Strategic communication and group formation

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Abstract

We present an informational theory of group formation. A set of individuals with heterogeneous, state-dependent preferences decide to allocate themselves into groups. Members of each group commit to taking a common action and may choose to share private information which becomes available only after the decision to join a group is made. Therefore, by joining a group, one gives up the freedom of deciding her own action in return of a possible informational benefit. In equilibrium, each group is composed blocks of individuals with preferences close to each other. The extent of diversity within each group is determined by the following tradeoff: additional individuals bring in more information, but too much diversity leads to incentive problems in disclosure. The size of each group in equilibrium is decreasing in the initial diversity of preferences in the population, and non-monotonic in the ex-ante probability of any individual being informed. An increase in the ex-ante probability of an individual being informed may lead to the society being fragmented in smaller groups, thereby lowering the overall probability of a random individual being informed in equilibrium.

Preliminary and incomplete. Do not quote.

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1 Introduction

Jackdaw percheth beside jackdaw.
- Diogenianus

The pervasive fact of homophily means that cultural, behavioral, genetic or material information that flows through networks will tend to be localized.
- McPherson, Smith-Lovin and Cook (2001)

Birds of a feather flock together. Most groups, whether social groups, political entitities or economic organizations have a concentration of certain characteristics that is different from the distribution of such characteristics in the population in general. People with similar demographics tend to get married to each other (Kalmijn 1998). Friendship networks have clusters of indivudals with similar demographics and similar attitudes (Lazarsfeld and Merton 1954, Benhabib, Bisin, and Jackson 2010). The phenomenon of homophily is, in the words of McPherson, Smith-Lovin, and Cook (2001), "a basic organizing principle of groups." Homophily is almost a matter of definition in groups that have a well-defined goal. The members of the Repulican party are by and large conservatives and those of the Democratic party are by and large liberals. The members of NATO are the capitalist nations ("the western bloc"), those that were the part of the Warsaw Pact were by and large socialist nations ("the eastern bloc"), and those engaged with the Non-Aligned movement were developing nations that were, as the name suggests, not aligned to either bloc.

While homophily is natural given the existence and the identity of these groups, it is important to notice that each of these groups admits a diversity among its own members, and the group identity itself reflects some aggregate of the somewhat diverse identities of the individuals that form the group. The question that interests us is the following: when a set of heterogeneous individuals organize themselves into groups, what are the different groups formed? What is the composition of the each group? In particular, how much diversity is there *within* each group?

We suggest that there is a basic tradeoff that drives the decision of every individual economic agent deciding whether to join a group. Being in a group offers the possibility of sharing information that is necessary for choosing the best course of action, and on the other hand, by signing up for a group one gives up the independence of deciding the best course of action for oneself. It is this tradeoff that determines the extent of diversity in a group. While more individuals bring in more potential informational benefit, a more diverse group has two problems: first, the common goal may be too far away from the optimal action of the fringe members and second, higher diversity poses incentive problems for information sharing between members.

In order to formally capture the above intuition, suppose there are a finite number n of individuals. Every individual's preferred action is sensitive to information about some state of the world, and given the same information, each individual prefers a different action. In particular, the state of the world θ can be either 0 or 1, and individual i prefers an action as close to $ib + \theta$ as possible. In our model, the only reason to join a group is to obtain information about θ. Each individual either starts a new group or joins an existing group. As a modelling device, we assume that individual $i = 1$ first announces her group, individual $i = 2$ then decides either to join the group or to form her own group, individual $i = 3$ then decides to join one of the existing groups or form her own group and so on till individual $i = n$. A group profile is any partition of the population, and each non-empty element of the partition is a group.

After the groups are formed, the individuals get private signals about the state. We assume a particularly simple information structure: with probability $p \in (0, 1)$, the state is revealed. Each individual has the option to share his information with the rest of her group. Finally, each group takes an action according to a prescribed rule–if the state of the world is disclosed, the group takes an action that maximizes the total payoff of its members conditional on the state; otherwise, the group takes an action that maximizes the total ex ante expected payoff of them. As a result, each group derives its "identity" from its members in the following way: the group acts like a representative individual with a preference parameter that is the average of the preference parameters of its members.

In equilibrium, there is a number k^* such that each consecutive block of k^* individuals form a distinct group, (and if n is not a multiple of k^*, there is one smaller group composed of the remainder). Therefore, we do find homophily as an eqilibrium phenomenon. However, groups do permit a certain extent of diversity. The size of the group is small enough so that no individual has an incentive to hide information: in this sense there is no incentive problem within the groups. However, information may remain localized within groups: some groups may be informed and some remain uninformed. In fact, the equilibrium group size is too small compared to what would have been socially optimal - this happens because each individual only cares about her own group members and not about other members in the society.

To see how the tradeoff between sharing of information and control over action determines the optimal diversity of groups, consider the incentives of the first member of a group (i.e. the one with the lowest index). As additional members join the group, while there is additional informational benefit, the group's average action moves away from the ideal action of the first member. At the optimal group size k^*, these two effects are exactly balanced for the first member. In fact, the consecutive group with k^* members maximizes the payoff for the first member among all possible compositions of the group. By symmetry, this is true of the last (i.e. k^*-th) member of the group. Since the extreme members of the group are also those with the lowest expected payoff among all group members, a group with k^* consecutive members has the property that it maximizes, among all possible group compositions, the payoff of the group member who achieves the lowest payoff in the group. Hence, we call such a group a *maxmin utility group*. In the same vein, a group profile where all groups are maxmin groups (except possibly for one smaller group containing the remainder) is called a *maxmin utility group profile.*

Our main result in section 2 is that when individuals are called upon to form or join groups in the "natural" order of their preference (*group formation stage*) and then each group takes its own action based on the information volunteered (*group decision stage*), there is an SPNE where the maxmin utility group profile is formed. However, in various applications, such an extensive form may not be the most appropriate one. In section 3, we take an agnostic position about the extensive form and consider whether the maxmin utility group profile would still be a stable profile. Notice that given any group profile induces an expected utility for every individual through the group decision stage. Since the ex-ante expected utility for agent depends only on the identities of her group members, we have what is known in the literature on co-operative game theory as a hedonic game (Drèze and Greenberg 1980, Banerjee, Konishi, and Sönmez 2001, Bogomolnaia and Jackson 2002). We show that the *maxmin utility group profile* satisfies both of the standard notions of stability used in hedonic games: Nash Stability and Core Stability.

In section 4, we study the comparative static properties of the maxmin utility group profile. If the initial population is very homogeneous, i.e. if b is small, each additional person will move the average away by a small amount: thus large groups will be formed. In an extreme case, all the N individuals will be part of a single group. On the other hand, if any two consecutive individuals have a large conflict of preference, i.e. if b is large, then the overall population will be fractured into many small groups. In an extreme case, each individual will be a group unto her own. The benefit of information sharing is large when p is moderate: if p is low, an additional individual brings little extra information; and if p is high, since the existing set of individuals is already informed with a high probability, there is little marginal net benefit from having another individual. Therefore, the groups will have low diversity (small size) if the probability of an individual being informed is either very high or very low. Thus, for a given p, larger groups are formed as b goes down, i.e. the initial population becomes more homogeneous. On the other hand, holding the preference parameter b constant, the group size is inverse U-shaped in the ex-ante likelihood p of any individual knowing the state.

Our model suggests that there will be limited information sharing in societies: in particular, information will be locked in homophilous groups. The probability that any individual will eventually learn the state is equal to the probability that any member of her group will be informed: thus the extent to which the population will be eventually informed of the state depends critically on the size of the group. In particular, excessive fragmentation (small optimal groups) leads to reduction in the likelihood that any individual will eventually learn the state. Our model suggests that a more homogeneous population (low b) is also a more informed population, given the same p. In fact, our model has a more perverse implication: an increase in p (the exogenous likelihood of any individual learning the state from nature) may lead to higher fragmentation (smaller k^*), which may reduce the endogenous likelihood that an individual in the population will eventually learn the state. When individuals are more likely to be informed, they will form smaller groups, and since information is locked locally in groups, we may end up with a less informed population.

In order to concentrate on our tradeoff, we abstract away from many other economic issues that affect the decision to form groups. First, in our model, the only benefit to the formation of groups is informational. Groups (in particular, organizations) often provide huge technological benefits to economic activity. For example, groups allow specialization of activities, and thus improve productivity. If we take into account the benefits of scale and scope by assuming that an activity when undertaken by a group is more efficient than when it is undertaken by an individual, then we will have larger groups in equilibrium. We have also assumed away any externalities across groups. Sometimes, the different groups formed compete or co-operate in some broader arena. Political parties engage in electoral competition, churches compete for membership. While we agree that the nature of economic interaction between groups do affect the composition of groups, such effects are very sensitive to the kind of interaction between the groups. While we have no doubt that such effects will modify the tradeoff we study in any particular application, we have chosen to abstract away from these effects in order to better capture what we think is one very important tradeoff in group formation.

The idea that choice of groups may itself be strategic is not new to economics. There is a literature on the theory of club formation which has already held this idea (see Wooders, Cartwright, and Selten (2006) for example). Baccara and Yariv (2013, 2008) study a set-up where a set of individuals with different priorities over two issues organize themselves into groups, and within each group, individuals acquire and share costly information about the issues. They use a notion of stable group composition: a composition that is optimal for all group members; and show that stable groups are formed of sufficiently similar members (much like we do). There are two important differences between our paper and Baccara and Yariv's. First, our basic focus is a tradeoff between independence and information, while theirs is on free-riding within the group. Moreover, in their paper, collected information is automatically shared within the group while we consider incentive problems in communication of (costless) information.

2 Model - Extensive form game

There is a set I of agents, with a typical agent indexed by i. If I is finite, then $|I| = n$. In this section, we also allow for the case where I is countably infinite. Each agent i takes an action y_i, which generates a payoff that is dependent on an underlying state θ. Formally, the state θ is drawn from $\{0, 1\}$, each equally likely. An agent is informed of the state with probability $p \in (0, 1)$. Each agent $i \in I$ has preferences that are represented by the quadratic-loss utility function

$$U(y_i, \theta, b_i) = -\left(y_i - (\theta + b_i)\right)^2,$$

where

$$b_i = ib, \text{ for some } b > 0.$$

The variable b_i measures agent i's "bias," as is customarily called in the strategic communication literature, i.e., in state θ, agent i's most preferred action is $\theta + b_i$. The parameter b is a measure of the heterogeneity of the population: the larger is b, the larger is the conflict of preference between any two given individuals.

Before the state is learnt, the agent has to decide whether or not to join a group. Members of each group have the option to share the information, and based on the information available in the group, members of each group take a common action according to a pre-committed decision rule. We explain the rules of group formation and decision-making in detail hereunder.

2.1 Timing of the game

The game takes place in two stages: the group-formation stage and the decision-making stage.

In stage 1, a group profile is formed, according to the following procedure. Individuals take turns, in ascending order of their indices, to form a new group or to propose joining an already existing group. If the proposal is accepted by the first member ("leader") of the said group, then the individual is admitted to the group. The individual $i = 1$, by definition forms a group. The individual $i = 2$ either joins $i = 1$ subject to her approval, or forms a new group. Then $i = 3$ applies to join one of the already existing groups or forms her own group and so on. A group is indexed by the index of its leader, i.e. the individual who formed the group.

Before the group profile is finalized, agents do not know the state of the world, nor do they know if they will be informed of the state. In the decision-making stage, each agent becomes privately informed of the state (independently) with probability $p \in (0, 1)$. Observation of the state remains private information. Each agent has the choice either to disclose the information or not to disclose it to the members of her group. If she does not disclose it, she simply claims to be uninformed. The sequence in which agents disclose information within a group does not matter. Conditional on the shared information, each member of the group takes a common action that is predetermined for the group. Information is not shared between individuals in different groups.

Each group has a preset rule of decision making: if anyone discloses the state to the other members of the group, all the group members commit to taking action

$$\theta + \bar{b},$$

where $\bar{b}$ is the average bias of the members of the group. If no one discloses the state, the group members commit to taking action

$$\frac{1}{2} + \bar{b}.$$

Notice that each group member has different preferences given the information (private or public). Thus, the common action of the group is a result of some bargaining protocol. The particular decision mechanism we assume satisfies the following condition. If the state of the world is disclosed, then it maximizes the sum of the payoffs of group members in that state; if the state of the world is not disclosed, then it maximizes the sum of the ex ante expected payoffs of group members. This is implied by the quadratic preferences of the group members. One could, of course, think of other modes of decision-making. We offer a discussion of alternative decision rules in Section 6.

2.2 Group profiles and group formation

Now, we introduce language that defines a group profile. We index each group by the member with the lowest index, i.e., the "leader" of the group. Then, a group profile can be represented by a mapping Γ from I to I that satisfies

1. $\Gamma(i) \leq i$,
2. if $\Gamma(i) < i$, then there does not exist $j \in I$ such that $\Gamma(j) = i$.

Thus, Γ assigns to the individual i the index of the group leader. Note that $\Gamma(i) = \Gamma(i')$ if and only if i and i' are in the same group. We will refer to Γ as the *group assignment function.* Furthermore, the collection of inverse images of the mapping Γ give the actual partition of the individuals into groups:

$$\{\Gamma^{-1}(\{i\})|i \in I, \Gamma^{-1}(\{i\}) \text{ is nonempty}\}.$$

For neatness of notation, henceforth, we will use $\Gamma^{-1}\{i\}$ instead of $\Gamma^{-1}(\{i\})$. Note that

1. $i \in \Gamma^{-1}\{i\}$ if $\Gamma^{-1}\{i\}$ is nonempty;
2. $\cup_{i\in I}\Gamma^{-1}\{i\} = I$ and $\Gamma^{-1}\{i\} \cap \Gamma^{-1}\{i'\} = \emptyset$ for $i, i' \in I$ and $i \neq i'$.

We say that a group profile satisfies *homophily* if its corresponding group assignment function Γ is nondecreasing. Intuitively, homophily refers to the phenomenon that every group member's two immediate neighbours are in the same group as her, except for the two extreme members.

Thus, the group formation stage can be represented in extensive form as follows. Agents move in ascending order of their index. Each agent can make a choice between applying to join a group that has already been formed and starting a new group. However, if she applies to join a group, her application has to be approved by the "group leader."[1] If it is rejected, then she has to

[1] We could assume other ways of approval of an individual's proposal to join the group. For instance, we might require unanimous approval of the existing group members. Such rules make a difference in a rather technical sense. Notice that given a population of n individuals, if the equilibrium group size is k^*, then the last group may have less than k^* members while all other groups will have the same number k^* members. Different approval mechanisms may lead to different kinds of strategic manipulation by the members in the penultimate group with effects rippling up the entire population, and the homophily equilibrium may even break down. We show such an example with the unanimity requirement later. We conjecture that if we have a countably infinite population of agents, then such approval mechanisms will not make a difference: in fact even if entry is free, we will get the same equilibrium outcome.

start a new group. The payoffs are determined by the information dsharing stage that follows.

Formally, each player $j \in I$ chooses an action $a_j \in A_j$, where $A_j \subset \{1, \ldots, j\}$. Denote the set of the first j individuals in the sequence by the set I_j. Let us denote by $\Gamma^j : I_j \to I_j$ as the group allocation function of individuals 1 through j, with properties 1 and 2 of group assignment functions as above, and we have $\Gamma^{|I|} = \Gamma$. In addition, let us define the operation "*extension* of Γ^{j-1}," $\mathcal{E}$, as follows:

$$\mathcal{E}(\Gamma^{j-1}, a) = \Gamma^j, \text{ where } \Gamma^j(i) = \Gamma^{j-1}(i) \text{ for } i = 1, \ldots, j-1 \text{ and } \Gamma^j(j) = a.$$

We may therefore represent the first-stage game in the following way:

1. At the beginning of the game, player 1 chooses $\Gamma^1(1) = 1$, and the game proceeds to $j = 2$.

2. Player j observes the group assignment Γ^{j-1}, and chooses $a_j \in A_j = \Gamma^{j-1}(I_{j-1}) \cup \{j\}$.

3. The player with index a_j decides to whether let agent j join the group. If so, $\Gamma^J = \mathcal{E}(\Gamma^{J-1}, a_j)$; otherwise, $\Gamma^J = \mathcal{E}(\Gamma^{J-1}, j)$.

4. The game proceeds to the next player, $j + 1$, and goes to step 2, unless $j = |I|$, in which case the group formation stage ends.

Note that the above group formation procedure can generate all the group profiles that are possible.

2.3 Strategies and Equilibrium

The action spaces is defined as follows. For every individual, there is a membership application action and a membership approval action in the group formation stage, and a disclosure action in the decision-making stage, defined as follows.

- Membership application action for individual j: Given Γ^{j-1}, choose $a_j \in A_j = \Gamma^{j-1}(I_{j-1}) \cup \{j\}$.

- Approval action p_j^i by individual j for the application of individual $i > j$: when $\Gamma^{i-1}(j) = j$ for some $i > j$ and $a_i = j$, j has to choose $p_j \in \{A, D\}$, where A means approval of i's application to join group j and D means denial.

- Given a group assignment Γ and information $\theta \in \{0, 1\}$ about the state, each informed member i has the choice whether to disclose her information to her group $\Gamma^{-1}\{\Gamma(i)\}$. Uninformed members have nothing to disclose. If $\Gamma^{-1}\{\Gamma(i)\}$. is a singleton, we assume that disclosure is automatic.

Define h^i as the history of application decisions a_j for $j = 1, 2, ..i - 1$ and corresponding approval actions. This history is observed by individual i before making her decisions. The strategy of each member j is therefore a choice of three actions: (i) application a_j contingent on h^j, (ii) acceptance/denial p_j^i of a possible application by agent i (contingent on h^i) for each $i > j$, and (iii) the disclosure action (contingent on Γ and state θ). Our equilibrium concept is simply subgame perfect Nash equilibrium (SPNE).

At the disclosure stage, given each group profile, a member j chooses to disclose or not depending on the composition of the group $\Gamma(j)$ and the disclosure strategies of every other member in $\Gamma(j)$. Notice that since there is no externality across groups, disclosure strategies of members in other groups does not affect one's payoff. SPNE demands that for every group in each subgame defined by some group assignment Γ, disclosure strategies of the members in each group constitute Nash equilibrium. At the group formation stage, when it is each agent's turn to move, she chooses the application and approval strategy that leads to the highest payoff, conditional on that every other agent that follows her would also do so, and that each group formed thus would play Nash equilibrium in disclosure strategies.

In the group formation stage, SPNE requires that when it is agent i's turn to move, she chooses the a_i and p_i that leads to the highest payoff for her, conditional on that every other agent that follows her would also act likewise.

3 Analysis

The group formation stage induces some group assignment function Γ. Given some Γ, every i such that $\Gamma^{-i}(i) \neq \phi$ uniquely indexes a group. To solve for the SPNE, we first look at the decision-making stage for a given group. First, notice that in equilibrium, even if I is countably infinite, we will never have any group that is infinitely large. To see that, suppose to the contrary there is some such group in equilibrium, and consider the member of that group with the lowest index. That individual's lowest possible payoff from staying alone is bounded below, and on the other hand, in equilibrium, since the average bias of the group is unboundedly large, her highest possible payoff is unboundedly low. Hence, she is strictly better off staying alone than playing her equilibrium strategy. Therefore, from now on we consider only groups of finite size.

3.1 Decision-making subgame

Consider any group with k members, where their indices are given by j_1 through j_k in the ascending order. SPNE requires that strategies of every individual forms a Nash equilibrium for the decision-making subgame in every group. Formally, we call strategies that constitute a Nash equilibrium of the decision-making subgame the optimal disclosure strategies. Denote the average bias as $\bar{b}$, where

$$\bar{b} = \sum_{l=1}^{k} \frac{b_{j_l}}{k} = \sum_{l=1}^{k} \frac{j_l b}{k}.$$

In the decision-making subgame, if an expert is not informed, she has nothing to report. If she is informed of the state, her report matters only in the event that no other expert has reported the state. In this event, she obtains an action $\bar{b} + \theta$ on reporting and $\bar{b} + \frac{1}{2}$ on not disclosing the state and pretending to be uninformed. Therefore, in an SPNE, an agent $i \in \{j_1, ...j_k\}$ reports state θ if and only if

$$U(\theta + \bar{b}, \theta, b_i) \geq U(\frac{1}{2} + \bar{b}, \theta, b_i).$$

Given the group decision rule, it may be better for an agent to not share "adverse" information with the group. For instance, the rightmost agent may not want to disclose the state 0 to the group, because her most preferred action in state 0 may be closer to the no-information action $1/2 + \bar{b}$ than the informed action $\bar{b}$. From simple calculations, we can easily verify that the optimal disclosure strategy of the agent $i \in \{j_1, ...j_k\}$ is the following.

Lemma 1. *When the state is* 0, *agent* i *reports the state if and only if* $b_i \leq \bar{b} + \frac{1}{4}$, *and when* $\theta = 1$, *she reports the state if and only if* $b_i \leq \bar{b} - \frac{1}{4}$.

In other words, in SPNE, an agent i reports truthfully iff $\bar{b} - \frac{1}{4} \leq b_i \leq \bar{b} + \frac{1}{4}$, reports only state 0 (and not state 1) iff $b_i < \bar{b} - \frac{1}{4}$ and reports only state 1 (and not state 0) iff $b_i > \bar{b} + \frac{1}{4}$. Notice that $\bar{b}$ depends on the group composition and hence on the observed history.

An implication of the above characterization of optimal strategies in the decision-making subgame is that if any member of the group discloses state 0 when she observes it, then every member to her left will disclose it as well. It is exactly the opposite for state 1, namely, a member is more likely to disclose

1 if she is more to the right. Formally, in a group consisting of agents j_1 through j_k, if agent j_l is willing to disclose 1, then every agent $j_{l'}$ is willing to disclose 1, if $l' \geq l$. Similarly, if agent j_l is willing to disclose 0, then every agent $j_{l'}$ is willing to disclose 0, if $l' \leq l$.

3.1.1 Consecutive groups

Next, we turn our attention to a group composition that is of special interest to us. We call a group *consecutive* if the indices of its members, arranged in an increasing order, are consecutive numbers. A consecutive group of size k has members with indices $i_0 + 1,\ i_0 + 2, ...i_0 + k$ where i_0 is any non-negative integer. The average bias of such a groups is $\bar{b} = \left(i_0 + \frac{k+1}{2}\right) b$.

In such a group, the optimal disclosure strategy of the j^{th} individual, i.e. the one with index $i_0 + j$ is to reveal the state 0 iff

$$(i_0 + j)b \leq \left(i_0 + \frac{k+1}{2}\right) b + \frac{1}{4} \Leftrightarrow j \leq \frac{(k+1)}{2b} + \frac{1}{4b},\ j = 1, 2, ..k.$$

Similarly, the j^{th} agent optimally reveals the state 1 iff

$$j \geq \frac{(k+1)}{2} - \frac{1}{4b},\ j = 1, 2, ..k.$$

Notice that the condition for all individuals in the group to report 0 is that the k^{th} individual reports state 0. By symmetry, this is also the condition for all individuals reporting state 1. Therefore, the condition for everyone in a consecutive group of size k to always report truthfully is

$$k \leq \frac{1}{2b} + 1,$$

which basically says that the group size should not be very large compared to the preference difference paraneter b. Notice also that as the population becomes more homogeneous in terms of preference, i.e. b goes down, the truthtelling constraint in consecutive groups is more relaxed.

3.2 Interim expected utility

In this section, we will study the interim expected utility, i.e. the expected utility of any individual after the group formation stage, but before the state is revealed and with the agents anticipating that they will use the optimal disclosure strategies. Consider an arbitrary group with agents indexed in ascending order from j_1 through j_k. The expected interim payoff EU_i of each agent $i \in \{j_1, ... j_k\}$ in the group can be written as

$$
\begin{aligned}
EU_i &= -\int_R \left(\bar{b} - b_i\right)^2 dF(\theta|R) - \int_{NR} \left[\left(\frac{1}{2} + \bar{b}\right) - (\theta + b_i)\right]^2 dF(\theta|NR), \\
&= -P(R)\left(\bar{b} - b_i\right)^2 - \int_{NR} \left[\left(\frac{1}{2} - E(\theta|NR) + \bar{b} - b_i\right) + (E(\theta|NR) - \theta)\right]^2 \\
&\qquad dF(\theta|NR), \qquad (1) \\
&= -\left(\bar{b} - b_i\right)^2 - P(NR)\left[\left(\frac{1}{2} - E(\theta|NR)\right)^2 + Var(\theta|NR)\right. \\
&\qquad \left. +2\left(\frac{1}{2} - E(\theta|NR)\right)\left(\bar{b} - b_i\right)\right],
\end{aligned}
$$

where R corresponds to the event that θ is revealed and NR that θ is not. Also, $F(\theta|R)$ and $F(\theta|NR)$ are the corresponding conditional distributions of θ, respectively. The above equation demonstrates that each agent's expected payoff can be decomposed into two parts: the loss caused by the difference between each group's average bias from her bias and the loss caused by the failure of the group to share information.

Notice that these distributions $F(\theta|R)$ and $F(\theta|NR)$ depend on the optimal disclosure strategies of the individuals in the group and therefore, on the group composition. For example, in a consecutive group of size k, we have $E(\theta|NR) = \frac{1}{2}$ and $Var(\theta|NR) = \frac{1}{4}$. Moreover, if $k \leq \frac{1}{2b} + 1$, everyone discloses their information, and the probability that the state is undisclosed is simply the probability that no one is informed, i.e. $P(NR) = (1-p)^k$. On the other hand, if $k > \frac{1}{2b} + 1$, it can be easily verified that $P(NR) = (1-p)^{\frac{k+1}{2} + \frac{1}{4b}}$. Therefore, in a consecutive group of size k, taking into account the optimal strategies, (1) yields that the interim expected utility of the j^{th} individual, i.e., the one with index $i_0 + j$, is

$$
EU_j = \begin{cases} -\left(\frac{k+1}{2}b - jb\right)^2 - \frac{1}{4}(1-p)^k & if \quad k \leq \frac{1}{2b} + 1 \\ -\left(\frac{k+1}{2}b - jb\right)^2 - \frac{1}{4}(1-p)^{\frac{k+1}{2} + \frac{1}{4b}} & if \quad k > \frac{1}{2b} + 1 \end{cases} \qquad (2)
$$

The following Lemma identifies the group size k^* that maximizes the leader's interim payoff in the class of all consecutive groups of size k. Notice that since the payoffs in a consecutive group is symmetric about the average bias, the leader's payoff is the same as the k^{th} member's payoff: and therefore, this group size also maximizes the rightmost member's payoff.

We denote by $\lceil x \rceil$ and $\lfloor x \rfloor$ respectively the smallest integer weakly greater than and the greatest integer weakly less than x for any $x \in R$.

Lemma 2. *Suppose agents play their optimal disclosure strategies. In the class of all consecutive groups, the group size that maximizes the leftmost and* *rightmost players' expected payoff, k^*, satisfies*

$$EU_{k^*} = \max\{EU_{\lfloor k' \rfloor}, EU_{\lceil k' \rceil}\}, \tag{3}$$

where k' is the solution to

$$2(k-1)b^2 + (1-p)^k \ln(1-p) = 0, \;\; k \in R_{++}.$$

Furthermore, in such a group, each group member discloses all her information to the group.

Proof. See Appendix. ∎

The above lemma characterizes the size of the group that maximizes the two extreme members' payoff. It shows that an agent is welcomed into the group consisting of her immediate preceding neighbours only if she finds it optimal to disclose all her information to the group. The condition that determines k^* reflects the tradeoff between bias loss and information loss-being the leftmost or rightmost player in a larger groups causes the group action to be far away from her most preferred action, but it also increases the chance of getting more information from other group members.

There are two steps in the proof of the Lemma. First, ignoring the integer constraint on k, we find value of k' where EU_k is maximized in the class of truthful consecutive groups, i.e. when $k \leq \frac{1}{2b} + 1$. Then we show that the largest truthful group obtains a better payoff for the leader compared to any group size larger than $\frac{1}{2b}+1$. The optimal group size k^* is the integer-corrected version of k'.

The next Proposition tells us that a consecutive group with k^* members is also the group composition that offers the highest payoff to the group leader among all arbitrary group compositions that would have the same individual as the group leader.

Proposition 1. *Fixing the identity of one individual, consider all possible group compositions with that individual as the group leader. When agents play their optimal disclosure strategies, the group composition that maximizes the leader's interim payoff is the consecutive group with k^* members. Moreover, in such a group, the leftmost and rightmost individual obtain the same interim expected utility which depends only on the parameters b and p, and not on their individual biases.*

This proposition extends the result in Lemma 2 to the case with arbitrary group composition in two steps. These two steps are presented as two lemmas below.

Lemma 3. *Consider all groups of any given size k. Fixing the bias of the group leader, if every member discloses all her information to the group, then the group leader's expected payoff is decreasing in the average bias of the group members.*

Proof. See Appendix. ∎

By (1), if every member discloses information, then the information loss is constant as long as the group size does not change. Therefore, the leader's expected payoff varies only due to bias loss, which increases as the average bias of the group increases, because it moves further away from her bias. This lemma implies that a group leader prefers to be in a consecutive group than in an arbitrary group, provided that everyone discloses information.

Lemma 4. *Suppose agents play their optimal disclosure strategies. In groups of any given size, the group leader's expected payoff is maximized when the group consists of consecutive members.*

Proof. See Appendix. ∎

The proof of the above lemma is by observing the following facts. There are three cases to consider: 1. everyone discloses information; 2. the leftmost player does not disclose all information but the rightmost player does (this means the bias composition of the group is skewed towards the right); 3. the rightmost player does not disclose information (either the group is too large or the bias composition of the group is skewed towards the left). The result for case 1 is implied by Lemma 3. For cases 2 and 3, the result is obtained by the observation that the information gain (if any at all) from having faraway members is always offset by the bias loss.

Lemmas 2, 3, 4 together establish the first part of the statement of proposition 1. To verify the second part, notice that in a consecutive group of size k, we have $EU_1 = EU_k$, and from equation (2), EU_1 depends only on k^* and b.

3.2.1 Maxmin utility group profiles

Denote by $\mathcal{G}$ the family of all possibe non-empty coalitions (i.e. groups) of I agents, and denote a generic element of $\mathcal{G}$ by g. Thus, g is any arbitrary group. Assuming that all agents use their optimal disclosure strategies, each group induces a profile of interim expected utilities, one for each of its members. Denote the minimum interim expected utility in group g by u_g. Observe first that according to (1), the agent i in the group with the largest value of $|b-b_i|$ obtains u_g. Therefore, the one with the lowest utility in the group is one of the two extreme members. Moreover, in any consecutive group, both the extreme members obtain the same interim utility. Therefore, proposition 1 tells us that in the class of all $g \in \mathcal{G}$, the group g that maximizes u_g is any group with k^* consecutive members. Since any group with k^* consecutive members maximizes the minimum payoff obtained by a group member, we call such groups *maxmin utility groups.* We denote by a *maxmin utility group profile,* a group profile where all (except possibly one smaller group) groups are *maxmin utility groups.* In particular, when I is countably infinite or if $|I| = n$ and n is a multiple of k^*, there is a unique maxmin utility group profile - it is the profile where each consecutive block of k^* individuals forms a separate group. When n is not a multiple of k^* and leaves a remainder $0 < r < k^*$, then a maxmin utility group profile is any profile with one group consisting of r consecutive members and all others with k^* consecutive members.

The most important feature of a maxmin utility group profile is that no individual can be strictly better off by leaving his group in a maxmin utility group profile and starting a new group. In the next section we show that there is an SPNE of the group formation game where a maxmin utility group profile is formed in equilibrium.

3.3 SPNE Characterization

We first present our main theorem and then proceed to prove it.

Theorem 1. *There exists an SPNE where, for some natural number k^*, the equilibrium group assignment function Γ^* is given by $\Gamma^*(i) = \left\lfloor \frac{i}{k^*} \right\rfloor k^* + 1$ for all $i \in I$. Therefore, Γ^* induces a maxmin utility group profile.*

According to the theorem, if the maxmin utility profile is unique, then it can be implemented in SPNE. If the maxmin profile is not unique, i.e. if n is not divisible by k^*, then a particular maxmin profile is implemented: only the last group is of size smaller than k^*. Since $\Gamma^*(i)$ is a weakly increasing function, i.e. groups are formed of adjacent members, homophily is the SPNE outcome. Moreover, the size of each group is small enough to permit full disclosure by informed individuals.

The proof of the theorem is constructive. We identify the strategies that lead to this equilibrium. The intuition, however, is simple. Any member that is not supposed to be a group leader, by deviating, can only start a new group. In that case, she gets at most the group leader's maximum payoff which is strictly lower than her equilibrium payoff. Any member that is supposed to be a group leader, by deviating, can become the $k^* + 1$-th member of a previous group, but then she gets at most the payoff of a group leader with $k^* + 1$

consecutive members: which is lower than what she gets in equilibrium. By accepting any proposal from any member beyond the k^* members that are supposed to be there in the group, the group leader will have her payoff lowered: therefore, there is no gain by deviating from the approval action either.

Now we provide the formal proof of the theorem.

3.3.1 Proof

Consider the following strategy:

- Proposal strategy for individual j : If $j = 1$, propose to start a new group. If $j > 1$, consider the group membership of individual $j - 1$. If the group has strictly less than k^* "adjacent members" (including just one member), propose to join the said group. If the previous group is non-adjacent or has k^* or more members, propose to start a new group.

- Approval strategy of individual j for proposal by individual i : Compare the payoff for individual j from two group compositions: one with all existing members except i and one with the existing members and individual i. Approve member i if including her in the group increases the payoff to member j.

- Disclosure strategy: For any group, suppose $\bar{b}$ is the average bias. An individual j informed that the state is 0 reveals state 0 iff $b_j \leq \bar{b} + \frac{1}{4}$. An individual j informed that the state is 1 reveals state 1 iff $b_j \geq \bar{b} - \frac{1}{4}$. An uninformed individual has nothing to reveal.

We have already shown that the above disclosure strategy constitutes the unique equilibrium of the disclosure game in every group.

First, consider any deviation from the proposal strategy. Suppose the previous member's group has strictly less than k^* members. By following the equilibrium strategy, individual j gets weakly greater than $V(k^*)$. By deviating and proposing to join some other group, if she is rejected, she gets $V(k^*)$. If she is accepted, she becomes the last member in that group (since the individual $j+1$ starts a new "chain"). From proposition 1, she gets strictly less than $V(k^*)$.

Now, consider subgames off the equilibrium path. If in a subgame, the individual $j - 1$ belongs to a consecutive group with less than k^* members, equilibrium strategy requires j to propose to join the group. The proposal will be accepted and she will get weakly greater than $V(k^*)$. If she deviates and proposes to join some other group, she will be the last member of the said group if she is accepted - then by Lemma 4, she makes strictly less than $V(k^*)$. If she is rejected, she makes $V(k^*)$. If she deviates and proposes to start a new group, she makes $V(k^*)$. Thus, she does not have a profitable deviation. Next, suppose that the individual $j-1$ belongs to a non-consecutive group or a group with at least k^* members. Then equilibrium strategy requires j to start a new group. Then she gets $V(k^*)$ following the equilibrium strategy. Suppose she deviates and proposes to join some group. If she is rejected, she still makes $V(k^*)$. If she is accepted, it must be the case that the group she had proposed to join was a non-consecutive group. Then, since the next member starts a new group, she will be the last member of her group which is non-consecutive. In that case, by Lemma 4, she makes strictly less than $V(k^*)$. Thus, she does not have a profitable deviation in this case either.

Second, consider the approval strategy. If an individual i proposes to join any consecutive group with less than k^* members, she is accepted by individual j since $V(k) > V(k-1)$ for $k \leq k^*$. If, off the equilibrium path, an individual i proposes to join any group when she is not supposed to do so, conditional on her acceptance, the member $i+1$ starts a new "chain". Therefore, it is the best response for j, the leader of the group who i has proposed to join, to accept or reject by comparing the marginal effect of member i alone.

4 Hedonic Game

In the previous section, we utilize a specific order in which the individuals are called upon to act while forming a group or joining one. While the extensive form may be appropriate in certain applications, it may not be so in others. In this section, we take an agnostic position about the extensive form at the group formation stage and show that the outcome we identified does not really depend on the extensive form assumed.

In this section, we simply assume that individuals form groups according to some mechanism, and then play the optimal disclosure strategies at the decision-making stage in each group. Notice that for each possible group assignment function Γ, the decision-making stage induces a profile of interim expected utilities, one for each agent in I. Moreover, for any Γ, an individual's interim expected utility depends only on the identities of the members of the group she belongs to. Formally, then we have what is known in the literature as a hedonic game.

In a hedonic game, there is a finite population of individuals who are partitioned into coalitions and each individual's preferences depend only on the identities of the other members in the coalition she belongs to. The objective of the game is to then find which partitions are stable, given the individuals' preferences. There are two standard definitions of coalitional stability in hedonic games: Nash stability and Core stability. A group profile is core stable if there is no possible coalition where everyone would receive at least as much and someone strictly higher payoff than their allocation in the initial group profile. A group profile is Nash stable if there is no individual and no group such that the individual would profit from deviating and joining the group, and the group would accept him.

Since the stability criteria in hedonic games are defined only over finite populations, we only consider the case where I is finite. Our main result in this section is that the homophily outcome identified in the previous section is a stable profile according to both these criteria. Moreover, even when I is countably infinite, the spirit of the argument goes through without any alteration.

Proposition 2. *Any maxmin utility group profile is both core stable and Nash stable.*

Proof. Consider any coalition of size $k \neq k^*$, who deviate to form a group of their own. Then, the leftmost and rightmost members' payoffs are lower than $V(k^*)$ by Lemma 4.

Now, consider any coalition of size k^* to form a new group. Then, if it is nonconsecutive, the leftmost and rightmost members' payoffs are lower than $V(k^*)$ by Lemma 4. If it is consecutive, then in order for some coalition member's payoff to strictly increase, it must be that she is moving from a more extreme position to a more intermediate one. But this means that there must be some other coalition member moving from a more intermediate position to a more extreme one.

So, there do not exist any blocking coalitions. Therefore, the equilibrium group profile is core stable.

Nash stability is straightforward, since if anyone deviates to join another group or stay alone, her payoff is going to be weakly less than $V(k^*)$. ∎

We must mention here that there can be other core or Nash stable group profiles, some of which may not satisfy homophily. For example., consider the case where $|I| = 8$ and b and p are such that $k^* = 4$. Then, $\{\{1, 2, 3, 5\}, \{4, 6, 7, 8\}\}$ may be also be core stable.

5 Social welfare and Comparative Statics

It is important to ask here whether groups too small or too large compared to the size that would maximize social welfare. We show that among the consecutive groups, the group size that mazimizes average (per person) payoff is a group size $\widehat{k}$ that is weakly larger than the maxmin utility group size k^*. This tells us that there is too little sharing in the society: information is locked in inefficiently small groups.

Remark 2. *Consider a consecutive group of arbitrary size k. The average payoff of members in such a group is maximized for $k = \widehat{k}$. The number $\widehat{k}$ is weakly greater than k^*, the maxmin utility group size. Each member in a consecutive group of size $\widehat{k}$ discloses information.*

Proof. Consider a consecutive group of arbitrary size k, and denote the average loss of members in such a group by $W(k)$. WLOG, the bias of the member j of that group is written as jb. First suppose that $k \leq \overline{k} = 1 + \frac{1}{2b}$, i.e. all group members disclose information. The expected loss of the member is $(jb - \overline{b})^2 + \frac{1}{4}(1-p)^k$. Therefore, the average payoff of the group members is given by

$$\begin{aligned} W(k) &= \frac{1}{k}\left[\sum_{j=1}^{k}\left((jb-\overline{b})^2 + \frac{1}{4}(1-p)^k\right)\right] \\ &= \frac{1}{k}\sum_{j=1}^{k}(jb-\overline{b})^2 + \frac{1}{4}(1-p)^k \\ &= \frac{b^2}{12}(k^2-1) + \frac{1}{4}(1-p)^k \end{aligned}$$

Next, we note that for $k > \overline{k}$, we must have $W(k) > W(\overline{k})$. For such a group, in state 0,individuals 1 through $\overline{k}$ will reveal information. And in state 1, individuals $k - \overline{k} + 1$ through k will reveal information. Therefore, the average payoff for $k > \overline{k}$ is given by

$$\begin{aligned} W(k) &= \frac{1}{k}\left[\sum_{j=1}^{k}\left((jb-\overline{b})^2 + \frac{1}{4}(1-p)^{\overline{k}}\right)\right] \\ &= \frac{b^2}{12}(k^2-1) + \frac{1}{4}(1-p)^{\overline{k}} > W(\overline{k}) \end{aligned}$$

Therefore, the value of k that minimizes the average loss is weakly less than $\overline{k}$, and is given by $\arg\min_{k \leq \overline{k}} W(k)$. By taking derivatives, we have

$$\begin{aligned} W'(k) &= \frac{1}{4}(1-p)^k \log(1-p) + \frac{b^2}{6}k \\ W''(k) &= \frac{1}{4}(1-p)^k \left[\log(1-p)\right]^2 + \frac{b^2}{6} > 0 \end{aligned}$$

Therefore, $\widehat{k}$ is given by the solution to

$$W'(k) = \frac{1}{4}(1-p)^k \log(1-p) + \frac{b^2}{6}k = 0$$

Now, we know that k^* satisfies

$$2(k^* - 1)b^2 + (1-p)^{k^*} \log(1-p) = 0$$

Therefore,

$$\begin{aligned} W'(k^*) &= \frac{1}{4}(1-p)^{k^*} \log(1-p) + \frac{b^2}{6}k^* \\ &= -\frac{1}{2}(k^* - 1)b^2 + \frac{b^2}{6}k^* \\ &= \frac{b^2}{6}[k^* - 3(k^* - 1)] = \frac{b^2}{6}[3 - 2k^*] < 0 \text{ if } k^* \geq \frac{3}{2} \end{aligned}$$

If $W'(k^*) < 0$, we must have $k^* < \widehat{k}$. Notice that the actual value of k^* is the "closest integer" to the solution of the above equation. So, the only exception to $W'(k^*) < 0$ can be when $k^* = 1$. Then, we must have $\widehat{k} = 1$. ∎

In the previous section we have analyzed the SPNE outcome of a group formation game given the initial heterogeneity b in the population and the ex-ante probability p of any individual being informed of the state. We have seen that the outcome is captured by k^*, the group size in equilibrium. A lower k^* would mean a more fragmented population. Notice first that the optimal group size k^* does not depend on the size n of the population. This is due to the local nature of the interaction and lack of externalities between groups. In what follows we shall assume that $|I|$ is large enough, so that $|I| > k^*$. If there is no group with size k^*, then comparative statics on k^* have no bite. The following proposition shows how the optimal group size depends on the model parameters.[2]

Proposition 3. *The equilibrium group size, k^* is*

1. *decreasing in b, the difference in bias between neighbours;*

[2]There may be other equilibria of the model. We provide comparative statics on the equilibrium we have identified.

2. for some $p_0 \in (0, 1 - 1/e)$, decreasing in p for $p > p_0$ and increasing in p for $p < p_0$.

Proof. See Appendix. ∎

Note that k^* is the number that optimally trades off, for the group leader, the gain in information from an additional person with the loss due to shifting of the average group bias by the marginal entrant. As the difference in bias between two adjacent individuals increases, the bias loss increases for every group size: therefore, the optimal group size goes down. This tells us that we will have large groups if neighbors are very similar in preferences. On the other hand, with very dissimilar neighbors, we will have very small groups - in fact, when $k^* = 1$, each individual will form her own isolated group. In this sense, preference heterogeneity breeds isolation.

On the other hand, the ex-ante likelihood p of an agent being informed has a non-monotonic effect on group size in equilibrium. When p is very low, the informational gain from an additional person is low. When p is very high, again the current group has a high likelihood of being informed, so the *marginal* information gain from the additional person is low. In either case, the optimal group size should be small, while for intermediate levels of probability of informedness, the optimal group size is bigger. Therefore, k^* is inverse U-shaped with respect to p. Therefore, we have severely fragmented populations when individuals are either very likely or very unlikely to be informed.

Next, we turn our attention to the extent of information-sharing in the entire population. Notice that while the ex-ante likelihood of any individual being informed is p, since information is shared, the equilibrium likelihood of an individual except possibly in the last group eventually learning the state is $q^* = 1 - (1-p)^{k^*}$. Fixing the parameters b and p, if we let n grow large, we will have fewer and fewer proportion of the total population in the last group. Thus, q^* would accurately describe the probability of an individual learning the state for an arbitrarily large proportion of individuals in the society by making n arbitrarily large. Thus, the next proposition has more significance for values of n that are large compared to k^*.

Clearly, due to sharing, we have $q^* > p$. While p is a parameter of the model, q^* is an equilibrium quantity, and it depends on other parameters of the model. In particular, q^* is increasing in the size of the group k^*, and thus decreasing in b. For any given p, as the population becomes more homogeneous, larger groups are formed, making it less likely that no one in the group will know the state. Notice that the non-monotonicity of k^* in p means that it is possible that an increase in an individual's likelihood p of knowing the state may reduce group size k^*. Then, we have opposing effects on q^* : while each group member is more likely to be informed, there are fewer members in the group formed in equilibrium. The following proposition shows that an increase in p may have ambiguous effects on q^*, the equilibrium probability of an individual getting to know the state.

Proposition 4. *Fix b and p and consider $n > k^*$. Denote, for an individual in any group other than that last group, the probability of learning the state in equilibrium by $q^* = 1 - (1-p)^{k^*}$. Now, consider a small change in p. There is some $b_0(p) > 0$ such that for $b > b_0(p)$, q^* is increasing in p, and for $b < b_0(p)$, q^* is decreasing in p.*

The above proposition goes to show that due to fragmentation of the population into groups, a higher likelihood of individuals being informed may lead to lower sharing, and in the process, a less informed society in general.

6 Discussion

In this section, we discuss the different modeling assumptions we have made.

First,.the assumption of a finite population makes no difference. All our results hold perfectly straightforwardly if there are a countably infinite number of members in the society, with the ideal action of the j-th member in sate θ being $jb + \theta$. In fact, our proof technique works equally well with the countably infinite population. In fact, our equilibrium outcome is completely independent of the size of the population. The main reason for this independence is our assumption of lack of externalities between groups. While some groups like hobby groups or church groups can be thought to satisfy this assumption, there are many other situations where this assumption is not satisfied. For example, political parties compete in elections, interest groups lobby for influencing policy by counteracting each other and so on. In fact, the goals (common action in our model) and identity (average preference in our model) of the group often take into account the nature of externalities that groups impose on each other. However, the nature such externalities vary from one application to another, and it would not be very difficult to include such interactions in the current model and study their effects on the composition of groups.

We have assumed a specific protocol for admission of a prospective member into the group. Our assumption that the "group leader" decides on every proposed entry may make sense in applications where we can take the term "group leader" literally for the member who starts the group. An alternative mode of approval could be a situation where all the existing members of have to be unanimous in their acceptance of a proposal for entry. If the size of the population is infinite, the SPNE outcome identified in the main theorem still holds with such a unanimity requirement. In fact, even if entry is "free", i.e. every entry proposal is accepted by default, the SPNE outcome identified above remains unchanged. However, if the population is finite, different entry protocols may lead to different outcomes. The reason is that in the last group the payoffs of the members are different from that in the other groups, and this leads to opportunities for manipulation by the members of the penultimate group. By backward induction, the manipulations lead to ripple effects throughout the population. We furnish an example below to make this point. In this example, there are 5 members of the population and $k^* = 4$. If the entry of every member is to be ratified only by the group leader, by Theorem 1, the first 4 members will form a group and the fifth one forms a separate group. If we change the game to require that the entry proposals need to be agreed upon unanimously by all existing members of a group, we no longer have the homophily outcome.

Example 1. *Let $p = 1/2$ and $b = \sqrt{6 \ln 2}/24 \approx 0.085$. Then, it is straightforward to verify that the optimal group size is $k^* = 4$, which satisfies (3). Suppose there are 5 individuals in the population. Theorem 1 suggests that individuals 1 through 4 should form a group and individual 5 stays in a separate group of her own. Now, suppose we change the game and require that the entry proposal of each prospective member be accepted unanimously by all existing members of the group. Now, the group composition suggested by Theorem 1 is no longer an SPNE. Instead, in an SPNE of the modified game, 1, 2, 3 and 5 are part of one group and individual 4 stays separate. Suppose that agents 1, 2, and 3 have formed a group. Now, suppose agent 4 wants to join their group. Then, 3 would have an incentive to reject 4's proposal, because she prefers that 5 joins the group, whose proposal will be accepted by both 1 and 2.*

The parameter values are such that each agent will disclose all her information to her group regardless of group composition. Thus, each agent's payoff is thus neatly decomposed into bias loss and information loss. Let us denote the collection of the first k individuals of the society by I_k. Then, for $j = 1, \ldots, k$,

$$
\begin{aligned}
E\left(U_j | \Gamma^{-1}\{1\} = I_k\right) &= -\left[\left(\bar{b}_{I_k} - jb\right)^2 + \frac{1}{4}(1-p)^k\right]; \\
E\left(U_j | \Gamma^{-1}\{1\} = I_k \cup \{k+2\}\right) &= -\left[\left(\bar{b}_{I_{k+1}} + \frac{b}{k+1} - jb\right)^2 + \frac{1}{4}(1-p)^{k+1}\right],
\end{aligned}
$$

where on the right hand side of each equation the first term corresponds to the *bias loss, and the second the information loss. It is straightforward to see that agent* 3 *prefers having* 5 *in the group to* 4*. Note that for agent* 1*, the change in bias loss from admitting agent* 5 *to the group is*

$$\left(\bar{b}_{I_{3+1}} + \frac{b}{3+1} - b\right)^2 - \left(\bar{b}_{I_3} - b\right)^2 = \frac{33}{16}b^2 = \ln 2/96 \approx 0.00722,$$

while the change in information loss is

$$\frac{1}{4}(1-p)^{3+1} - \frac{1}{4}(1-p)^3 = \frac{1}{4}(1-p)^3 p = -\frac{1}{64} = 0.015625.$$

Thus, the reduction in information loss dominates the increase in bias loss. So agent 1 *will vote to approve* 5*'s request to join the group. For agent* 5*, the decision is between joining the group* I_3 *or* $\{4\}$*. It is straightforward to verify that joining* I_3 *is better for him.*

Finally, one may consider alternative decision rules for groups. One possibility is that the median member's optimal action is taken, and if there are two median members, then the mean of their optimal actions is taken. However, in our current setup, the decision made in a homophily equilibrium is indeed the same as the median member's best action. We conjecture that adopting this rule will not change our result.

Another possibility is to change the rule such that when no information is revealed, instead of the $1/2+\bar{b}$ (the "1/2 rule"), the action to be taken is $\bar{\theta}+\bar{b}$ (the "$\bar{\theta}$ rule"), where $\bar{\theta}$ is the conditional expectation of the state of the world given that no information is revealed. A potential issue of using such a rule is for certain group compositions, there might exist multiple information sharing equilibria, which makes comparisons between different group compositions less clear-cut. On the other hand, in groups consisting of consecutive members, the disclosure strategies in an information sharing equilibrium of our model under the 1/2 rule continues to be an equilibrium under the $\bar{\theta}$ rule. So, it is still possible for our result to continue to hold. Finally, in practical terms, the 1/2 rule is arguably simpler to articulate and implement than the $\bar{\theta}$ rule in an organizational and institutional setting. We will explore these variations of the rules in our future research.

7 Appendix

Proof of Lemma 2. First note that for groups consisting of adjacent members, $E(\theta|NR) = 1/2$ and $Var(\theta|NR) = 1/4$. By (1),

$$EU_i = \left(\bar{b} - b_i\right)^2 + \frac{1}{4}P(NR).$$

Case 1: $k \leq 1/(2b) + 1$. In this case, each group member discloses all her information to the group. Therefore,

$$EU_k = -\left[\left(\frac{k-1}{2}b\right)^2 + \frac{1}{4}(1-p)^k\right].$$

The derivative with respect to k is

$$\frac{dEU_k}{dk} = -\frac{1}{4}\left[2(k-1)b^2 + (1-p)^k \ln(1-p)\right].$$

The second order derivative is

$$\frac{d^2 EU_k}{dk^2} = -\frac{1}{4}\left[2b^2 + (1-p)^k [\ln(1-p)]^2\right] < 0.$$

Case 2: $k > 1/(2b) + 1$. In this case, only some group member discloses each state, as stated above. Therefore,

$$EU_k = -\left[\left(\frac{k-1}{2}b\right)^2 + \frac{1}{4}(1-p)^{\frac{k+1}{2}+\frac{1}{4b}}\right].$$

The derivative with respect to k is

$$\frac{dEU_k}{dk} = -\frac{1}{4}\left[2(k-1)b^2 + \frac{1}{2}(1-p)^{\frac{k+1}{2}+\frac{1}{4b}} \ln(1-p)\right].$$

The second order derivative is

$$\frac{d^2 EU_k}{dk^2} = -\frac{1}{4}\left[2b^2 + \frac{1}{4}(1-p)^{\frac{k+1}{2}+\frac{1}{4b}} [\ln(1-p)]^2\right] < 0.$$

Let us find the k that maximizes EU_k. In both cases, the second order derivatives with respect to k is negative, so in each case, the function is strictly concave in k. Furthermore, note that the first function is greater than the second for $k > 1/(2b) + 1$. Hence, if the first function reaches a maximum at $k^* < 1/(2b) + 1$, then EU_k reaches its maximum at k^*.

Note that, at $k = 1/(2b) + 1$,

$$\frac{dEU_k}{dk} = -\frac{1}{4}\left[b + (1-p)^{\frac{1}{2b}+1} \ln(1-p)\right] \equiv G(b).$$

As $b \to 0$, $G(b)$ converges to 0; as $b \to \infty$, it converges to $-\infty$. Now, we show $G(b) < 0$ for all b. This implies that EU_k reaches its maximum at $k^* < 1/(2b) + 1$.

Note that $G(b) < 0$ is equivalent to the following claim.

Claim. The inequality

$$\frac{b}{(1-p)^{\frac{1}{2b}+1}} > -\ln(1-p)$$

holds for all b.

Proof of claim. Note that

$$\begin{aligned}
\frac{d\frac{b}{(1-p)^{\frac{1}{2b}+1}}}{db} &= \frac{(1-p)^{\frac{1}{2b}+1} - b(1-p)^{\frac{1}{2b}+1}\ln(1-p)\cdot\frac{-1}{2b^2}}{\left[(1-p)^{\frac{1}{2b}+1}\right]^2}, \\
&= \frac{1+\frac{1}{2b}\ln(1-p)}{(1-p)^{\frac{1}{2b}+1}}.
\end{aligned}$$

So $b/(1-p)^{\frac{1}{2b}+1}$ reaches a minimum at $b = -\frac{1}{2}\ln(1-p)$. Therefore,

$$\frac{b}{(1-p)^{\frac{1}{2b}+1}} \geq \frac{-\frac{1}{2}\ln(1-p)}{(1-p)^{-\frac{1}{\ln(1-p)}+1}}.$$

Note that

$$\ln(1-p)^{-\frac{1}{\ln(1-p)}+1} = -1+\ln(1-p) < -1.$$

So,

$$\frac{b}{(1-p)^{\frac{1}{2b}+1}} \geq \frac{-\frac{1}{2}\ln(1-p)}{1/e} > -\ln(1-p). \blacksquare$$

The above claim completes the proof the lemma. $\blacksquare$

Proof of Lemma 3. Note that

$$EU_1 = -\left(\bar{b}-b_1\right)^2 - P(NR)Var(\theta|NR).$$

Since information is always disclosed to the group,

$$\begin{aligned}
P(NR) &= (1-p)^k; \\
Var(\theta|NR) &= \frac{1}{4},
\end{aligned}$$

which is independent of bias of the group members. Thus, the group leader's expected payoff decreases with $\bar{b}$. $\blacksquare$

Proof of Lemma 4. Let a group of size k consist of individuals $j_1, j_2, \ldots, j_k$, where $j_1 = 1$. Note that $j_i - i$ is weakly increasing in i. Furthermore, $j_l > l$ for all $l > i$ if $j_i > i$ for some i.

(1) Suppose all players disclose information, by Lemma 3, player 1's payoff is maximized in the consecutive group I_{k+1} (it is straightforward to show that all agents disclose information in group I_{k+1}).

(2) Suppose j_k discloses all her information to the group, but 1 does not disclose 1. This means that

$$j_k b \leq \bar{b} + \frac{1}{4}; \tag{4}$$

$$b \leq \bar{b} - \frac{1}{4}. \tag{5}$$

Now consider the payoff of player 1 in group I_k. Let $\bar{b}'$ be the average bias of the group I_k. Note that (4) implies

$$kb \leq \bar{b}' + \frac{1}{4},$$

as

$$\bar{b} - \bar{b}' = \frac{\sum_{i=1}^{k}(j_i - i)}{k} b \leq (j_k - k)b.$$

By symmetry

$$b \geq \bar{b}' - \frac{1}{4}. \tag{6}$$

This implies in group I_k, all players disclose all information to the group.

So, going from group $\{j_1, \ldots, j_k\}$ to I_k, the only possible changes in actions are from $1 + \bar{b}$ to $1 + \bar{b}'$ in state 1, $0 + \bar{b}$ to $0 + \bar{b}'$ in state 0, and $1/2 + \bar{b}$ to $1 + \bar{b}'$ in state 1. Suppose there are l agents who switch from not disclosing 1 to disclosing it. To compute the difference in payoff for player 1, observe agent 1's expected payoff in group $\{j_1, \ldots, j_k\}$

$$\begin{aligned} EU_1 &= -\left[1 - \frac{1}{2}(1-p)^k - \frac{1}{2}(1-p)^{k-l}\right](b - \bar{b})^2 \\ &\quad -\frac{1}{2}(1-p)^k \left(\left(\frac{1}{2} + \bar{b}\right) - (0 + b)\right)^2 - \frac{1}{2}(1-p)^{k-l}\left(\left(\frac{1}{2} + \bar{b}\right) - (1 + b)\right)^2, \end{aligned}$$

and agent 1's expected payoff in group I_k

$$\begin{aligned} EU_1' &= -\left[1 - \frac{1}{2}(1-p)^k - \frac{1}{2}(1-p)^k\right](b - \bar{b}')^2 \\ &\quad -\frac{1}{2}(1-p)^k \left(\left(\frac{1}{2} + \bar{b}'\right) - (0 + b)\right)^2 - \frac{1}{2}(1-p)^k\left(\left(\frac{1}{2} + \bar{b}'\right) - (1 + b)\right)^2. \end{aligned}$$

So

$$\begin{aligned} EU_1' - EU_1 &= \left[1 - \frac{1}{2}(1-p)^k - \frac{1}{2}(1-p)^{k-l}\right]\left[(b - \bar{b})^2 - (b - \bar{b}')^2\right] \\ &\quad + \left[\frac{1}{2}(1-p)^{k-l} - \frac{1}{2}(1-p)^k\right]\left[\left(\left(\frac{1}{2} + \bar{b}\right) - (1 + b)\right)^2 - (b - \bar{b}')^2\right] \\ &\quad + \frac{1}{2}(1-p)^k \left[\left(\left(\frac{1}{2} + \bar{b}\right) - (0 + b)\right)^2 - \left(\left(\frac{1}{2} + \bar{b}'\right) - (0 + b)\right)^2\right. \\ &\qquad \left. + \left(\left(\frac{1}{2} + \bar{b}\right) - (1 + b)\right)^2 - \left(\left(\frac{1}{2} + \bar{b}'\right) - (1 + b)\right)^2\right], \end{aligned}$$

which simplifies into

$$\begin{aligned} EU_1' - EU_1 &= \left[1 + \frac{1}{2}(1-p)^k - \frac{1}{2}(1-p)^{k-l}\right]\left[(b-\bar{b})^2 - (b-\bar{b}')^2\right] \\ &\quad - \left[\frac{1}{2}(1-p)^{k-l} - \frac{1}{2}(1-p)^k\right]\left[(b-\bar{b}')^2 - \left(\left(\frac{1}{2}+\bar{b}\right) - (1+b)\right)^2\right]. \end{aligned}$$

Note that

$$\begin{aligned} & \left[(b-\bar{b})^2 - (b-\bar{b}')^2\right] - \left[(b-\bar{b}')^2 - \left(\left(\frac{1}{2}+\bar{b}\right) - (1+b)\right)^2\right] \\ = \;& (b-\bar{b})^2 + \left(b - \bar{b} + \frac{1}{2}\right)^2 - 2(b-\bar{b}')^2 \\ \geq \;& 2 \cdot \left(-\frac{1}{4}\right)^2 - 2(b-\bar{b}')^2 \\ \geq \;& 0, \end{aligned}$$

the last step of which is implied by (6) and $b < \bar{b}'$. Observe also

$$1 + \frac{1}{2}(1-p)^k - \frac{1}{2}(1-p)^{k-l} \geq \frac{1}{2}(1-p)^{k-l} - \frac{1}{2}(1-p)^k.$$

Therefore, we conclude

$$EU_1' - EU_1 \quad \geq \quad 0,$$

(3) Suppose j_k does not disclose 0. This means that

$$j_k b \quad \geq \quad \bar{b} + \frac{1}{4}. \tag{7}$$

$$b \quad \geq \quad \bar{b} - \frac{1}{4}. \tag{8}$$

Let $j_i^0 = j_i$, $i = 1, \ldots, k$. Then, consider the following chain of changes to the group members.

Step l: Let $j_{l+1}^l = l+1$, $j_k^l = j_k^{l-1} + j_l^{l-1} - (l+1)$, and $j_i^l = j_i^{l-1}$ for $i = 1, \ldots, k$ and $i \neq l$ or k, where $l = 1, \ldots, k-2$.

In the process, we keep $\bar{b}$ fixed, so no player among $j_1, \ldots, j_{k-1}$ changes her strategy. Neither does the rightmost player, whose bias is increased. So player 1's payoff remains unchanged in this process. But the result of the changes is $\left\{1, 2, \ldots, k-1, j_k^k\right\}$, which gives player 1 a worse payoff than the group $\{1, 2, \ldots, k\}$. ∎

Proof of Proposition 3. Apply the implicit function theorem on (3). Suppose $k^* > 1$ is given by (3):

$$2(k-1)b^2 + t^k \ln t = 0$$

where $t = (1-p) \in (0,1)$.

By the implicit function theorem,

$$\frac{\partial k^*}{\partial b} = -\frac{4(k-1)b}{2b^2 + t^k(\ln t)^2} < 0.$$

as both the denominator and the numerator are strictly positive. To see how k^* depends on t, we have

$$\frac{\partial k^*}{\partial t} = -\frac{t^{k-1}(k \ln t + 1)}{2b^2 + t^k(\ln t)^2},$$

which means that

$$k'(t) \lesseqgtr 0 \text{ according as } k(t) \lesseqgtr -\frac{1}{\ln t} \tag{9}$$

Since $t \in (0,1)$, $\ln t < 0$. Thus, $-\frac{1}{\ln t} > 0$. Note that this implies that if $\ln t < -1$, or $t < 1/e$, then k^* is increasing in t. In other words, when $p > 1 - 1/e$, k^* is decreasing in p.

Claim: There exists some $p_0 \in (0, 1 - 1/e)$ such that k^* is increasing in p for $p < p_0$, decreasing in p for $p > p_0$, and has zero derivative with respect to p at $p = p_0$.

Proof of claim: Suppose $k(t)$ intersects $-\frac{1}{\ln t}$ for some $t = t^*$. We show that it must be the case that $k(t)$ intersects $-\frac{1}{\ln t}$ from above.

Suppose not. Then, for all ϵ small enough, $k(t) < -\frac{1}{\ln t}$ for $t = t^* - \epsilon$ and $k(t) > -\frac{1}{\ln t}$ for $t = t^* + \epsilon$. Notice that $-\frac{1}{\ln t}$ is increasing in t. Then, we must have $k(t^* - \epsilon) < k(t^*) < k(t^* + \epsilon)$. However, since, by assumption, $k(t)$ intersects $-\frac{1}{\ln t}$ from below, $k(t^* - \epsilon) < -\frac{1}{\ln(t^* - \epsilon)}$, which implies that for small enough ϵ, we must have from (9), $k(t^* - \epsilon) > k(t^*)$, which is a contradiction.

Since $k(t)$ must intersect $-\frac{1}{\ln t}$ from above, by continuity of $k(t)$ and $-\frac{1}{\ln t}$, there must be at most one intersection t_0. Note when $t \to 1$, $k(t) \to 1$, so $k(t) < -1/\ln t$, hence $\partial k^*/\partial t < 0$. In other words, when p is relatively small, $\partial k^*/\partial p > 0$. By continuity of $\partial k^*/\partial p$ in p, there exists $p_0 \in (0, 1 - 1/e)$ that satisfies the condition in the claim. ∎

Proof of Proposition 4. Define $q^*(t) = 1 - t^{k^*(t)}$, where $t = 1 - p$.

Now, $\frac{d}{dt}(1 - t^{k^*(t)}) = -t^{k-1}(k' t \ln t + 1)$, where $k' = \frac{dk^*(t)}{dt}$.

Therefore, $\frac{dq^*}{dt} \gtrless 0 \Leftrightarrow$. $k' \gtrless -\frac{1}{t\ln t}$. Using the expression for k' from the proof of Proposition 3, we have

$$\begin{aligned} \frac{dq^*}{dt} &\gtrless 0 \\ &\Leftrightarrow -\frac{t^{k^*-1}(k^*\ln t+1)}{2b^2+t^{k^*}(\ln t)^2} \gtrless -\frac{1}{t\ln t} \\ &\Leftrightarrow t^{k^*}\ln t[(k^*-1)\ln t+1] \gtrless 2b^2 \end{aligned}$$

From the definition of k^* (3), we have $t^{k^*}\ln t = -2(k^*-1)b^2$. Hence, $\frac{dq^*}{dt} \gtrless 0$

$$\begin{aligned} &\Leftrightarrow -2(k^*-1)b^2[(k^*-1)\ln t+1] \gtrless 2b^2 \\ &\Leftrightarrow \frac{(k^*-1)^2}{k^*} \gtrless -\frac{1}{\ln t} \end{aligned}$$

Notice that $\frac{(k^*-1)^2}{k^*}$ is strictly increasing in k^*. This expression is less than k^* and hence less than n. Now, for each $p \in (0,1)$, by Proposition 3, k^* is strictly decreasing in b. Also, k^* can be made arbitrarily large as b changes. Consider b_0 such that $\frac{(k^*(b_0)-1)^2}{k^*(b_0)} = -\frac{1}{\ln t}$, and the result is immediate. ∎

References

Baccara, Mariagiovanna and Leeat Yariv, "Similarity and polarization in groups," *Working Paper*, 2008.

— **and** — , "Homophily in Peer Groups," *American Economic Journal: Microeconomics*, 2013, *5* (3), 69–96.

Banerjee, Suryapratim, Hideo Konishi, and Tayfun Sönmez, "Core in a simple coalition formation game," *Social Choice and Welfare*, 2001, *18* (1), 135–153.

Benhabib, Jess, Matthew O. Jackson, and Alberto Bisin, *Handbook of social economics*, Vol. 1, North-Holland, 2010.

Bogomolnaia, Anna and Matthew O. Jackson, "The Stability of Hedonic Coalition Structures," *Games and Economic Behavior*, 2002, *38* (2), 201 – 230.

Drèze, Jacques H. and Joseph Greenberg, "Hedonic Coalitions: Optimality and Stability," *Econometrica*, May 1980, *48* (4), 987–1003.

Kalmijn, Matthijs, "Intermarriage and homogamy: Causes, patterns, trends," *Annual review of sociology*, 1998, pp. 395–421.

Lazarsfeld, Paul F. and Robert K. Merton, "Friendship as a social process: A substantive and methodological analysis," *Freedom and control in modern society*, 1954, *18*, 18–66.

McPherson, Miller, Lynn Smith-Lovin, and James M. Cook, "Birds of a feather: Homophily in social networks," *Annual review of sociology*, 2001, pp. 415–444.

Wooders, Myrna, Edward Cartwright, and Reinhard Selten, "Behavioral conformity in games with many players," *Games and Economic Behavior*, 2006, *57* (2), 347–360.