†	0.354	0.262	0.321	0.113	0.312	0.375
S.A. after†	0.329	0.254	0.331	0.131	0.291	0.332
	C_t/GDP_t	x_t	e_t	$e_t w_t/a_{t+1}^y$	$GDP_t pc$	$GNP_t pc$
US 1990	0.671	0.082	0.132	0.420	5.375	0.493
US after	0.667	0.078	0.038	0.076	4.511	0.441
S.A. 1990*	0.651	0.069	0.035	0.076	3.739	0.318
S.A. after*	0.644	0.065	0.000	0.000	3.412	0.297
S.A. 1990†	0.611	0.065	0.032	0.063	3.629	0.321
S.A. after†	0.604	0.0613	0.000	0.000	3.310	0.299

* US calibration, except for country specific mortality rates, and population growth

† US calibration, except for country specific mortality rates, population growth, and bequest parameter ϕ

5 Conclusion

Using a dynamic model of portfolio choice between human and physical capital investment and lifetime uncertainty, we have studied the distributional and aggregate effects of an epidemic such as the HIV/AIDS crisis. While the aggregate effects are unambiguously, and strongly, negative, the cohort-specific effects differ. Lower survival rates make, on the one hand, everybody more productive at the margin because of factor scarcity. On the other, adults are the ones that make the investment decisions in physical and human capital. Which effect dominates depends on which group is affected more strongly by the disease. If the epidemic mainly affects children, the economy ends up better off economically, whereas it is worse off if the young and middle-aged die. Using survival rates extracted from the U.S. and South Africa's Life Tables, we find that both economies end up worse off after the HIV shock, the US more than South Africa because of a stronger human capital margin.

Appendix

Computational steps

Unfortunately, because we are dealing with a nonlinear system, there is no pre-established method to approximate the solutions. The procedure will try to build progressively the time path followed by the variables from initial coordinates until the steady state is reached – although we have not proved yet that the latter exists. To achieve this goal, we can do the following:

1. Fix values for s_t^t, s_{t+1}^t and s_{t+2}^t and assume they remain constant.
2. Initial coordinates. Pick values for the stocks: $a_1^{-3}, a_1^{-2}, a_1^{-1}$, and $h_1^0 = h_1^{-1}$. Assume $a_1^{-3} = 0$, that is, the old does not leave bequests.
3. From those stocks
   (a) Compute endogenous prices w_1 and r_1, and endogenous bequest b_1
   (b) And to give an initial guess to their future values, assume that the remain constant for three periods: $w_1 = w_2 = w_3$, $r_1 = r_2 = r_3$, and $b_1 = b_2 = b_3$.
4. Compute the stock variables' optimal values for next period: $a_2^{-2}, a_2^{-1}, a_2^0, h_2^1, h_2^0$.
5. Repeat steps 3 and 4 until n periods of optimal values of the stocks have been generated for given prices and bequests.
6. Using the initial coordinates in 1 and the sequence of values r_t, w_t and b_t given by the "optimal" stocks each period, regenerate $a_t^{t-4}, a_t^{t-3}, a_t^{t-2}, h_t^{t-1}, h_t^{t-2}$, with $t = 1, .., n$.
7. Compute r_t^*, w_t^*, b_t^* with expressions (15), (16) and (34), respectively, with $t = 1, .., n$.
8. Choose $\theta \in (0,1)$ and compute $\hat{r}_t = (1-\theta)r_t + \theta r_t^*$, $\hat{w}_t = (1-\theta)w_t + \theta w_t^*$, and $\hat{b}_t = (1-\theta)b_t + \theta b_t^*$,with $t = 1, .., n$. Those values $\hat{r}_t, \hat{w}_t, \hat{b}_t$ will represent the next guess for the needed future endogenous variables.
9. Equalize $r_t = \hat{r}_t$, $w_t = \hat{w}_t$ and $b_t = \hat{b}_t$, and repeat 6 to 8 until $r_t - r_t^*$, $w_t - w_t^*$, and $b_t - b_t^*$ is sufficiently close to zero.
10. Increase progressively n as needed until the steady state is found. Each time n rises, the whole thing is repeated using the previously found fixed-point values for $\{r_t, w_t, b_t\}_{t=1}^n$.

Large Preliminary Simulations

- Solve for an individual's problem in partial equilibrium.
- Use log preferences and pick other parameter values close to De Nardi (2004) who uses

$$
\begin{aligned}
&n = 1.2\% \text{ (net US population growth)} \\
&\alpha = 1/3 \\
&\beta = (0.99)^{25} = 0.78 \\
&\sigma = 1.5 \\
&\kappa = 1 \\
&\phi_1 = -9.5 \\
&\phi_2 = 11.6
\end{aligned}
$$

  The last two parameters are fitted to match features of US and Swedish data.

  These parameters may have to be later adjusted for a high mortality/developing country.

- Start with an invariant life table, that is parents and children have the same survival rates in the four periods of life.
- Choose a benchmark $\mathbf{s}$, draw individual probabilities randomly, and simulate c, a and h paths.
  - Repeat exercise by changing each survival rate one at a time and track how (h_t^c, e_t) and (a_{t+1}, a_{t+2}) change.
  - We expect that an increase in s_t^{t-1} will raise the human-to-physical capital investment ratio e_t/a_{t+1}.
  - We also expect $s_t^c \equiv s_t^t$ should make no difference to h_t^c since parents care about all of their children's human capital.
  - Unclear at this stage how $\{s_{t+1}^{t-1}, s_{t+2}^{t-1}\}$ change the investment propensity e_t/a_{t+1} but we expect them to positively affect a_{t+2}.

References

[1] Acemoglu, D., Johnson, S., (2007), “Disease and Development: The Effect of Life Expectancy on Economic Growth”, *Journal of Political Economy* 115, 925-985.

[2] Aghion, P., P. Howitt and F. Murtin (2011). The Relationship Between Health and Growth: When Lucas Meets Nelson-Phelps. *Review of Economics and Institutions*, 2(1).

[3] Bell, C., D. S. Devarajan and H. Gersbach (2006). The Long-Run Economic Costs of AIDS: A Model with an Application to South Africa. *World Bank Economic Review*, 20(1): 55-89.

[4] Chakraborty, S., Papageorgiou, C., Perez-Sebastian, F., (2016), “Health Cycles and Health Transitions”, *Macroeconomic Dynamics*, 20: 189-213.

[5] de la Croix, D., and M. Doepke (2003). Inequality and Growth: Why Differential Fertility Matters. *American Economic Review*, 93(4): 1091-1113.

[6] De Nardi, Mariacristina (2004), “Wealth Inequality and Intergenerational Links”, *Review of Economic Studies*, 71, 743-768.

[7] Durevall D. and A. Lindskog (2011), “Uncovering the impact of the HIV epidemic on fertility in sub-Saharan Africa: the case of Malawi”, *Journal of Population Economics*, 24: 629-655.

[8] Galor, Oded and Omer Moav (2004), “From Physical to Human Capital Accumulation: Inequality and the Process of Development”, *Review of Economic Studies*, 71, 1001-1026.

[9] Hart, Oliver and John Moore (1994), “A Theory of Debt Based on the Inalienability of Human Capital”, *Quarterly Journal of Economics*, 109(4), 841-879.

[10] Heckman, James J. (2000), “Policies to foster human capital,” *Research in Economics*, Elsevier, Elsevier, vol. 54(1), pages 3-56, March.

[11] Kalemli-Ozcan, Sebnem and Belgi Turan (2011), “HIV and fertility revisited,” *Journal of Development Economics*, 96(1): 61-65.

[12] Krebs, Tom (2003), “Human Capital Risk and Economic Growth” *Quarterly Journal of Economics*, 118(2), 709-744.

[13] Razin, Assaf (1976), “Lifetime Uncertainty, Human Capital and Physical Capital”, *Economic Inquiry*, 16: 439-448.

[14] Wang, Ruixing, Yang Zhou and Erwin Bulte (2013), “Fertility in Times of AIDS: Adult Mortality, Portfolio Choice and Human Capital Investment”, CentER Discussion Paper, 2013-051.

[15] Weil, D.N., (2007), “Accounting for the Effect of Health on Economic Growth”, *Quarterly Journal of Economics* 122, 1265-1306.

[16] Young, A., (2005), “The Gift of the Dying: The Tragedy of AIDS and the Welfare of Future African Generations”, *Quarterly Journal of Economics* 120, 243-266.