Contracting for Innovation under Ambiguity

Swagata Bhattacharjee[1]

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Outsourcing of research is a large and growing trend in knowledge-intensive industries such as biotech, software industries. The smaller research-oriented contractees specialize in handling research specific uncertainties and ambiguities, and the contracts are typically very short term. I model innovation as an ambiguous stochastic process, and assume that the commercial firms and research labs differ in their attitude towards ambiguity. I characterize the sequence of short-term contracts between the ambiguity averse contractor and the ambiguity neutral contractee doing the research, and examine how the special features of the optimal contract facilitate ambiguity sharing. In this model, the commercial firm's ambiguity aversion acts as a commitment device and mitigates the dynamic moral hazard problem. This results in monotonically decreasing investment flow and prevents equilibrium delay. Also, experimentation stops earlier than the policymaker deems optimal, and there is a range of posterior beliefs for which the contracting parties choose to liquidate the project even after being granted a patent. I discuss the policy implications of these results, examining how patent law affects innovation produced in these research alliances.

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[1]Department of Economics, Ashoka University. Email: swagata.bhattacharjee@ashoka.edu.in.
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1. INTRODUCTION

Outsourcing of research is a growing and prevalent trend in knowledge intensive sectors (e.g. Biotechnology, Information Technology, and Software sectors). In these industries, big commercial firms often outsource their research to smaller research oriented firms. These inter-organizational research alliances are generally voluntary agreements between firms involving exchange, sharing or co-development of products, technologies, or services, and play an important role in organizing R&D in the innovation-intensive industries. For example, in Biotechnology sector, 650 new alliances formed in 2006 alone, with related financial commitments of over $90 billion (Edwards, 2007). During 1996-2007, the industry-university strategic partnerships alone resulted in $457.1 billion worth of patented innovations (Sytch and Philipp, 2008). In Pharmaceutical industry, more than 70% of the U.S. companies are involved in research partnerships, and each year on average 25% of the 26bn industry-financed R&D is invested in research alliances (Biopharmaceutical Research Industry Profile, 2013). Information technology sector, accounting for 37% of all strategic research partnerships, registered 254 technology agreements in the year 1996 alone (Reddy, 2001, Hagedoorn et.al., 1992). This paper studies these research partnerships and evaluates them as modes of organizing research.

In the context of innovation, the projects in question are unique in nature. So, sufficient amount of data from very similar situations are generally not available to form a reliable estimate of the true profitability of the project. Thus, it is often difficult to form a unique single-valued probability measure about the profitability. Such situations can be modeled as "Knightian Uncertainty", or, "Ambiguity", using Knight's definition (Knight, 1921):

> "The practical difference between the two categories, risk and uncertainty, is that in the former the distribution of the outcome in a group of instances is known (either through calculation a priori or from statistics of past experience), while in the case of uncertainty this is not true, the reason being in general that it is impossible to form a group of instances, because the situation dealt with is in a high degree unique."

In innovation contexts, then, we can assume that the researching entities know only a partial description of the underlying probability distribution associated with the choices. Here innovation is modelled as a stochastic ambiguous process, with the research labs, specialized in dealing with ambiguity are less ambiguity averse than the commercial firms. The strategic partnerships between the commercial firms and the research firms aim to exploit the gains from this specialization to deal with ambiguity.

Given the importance of research alliances in innovation-based industries as demonstrated above, it is important to examine how these alliances optimally organize R&D. To this end, this paper provides a theoretical model to analyze the strategic partnerships carrying out innovation in ambiguous environment. The main focus is on the dynamic contracts that govern these partnerships.

The questions that we can address in the present framework are: what is the optimal sequence of short term contracts governing innovation in these strategic partnerships? How does the optimal investment in the project evolve over time? When does the research alliance stop experimenting? Assuming that the Policymaker is a risk and ambiguity neutral entity and cares only for the payoffs the project generates, we analyze how the Policymaker sets the Patent Law. Then, the natural question is: how does the optimal contractual outcome in the strategic partnerships compare to the Policymaker's desired optimal outcome? Also, is it possible to re-design the patent laws so as to implement the Policymaker's desired optima?

We consider a dynamic principal-agent framework to address these questions. In particular, we examine a sequence of short term contracts where the contractees differ in their attitude towards ambiguity.

We characterize the optimal sequence of short term contracts conducting the innovation, and show how the contractual terms facilitate ambiguity sharing. However, the contractual optimal outcome diverges from the desired outcome by the Policymaker: the strategic alliance stops experimenting earlier than the Policymaker deems optimal, sometimes liquidates the project even after being granted a patent, also invests less in the project. We can show that it is never possible to implement the Policymaker's optima by restructuring the patent law.

The paper is organized as follows. The remainder of this section (subsection 1.1) discusses some examples of strategic partnerships. Section 2 describes the existing body of literature related to the questions addressed in this paper. Section 3 develops the model and analyzes the main results of the paper. Section 4 provides a comparison between the contractual outcome and the Policymaker's optima and discusses the policy implications of the results of this paper. In section 5 I consider some generalizations and robustness checks of the model. Section 6 reflects on the general implications of the results. The last section summarizes the findings of this paper and concludes.

1.1. Motivating Examples

The contracts within the research partnerships take a special form: they are generally of short duration, designed to overcome the problems that may arise in inter-organizational collaborations and use a mix of explicit (legally enforceable) and implicit (legally unenforceable, e.g., allocation of decision rights, property rights, etc.) terms

(Gilson et al, 2003). In this subsection, we will study a contracts governing a research partnership. From this case study, we make note of the contractual features, so that in the theoretical model, we can retain these properties and show how they help organizing research in this context.

Example 1: Warner- Lambert-Ligand agreement

Let us examine the "Warner- Lambert-Ligand agreement" (September 1, 1999) (henceforth W-L): a research, development, and license agreement between Warner-Lambert, a large pharmaceutical company, and Ligand Pharmaceuticals, a much smaller biotech company.

The W-L partnership was engaged in directed research to discover and design small-molecule compounds that act through the estrogen receptors, to develop those compounds into pharmaceutical products, and to take those products through the FDA approval process and through commercialization (Warner-Lambert & Ligand Agreement, 1999). They started off with almost 10,000 compounds, out of which only 250 compounds reached the pre-clinical stage[2]. During the research stage, Ligand engaged in directed research, with Warner-Lambert providing the bulk of the funding[3]. The research stage consisted of three periods with duration of fifteen months to three years, after each of the periods Warner-Lambert had the option of unilaterally abandoning the project with little or no direct cost.

Once a successful compound was identified, the project moved from the research to the development stage, and regulatory and market experience became more important. The cost of the project, all of which will be borne by Warner-Lambert, also increased exponentially. As a result, both responsibility and decision making shifted to Warner-Lambert, who had the option to develop the project[4].

[2] A brief description of drug-development process: the initial screening of compounds and pre-clinical work takes, on average, three to six years. During that period, the number of compounds under consideration is winnowed from 5,000-10,000 down to a quite small number through scientific and animal testing. At that point, an application for an Investigational New Drug is filed with the FDA. If the FDA approves, the drug can move to clinical testing on humans. Clinical testing takes another six to seven years. If the drug surmounts these hurdles, the sponsoring company submits a New Drug Application (NDA) with supporting documentation. FDA review of the NDA can take another six months to two years. If the FDA approves, the drug can be brought to market. Estimates are that out of 5,000 to 10,000 compounds, only 250 enter pre-clinical testing, and only about twenty percent of drugs that begin phase one testing are ultimately approved by the FDA. Only upon approval does the pharmaceutical company discover whether the drug will be successful commercially.

[3] If the project ultimately succeeds, only a small fraction of costs would be associated with the research phase. The major costs of bringing a drug to market are incurred in the later stages, in which the manufacturer must prove efficacy and safety through clinical studies in the FDA approval process.

[4] In the contract, Warner-Lambert promises to "use diligent efforts to pursue the Clinical Development and commercialization of each Collaboration Lead Compound at its own expense"; however, it "shall have the sole discretion to determine (a) which Products to develop or market or to continue to develop or market, (b) which Products to seek regulatory approval for, and (c) when and where and how and on what terms and conditions, to market such Products in the Territory."

The gap between contract formation and the appearance of a marketable drug was more than a decade. So, Ligand's compensation was carefully structured. First, it was paid for some fraction (perhaps all) of the resources assigned to the task. Second, the agreement established a number of specific *milestones*, and, upon reaching each milestone, Ligand received an additional payment. Finally, after the research produced marketable products, Ligand received royalty payments on sales. However, if Warner-Lambert chose to abort the project at any time, they retained the property rights.

This example illustrates the unique features of a typical contract governing a strategic alliance that operates in an innovation-intensive industry. Our model retains these features as well.

Modelling the Dynamic Contracts:

- *Short Term Contracting:* In W-L agreement, each contracting phase lasted for fifteen months up to three years, whereas the partnership lasted for more than a decade. Likewise, many of the collaborative R&D ventures are governed by short term contracts, with the contracting terms being renegotiated after every contracting phase. This paper studies the optimal sequence of short term contracts with the contractees having no commitment power.

- *Rich forms of collaborating:* The W-L agreement, containing rich braiding of explicit and implicit terms, shows that often the contracts governing innovation process are quite complex in structure. On one hand there is an elaborate description of the payments under various possible contingencies (e.g., the milestone bonuses, the royalty rate), which are legally enforceable. On the other hand, the contract specifies the control rights and property rights, which gives unilateral decision power to one of the contracting parties. To mimic this interesting blend of explicit and implicit contracting terms, the present model assumes a contract structure containing both the state contingent payment structure and the movement of unilateral decision power.

- *Learning about the Project's Prospects:* The project started off with almost 10000 possible candidates for the molecule to be developed into a commercial drug. Only through a series of experiments the true potential of the project is learned. At each contracting phase, Ligand conducts a series of experiments on a particular subset of molecules, at the end of which a report summarizes the results: if there is a molecule fit to be taken to the clinical trials. The present model considers innovation as a learning process, where at the end of each period, a binary signal is publicly realized which contains information about the true state of the project.

- *Moral Hazard:* In the R&D conducted by W-L partnership, the public signal depends on the resources devoted to the project. For example, if Ligand does not carry out the experiments using the expensive laboratory testing procedure, and instead, to save time and money, uses some cheaper and unreliable methods of testing, then it is unlikely that they will find a molecule suitable for clinical trial among the subset of molecules to be tested at that period. This possible diversion of resources to cross-subsidize other projects or used for personal gain underlines the existing moral hazard concern in this context. Since Warner-Lambert cannot perfectly monitor Ligand's activity, such cross-subsidization possibility gives rise to potential moral hazard problem in the contractual relationship.

  In the dynamic relationship between the two firms, the moral hazard problem is more severe. Apart from the one-time gain by diverting resources, the researching party can also appropriate a dynamic gain from diversion. Once Ligand diverts resources, the test results turn out to be negative. Observing this public signal, Warner-Lambert's perception about the project's profitability changes accordingly. However, Ligand, who privately observed its own action, disregards this signal as it contains no information. Thus, following a diversion of resources, the learning paths for the two firms diverge. Warner-Lambert, who could not observe the diversion, updates its beliefs about the project's prospects differently than Ligand. Hence Ligand evaluates the next period's contracting terms using a different, and more optimistic, belief. This gives rise to a further incentive to cheat and is referred to in the literature as the "dynamic moral hazard" problem. In this model we consider dynamic contracting environment, so dynamic moral hazard problem arises here.

- *Innovation as an Ambiguous Process:* Finally, we discuss why the innovation activity carried out in W-L agreement can be considered an ambiguous, rather than risky process.

  In the strategic partnership between Warner-Lambert and Ligand, the research could have ended in one of the three possible ways:

  (a) They could have found a molecule which passes all the clinical trials and is found fit to be developed into a drug. This can be modeled as the case when the true state (or, profitability) of the project is "*Good*."

  (b) They could have failed to find a suitable molecule even after testing all the candidate molecules. This case can be modeled as the true state being "*Bad*."

  (c) Apart from these two states, the research could have ended in finding a molecule which is capable to work through the estrogen receptors, but, given the state of the present pharmaceutical technology, can not be developed into a drug. If the research finds such a molecule, it is not presently known if in the future the pharmaceutical technology will ever improve and the molecule can be developed into a drug. So, in this case, even after conducting the decade-long research, we stumble upon an "Open question". We model this case as a new epistemic state and call it "*Unknowable*" or "*Amalgamated*", because if the research ends up here, the true profitability of the project is simply not known.

We follow the ambiguity framework developed in Dumav and Stinchcombe (2013), which shows that this new state captures the idea of ambiguity. It can be considered as an alternative interpretation of the multiple prior model. Appendix B contains the preliminaries of this framework.

In the present model, the binary signal observed at each contracting term reveals information about the true state, which can be "*Good,*" "*Bad,*" or, "*Unknowable.*" For example, if at any period, Ligand finds that a molecule among the ones being tested is suitable for conducting clinical trial, that may indicate that it is more likely that the true state is "*Good*" or "*Unknowable,*" rather than "*Bad.*" We also assume that Ligand, being a research firm, prefers this "*Unknowable*" state more than Warner-Lambert. For Ligand, this presents an opportunity to work on developing new pharmaceutical technology which might earn them revenue in future, but for Warner-Lambert, reaching the "*Unknowable*" state does not generate any immediate payoff.

Let us look at another example to illustrate the interpretation of ambiguity we will deal with in this paper.

Example 2: Cancer Genome Anatomy Project (CGAP)

Cancer Genome Sequencing refers to the laboratory method of characterization and identification of genetic sequencing of cancer cells. Funded in 1997, the Cancer Genome Anatomy Project (CGAP) published their first Cancer Genome Sequencing report in 2003, which enables identification and characterization of all the genetic and epigenetic mutational changes that happen in the process of tumorigenesis. Before the CGS, such an exhaustive list of all possible variants of cancer cells was not available, thus different variants and subtypes of cancer were not identified (Cancer Genome Sequencing Report, 2003).

Now, let us consider a Biotechnology research venture aiming to find a medicine to treat Acute Myeloid Leukemia (AML), a particular type of cancer, before this CGS report was made available . The CGS identified several new subtypes of variants of carcinogenic mutational changes associated with AML. Before CGS, then, the research could have ended in one of the three states:

(a) The research venture could have found a medicine which can treat one of the already identified subtype of carcinogenic cells, which can be considered as the case when the true state (or, profitability) of the project is "*Good.*"

(b) The project could have ended in discovering that the medicine is not even biologically active on the epigenetic mutational changes. This case can be identified with the true state being "*Bad.*"

(c) The research could have found a medicine which is biologically active, but can not treat any identified variant of cancer. However, it could have been possible that there are epigenetic changes in cancer cells which are not yet identified (before CGS), and the medicine might be useful to treat those not-yet-identified variants. This state can be considered as the "*Unknowable*" or "*Amalgamated*" state, where the true profitability of the research venture is yet unknown[5].

Thus, from the two examples, it can be seen that in the innovation-intensive sectors, we can consider a new epistemic state: "Unknowable," which captures the idea that the true probability distribution associated with the choices may not be completely known, so innovation can be considered to be an ambiguous process. This paper provides a model of how these research alliances operate under ambiguity and examines the contractual structures that govern these inter-organizational research partnerships.

Specifically, we consider innovation to be an ambiguous process where investing in research every period generates informative signals which enable the researching parties to learn about the true nature of the project. This process is organized in a research alliance through a sequence of short term contracts with both explicit and implicit contracting terms, which take care of the existing moral hazard problem. In this set up, we characterize the optimal contract, analyze its properties, and show how this research alliances fail to implement the Policymaker's desired optima.

2. RELATED LITERATURE

This paper is primarily related to the literature discussing *Optimal Contracts for Innovation.* It is most closely related to the seminal work by Bergemann and Hege (Bergemann and Hege, 1998; Bergemann and Hege, 2005), which characterize the optimal contract for experimentation under risk. These two papers model innovation as a risky stochastic optimal stopping time problem, where an entrepreneur and a capitalist invest funds every period to learn about the project's true profitability and if the project

[5]Indeed, much later, after the CGS report was available, targeted drugs like vemurafenib (ZELBORAF®) were discovered (approved by the Food and Drug Administration (FDA) in 2011) for the treatment of some specific mutation in the BRAF gene as detected by an FDA-approved test using CGS.

succeeds, the game ends immediately. In this framework, the authors document the potential dynamic moral hazard problem and how it makes the funding conditions more stringent in the earlier rounds. In their setting, they find the possibility of in-equilibrium delay of funding (in finite horizon) and in the infinite horizon, they find that the investment volume may increase over time. Hörner and Samuelson (2013) examine a similar framework of experimentation and characterize all possible equilibria.

There are two significant differences between these papers and ours. Firstly, here we consider innovation as an ambiguous process, rather than a risky one. Thus, the central problem of this paper is the characterization of the optimal contract in presence of ambiguity. We show that the introduction of ambiguity and the different attitudes towards ambiguity among the contractees alleviate the dynamic moral hazard problem, preventing in-equilibrium delay in funding in the finite horizon case, and in the infinite horizon this leads to a monotonically decreasing level of investment. Also, in the current paper we model innovation as a two stage game, where at the first stage, in each period the firms experiment to observe an informative binary signal, and depending on the signal realization, may enter the development stage, where the true quality of the project is finally revealed. This modelling framework with non-conclusive signals gives rise to a positive option value of waiting and changes the optimal contract structure. It illustrates the role of patent laws, which enables us to analyze the role of government policies in innovation.

Bonatti and Horner (2009) and Campbell et al. (2013) study experimentation in teams with unobservable actions and they also find the possibility of delay. In a two period model with a the principal with commitment power, Manso, 2011, Ederer and Manso (2013) , show that the contracts that foster experimentation greatly differ from standard pay-for-performance contracts. Halac, Liu, and Kartik (2013) examine long term contracting for experimentation with moral hazard and adverse selection, and show that the optimal contract implements low effort from the low ability agent. 1 develops a model in which the principal and the agent disagree about the resolution of uncertainties and shows that this disagreement risk sharing leads to an endogenous regime shift. He, Wei and Yu (2014) introduces uncertainty in the seminal work by Holmstrom and Milgrom (Holmstrom and Milgrom, 1991), shows that the optimal contract displays a front-loading pattern. Optimal contracting for experimentation under moral hazard or adverse selection concerns has been studied in a growing body of literature (Poblete and Spulber, 2014. ; Gomes, Gottlieb and Maestri, 2013 ; Gerardi and Maestri, 2012).

In contrast, this paper studies innovation under ambiguity and in an infinite horizon stopping time problem, characterizes the dynamic contract organizing the research activities.

This paper is part of the literature examining the impact of *ambiguity in the con-*

tracting environment. Similar to this paper, the paper by Besanko, Tong and Wu, 2012 considers delegated experimentation under ambiguity. However, while their paper examines the adverse selection problem in the experimentation context and using maximum likelihood updating, shows the optimality of a pooling contract in a perfect objectivist equilibrium, my paper analyzes the moral hazard problem.

In a static context, Lopomo, Rigotti and Shannon, 2011 examine moral hazard problem under ambiguity and show how simple contract structures turn out to be optimal. In a static general equilibrium framework, Amarante, Ghossoub and Phelps (2015) discuss the effects of ambiguity and heterogeneous belief among the decision makers and the entrepreneur. Rigotti, Ryan and Vaithianathan (2011) characterize the diffusion profile of a new technology under ambiguity. Byun (2014) characterizes the optimal incentive scheme for innovation in a static game. In contrast, we analyze ambiguity in a dynamic environment and using dynamically consistent Bayesian updating, we show how the optimal contract structure facilitates ambiguity sharing.

There is also a growing strand of literature that analyzes *dynamic contracts* and mechanism design problem and illustrates the importance of dynamic agency costs. This paper, discussing the dynamic agency cost under ambiguity, is related to that strand of literature as well. Bergemann and Pavan (2015) contain a detailed survey of this literature. The importance of dynamic agency cost has been well documented in literature using both the continuous time framework (DeMarzo and Sannikov, 2011; Sannikov, 2008; DeMarzo and Sannikov, 2006; Biais et al., 2010; Fuchs, 2006) and discrete time models (Bhaskar, 2014; Bhaskar, 2012; Kwon, 2011). In this paper, we analyze the dynamic agency cost arising from the diversion of resources by the researcher and show that the presence of ambiguity and difference in attitude towards ambiguity among the contracting parties alleviate the dynamic moral hazard problem.

Following Gilboa and Schmeidler's seminal work on ambiguity, (Gilboa and Schmeidler, 1989), multiple prior models of ambiguity have been applied to various dynamic decision making contexts. However, with multiple priors, prior-by-prior updating of belief using Bayes rule usually leads to dynamic inconsistency. There are different approaches to modelling ambiguity averse preferences in dynamic setting. Some papers take the approach that deals with recursive extensions (e.g., Epstein and Schneider, 2003, Maccheroni, Marinacci, and Rustichini, 2006, Klibanoff, Marinacci and Mukherji, 2009), others posit dynamic inconsistency and adopt assumptions, such as backward induction or naive ignorance of the inconsistency, to pin down behavior (e.g. Siniscalchi, 2008) , yet another approach uses non-consequentialist updating rules (Machina and Schmeidler, 1992)[6]. In this paper, we use the ambiguity framework developed in Dumav and Stinchcombe (2013), which characterizes a vNM approach to ambiguity and

[6]For a more complete survey, refer to Etner, Jeleva and Tallon, 2012.

uses Bayes rule to obtain dynamically consistent updating of beliefs. Thus, this paper fits in the literature of *decision making with ambiguity* in a dynamic framework.

Apart from these strands of literature, there is a vast body of literature in Economics, Management, Law and Organization design that discusses the *strategic partnerships*, their governance structure, and the role of government policies in innovation. Gilson, Sabel and Scott (2011) analyzes the specific features of strategic partnerships and underlines the importance of Knightian uncertainty in innovation context. Baker et al, 2008 show how all possible governance structures may emerge in such contexts. Van de Ven (1986) discusses how the management of innovation can overcome the problems associated with the innovation process. Lerner and Malmendier (2010) show how incomplete contracts can be used as optimal contractual design to solve the problem of moral hazard in Biotechnology research partnerships. Hagedoorn, Link, and Vonortas (2000) underline the importance of research partnerships and suggest that the patent granting authority should be aware of the benefits and shortcomings of these partnerships in conducting R&D. Papers by Hagedoorn, 2002, Hagedoorn et.al., 1992, Reddy, 2001, Sytch and Philipp, 2008, Biotechnology Industry Report (2009) discuss various issues of research partnerships in different industries. This paper, analyzing the research alliances from a theoretical point of view and showing how the observed contract structure optimally organizes innovation, fits in this strand of literature as well.

3. MODEL AND ANALYSIS

3.1. General Set-up

States: The innovation activity is centered around a project, success of which depends on the true state or true profitability of the project: $\theta \in \Theta$. The true state is not known; moreover, it is not possible to form a single probabilistic assessment about it. In a multiple prior setting,

$$\begin{aligned} \Theta &= \{Good, Bad\} \\ \Pr(\theta &= Good) = [r_0, s_0] \ \ ; 0 \leq r_0 < s_0 \leq 1. \end{aligned}$$

Using the framework of ambiguity developed in Dumav and Stinchcombe (2013) (described in greater detail in Appendix B of this paper), we observe that the interval $[r_0, s_0]$ has a unique representation as a convex combination of extreme sets given by $\Theta' = \{Good, Bad, Unknowable\}$, where the new epistemic state "*Unknowable*" is motivated in subsection 1.1.

Thus, each $[r_0, s_0]$ is represented as:

$$[r_0, s_0] = r_0[1,1] + (1 - s_0)[0,0] + (s_0 - r_0)[0,1]$$

The state *Unknowable* is represented as $[0,1]$, the state at which the decision maker knows only that the probability of $\theta = Good$ is someplace between 0 and 1.

Thus, in this framework, we can alternatively represent this set-valued prior by a three state expected utility model, where the true state of the project lies in Θ' :

$$\begin{aligned}
\Theta' &= \{Good, Bad, Unknowable\} \\
\Pr(\theta &= Good) = r_0 \\
\Pr(\theta &= Bad) = 1 - s_0 \\
\Pr(\theta &= Unknowable) = s_0 - r_0 \\
0 &\leq r_0 < s_0 \leq 1.
\end{aligned}$$

That is, with probability r_0, at the end it will be revealed that the project is profitable, with probability $1 - s_0$ it will be revealed that the project is not profitable, but with probability $s_0 - r_0$, the true profitability of the project will turn out to be "*Unknowable*", or, Not Yet Known, depending on the current state of technology and knowledge. Notice that $s_0 - r_0$ captures the idea that the decision maker knows only a partial description about the underlying distribution; if $r_0 = s_0$ then we are back to the "risky" context.

If the payoff for $\theta = Good$ is u_G, for $\theta = Bad$ is $u_B < u_G$, then the payoff associated with the new state $\theta = Unknowable$ is computed as:

$$u(\theta = Unknowable) = \frac{1}{2}(u_G + u_B) - \frac{v}{2}(u_G - u_B);$$

where the ambiguity aversion parameter v captures the attitude towards ambiguity. $v > 0$ refers to the decision maker being ambiguity averse. The higher v is, the more the decision maker dislikes the state $\theta = Unknowable$, hence can be considered as more ambiguity averse. Here, I assume $v \in (0,1)$.

Innovation Time line: To finish the project, one must go through two distinct stages:

1. *Experimentation stage:* At this stage of innovation, at every period t, some fund $K_t \in [0, \overline{K}] \subset [0,1)$ is invested in the project and at the end of the period an informative signal S_t is realized. The signal is binary: $S_t \in \{s_H, s_L\}$, with the distribution to be specified below. Only if the signal is "high enough", i.e., it surpasses the quality threshold determined by the patent-granting authority, the project is allowed to move to the next stage: the Development stage. This thresh-

old can be interpreted as the Patent Law or the FDA approval criterion. If the signal fails to clear the threshold, the researching authorities may continue experimenting (move to period $t+1$ in the experimentation stage), or abandon the project forever (gross return$=0$).

2. *Development stage:* If the signal is high enough to clear the patenting threshold, the project is enters the Development stage. Here, the researcher(s) can choose to develop the project by making a fixed investment of the amount $I > 0$, after which the true state will be revealed. If the true state is $\theta = Good$, the project yields a return of $R > I$, otherwise the gross return is 0. However, instead of investing I to reveal the true state, the researching authority may want to liquidate the project as well, collecting a liquidation value of $L > 0$.

The general time line is represented in Figure 1.

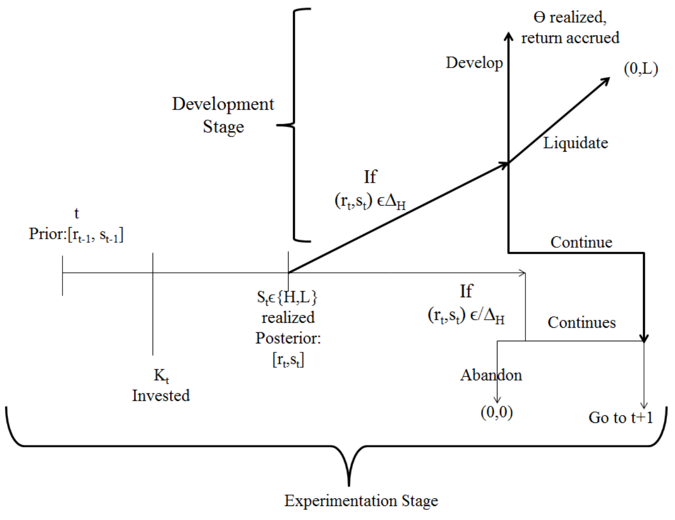

FIG. 1 Innovation Process

Signal Structure: The signal structure assumed throughout this paper is given below. At any period t, the signal is conditionally independent and jointly distributed with the state $\theta \in \Theta'$.

At any period t, investment flow increases signal precision.

$$
\begin{aligned}
\Pr(S_t &= s_H|\theta = G) = \lambda_G(K_t) \\
\Pr(S_t &= s_H|\theta = U) = \lambda_U(K_t) \\
\Pr(S_t &= s_H|\theta = B) = \lambda_B(K_t) \qquad (1)
\end{aligned}
$$

The parametric restrictions we impose on the signal structure are:

***Assumption*1 :**

$$1 > \lambda_G(K_t) > \lambda_U(K_t) > \lambda_B(K_t) \geq 0 \quad \forall K_t \in [0, \overline{K}]$$

***Assumption*2 :**

$$\lambda'_G(K_t) > \lambda'_U(K_t) > \lambda'_B(K_t) \quad \forall K_t \in [0, \overline{K}]$$

***Assumption* 3 :**

$$\frac{\lambda_G(K_t)}{1-\lambda_G(K_t)} > \frac{\lambda_U(K_t)}{1-\lambda_U(K_t)} > \frac{\lambda_B(K_t)}{1-\lambda_B(K_t)} \quad \forall K_t \in [0, \overline{K}] \qquad \text{(MLRP)}$$

While the first assumption ensures that $\lambda_\theta(K_t)$ is a valid probability measure defined on Θ', the second assumption states that higher investment increases the signal precision. The third assumption is called the Monotone Likelihood Ratio Property and is defined as follows:

Definition 1 (Monotone Likelihood Ratio Property). The signal structure satisfies Monotone Likelihood Ratio Property (MLRP) if the probability of observing $S_t =$

s_H relative to that of observing $S_t = s_L$ is increasing in the true state, when the states are ordered *Good* $\succ$ *Unknowable* $\succ$ *Bad*. Mathematically, it is captured by equation MLRP.

Now, the conditional distribution associated with this binary signal is characterized below:

Signal Structure

S_t	s_H	s_L	
$\theta = G\ (1,1)$	$r_{t-1}\lambda_G(K_t)$	$r_{t-1}(1-\lambda_G(K_t))$	r_{t-1}
$\theta = Unknowable\ (0,1)$	$(s_{t-1}-r_{t-1})\lambda_U(K_t)$	$(s_{t-1}-r_{t-1})(1-\lambda_U(K_t))$	$s_{t-1}-r_{t-1}$
$\theta = B(0,0)$	$(1-s_{t-1})\lambda_B(K_t)$	$(1-s_{t-1})(1-\lambda_B(K_t))$	$1-s_{t-1}$
	μ_t	$1-\mu_t$	1

So that,

$$\begin{aligned}\Pr(S_t &= s_H) = \mu_t(K_t) \\ &= r_{t-1}\lambda_G(K_t) + (s_{t-1} - r_{t-1})\lambda_U(K_t) + (1 - s_{t-1})\lambda_B(K_t)\end{aligned}$$

After observing the binary signal, at the end of each period, the beliefs are updated using Bayes Law.

After observing a high signal $S_t = s_H$, the updated posterior puts weight on the three states as follows:

$$\begin{aligned}\Pr(\theta &= G|S_t = s_H) = \frac{r_{t-1}\lambda_G}{\mu_t} = r_t^H \\ \Pr(\theta &= B|S_t = s_H) = \frac{(1 - s_{t-1})\lambda_B}{\mu_t} = 1 - s_t^H \\ \Pr(\theta &= U|S_t = s_H) = \frac{(s_{t-1} - r_{t-1})\lambda_B}{\mu_t} = s_t^H - r_t^H\end{aligned}$$

Thus, in the multiple prior interpretation, the set valued posterior after observing a high signal $S_t = s_H$ is:

$$\Pr(\theta = G)|_{S_t = s_H} = [r_t^H, s_t^H] = \left[\frac{r_{t-1}\lambda_G(K_t)}{\mu_t}, 1 - \frac{(1 - s_{t-1})\lambda_B(K_t)}{\mu_t}\right]$$

Similarly, after $S_t = s_L$, posterior becomes:

$$\Pr(\theta = G)|_{S_t = s_L} = [r_t^L, s_t^L] = \left[\frac{r_{t-1}(1 - \lambda_G(K_t))}{1 - \mu_t}, 1 - \frac{(1 - s_{t-1})(1 - \lambda_B(K_t))}{1 - \mu_t}\right]$$

To save on notation, let us define the average of the posterior belief as the posterior mean:

$$\textit{posterior mean} = \frac{r_t + s_t}{2} = p_t$$

and the average spread of the posterior belief as the posterior ambiguity:

$$\textit{posterior ambiguity} = \frac{s_t - r_t}{2} = q_t$$

Note that, by $MLRP$, after observing $S_t = s_H$, posterior mean p_t increases and posterior ambiguity q_t decreases; and after $S_t = s_L$, p_t decreases and q_t increases.

Intuitively, the signals can be thought of as random draws from a Bernoulli distribution:

$$
\begin{aligned}
S_t &\sim Bernoulli(\lambda_G(K_t)) \;\text{ if } \theta = Good \\
S_t &\sim Bernoulli(\lambda_U(K_t)) \;\text{ if } \theta = Unknowable \\
S_t &\sim Bernoulli(\lambda_B(K_t)) \;\text{ if } \theta = Bad
\end{aligned}
$$

Then, after observing each binary signal, the decision maker updates his belief about the true parameter. The following graph (Figure 2) depicting 30 simulations of signals for each of the three true states (with parameters: $\lambda_G = 0.7, \lambda_U = 0.5, \lambda_B = 0.1, \overline{K} = 1$) shows how repeated sampling for a long time eventually reveals the state, as the posterior converges to one of the states with almost certainty. However, due to the positive cost of experimenting, it is not optimal to experiment forever. Then the problem for the decision maker becomes an optimal stopping problem: the decision maker has to follow an optimal rule about when to stop experimenting, depending on the observed sequence of signals.

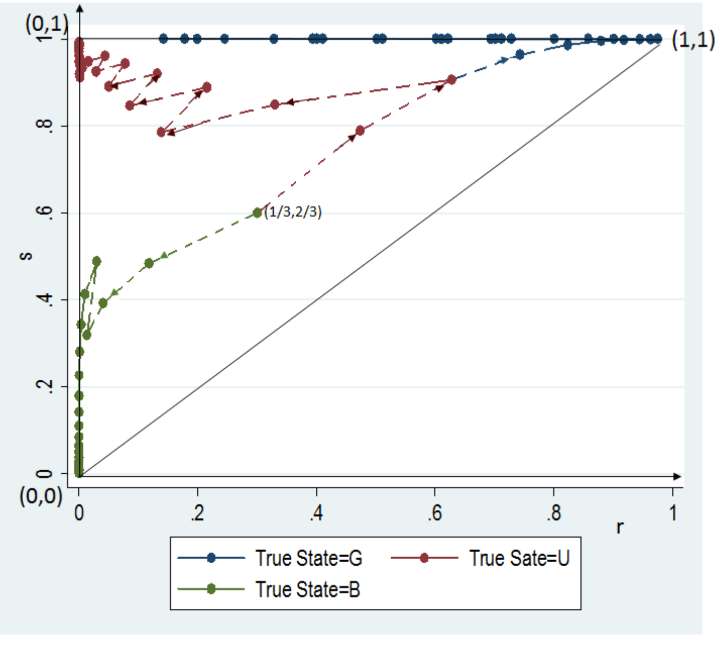

FIG. 2 Evolution of Beliefs for 30 Consecutive Signals

In the main body of this paper, we will assume linear signal structure, i.e.,

$$
\begin{aligned}
\Pr(S_t &= s_H|\theta = G) = \lambda_G(K_t) = \lambda_G K_t \\
\Pr(S_t &= s_H|\theta = U) = \lambda_U(K_t) = \lambda_U K_t \\
\Pr(S_t &= s_H|\theta = B) = \lambda_B(K_t) = \lambda_B K_t
\end{aligned} \tag{2}
$$

with

$$
\begin{aligned}
\Pr(S_t &= s_H) = \mu_t = K_t\lambda_t \\
&= K_t \underbrace{[r_{t-1}\lambda_G + (1-s_{t-1})\lambda_B + (s_{t-1}-r_{t-1})\lambda_U]}_{\lambda_t}
\end{aligned} \tag{3}
$$

In section 6, we will discuss the case with general non-linear signal structure and show that qualitatively the results hold in that case.

The next subsection discusses how the patent law is set, depending on the signal structure described above.

3.2. Patent Law

Assume that the patent law is set by the Policymaker (the patent-granting authority, or the regulatory agency), who is a risk and ambiguity neutral entity. The Policymaker values the "open questions", or the "*Unknowable*" state more than the commercial firms do, hence is less ambiguity averse (for simplification, I assume ambiguity neutrality). Assume that the Policymaker cares only for the payoffs generated from the project[7]. The Policymaker sets the patent law to reflect his own desired outcome: the "Policymaker's Optima" , or, the "*Risk and Ambiguity Neutral Optima* (*RAN Optima*)" .

After observing the signal at the end of each period, the Policymaker chooses whether to develop ($a_t^{RAN} = Dev$), or to liquidate the project ($a_t^{RAN} = Liq$), or to continue experimenting further ($a_t^{RAN} = Continue$).

The payoffs associated with the actions are:

	a_t^{RAN} =Dev	a_t^{RAN} =Liq
$\theta = Good$	$R - I$	L
$\theta = Bad$	$-I$	L
$\theta = Unknowable$	$\frac{1}{2}R - I$	L

[7]It might be argued that it is more natural to assume that the Policymaker would internalize the positive externalities the project might generate as well. However, to make the comparison between the contractual outcome and the outcome desired by the Policymaker, here I do not consider the externalities. In Section 5, I discuss how including the externalities make the contractual outcome diverge further from the risk and ambiguity neutral benchmark outcome.

Payoffs

Thus, after observing a signal at period t, with the updated posterior $[r_t, s_t]$, the expected payoff to the Policymaker from choosing action $a_t^{RAN} = Dev$ is:

$$\begin{aligned} Eu_t^{RAN}(a_t^{RAN} &= Dev, (r_t, s_t)) \\ &= r_t(R-I) + (1-s_t)(-I) + (s_t - r_t)(\frac{1}{2}R - I) \\ &= \frac{r_t + s_t}{2}R - I = p_t R - I \end{aligned}$$

The expected payoff from choosing $a_t^{RAN} = Liq$ is L.

The Policymaker's optimal stopping rule identifies the regions of posterior beliefs where it is optimal to stop experimenting and develop the project: Δ_H, and the region where it is optimal to stop experimenting and liquidate the project: Δ_S. Then, at the beginning of each period, the problem can be formulated recursively using the optimality equation or Bellman equation:

$$\begin{aligned} V_t^{RAN}(r_{t-1}, s_{t-1}) &= \max_{\Delta_H, \Delta_S, K_t^{RAN}} \Pr_t((r_t, s_t) \in \Delta_H)(p_t R - I) + \Pr_t((r_t, s_t) \in \Delta_S) L - K_t \\ &\quad + \delta E_t V_{t+1}^{RAN}(r_t, s_t) \qquad (RAN\ ) \end{aligned}$$

where the regions Δ_H, Δ_S are defined as follows:

$$\begin{aligned} \Delta_H &= \{(r_t, s_t) \in \mathbb{K}_{\Delta[0,1]} |\ a_t^{RAN} = Dev\} \\ \Delta_S &= \{(r_t, s_t) \in \mathbb{K}_{\Delta[0,1]} |\ a_t^{RAN} = Liq\} \end{aligned}$$

LEMMA 1. *There exists a unique solution to the RAN optimization problem.*

Proof. The proof involves showing that the optimality equation satisfies the Blackwell sufficiency conditions, hence is a contraction. Then a direct use of the Contraction Mapping Theorem gives the existence and uniqueness of the result. Details in Appendix A. ∎

Now, let us examine the optimal stopping rule. After observing the signal, based on the updated posterior $[r_t, s_t]$, the expected payoff is:

$$\max\{p_t R - I, L, \delta E_t V_{t+1}^{RAN}(r_t, s_t)\}$$

In order to solve for the RAN optima, let us define:

$$\begin{aligned} F_j(r_t, s_t) &= \text{based on } [r_t, s_t], \text{ the maximum expected value if experimentation stops at } j \\ &= E_t\left[\delta^{j-t}\max\{p_j R - I, L\} - \sum_{s=t}^{j-1}\delta^{s-t}K_s\right] \end{aligned} \tag{4}$$

Define:

$$A_t = \{F_t > (F_{t+1}|(r_t, s_t)\} \quad t = 1, 2, ..$$

we show that A_t s form a monotone sequence.

LEMMA 2. *If $F_t(r_t, s_t) \geq F_{t+1}(r_t, s_t)$, then $F_{t+1}(r_t, s_t) \geq F_{t+2}(r_t, s_t)$, i.e., $A_1 \subset A_2 \subset .. \cup_1^\infty A_n$, hence the region where stopping immediately is optimal forms a monotone sequence.*

Proof. In Appendix A. ∎

Then, the "One-stop ahead" rule is optimal, i.e., if stopping the experimentation process today is better than continuing experimenting for exactly one more period, then it is always optimal to stop today (Chow, Robbins, Siegmund (1971)). Using that, we obtain the optimal stopping rule, given in the next proposition.

PROPOSITION 1. *The RAN optima, or, the "Policymaker's Optima" is given by the stopping rule*

$$a_t^{RAN}(r_t, s_t) = \begin{cases} Dev & if\ (r_t, s_t) \in \Delta_H \\ Liq & if\ (r_t, s_t) \in \Delta_S \\ Continue & otherwise \end{cases}$$

where the optimal stopping thresholds are:

$$\begin{aligned} \Delta_H : &= \{(r_t, s_t)|\beta_{H1}r_t + \beta_{H2}s_t \geq \beta_{H3}\}; \\ \Delta_S : &= \{(r_t, s_t)|\beta_{S1}r_t + \beta_{S2}s_t < \beta_{S3}\} \end{aligned}$$

The stopping time is:

$$T_{RAN} = \inf\{t|(r_t, s_t) \in \Delta_H \cup (r_t, s_t) \in \Delta_S\}$$

Also, the project receives full funding in every period it is continued.

$$K_t = \overline{K} \quad \forall t \leq T_{RAN}$$

Proof. In Appendix A. ∎

Thus, the Policymaker's value from this innovation project becomes:

$$V_0^S = E_0\left[\sum_{t=1}^{T_S}\delta^{t-1}\left(\Pr_t((r_t,s_t)\in\Delta_H)(p_tR-I)+\Pr_t((r_t,s_t)\in\Delta_S)L-\overline{K}\right)\right] \qquad (5)$$

The Policymaker sets the region Δ_H as the patent threshold. According to the patent law, the project has to clear this threshold in order to be granted a patent. Only after the patent is granted, the property rights are recognized; hence the project can be liquidated for a positive liquidation value $L > 0$[8].

The patent law threshold is depicted in the Figure 3.

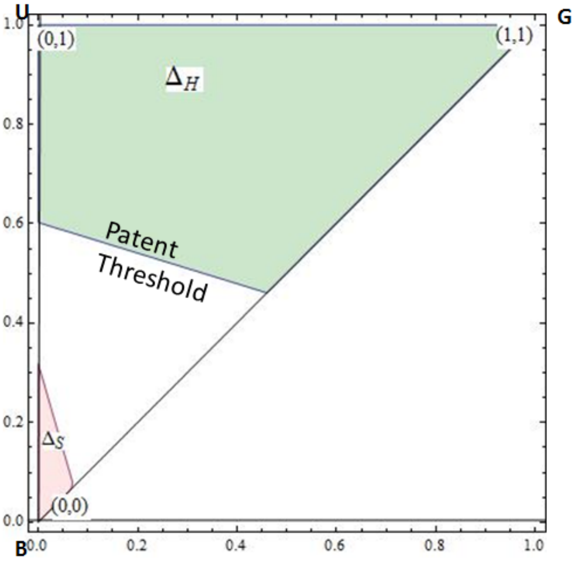

FIG. 3 Policymaker's Optima and Patent Threshold

Note that, once the posterior belief $[r_t, s_t] \in \Delta_H$, so that the project is granted a patent, according to the RAN optima, it is optimal to stop experimenting and develop the project. However, we will see in the next sections that the contractual outcome between an ambiguity neutral research lab and a ambiguity averse commercial firm may differ from this RAN optimal stopping rule.

[8] The patent law mandates that before clearing the patenting threshold, the project is not worth any positive value. This loss of value associated with the patent law reflects the social cost of granting monopoly power to the patent owners.

3.3. Contractual Outcome

Given the patent law set by the Policymaker, now let us focus on the contractual problem. The two parties forming the research alliance are: a big commercial firm (henceforth CF) and the smaller research-oriented firm or research lab (henceforth RL). Both the parties are risk-neutral and initially share a common prior about the true profitability of the project:

$$
\begin{aligned}
\Pr(\theta &= Good) = [r_0, s_0]; \quad 0 \le r_0 < s_0 \le 1. \\
[r_0, s_0] &\notin \Delta_H
\end{aligned}
$$

RL owns the project, but is liquidity constrained, so CF funds the project. At the experimentation phase, RL conducts the research activities, but after the project moves to the development phase, CF takes over the clinical trial and/or commercialization process ("development of the project").

The two parties, however, differ in their attitude towards ambiguity. RL likes the "open questions", or the "*Unknowable*" state more than the commercial firm, so is less ambiguity averse than CF. It can be justified by arguing that identifying open questions can open up the avenue of further research and help RL, or, "learning by doing" might add to the existing knowledge base of RL, whereas the commercial firm, which cares only for current profits, dislikes this state more, because the project does not yield a stream of payoffs if the true state is "*Unknowable*". To simplify, we assume that RL is ambiguity neutral while CF is ambiguity averse[9].

Now, let us describe the contracting time line, as captured in the figures **??** and 4 below. At the beginning of each period t, RL makes a take-it-or-leave-it offer to CF specifying

(a) $x_{t:}$: the proportional share of the final return RL receives, if the project is developed till the end

(b) b_t : the bonus that RL gets once the project clears the threshold, i.e., is granted a Patent, and,

(c) K_t : amount of investment to be disbursed in the t^{th} period[10].

CF accepts or rejects the offer. If accepted, the funds are disbursed and then RL privately decides whether to invest the fund or divert it for personal benefit (or cross-subsidization). At the end of the period, the signal S_t is publicly realized and beliefs

[9] In section 5, I discuss how the ability to write a contract on the knowledge generated from the research can change the ambiguity attitude of the two firms.

[10] Here, it is assumed that the research lab owns the project and faces a competetive market of commercial firms for that project, hence enjoys all the bargaining power. In real life, such contexts feature multiple commercial firms as well as research labs, so in any contracting environment, no party enjoys the full extent of the bargaining power. However, this assumption, while simplifying the calculations, does not qualitatively change the results.

are accordingly updated. If the signal is high enough, i.e., $[r_t, s_t] \in \Delta_H$, then the project is allowed to move to the Development Stage. In the Development stage, CF unilaterally decides whether to continue developing the product, liquidate the project, or keep experimenting further. If the project is continued till the end, after investing the fixed amount I, the true state θ is realized and returns accrue to the contracting parties. If the project is liquidated, CF appropriates the property rights, therefore obtains the liquidation value $L > 0$.

If the signal is not high enough , i. e. , $[r_t, s_t] \notin \Delta_H$, then CF decides whether to continue experimenting at period $t+1$ with updated beliefs, or to abandon the project, earning a return of 0 forever. The time line is depicted below.

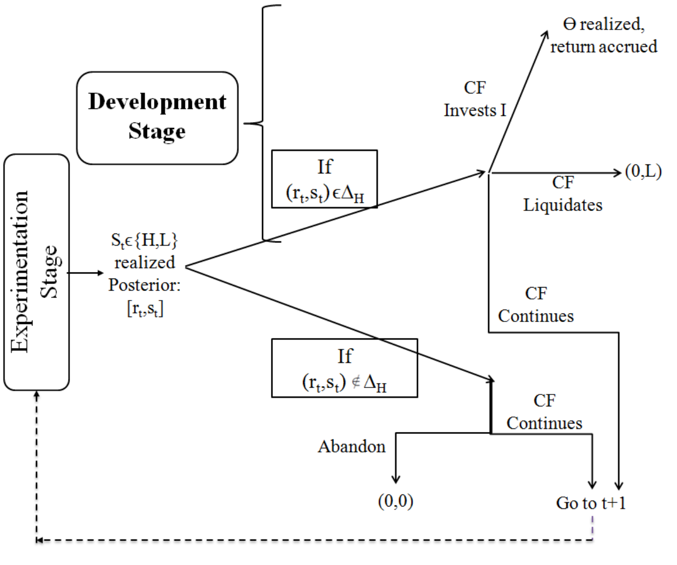

FIG. 4 Contracting Timeline: Development Stage

After observing the signal, with posterior$[r_t, s_t]$, the expected payoffs for the contracting parties are:

Payoffs of RL

	a_t^{CF} =Dev	a_t^{CF} =Liq
$\theta = Good$	Rx_t	b_t
$\theta = Bad$	0	b_t
$\theta = Unknowable$	$\frac{1}{2}Rx_t$	b_t

Payoffs of CF

	a_t^{CF} =Dev	a_t^{CF} =Liq
$\theta = Good$	$R(1-x_t) - I$	$L - b_t$
$\theta = Bad$	$-I$	$L - b_t$
$\theta = Unknowable$	$\frac{1}{2}R(1-x_t)(1-v) - I$	$L - b_t$

Thus, the expected payoffs:

$$
\begin{aligned}
CF &: \\
Eu_t^{CF}(a_t^{CF}) &= Dev, (r_t, s_t)) = (p_t - vq_t)R(1-x_t) - I \\
Eu_t^{CF}(a_t^{CF}) &= Liq, (r_t, s_t)) = L - b_t \\
RL &: \\
Eu_t^{RL}(a_t^{CF}) &= Dev, (r_t, s_t)) = p_t R x_t \\
Eu_t^{RL}(a_t^{CF}) &= Liq, (r_t, s_t)) = b_t
\end{aligned}
$$

The contracting parties do not have the power to commit to a long term contract. Then, RL, who has the full bargaining power in this model, always offers a contract that ensures CF only the minimum payment required to keep investing, so CF always breaks even. After observing $[r_t, s_t] \in \Delta_H$, CF obtains a payoff of $p_t R(1-x_t) - I$ if he develops the project, $L - b_t$ if he liquidates, and an expected payoff of 0 from future experimentation. Clearly, CF always chooses to stop experimentation as soon as $[r_t, s_t] \in \Delta_H$[11]. Thus, at any period t, if the observed signal induces a posterior belief higher than the patenting threshold, CF never continues experimentation.

Before discussing the infinite horizon model, let us first analyze the two period contracting game, which will illustrate the intuitions behind the main results of this paper. The findings from this two period example are readily extendable to the finite horizon contracting problem, and they will provide the intuitive understanding about the model in the general infinite horizon setting.

3.4. Two Period Example

In this example, the project is exogenously terminated after $t = 2$. Let us first describe the problem, then using backward induction, we will analyze the optimal contract.

If the project is continued till $t = 2$, at the beginning of the last period, RL chooses the contractual term considering $CF's$ optimal action choice after the signal clears the patent threshold: $a_2^{CF}|_{[r_2,s_2]\in\Delta_H.} \in \{Dev, Liq\}$.

[11] If we relax the assumption that RL has limited liability, then RL can make a payment to CF in order to continue experimenting even after clearing the patenting threshold. I discuss this case in section 6 and show that qualitatively the results do not change.

At $t = 2$, the state variables on the equilibrium path are $[r_1, s_1]$, the updated belief after observing last period's signal. RL solves:

$$V_2(r_1, s_1) = \max_{a_2^{CF}} \{V_2^{Dev}, V_2^{Liq}\}$$

where

$$\begin{aligned}
V_2^{Dev} = \; & \begin{array}{c} RL'\text{s expected payoff from period 2 if, given the contractual terms,} \\ \text{CF develops the product after reaching } \; \Delta_{H.}(a_2^{CF} = Dev) \end{array} \\
V_2^{Liq} = \; & \begin{array}{c} RL's \text{ expected payoff from period 2 if, given the contractual terms,} \\ \text{CF liquidates the product after reaching } \Delta_{H.}(a_2^{CF} = Liq) \end{array}
\end{aligned} \tag{6}$$

Now,

$$V_2^{Dev} = \max_{x_2, b_2, K_2} \Pr((r_2, s_2) \in \Delta_H)[Rp_2 x_2]$$

$$\Pr((r_2, s_2) \in \Delta_H)[Rp_2|_{(r_2,s_2)\in\Delta_H} x_2] \geq K_2 \tag{7}$$

$$\Pr((r_2, s_2) \in \Delta_H)[R(p_2 - vq_2)|_{(r_2,s_2)\in\Delta_H}(1 - x_2) - I] \geq K_2 \tag{PC_2^{CF}}$$

$$R(p_2 - vq_2)|_{(r_2,s_2)\in\Delta_H}(1 - x_2) - I \geq L - b_2 \tag{8}$$

$$x_2 \in [0, 1]; b_2 \geq 0; K_2 \in [0, \overline{K}]$$

And,

$$V_2^{Liq} = \max_{x_2, b_2, K_2} \Pr((r_2, s_2) \in \Delta_H)[b_2] \tag{9}$$

$$\Pr((r_2, s_2) \in \Delta_H)[b_2] \geq K_2 \qquad IC_{2,Liq}^{RL}$$

$$\Pr((r_2, s_2) \in \Delta_H)[L - b_2] \geq K_2 \tag{$PC_{2,Liq}^{CF}$}$$

$$R(p_2 - vq_2)|_{(r_2,s_2)\in\Delta_H}(1 - x_2) - I \leq L - b_2 \tag{$IC_{2,Liq}^{CF}$}$$

$$x_2 \in [0, 1]; b_2 \geq 0; K_2 \in [0, \overline{K}]$$

Let us take a closer look at the constraint set. The first constraint is the standard incentive compatibility constraint for RL, which ensures that the expected payoff for RL at $t = 2$ has to be greater than or equal to the static gain that RL might enjoy by diverting the investment, thereby implementing no diversion on the equilibrium path. Notice that, in this setting, if any partial diversion is beneficial, so is the full diversion, that is why it is sufficient to consider the incentive constraint only for the full diversion case. The second constraint is the participation constraint for CF, guaranteeing CF an expected return to cover the investment cost. Without loss of generality, $CF's$ outside option is normalized to 0. The last constraint shows that after the signal realization, it is sequentially optimal for CF to develop the project in the first case and liquidate in the second.

Solving the problem, we get three regions of posterior belief: Δ_D, Δ_L , such that

$$\begin{aligned}
\Delta_D &= \{(r_t, s_t) \in \Delta_H | \ a_t^{CF}|_{\Delta_H} = Dev \\
&\quad \textit{i.e.}, CF \text{ chooses to develop the project once being granted a patent}\} \\
\Delta_L &= \{(r_t, s_t) \in \Delta_H | \ a_t^{CF}|_{\Delta_H} = Liq \\
&\quad \textit{i.e.}, CF \text{ chooses to liquidate the project once being granted a patent}\}
\end{aligned}$$

If the project is funded till $t = 2$, it must be that $(r_2, s_2)|_{S_2=s_H} \in \Delta_H$, but $(r_2, s_2)|_{S_2=s_L} \notin \Delta_H$[12]. So,

$$\begin{aligned}
\Pr((r_2, s_2) &\in \Delta_H) = \Pr(S_2 = s_H) = \mu_2 \\
&= K_2\lambda_2 \qquad \text{(under linear signal)}
\end{aligned}$$

At every t, the participation constraint of CF holds with equality, so, if at $t = 2$, if $(r_2, s_2) \in \Delta_D$,

$$x_2 = 1 - \frac{1}{R((p_2 - vq_2))}\left[\frac{1}{\lambda_2} + I\right]$$

and if $(r_2, s_2) \in \Delta_L$,

$$b_2 = L - \frac{1}{\lambda_2}$$

Remark 1 (Ambiguity Sharing). Observe that, as v increases, i.e., CF becomes more ambiguity averse, the share he receives, $1 - x_2$, goes up. Thus, the contractual payment

rule effectively shares ambiguity.

Remark 2 ((Evolution of Share)). As experimentation continues, the contracting parties grow more pessimistic as posterior belief declines. The share CF demands goes up accordingly over time to compensate.

Thus, RL solves:

$$V_2 = \lambda_2 \max_{K_2^{Dev}, K_2^{Liq}} \left\{K_2^{Dev} p_2 \left(R - \frac{1}{p_2 - vq_2}\left(I + \frac{1}{\lambda_2}\right)\right), K_2^{Liq}\left(L - \frac{1}{\lambda_2}\right)\right\} \tag{10}$$

[12] If $(r_2, s_2)|_{S_2=s_L}, (r_2, s_2)|_{S_2=s_H} \in \Delta_H$, then it must be that $(r_1, s_1) \in \Delta_H$. Then, experimentation should have stopped after $t = 1$. Again, if $(r_2, s_2) \notin \Delta_H$ for $S_2 = s_H$ or $S_2 = s_L$, then the project is not funded in $t = 2$.

subject to the constraint:

$$
\begin{aligned}
K_2^{Dev} p_2\left(R-\frac{1}{p_2-vq_2}\left(I+\frac{1}{\lambda_2}\right)\right) &\geq \frac{K_2^{Dev}}{\lambda_2} \quad \text{if } (r_2,s_2)\in\Delta_D\\
K_2^{Liq}\left(L-\frac{1}{\lambda_2}\right) &\geq \frac{K_2^{Liq}}{\lambda_2} \quad \text{if } (r_2,s_2)\in\Delta_L
\end{aligned}
$$

So, $K_2^{Dev}=\overline{K}$, and $K_2^{Liq}=\overline{K}$, if

$$
\max\left\{p_2\left(R-\frac{1}{p_2-vq_2}\left(I+\frac{1}{\lambda_2}\right)\right), L-\frac{1}{\lambda_2}\right\}\geq\frac{1}{\lambda_2} \tag{11}
$$

If this condition is satisfied, the expected value to RL from $t=2$ is:

$$
V_2(r_1,s_1)=\lambda_2\overline{K}\left\{p_2\left(R-\frac{1}{p_2-vq_2}\left(I+\frac{1}{\lambda_2}\right)\right),\left(L-\frac{1}{\lambda_2}\right)\right\} \tag{12}
$$

The regions where the project is developed till the end, and where it is liquidated are identified as:

$$
\Delta_D = \left\{(r_2,s_2)\in\Delta_H|\ p_2\left(R-\frac{1}{p_2-vq_2}\left(I+\frac{1}{\lambda_2}\right)\right)\geq\left(L-\frac{1}{\lambda_2}\right)\right\} \tag{13}
$$

$$
\Delta_L = \left\{(r_2,s_2)\in\Delta_H|\ p_2\left(R-\frac{1}{p_2-vq_2}\left(I+\frac{1}{\lambda_2}\right)\right)<\left(L-\frac{1}{\lambda_2}\right)\right\} \tag{14}
$$

Remark 3 (Patent Troll). Observe that, in the absence of ambiguity, or, if both the parties were ambiguity neutral ($v=0$), then

$$
p_2R-I>L \quad \forall(r_t,s_t)\in\Delta_H,
$$

so, $\Delta_L=\phi$.

In ambiguous context, however, there exists v_m such that for $v\in(v_m,1)$ [13], $\Delta_L=\Delta_H\backslash\Delta_D\neq\phi$.

This region resembles Patent Troll[14] behavior, where even after being granted a patent, the research alliance liquidates the project. Patent troll happens because of the ambiguity aversion of CF, who acts more pessimistically after observing each low signal. So, even if the posterior ensures that a risk and ambiguity neutral entity would

[13] $v_m=\frac{(r_0+s_0)((r_0+s_0)R-L-I)\lambda_0}{\lambda_0[(s_0-r_0)((r_0+s_0)R-L]+1)}$

[14] Technically, the term "patent troll" refers to the entities which obtain and enforce patent rights but do not manufacture products or supply services based upon the patent in question, thus engaging in economic rent-seeking.

optimally choose to develop the project, CF decides to liquidate.

Now, let us go one step backward at $t = 1$.

At $t = 1$, RL solves:

$$V_1(r_0, s_0) = \max_{a_1^{CF}} \{V_1^{Dev}, V_1^{Liq}\}$$

where

$$\begin{aligned}
V_1^{Dev} = \max_{x_1, b_1, K_1} & \; K_1\lambda_1[Rp_1x_1] + \delta(1 - K_1\lambda_1)E_1V_2(r_1, s_1) \\
& K_1\lambda_1[Rp_1x_1] + \delta(1 - K_1\lambda_1)E_1V_2(r_1, s_1) \\
& \qquad \geq K_1 + \delta E_1V_2(r_1, s_1; r_0, s_0) \qquad (IC_1^{RL}) \\
& K_1\lambda_1[R(p_1 - vq_1)(1 - x_1) - I] \geq K_1 \qquad (PC_1^{CF}) \\
& R(p_1 - vq_1)(1 - x_1) - I \geq L - b_1 \qquad (IC_1^{CF}) \\
& x_1 \in [0, 1]; b_1 \geq 0; K_1 \in [0, \overline{K}]
\end{aligned}$$

And,

$$\begin{aligned}
V_1^{Liq} = \max_{x_1, b_1, K_1} & \; K_1\lambda_1b_1 + \delta(1 - K_1\lambda_1)E_1V_2(r_1, s_1) \\
& K_1\lambda_1b_1 + \delta(1 - K_1\lambda_1)E_1V_2(r_1, s_1) \\
& \qquad \geq K_1 + \delta E_1V_2(r_1, s_1; r_0, s_0) \qquad (IC_{1,Liq}^{RL}) \\
& K_1\lambda_1[L - b_1] \geq K_1 \qquad (PC_{1,Liq}^{CF}) \\
& R(p_1 - vq_1)(1 - x_1) - I \leq L - b_1 \qquad (IC_{1,Liq}^{CF}) \\
& x_1 \in [0, 1]; b_1 \geq 0; K_1 \in [0, \overline{K}]
\end{aligned}$$

In period 1, compared to the problem at $t = 2$, the participation constraint for CF remains same with the corresponding posterior belief at $t = 1$; however the incentive constraint for RL requires a closer look. The incentive constraints $(IC_{1,Liq}^{RL})$ and (IC_1^{RL}) highlight the two sources of gain from cheating: the static gain and the dynamic gain. The static gain is similar as in the second period, stemming from the benefit RL derives by diverting the investment amount (K_1), so the IC at $t = 1$ has to ensure that $RL's$ expected payoff from $t = 1$ has to be greater than the investment. However, there is a dynamic gain from cheating as well, captured by the dynamic cheating value: which arises from the fact that following a diversion of funds at $t = 1$, the posterior belief of

RL and CF diverge. Because of the diversion, the signal S_1 is always s_L, observing which CF is prompted to update his belief to $[r_1, s_1]|_{S_1=s_L}$, with posterior mean p_1 and ambiguity q_1. The next period's contract will then be based on this public belief $[r_1, s_1]$. However, RL has perfectly observed his own action, so even after the low signal he does not update his belief and evaluates the future contracting terms using his private belief $[r_0, s_0]$. This constitutes the dynamic agency cost:

$$
\begin{aligned}
DAC_2 &= \delta[V_2(cheat) - V_2(no\ cheat)] \\
&= \delta[E_1 V_2(r_1, s_1; r_0, s_0) - (1 - K_1\lambda_1)E_1 V_2(r_1, s_1)] \\
&= \delta \begin{cases} \left[\frac{\lambda_1 p_1}{\lambda_2 p_2} - (1 - K_1\lambda_1)\right] V_2(r_1, s_1) & \text{if } (r_2, s_2) \in \Delta_D \\ \left[\frac{\lambda_1}{\lambda_2} - (1 - K_1\lambda_1)\right] V_2(r_1, s_1) & \text{if } (r_2, s_2) \in \Delta_L \end{cases} \\
&> 0
\end{aligned}
$$

Under some parametric conditions, the dynamic agency cost leads to delay in funding as well, so that it is optimal for the project to receive funding at $t = 2$ but no contract with positive funding satisfies both the participation and moral hazard constraints. Let us analyze all possible cases separately to see the region of posteriors where in-equilibirum delay might occur.

Case 1: $(r_1, s_1) \in \Delta_D$ and $(r_2, s_2) \in \Delta_D$:

With $\delta = 0$, delay never occurs, since:

$$
\lambda_1 \left(p_1 R - \frac{p_1}{(p_1 - vq_1)\lambda_1} - \frac{Ip_1}{p_1 - vq_1} \right) \geq \lambda_2 \left(p_2 R - \frac{p_2}{(p_2 - vq_2)\lambda_2} - \frac{Ip_2}{p_2 - vq_2} \right) \geq 1
$$

However, if $\delta > 0$, dynamic moral hazard makes funding the project at $t = 1$ more difficult than at $t = 2$. As a result, in-equilibrium delay happens if

$$
\begin{aligned}
& \frac{1 + \lambda_1 p_1 \left(\frac{1}{(p_1 - vq_1)\lambda_1} + \frac{I}{p_1 - vq_1} \right) - \delta \left(\lambda_1 p_1 - (1 - \mu_1)\lambda_2 p_2 \left(\frac{1}{(p_2 - vq_2)\lambda_2} - \frac{I}{p_2 - vq_2} \right) \right)}{\lambda_1 p_1 - \delta \left(\lambda_1 p_1 - (1 - \mu_1)\lambda_2 p_2 \right)} \\
> \quad & R \geq \frac{1 + \lambda_2 p_2 \left(\frac{1}{(p_2 - vq_2)\lambda_2} + \frac{I}{p_2 - vq_2} \right)}{\lambda_2 p_2} \qquad (15)
\end{aligned}
$$

The possibility of in-equilibrium delay due to dynamic agency cost is well documented in the literature of dynamic contracts (Bergemann and Hege, 1998; Bonatti and Horner (2009)). In this paper, we find that in the presence of ambiguity, the commercial firm's ambiguity aversion reins in this dynamic moral hazard problem. Intuitively, CF, being ambiguity averse, becomes much more cautious and pessimistic after each low signal. So, following a low signal, CF has to be guaranteed a greater share of the final return in order to keep investing. This ambiguity sharing agreement disciplines

RL and lowers his dynamic expected value from cheating (DAC_2) which, in turn, eases the funding constraint at $t = 1$ and possibility of in-equilibrium delay falls.

The next proposition summarizes the finding that, in this two period context, under *Case 1,* the dynamic value of cheating decreases with v and in-equilibrium delay happens for a smaller range of R , and, in fact if $v \geq \widetilde{v}$, where $\widetilde{v} \in (0, 1)$ is characterized below, then delay in funding does not happen on the equilibrium path.

PROPOSITION 2. *For discount rate* $\delta \leq \overline{\delta}$, $\exists \widetilde{v} \in (0, 1)$, *such that* $\forall v \geq \widetilde{v}$, *in-equilibrium delay never happens in the basic two period model.*

Proof. In Appendix A. ∎

Also, in this case, if funding condition is met at $t = 1$, full funding is disbursed, because of the linearity of signal structure.

In the next two cases, there is no possibility of in-equilibirum delay.

Case 2: $(r_1, s_1) \in \Delta_D$ *and* $(r_2, s_2) \in \Delta_L$:

Here, funding at $t = 2$ requires

$$L - \frac{2}{\lambda_2} \geq 0 \tag{16}$$

Now, at $t = 1$,

$$\begin{aligned} & p_1 \left[R - \frac{1}{p_1 - vq_1} \left(I + \frac{1}{\lambda_1} \right) \right] - \left(\frac{\lambda_1}{\lambda_2} - (1 - K_1 \lambda_2) \right) \left(L - \frac{2}{\lambda_2} \right) \\ > & \left(L - \frac{2}{\lambda_2} \right) \left[1 - \left(\frac{\lambda_1}{\lambda_2} - (1 - K_1 \lambda_2) \right) \right] \\ \geq & \ 0 \end{aligned} \tag{17}$$

so, full funding is always available at $t = 1$ is always met if 16 is satisfied.

Similarly, in *Case 3:* $(r_1, s_1) \in \Delta_D$ *and* $(r_2, s_2) \in \Delta_L$,

since

$$\begin{aligned} & L - \frac{2}{\lambda_1} - \left(\frac{\lambda_1}{\lambda_2} - (1 - K_1 \lambda_2) \right) \left(L - \frac{2}{\lambda_2} \right) \\ > & \left(L - \frac{2}{\lambda_2} \right) \left[1 - \left(\frac{\lambda_1}{\lambda_2} - (1 - K_1 \lambda_2) \right) \right] \\ \geq & \ 0 \end{aligned}$$

there is no possibility of in-equilibrium delay.

The contractual terms at $t = 1$ are otherwise similar to those at $t = 2$.

Thus, from analyzing this two period problem, we observe that

Remark 4 (Result 1:). With ambiguity averse CF and ambiguity neutral RL, dynamic moral hazard problem is alleviated. As a result, under some parametric restrictions, in-equilibirium delay does not happen.

Remark 5 (Result 2:). The research alliance may liquidate the project even after being granted a patent.

3.5. Infinite Horizon Model

In this section we analyze the infinite horizon sequential contracting game between CF and RL and derive the equilibrium contractual outcome. Let us first formally define the equilibrium.

At any period t, the observable, or, public history consists of the past contracts offered, the past realizations of signals and $CF's$ decision whether to develop, liquidate or continue the project. Potentially, this public history can be different than the private history of RL, who observes his own decision to divert the fund as well.

Formally, let H_t^P denote the set of all possible public histories up to, but not including, period t. Each element $h_t^P \in H_t^P$ contains

(a) the past contractual terms: $\{x_j, b_j, K_j\}_{j=1}^{t-1}$

(b) past strategic choices of CF to accept or reject the contract offered at each period: $\{\zeta_j\}_{j=1}^{t-1}$ ($\zeta_t = 1$ if CF accepts an offer at period t, 0 otherwise)

(c) past realized values of the signals: $\{S_j\}_{j=1}^{t-1}$

(d) past strategic choices of CF after observing the signal realizations at every period: $\{a_j^{CF}\}_{j=1}^{t-1}$.

In contrast, the set of possible private histories is denoted by H_t, where each element $h_t \in H_t$, in addition to h_t^P, contains $\{d_j\}_{j=1}^{t-1}$, the past realizations of the strategic choices of RL whether to divert the fund ($d_t = 1$ if the fund is invested in period t and 0 if diverted).

The true history leads to the posterior belief formed by RL at the beginning of period $t : [r_{t-1}, s_{t-1}] : H_t \to \mathbb{K}_{\Delta_{[0,1]}}$. In consequence, CF also has a belief about the true history, captured by the belief about the true posterior formed by $CF : [r'_{t-1}, s'_{t-1}] : H_t^P \times D'_t \to \mathbb{K}_{\Delta_{[0,1]}}$, which depends on the public history as well as the belief CF has about $RL's$ past investment behavior: $\{d'_j\}_{j=1}^{t-1}$. D'_t contains the set of all beliefs $\{d'_j\}_{j=1}^{t-1}$.

Then, a contract (x_t, b_t, K_t) by RL is a mapping from the true history H_t into the sharing rule x_t , bonus rule b_t and investment flow K_t.

$$\begin{aligned} x_t &: H_t \to [0,1] \\ b_t &: H_t \to \mathbb{R}_+ \\ K_t &: H_t \to [0,\overline{K}] \subset [0,1] \end{aligned}$$

A decision rule by CF whether to accept or reject the contract is then a mapping from the perceived history: $\{x_j, b_j, K_j, \zeta_j, a_j^{CF}, d_j'\}_{j=1}^{t-1}$, and the contract proposed, into a binary decision to reject or accept the contract:

$$\zeta_t : H_t^P \times [0,1] \times \mathbb{R}_+ \times [0, \overline{K}] \rightarrow \{0,1\}$$

An investment policy by RL is:

$$d_t : H_t \times [0,1] \times \mathbb{R}_+ \times [0, \overline{K}] \times \{0,1\} \rightarrow \{0,1\}$$

A decision rule by CF after observing the signal at the end of period t is a mapping from the public history, contractual terms, perceived belief about diversion strategy of RL given the incentives provided by the contract, and the realized signal $S_t \in \{s_H, s_L\}$ into the choice to develop, liquidate, continue, or abandon the project at the end of period t.

$$a_t^{CF} : H_t^P \times [0,1] \times \mathbb{R}_+ \times [0, \overline{K}] \times \{0,1\} \times \mathbb{K}_{\Delta_{[0,1]}} \rightarrow \{Dev, Liq, Abandon, Cont\}$$

In this model, we are in a Markovian world, because all the payoff relevant history can be captured by the four state variables: $(r_{t-1}, s_{t-1}, r'_{t-1}, s'_{t-1})$: the true posterior belief held by $RL : [r_{t-1}, s_{t-1}]$ and the belief of CF about the true posterior:$[r'_{t-1}, s'_{t-1}]$. In this context, let us define the suitable Markov equilibrium concept.

Definition 2 (Markov Sequential Equilibrium). A Markov sequential equilibrium is a sequential equilibrium $\{x_t, b_t, K_t, \zeta_t, a_t^{CF}, d_t\}_{t=1}^{\infty}$, if

$$(r_{t-1}, s_{t-1})(h_t) = (r_{t-1}, s_{t-1})(\hat{h}_t) \quad \Longrightarrow \quad \begin{array}{c} x_t(h_t) = x_t(\hat{h}_t) \\ b_t(h_t) = b_t(\hat{h}_t) \\ K_t(h_t) = K_t(\hat{h}_t) \end{array}$$

$$\left.\begin{array}{c} (r'_{t-1}, s'_{t-1})(h_t^P) = (r'_{t-1}, s'_{t-1})(\hat{h}_t^P) \\ (x_t, b_t, K_t) = (\hat{x}_t, \hat{b}_t, \hat{K}_t) \end{array}\right\} \Longrightarrow \quad \zeta_t(h_t^P, x_t, b_t, K_t) = \zeta_t(\hat{h}_t^P, \hat{x}_t, \hat{b}_t, \hat{K}_t)$$

$$\left.\begin{array}{c} (r_{t-1}, s_{t-1})(h_t) = (r_{t-1}, s_{t-1})(\hat{h}_t) \\ (x_t, b_t, K_t) = (\hat{x}_t, \hat{b}_t, \hat{K}_t) \\ \zeta_t = \hat{\zeta}_t \end{array}\right\} \Longrightarrow \quad d_t(h_t, x_t, b_t, K_t, \zeta_t) = d_t(\hat{h}_t, \hat{x}_t, \hat{b}_t, \hat{K}_t, \hat{\zeta}_t)$$

$$\left.\begin{array}{c} (r_{t-1}, s_{t-1})(h_t) = (r_{t-1}, s_{t-1})(\hat{h}_t) \\ (x_t, b_t, K_t) = (\hat{x}_t, \hat{b}_t, \hat{K}_t) \\ \zeta_t = \hat{\zeta}_t \\ d_t = \hat{d}_t \end{array}\right\} \Longrightarrow \quad a_t^{CF}(h_t^P, x_t, b_t, K_t, \zeta_t, d_t) = a_t^{CF}(\hat{h}_t^P, \hat{x}_t, \hat{b}_t, \hat{K}_t, \hat{\zeta}_t, \hat{d}_t)$$

$$\forall h_t \in H_t; \forall h_t^P \in H_t^P; \forall \hat{h}_t \in \hat{H}_t; \forall \hat{h}_t^P \in \hat{H}_t^P; \forall (x_t, b_t, K_t), (\hat{x}_t, \hat{b}_t, \hat{K}_t); \forall \zeta_t, \hat{\zeta}_t, \forall d_t, \hat{d}_t$$

The Markovian sequential equilibrium ensures that the continuation strategies are time consistent and identical after any history with identical updated true posterior belief $[r_{t-1}, s_{t-1}]$ and $CF's$ belief about the posterior: $[r'_{t-1}, s'_{t-1}]$. It imposes that on the equilibrium path CF has the true belief given the incentives, i. e., on the equilibrium path $[r_{t-1}, s_{t-1}] = [r'_{t-1}, s'_{t-1}]$, but allows for the possibility of divergence of posterior beliefs off the equilibrium path.

The stopping regions are defined as before:

$$\begin{aligned}
\Delta_D &= \{(r_t, s_t) \in \Delta_H | \ a_t^{CF} = Dev\} \\
\Delta_L &= \{(r_t, s_t) \in \Delta_H | \ a_t^{CF} = Liq\} \\
\Delta_S^C &= \{(r_t, s_t) \in \mathbb{K}_{\Delta_{[0,1]}} | \ a_t^{CF} = Abandon\}
\end{aligned}$$

Now, at every period t, RL solves:

$$\begin{aligned}
&V_t(r_{t-1}, s_{t-1}) \\
&= \max_{\Delta_D, \Delta_L, \Delta_S^C, (x_t, b_t, K_t) \in \mathbb{C}_t} \Pr_t((r_t, s_t) \in \Delta_D)(p_t R x_t) \\
&\quad + \Pr_t((r_t, s_t) \in \Delta_L) b_t \\
&\quad + \delta(1 - \Pr_t((r_t, s_t) \in \Delta_D) - \Pr_t((r_t, s_t) \in \Delta_L) - \Pr_t((r_t, s_t) \in \Delta_S^C)) E_t V_{t+1}(r_t, s_t)
\end{aligned} \tag{18}$$

where the contract space $\mathbb{C}_t$ is given by:

$$\mathbb{C}_t = \{(x_t, b_t, K_t) \in [0,1] \times \mathbb{R}_+ \times [0, \overline{K}] |$$

$$\begin{aligned}
&\Pr_t((r_t, s_t) \in \Delta_D)(p_t R x_t) + \Pr_t((r_t, s_t) \in \Delta_L) b_t \\
&+ \delta(1 - \Pr_t((r_t, s_t) \in \Delta_D) - \Pr_t((r_t, s_t) \in \Delta_L) - \Pr_t((r_t, s_t) \in \Delta_S^C)) E_t V_{t+1}(r_t, s_t) \\
&\geq K_t + \delta E V_{t+1}(r_{t-1}, s_{t-1}, r_t, s_t)
\end{aligned} \tag{IC_t^{RL}}$$

$$\begin{aligned}
&\Pr_t((r_t, s_t) \in \Delta_D)[(p_t - v q_t) R(1 - x_t) - I] + \Pr_t((r_t, s_t) \in \Delta_L)(L - b_t) \\
&\geq K_t
\end{aligned} \tag{PC_t^{CF}}$$

$$\begin{aligned}
\text{if } (r_t, s_t) &\in \Delta_D, (p_t - v q_t) R(1 - x_t) - I \geq L - b_t \\
\text{if } (r_t, s_t) &\in \Delta_L, (p_t - v q_t) R(1 - x_t) - I < L - b_t
\end{aligned} \tag{IC_t^{CF}}$$

Now, by the same logic as in the two period example, we observe that the experimentation stops the first time $(r_t, s_t) \in \Delta_H$. Thus, the problem can be simplified as:

$$V_t(r_{t-1}, s_{t-1}) = \max_{\Delta_D, \Delta_L, \Delta_S^C, (x_t, b_t, K_t) \in \mathbb{C}_t} \mu_t \mathbf{1}_t((r_t, s_t) \in \Delta_D)(p_t R x_t) + \mu_t \mathbf{1}_t((r_t, s_t) \in \Delta_L) b_t$$
$$+ \delta(1-\mu_t) E_t V_{t+1}(r_t, s_t)$$

where the contract space $\mathbb{C}_t$ is:

$$\begin{aligned}
&\mathbb{C}_t = \{(x_t, b_t, K_t) \in [0,1] \times \mathbb{R}_+ \times [0, \overline{K}] | \\
&\qquad if \ (r_t, s_t) \in \Delta_D \\
&\qquad \mu_t p_t R x_t + (1-\mu_t)\delta E V_{t+1}(r_t, s_t) \\
&\qquad \geq K_t + \delta E V_{t+1}(r_{t-1}, s_{t-1}, r_t, s_t) && (IC_t^{RL}(Dev)) \\
&\mu_t[(p_t - v q_t) R (1 - x_t) - I] \geq K_t && (PC_t^{CF}(Dev)) \\
&(p_t - v q_t) R (1 - x_t) - I \geq L - b_t && (IC_t^{CF}(Dev)) \\
\\
&\qquad if \ (r_t, s_t) \in \Delta_L \\
&\qquad \mu_t b_t + (1-\mu_t)\delta E V_{t+1}(r_t, s_t) \\
&\qquad \geq K_t + \delta E V_{t+1}(r_{t-1}, s_{t-1}, r_t, s_t) && (IC_t^{RL}(Liq)) \\
&\qquad \mu_t[L - b_t] \geq K_t && (PC_t^{CF}(Liq)) \\
&\qquad L - b_t \geq (p_t - v q_t) R (1 - x_t) - I && (IC_t^{CF}(Liq)) \\
\\
&\qquad if \ (r_t, s_t) \in \Delta_S^C \\
&\qquad E_t V_{t+1}(r_t, s_t) = 0
\end{aligned}$$

Lemma 3. *There exists a unique Markov sequential equilibrium in the dynamic contracting game.*

Proof. Similar to Lemma 1, the Bellman equation satisfies monotonicity and discounting properties with the discount factor $\delta(1-\mu)$, hence is a contraction mapping by Blackwell's sufficiency conditions (Theorem 3.3 in Stokey, 1989) . Then, by contracting mapping theorem (Theorem 3.2 in Stokey, 1989), it has a unique solution. ∎

Now let us find the optimal contracting terms.

At every period, by the same logic as in the two period example, the participation constraint for CF holds as an equality, so

$$\begin{aligned} if\ (r_t, s_t) &\in \Delta_D \\ x_t &= 1 - \frac{1}{R(p_t - vq_t)}\left(I + \frac{1}{\lambda_t}\right); \\ b_t &\geq L - \frac{1}{\lambda_t} \end{aligned} \tag{19}$$

and

$$\begin{aligned} if\ (r_t, s_t) &\in \Delta_L \\ b_t &= L - \frac{1}{\lambda_t} \\ x_t &\geq 1 - \frac{1}{R(p_t - vq_t)}\left(I + \frac{1}{\lambda_t}\right); \end{aligned} \tag{20}$$

From 19, we can observe how the contracting terms facilitate ambiguity sharing among the ambiguity neutral RL and ambiguity averse CF.

Then, the optimal stopping regions are[15] given by the following proposition.

PROPOSITION 3. *The strategic alliances develop the project after being granted patent if $(r_t, s_t) \in \Delta_D$, liquidate the project after being patented if $(r_t, s_t) \in \Delta_L$, and abandon the project forever if $(r_t, s_t) \in \Delta_S^C$, where*

$$\begin{aligned} \Delta_D &= \left\{(r_t, s_t) \in \Delta_H |\ \left[p_t R - \frac{p_t}{(p_t - vq_t)}\left(I + \frac{1}{\lambda_t}\right)\right] \geq L - \frac{1}{\lambda_t}\right\} \\ \Delta_L &= \left\{(r_t, s_t) \in \Delta_H |\ \left[p_t R - \frac{p_t}{(p_t - vq_t)}\left(I + \frac{1}{\lambda_t}\right)\right] < L - \frac{1}{\lambda_t}\right\} \\ \Delta_S^C &= \left\{(r_t, s_t) \in \mathbb{K}_{\Delta_{[0,1]}} |\ L < \frac{2}{\lambda_t}\right\} \end{aligned}$$

Let T be the optimal stopping time:

$$T := \inf\{t | (r_t, s_t) \in \Delta_H \cup (r_t, s_t) \in \Delta_S^C\}$$

Proof. In Appendix A. ∎

Now we will turn to the funding pattern. We need to characterize the optimal investment schedule to answer the questions:

a) is it possible that the project will obtain full funding till the end, i.e., till the time the posterior $(r_t, s_t) \in \Delta_S^C$,

[15] Note that due to the linearity of the signal structures, the stopping decision does not depend on the investment amount at the last period.

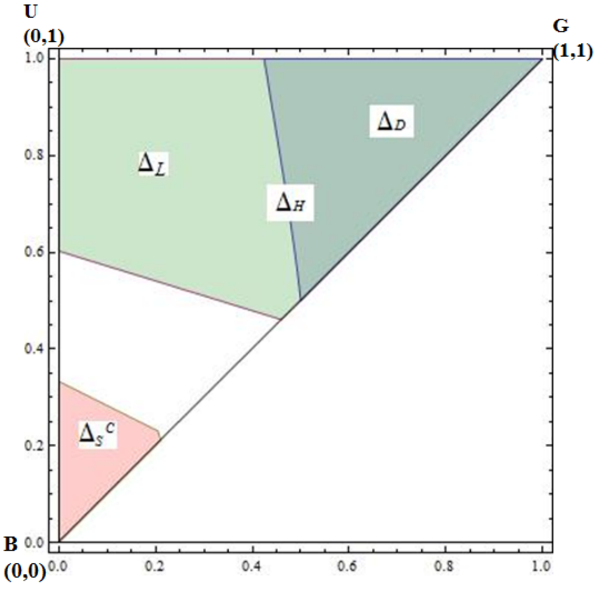

FIG. 5 Contractual Equilibrium

b) if full funding is not available at all times, how does the funding flow evolve over time?

To examine the funding flow, first let us look at the incentive constraint RL faces at any t.

If $(r_t, s_t) \in \Delta_D$, the dynamic incentive constraint is:

$$\mu_t p_t R x_t + (1 - \mu_t)\delta EV_{t+1}(r_t, s_t) \geq K_t + \delta EV_{t+1}(r_{t-1}, s_{t-1}, r_t, s_t)$$

Substituting for the optimal share x_t from 19, rewrite it as:

$$\mu_t p_t \left(R - \frac{1}{p_t - vq_t} \left(I + \frac{1}{\lambda_t} \right) \right) + (1 - \mu_t)\delta EV_{t+1}(r_t, s_t) \\ \geq K_t + \delta EV_{t+1}(r_{t-1}, s_{t-1}, r_t, s_t)$$

Now, the dynamic expected payoff to be collected by RL in future periods following a diversion can be expressed as:

$$EV_{t+1}(r_{t-1}, s_{t-1}, r_t, s_t) = \frac{\lambda_{t-1} p_{t-1}}{\lambda_t p_t} EV_{t+1}(r_t, s_t)$$

So, the dynamic IC can be rewritten as:

$$\mu_t p_t \left(R - \frac{1}{p_t - vq_t} \left(I + \frac{1}{\lambda_t} \right) \right) - K_t \geq \delta \left[\frac{\lambda_{t-1} p_{t-1}}{\lambda_t p_t} - (1 - \mu_t) \right] EV_{t+1}(r_t, s_t) \tag{21}$$

where the RHS captures the dynamic agency cost.

Similarly, if $(r_t, s_t) \in \Delta_L$, the dynamic incentive constraint can be rewritten as:

$$\mu_t \left(L - \frac{1}{\lambda_t} \right) - K_t \geq \delta \left[\frac{\lambda_{t-1}}{\lambda_t} - (1 - \mu_t) \right] EV_{t+1}(r_t, s_t) \tag{22}$$

and it does not depend on $CF's$ ambiguity aversion.

We show that under a sufficient condition on the initial parameters, the project will never receive full funding till the end. In that case, there will be a switching point, captured by a range of posterior beliefs such that if the posterior belief lies below that locus then full funding is no longer available. Then, we show that for the range of posteriors where full funding is not available, the funding volume decreases with posterior belief over time. Also, as CF becomes more ambiguity averse, the dynamic moral hazard problem is alleviated, resulting in a longer horizon of full funding. The investment pattern is characterized by the following proposition.

PROPOSITION 4. *The project does not receive full funding till the end if:*

$$\lambda_0 < \frac{2 - \delta}{L\left(1 - \frac{\delta}{2}\right)} \tag{23}$$

If 23 holds, then there is a region of posterior beliefs Δ_F where the project does not receive full funding:

$$\begin{aligned} \Delta_F \quad : \quad &= \{(r_t, s_t) \in \mathbb{K}_{\Delta_{[0,1]}} \backslash \Delta_S^C | \\ &\quad \textit{the region of posterior beliefs where full funding is not available}\} \end{aligned} \tag{24}$$

Then, there exists a δ_L such that

a) If $\frac{\lambda_0 L - 2}{\lambda_0\left(\frac{L}{2} + \overline{K}\right) - 1} \leq \delta \leq \delta_L, \Delta_D \cap \Delta_F = \phi$; so full funding is available for all $(r_t, s_t) \in \Delta_D$.

b) If $1 > \delta \geq \delta_L, \Delta_D \cap \Delta_F \neq \phi$; the project does not receive full funding for all $(r_t, s_t) \in \Delta_D$. In this case, as v increases, the project receives full funding for a longer time horizon, i.e., $\Delta_D \backslash \Delta_F$ expands.

After full funding stops, investment volume monotonically decreases over time.

Proof. In Appendix A. ∎

From the proposition 4, we observe how the different components of the model interact with each other to determine the investment level.

1. Discount factor (δ) : For higher discount factor, $\delta \geq \delta_L$ full funding horizon shrinks. There exists a range of posteriors for which the project is developed after being patented, still only restricted funding is available. This is intuitive because as RL becomes more patient, he values the future gains more, so the dynamic moral hazard problem is more severe and the incentive constraint is more difficult to hold. As a result, only partial funding is available for a large range of posterior belief.

2. Prior belief (r_0, s_0) : If the prior belief that the true state $\theta = Good$ is high, i.e., initially the belief about the profitability of the project is favorable enough, the project can receive full funding till the end.

3. Ambiguity aversion coefficient (v): If CF is more ambiguity averse (v increases) the dynamic moral hazard problem is alleviated. The intuition is similar to the two period example. CF, being ambiguity averse, becomes much more cautious and pessimistic after each low signal. So, following a low signal, CF has to be guaranteed a greater share of the final return in order to keep investing. Thus, the contractual terms sharing ambiguity also discipline RL and lower his dynamic expected value from cheating which, in turn, eases the funding constraint towards the beginning. Thus, if the project receives full funding till $\Phi_D(r_t, s_t) = 0$, as v increases, full funding horizon increases. After the project stops receiving full funding, the investment flow is monotonically decreasing over time. This result is in contrast with the result in Bergemann and Hege, 2005, where it is possible to have monotonically increasing investment pattern over time due to the severity of the dynamic agency problem.

4. POLICY RECOMMENDATIONS

In this section, we will compare the equilibrium outcome of the strategic partnerships to the Policymaker's optima derived in section 3.2. Notice that in the contractual scenario, there are three possible sources of deviation from the RAN outcome, i.e. the risk and ambiguity neutral Policymaker's preferred outcome. Firstly, the static and dynamic moral hazard can potentially distort the incentives and make it harder for the project to obtain funding at every period, thereby creating a divergence from the optima the Policymaker intends to implement. Also, the presence of ambiguity and $CF's$ ambiguity aversion creates a divergence in preferences among the strategic alliance and the Policymaker, thus contributing to the difference from the RAN optima. Lastly, the short term contracting and lack of commitment can result in the contractual outcome being different that the RAN optima. Let us first examine how the two outcomes differ and then we will analyze the effects of each of these possible sources of inefficiencies.

The Policymaker's optimal value from the project is given by:

$$V_0^S = E_0 \left[\sum_{t=1}^{T_S} \delta^{t-1} \left(\Pr_t((r_t, s_t) \in \Delta_H)(p_t R - I) + \Pr_t((r_t, s_t) \in \Delta_S) L - \overline{K} \right) \right] \tag{25}$$

whereas the Policymaker's value from the project carried out by the strategic partnership is given by:

$$V_0^{SC} = E_0 \left[\begin{array}{c} \sum_{t=1}^{T} \delta^{t-1} [\Pr_t((r_t, s_t) \in \Delta_D)(p_t R - I) + \Pr_t((r_t, s_t) \in \Delta_L) L \\ -(1 - \Pr_t((r_t, s_t) \in \Delta_F))\overline{K} - \Pr_t((r_t, s_t) \in \Delta_F) K_t] \end{array} \right] \tag{26}$$

The contractual outcome diverges from the Policymaker's outcome in three ways:

(a) *Patent Troll:* If the posterior belief $(r_t, s_t) \in \Delta_L \subset \Delta_H$, the risk and ambiguity neutral Policymaker finds it optimal to develop the product, but because of $CF's$ ambiguity aversion $v > 0$, the strategic partnership liquidates the product even after being granted patent. So, every time the posterior lies in this region, there is a loss of value $p_t R - I - L > 0$ to the Policymaker. This loss is attributed to the difference in ambiguity attitude of the Policymaker and CF.

(b) *Less experimentation:* The Policymaker optimally stops experimentation and abandons the project as soon as the posterior belief enters Δ_S, while the research alliance abandons it when the posterior lies in Δ_S^C. Algebraically, it can be shown that $\Delta_S \subset \Delta_S^C$, so the research alliance abandons the project for a larger range of posterior beliefs, compared to the Policymaker. This result is due to the short termism, lack of commitment power of the research alliance, and the moral hazard problem.

(c) *Partial Funding:* The Policymaker optimally invests the maximal funding in the project till the end, whereas the research partnership, if the prior belief is not too high (if 23 is not satisfied), does not receive full funding till the end. The lower investment flow is driven by the static and dynamic moral hazard problem, which makes the incentive constraints harder to satisfy. However, as we have noted in Proposition 4, dynamic moral hazard problem is alleviated as v goes up, causing the project to receive maximal funding for a longer time horizon.

The next proposition captures how the equilibrium contractual outcome diverges from the Policymaker's optimal outcome.

PROPOSITION 5. *Compared to the Policymaker's optima, the equilibrium contracts governing the research alliances result in (a) liquidation of the project even after being patented, (b) less experimentation, and (c) lower investment flow.*

Proof. In Appendix A. ∎

The following figure illustrates the difference between the two outcomes.

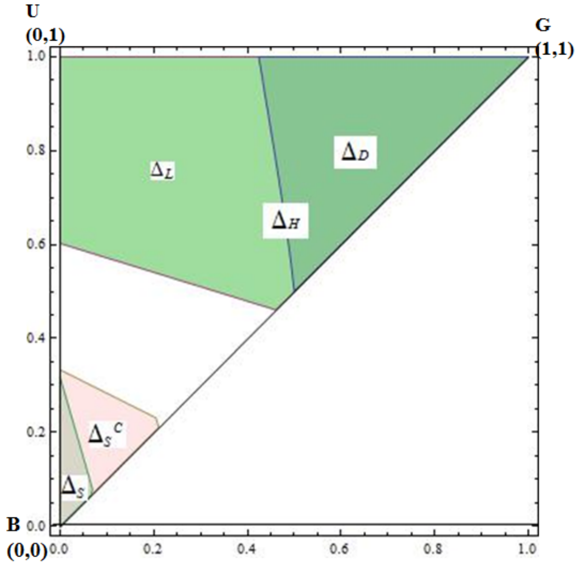

FIG. 6 Comparison Between Contractual Outcome and Policymaker's Outcome

Given that the contracts governing the strategic partnerships fail to implement the Policymaker's optima, next we examine if the Policymaker can restructure the patent law in order to implement its desired optima. Specifically, if the patent law is designed to internalize the possible response from the research alliances, is it possible to alleviate the three sources of inefficiency discussed above? Analyzing the effects of changing the patent law, we find that if the patent law is made stricter, i.e., Δ_H is set at a higher level, it will shrink Δ_L, so it is less likely that the project will be liquidated after being granted patent. However, this would lower the incentive to experiment as well, because $\Pr_t((r_t, s_t) \in \Delta_H)$ decreases, causing the research alliance to abandon the project even earlier (for a larger range of posteriors) than before. In fact, setting $\Delta_H = \Delta_D$ eliminates the possibility of patent troll, but increases the range of posteriors for which the project is abandoned forever; i. e. , Δ_S^C expands.

On the other hand, if the patent policy is relaxed, that boosts the incentive to invest in the project, increasing $\Pr_t((r_t, s_t) \in \Delta_H)$ at every period, and results in longer experimentation and higher level of investment. However, it also results in an expansion of Δ_L, so patent troll problem becomes more severe. Thus, changing the patent law can never fully implement the Policymaker's optima and eliminate all three sources of efficiency. If initially Δ_L is large, i.e., patent troll is a severe problem to start off with, then making the patent law more stringent benefits the Policymaker more, whereas if the inefficient stopping proves to be a more severe concern, then relaxing the patent policy would be beneficial. So, depending on the initial parameter values, the patent policy should be redesigned to consider the possible effects on the innovation conducted in the strategic alliances.

5. GENERALIZATIONS

First, we will discuss how the model behaves under a few possible extensions and alternative assumptions.

5.1. *Non-linear signal*

In this model, we used the simplifying assumption of linearity in the signal structure. This resulted in the Policymaker's optima characterized by full funding at all times.

With a more general signal structure satisfying only the Assumptions 1-3, we can characterize the optimal contractual outcome as well as the Policymaker's optima using similar technique. Instead of full funding, the optimal outcomes are characterized by a partial investment flow that decreases over time for the Policymaker as well as the strategic partnership. The regions Δ_D, Δ_L, and Δ_S^C can be characterized likewise. The main results qualitatively stays the same:

(a) $\Delta_L \neq \phi$, so Patent Troll happens if posterior lies in Δ_L.

(b) Optimal funding in strategic alliances decreases with time. As v increases, dynamic moral hazard is alleviated.

(c) Restructuring the patent law can not implement the Policymaker's outcome.

It is also interesting to examine a more general signal structure instead of the binary signal discussed in this paper. Indeed, in some real life contexts, the information flow that arrives at each period of experimentation can not be encoded into a simple binary signal. For example, assuming a continuous signal structure will generalize the model and consequently change the optimal contract structure.

5.2. *No Limited Liability of* RL

In the present model, the research lab is assumed to be liquidity constrained, thus always requires non-negative payment in each period. However, in many real life scenario, the research based firms, though smaller in comparison to the commercial giants, can afford to put forth some investment, in the form of collateral , in order to continue experimentation even after clearing the patent thresholds.

Under this assumption, experimentation continues even after clearing the patenting threshold, the patent troll region shrinks, and the alliance experiments longer.

5.3. *Long Term Contracts*

In some situations, firms can attain commitment power through brand reputations, press releases and a variety of other ways. If the contracting parties can commit to long term relations, the participation constraint of CF will not have to be met in every period, so intertemporal transfer of payments will be possible. This relaxes the funding condition at every period and results in longer experimentation. In this case, experimentation may continue even after being granted a patent and the patent troll region shrinks. It is interesting to compare the optimal outcome in long term contracting with the one in this paper and analyze the effect of commitment.

5.4. *Partially Observable Signal*

In many scenario, the informative signal is not publicly revealed. Sometimes, the financing firm hires experts to evaluate the reports given by the research firm, whose evaluation criteria varies from the research firm. It is also possible that the results from the experimentation can be mis-reported. In these cases, the assumption that the signal at each period is publicly observed breaks down. A very interesting question will be to characterize the contract under this partial observability and possible mis-reporting of the signals, using a mechanism design approach to this contracting environment.

6. DISCUSSION

In the innovation intensive industries, we observe that research partnership is increasingly becoming an important mode of organizing research. The results from this paper suggest that the policy making organizations should recognize the fact and be aware of how the innovation activity conducted in the research alliances is affected by the patent policy. Using the predictions from the theoretical model, we observe that relaxing the patent criteria is likely to result in longer experimentation, but at the same time the possibility of patent troll like cases increases; whereas if the patent law is made more stringent then the patented projects are more likely to be developed, but the research alliances stop experimenting inefficiently early. This result suggests that studying the present state of the industry, the patent authority should decide on the patent criterion.

Also, comparing the optimal contractual outcome and the Policymaker's optima, we can see that it is never possible to implement the Policymaker's optima. As the contextual ambiguity associated project increases, the divergence of the contractual outcome and the desired outcome increases. This suggests that the projects with high level of ambiguity can not be satisfactorily organized by research partnerships. Indeed, there can be projects, which the Policymaker deems profitable enough to invest in, that can be never funded in a research partnership. In innovative industries, the concern about important innovations not being carried out has long been voiced (Clayton Christensen, ITExpo, 2011). The industry's Internal Rate of Return Criterion and lack of foresight, are often blamed for not investing in innovative technologies.

This suggests a potential role of a regulatory body or the "State" as an entrepreneur. State intervention in innovation in the form of funding programs for smaller research oriented firms can support innovation organized in research firms. State programs for Small and Medium Enterprises (SMEs) and New Biotechnology Firms (NBFs) like Small Business Innovation Research (SBIR), 1982, Small Business Technology Transfer (STTR), 1992 have been able to fund numerous ventures by smaller research firms and touted as success(SBIR/STTR Impact Report, 2012). In the US, 57% of "basic research" is supported through Federal funding (2008) (source: NSF report, 2008). Programs such as these, providing funds to the research oriented smaller firms, lead to the development of the projects not otherwise funded (Mazzucato, 2013).

Another mode of organizing innovation when the research alliances can not efficiently carry it out is direct state initiative. There are several examples where State as an entrepreneur has participated in innovation and led to successful development of projects. In UK, Medical Research Council (MRC), funded by the Department for Business, Innovation and Skills (BIS) has been leading the Pharmaceutical innovation and was behind the development of monoclonal antibodies, widely used in Pharmaceutical industry since then. In the US, National Institute of Health (NIH) has been key funding source for research in Biotechnology, spending $30.9 bn in 2012 alone. Another example of State's entrepreneurial venture is National Nanotechnology Initiative (NNI), which, funded in 2000, strives to engage in cutting edge research in Nanotechnology. According to the famous adage by Polanyi (1944):

> "The road to the free market was opened and kept open by an enormous increase in continuous, centrally organized and controlled interventionism."

7. SUMMARY AND CONCLUSION

Research alliances are responsible for a major share of innovation activity in the research-intensive industries. The innovation processes they undertake is often characterized by ambiguity rather than risk. Given the prevalence of these research alliances in these sectors, it is important to examine the optimal research outcome that is generated in these R&D partnerships, understand the strategic incentives of the contracting parties and how these interact to shape the optimal choices, and to evaluate the research alliance as a mode of organizing research in the ambiguous environment. This paper provides a theoretical model to analyze these partnerships and compare it to the optimal outcome that a risk and ambiguity neutral Policymaker wants to implement.

In this paper, we consider a dynamic principal-agent model with moral hazard where the contracting parties differ in their attitude towards ambiguity. The contractees use short term contracts to organize innovation in the research alliances. To model the ambiguous preference, I follow a dynamically consistent framework of ambiguity that uses Bayes rule to consistently update ambiguous belief. We focus on Markov sequential equilibrium to characterize the optimal contract in this model of strategic experimentation with moral hazard.

Analyzing the optimal sequence of short term contracts, we find that the contractual terms facilitate ambiguity sharing and thus prevents in-equilibrium delay. The investment flow that the project receives decreases over time. We have shown that the Policymaker's optimal outcome can never be implemented in the research alliances. This leads us to suggest policy recommendations regarding the patent law.

Apart from the different extensions and robustness issues mentioned in the previous section, this research can open up the path of further research on strategic partnerships. It will be interesting to study multi-lateral strategic partnerships in the innovation-based industries as networks and examine the optimal network structure that emerges under ambiguity with different parametric assumptions. Also, analyzing different patent policies in this context under ambiguity constitutes another interesting direction for future research.

8. APPENDIX

8.1. Appendix A: Proofs

Proof of Lemma 1. The Policymaker's problem is recursively written as:

$$\begin{aligned} V^{RAN}(r,s) &= \max_{\Delta_H,\Delta_S,K^{RAN}} (\Pr((r',s')\in\Delta_H)(pR-I)+\Pr((r',s')\in\Delta_S)L-K) \\ &\quad +\delta E V^{RAN}(r',s') \end{aligned}$$

We can define the operator $T:\boldsymbol{C}(\mathbb{K}_{\Delta_{[0,1]}})\to\boldsymbol{C}(\mathbb{K}_{\Delta_{[0,1]}})$ as:

$$\begin{aligned} T(V^{RAN}) &= \max_{\Delta_H,\Delta_S,K^{RAN}} (\Pr((r',s')\in\Delta_H)(pR-I)+\Pr((r',s')\in\Delta_S)L-K) \\ &\quad +\delta E V^{RAN}(r',s') \end{aligned}$$

As $V^{RAN}(r,s)\le V^1(r,s)$ $\forall(r,s)\in\mathbb{K}_{\Delta_{[0,1]}}$, $T(V^{RAN})\le T(V^1)$ for all $(r,s)\in\mathbb{K}_{\Delta_{[0,1]}}$

as well.

Also, the discount factor $\delta \in (0, 1)$ ensures that

$$[T(V^{RAN} + a)](r, s) \leq T(V^{RAN})(r, s) + \delta a$$

for all $V^{RAN}, a \geq 0, (r, s) \in \mathbb{K}_{\Delta_{[0,1]}}$.

By Theorem 3.3 in Stokey, 1989, T satisfies Blackwell's sufficiency conditions: monotonicity and discounting, so it is a contraction. Then, by Contraction Mapping Theorem (Theorem 3.2 in Stokey, 1989), T has exactly one fixed point V^{RAN} that solves the Policymaker's problem. ∎

Proof of Lemma 2. Suppose $F_t(r_t, s_t) \geq F_{t+1}(r_t, s_t)$. Consider if $(r_t, s_t) \in \Delta_H$.

$$F_t(r_t, s_t) = p_t R - I$$

If $(r_{t+1}, s_{t+1}) \in \Delta_H$ for both $S_t = s_H$ and $S_t = s_L$, then for all j,

$$F_{t+j}(r_t, s_t) = \delta^j \left[p_t R - I - \sum_{s=t+1}^{t+j} K_s \right] \leq F_t$$

so the result follows.

If $(r_{t+1}, s_{t+1})|_{S_t = s_L} \in \Delta_S$ and $(r_{t+1}, s_{t+1})|_{S_t = s_H} \in \Delta_H$,

$$\begin{aligned}
F_{t+1}(r_t, s_t) &= \delta[\mu_{t+1}(p_{t+1}R - I) + (1 - \mu_{t+1})L - K_{t+1}] \\
&\leq p_t R - I \\
&\iff (1 - \delta)\mu_{t+1}(p_{t+1|H}R - I) \\
&\geq \delta(1 - \mu_{t+1})[L + p_{t+1|L}R - I] - K_{t+1}]
\end{aligned} \tag{27}$$

then,

$$F_{t+2}(r_t, s_t) = E_t \left[\delta^2 \max\{p_{t+2}R - I, L\} - \delta^2 K_{t+2} - \delta K_{t+1} \right]$$

Thus,

$$
\begin{array}{rl}
& F_{t+2}(r_t, s_t) - F_{t+1}(r_t, s_t) \\
= & E_t\left[\delta^2 \max\{p_{t+2}R - I, L\} - \delta^2 K_{t+2}\right] - \delta[\mu_{t+1}(p_{t+1}R - I) + (1-\mu_{t+1})L] \\
\leq & \delta \left[\begin{array}{c} \delta\mu_{t+2}\mu_{t+1}(p_{t+2}|_{HH}R - I) + 2\delta(1-\mu_{t+1})\mu_{t+2}(p_{t+2}|_{LH}R - I) \\ +\delta(1-\mu_{t+1})(1-\mu_{t+2})L \\ -[\mu_{t+1}(p_{t+1}R - I) + (1-\mu_{t+1})L] \end{array} \right] - \delta^2 K_{t+2} \\
= & \delta \left[\begin{array}{c} \delta\mu_{t+2}\mu_{t+1}(p_{t+2}|_{HH}R - I) + 2\delta(1-\mu_{t+1})\mu_{t+2}(p_{t+2}|_{LH}R - I) \\ +\delta(1-\mu_{t+1})(1-\mu_{t+2})L \\ -\mu_{t+2}\mu_{t+1}(p_{t+2}|_{HH}R - I) - (1-\mu_{t+2})\mu_{t+1}(p_{t+2}|_{LH}R - I) \\ -(1-\mu_{t+1})L \end{array} \right] - \delta^2 K_{t+2} \\
= & \delta \left[\begin{array}{c} (1-\mu_{t+1})\mu_{t+2}(p_{t+2}|_{LH}R - I)(2\delta - 1) \\ -(1-\delta)\mu_{t+2}\mu_{t+1}(p_{t+2}|_{HH}R - I) \\ -(1-\mu_{t+1})(1-\mu_{t+2})(1-\delta)L \end{array} \right] - \delta^2 K_{t+2} \\
= & \delta \left[\begin{array}{c} (1-\mu_{t+1})\mu_{t+2}(p_{t+2}|_{LH}R - I)\delta \\ -(1-\delta)\mu_{t+1}(p_{t+1}|_{H}R - I) \\ -(1-\mu_{t+1})(1-\mu_{t+2})(1-\delta)L \end{array} \right] - \delta^2 K_{t+2} \\
\leq & \delta \left[\begin{array}{c} -\delta(1-\mu_{t+1})((p_{t+1}|_{H} - p_{t+2}|_{LH})R - I) \\ -L(1-\delta)(1-\mu_{t+1})(1-\mu_{t+2}) \\ -K_{t+1} \end{array} \right] - \delta^2 K_{t+2} \\
& \text{(using 27)}
\end{array}
$$

Similarly, we can prove for $(r_t, s_t) \in \Delta_L$. ∎

Proof of Proposition 1. By Lemma 2, $A_t s$ form a monotone sequence, by Theorem 3.3 from Chow, Robbins, Siegmund (1971), the "One-stop ahead" rule is optimal, i.e., if stopping the experimentation process today is better than continuing experimenting for exactly one more period, then it is always optimal to stop today . Then, the optimal stopping rules are found by equating F_t and F_{t+1}.

If $p_t R - I \geq L$,

$$F_t(r_t, s_t) = F_{t+1}(r_t, s_t)$$

yields the equation:

$$\beta_{H1} r_t + \beta_{H2} s_t = \beta_{H3}$$

and if $p_t R - I < L$, we obtain:

$$\beta_{S1} r_t + \beta_{S2} s_t = \beta_{S3}$$

where:

$$
\begin{aligned}
\beta_{H1} &= R[1-\delta(2\lambda_G-\lambda_U)]+\delta 2\overline{K}(I+L)(\lambda_G-\lambda_U) \\
\beta_{H2} &= R[1-\delta\lambda_U]+\delta 2\overline{K}(I+L)(\lambda_U-\lambda_B) \\
\beta_{H3} &= 2I+2\overline{K}\delta(1-\lambda_B(I+L)) \\
\beta_{S1} &= \delta[R(2\lambda_G-\lambda_U)-2\overline{K}(I+L)(\lambda_G-\lambda_U)] \\
\beta_{S2} &= \delta[R\lambda_U-2\overline{K}(I+L)(\lambda_U-\lambda_B)] \\
\beta_{S3} &= 2L(1-\delta)+2\overline{K}\delta\lambda_B(I+L)
\end{aligned}
$$

∎

Proof of Proposition 2. Using a few lemmata leads us to the main result of the two period example, captured in Proposition 2.

Let us, for the sake of brevity, define:

$$
\begin{aligned}
T_1 &= \frac{1+\lambda_1 p_1\left(\frac{1}{(p_1-vq_1)\lambda_1}+\frac{I}{p_1-vq_1}\right)-\delta\left(\lambda_1 p_1-(1-\mu_1)\lambda_2 p_2\left(\frac{1}{(p_2-vq_2)\lambda_2}-\frac{I}{p_2-vq_2}\right)\right)}{\lambda_1 p_1-\delta\left(\lambda_1 p_1-(1-\mu_1)\lambda_2 p_2\right)} \\
T_2 &= \frac{1+\lambda_2 p_2\left(\frac{1}{(p_2-vq_2)\lambda_2}+\frac{I}{p_2-vq_2}\right)}{\lambda_2 p_2}
\end{aligned}
$$

The first step identifies the values of ambiguity aversion coefficient v for which (T_1-T_2) decreases with v.

Lemma 4. *If the discount factor is not too high, $\delta \leq \overline{\delta} < 1$, for all $v \in [0,1]$, as v increases, T_1-T_2 falls, where $\overline{\delta}$ is given by:*

$$
\overline{\delta} = 1-\left(\frac{p_2-vq_2}{p_1-vq_1}\right)^2\frac{q_1}{q_2}\left(\frac{\frac{1}{\lambda_1}+I}{\frac{1}{\lambda_2}+I}\right)
$$

The proof follows directly from taking derivatives. Next, we show that if CF is ambiguity neutral, then there is a possibility of delay.

Lemma 5. *For $v=0$, i. e. , if the principal is ambiguity neutral, then*

$$
T_1 > T_2
$$

So, in equilibrium delay happens whenever $T_1 > R \geq T_2$.

Proof. If $v = 0$,

$$\begin{aligned} T_1 &= \frac{2 + \lambda_1 I - \delta\left(\lambda_1 p_1 - (1-\mu_1)\lambda_2 I\right)}{\lambda_1 p_1 - \delta\left(\lambda_1 p_1 - (1-\mu_1)\lambda_2 p_2\right)} \\ T_2 &= \frac{2 + \lambda_2 I}{\lambda_2 p_2} \end{aligned}$$

Hence,

$$\begin{aligned} T_1 - T_2 &= \frac{[I\lambda_2 p_2 \lambda_1 \delta + \lambda_1 p_1(\delta - (1-\delta)\lambda_2 I)]}{(\lambda_1 p_1 - \delta\left(\lambda_1 p_1 - (1-\mu_1)\lambda_2 p_2\right))(\lambda_2 p_2)} \\ &\geq 0 \end{aligned}$$

∎

Next, we prove the existence of a threshold value of $v = \widetilde{v}$ for which delay does not happen.

Lemma 6. *There exists $\widetilde{v} \in (0,1)$ for which $T_1 = T_2$.*

Proof.

$$T_1 - T_2 = \frac{1}{(p_2 - vq_2)(p_1 - vq_1)} \left[\begin{array}{c} (p_2 - vq_2)\left(\frac{1}{\lambda_1} + I\right) - (1-\delta)(p_1 - vq_1)\left(\frac{1}{\lambda_2} + I\right) \\ -(p_1 - vq_1)(p_2 - vq_2)\left\{\lambda_1 p_1(1-\delta) + \lambda_2 p_2[\delta\lambda_1 p_1 + \delta(1-\mu_1) - 1]\right\} \end{array} \right]$$

For $v = 1$,

$$T_1 - T_2|_{v=1} = \frac{1}{(p_2 - q_2)(p_1 - q_1)} \left[\begin{array}{c} (p_2 - q_2)\left(\frac{1}{\lambda_1} + I\right) - (1-\delta)(p_1 - q_1)\left(\frac{1}{\lambda_2} + I\right) \\ -(p_1 - q_1)(p_2 - q_2)\left\{\lambda_1 p_1(1-\delta) + \lambda_2 p_2[\delta\lambda_1 p_1 + \delta(1-\mu_1) - 1]\right\} \end{array} \right]$$

Now,

$$(p_2 - q_2)\left(\frac{1}{\lambda_1} + I\right) - (1-\delta)(p_1 - q_1)\left(\frac{1}{\lambda_2} + I\right) \leq 0$$

$$\Leftrightarrow \delta \leq 1 - \frac{p_2 - q_2}{p_1 - q_1} \frac{\frac{1}{\lambda_1} + I}{\frac{1}{\lambda_2} + I} \tag{28}$$

And

$$\begin{aligned} & \lambda_1 p_1(1-\delta) + \lambda_2 p_2[\delta\lambda_1 p_1 + \delta(1-\mu_1) - 1] \\ = \; & (1-\delta)(\lambda_1 p_1 - \lambda_2 p_2) + \lambda_2 p_2 \delta[\lambda_1 p_1 - \mu_1] > 0 \end{aligned}$$

Since

$$
\begin{aligned}
1 - \frac{p_2 - q_2}{p_1 - q_1} \frac{\frac{1}{\lambda_1} + I}{\frac{1}{\lambda_2} + I} &> \overline{\delta} = 1 - \left(\frac{p_2 - q_2}{p_1 - q_1}\right)^2 \frac{q_1}{q_2} \left(\frac{\frac{1}{\lambda_1} + I}{\frac{1}{\lambda_2} + I}\right), \\
\forall \delta &\leq \overline{\delta}, T_1 - T_2|_{v=1} < 0
\end{aligned}
$$

So, $T_1 - T_2$ continuous in v and it decreases as v increases, $T_1 - T_2|_{v=0} > 0$ and $T_1 - T_2|_{v=1} < 0$, hence there must exist a $\widetilde{v} \in (0,1)$, for which $T_1 = T_2$. ∎

Proof of Proposition 3. Since CF stops experimenting the first time the posterior crosses the patenting threshold, RL only chooses the contract to offer depending on whether developing the project after being patented is more beneficial than liquidating. Thus, whenever $RL's$ expected payoff if CF develops the product: $\mu_t p_t \left[R - \frac{\left(I + \frac{1}{\lambda_t}\right)}{p_t - v q_t}\right]$ is greater than the expected payoff if CF liquidates: $L - \frac{1}{\lambda_t}$, he chooses

$$
x_t = 1 - \frac{\left(I + \frac{1}{\lambda_t}\right)}{R(p_t - v q_t)}, b_t \geq L - \frac{1}{\lambda_t}
$$

and the reverse otherwise. This gives us Δ_D, Δ_L.

The project is abandoned when no contract satisfying both the incentive constraint for RL and the participation constraint for CF can be offered. Combining both the constraints, it is most difficult to hold if $(r_t, s_t) \in \Delta_L$:

$$
L - \frac{1}{\lambda_t} \geq \frac{1}{\lambda_t}
$$

So, the project is abandoned if

$$
(r_t, s_t) \in \Delta_S^C = \{(r_t, s_t) | L < \frac{2}{\lambda_t}\}
$$

∎

∎

Proof of Proposition 4. The first lemma finds the sufficient conditions under which the project receives full funding till the end.

Lemma 7. *Sufficient condition for the project to obtain full funding till the end is:*

$$
\lambda_0 \geq \frac{2 - \delta}{L\left(1 - \frac{\delta}{2}\right)}
$$

∎

Let us look at the last period T, after which the project is abandoned forever. At T^{th} period, the incentive constraint binds:

$$\mu_T\left[L-\frac{1}{\lambda_T}\right]=\frac{1}{\lambda_T}$$

So,

$$EV_T(r_{T-1},s_{T-1})=K_T$$

At the penultimate period, the dynamic IC is:

$$\begin{aligned}\mu_{T-1}(L-\frac{1}{\lambda_{T-1}})-K_{T-1} \quad &\geq \quad \delta\left[\frac{\lambda_{T-1}}{\lambda_T}-(1-\mu_{T-1})\right]EV_T(r_{T-1},s_{T-1})\\ &\iff \quad \mu_{T-1}(L-\frac{1}{\lambda_{T-1}})-K_{T-1}\geq\delta\left[\frac{\lambda_{T-1}}{\lambda_T}-(1-\mu_{T-1})\right]K_T\end{aligned}$$

Clearly, this incentive constraint is most difficult to satisfy if $K_T=\overline{K}$. Thus, the project receives full funding till the end if:

$$\delta\leq\frac{\lambda_{T-1}L-2}{\lambda_{T-1}\left(\frac{L}{2}+\overline{K}\right)-1}$$

The sufficient condition becomes:

$$\lambda_0\geq\frac{2-\delta}{L\left(1-\frac{\delta}{2}\right)} \tag{4}$$

If 4 is violated, the project may not receive full funding till the end. Then, we want to characterize the switching point, i. e. the posterior beliefs for which the investment flow switches from full funding to partial funding. To characterize the equilibrium switching point, we derive the difference equation for $CF's$ funding decision, provided the IC_t^{RL} is binding under restricted funding.

Denote Δ_F = the region of posterior belief where the project does not receive full funding.

There are two cases: one when the switching point lies in the region of posterior beliefs where after being patented, the project is liquidated, i.e. $\Delta_F\cap\Delta_D=\phi$; and the other when at the switching point, after being granted a patent, the project is developed till the end, i.e., $\Delta_F\cap\Delta_D\neq\phi$.

First, let us focus on the case where at the switching point after being granted patent it is optimal to liquidate the project.

Lemma 8. *If $\Delta_F\cap\Delta_D=\phi$, then the switching point can be given as a quadratic* *equation in* (r_t, s_t) :

$$\Phi_L(r_t, s_t) = \gamma_{L1} r_t^2 + \gamma_{L2} s_t^2 + \gamma_{L3} r_t s_t + \gamma_{L1} = 0 \tag{29}$$

and

$$\begin{aligned} \Delta_F \quad &: \quad = \{(r_t, s_t) | \Phi_L(r_t, s_t) < 0\} \\ &= \quad \textit{the region of posteriors where the project does not receive full funding.} \end{aligned}$$

Proof. The expected value of RL along the equilibrium path can be represented as:

$$EV_t(r_{t-1}, s_{t-1}) = \mu_t(L - \frac{1}{\lambda_t}) + \delta(1 - \mu_t) EV_{t+1}(r_t, s_t) \tag{30}$$

Now, with restricted funding, IC_t^{RL} binds on the equilibrium path, so:

$$\mu_t(L - \frac{1}{\lambda_t}) - K_t = \delta \left[\frac{\lambda_t}{\lambda_{t+1}} - (1 - \mu_t) \right] EV_{t+1}(r_t, s_t)$$

∎

Using the Bayesian updating:

$$\begin{aligned} \lambda_{t+1} &= \frac{(\lambda_G - \lambda_U) r_{t-1}(1 - K_t \lambda_t) + 1 - \lambda_t K_t - (1 - s_{t-1})(1 - \lambda_B K_t)(\lambda_U - \lambda_B)}{1 - \lambda_t K_t} \\ &= \frac{A_t - B_t K_t}{1 - \mu_t} \end{aligned}$$

where A_t and B_t are expressions involving r_{t-1} and s_{t-1} and do not depend on K_t.

$$\begin{aligned} EV_{t+1}(r_t, s_t) &= \frac{\left(\mu_t(L - \frac{1}{\lambda_t}) - K_t \right)(A_t - B_t K_t)}{\delta(1 - \mu_t)\left[\lambda_t - (1 - \mu_t)(A_t - B_t K_t)\right]} \\ &= h_L(K_t) \end{aligned} \tag{31}$$

Taking derivatives, it can be shown that

$$\frac{\partial h_L}{\partial K_t} \leq 0 \tag{32}$$

Substituting 31 into 30, we obtain:

$$\begin{aligned} EV_t(r_{t-1}, s_{t-1}) &= \mu_t(L - \frac{1}{\lambda_t}) + \delta(1-\mu_t)h_L(K_t) \\ &= \mu_t(L - \frac{1}{\lambda_t}) + \frac{\left(\mu_t(L - \frac{1}{\lambda_t}) - K_t\right)(A_t - B_t K_t)}{\lambda_t - (1-\mu_t)(A_t - B_t K_t)} \end{aligned}$$

Moving it one period forward, an alternative expression for $EV_{t+1}(r_t, s_t)$ is found:

$$\begin{aligned} EV_{t+1}(r_t, s_t) &= \mu_{t+1}(L - \frac{1}{\lambda_{t+1}}) \\ &\quad + \frac{\left(\mu_{t+1}(L - \frac{1}{\lambda_{t+1}}) - K_{t+1}\right)(A_{t+1} - B_{t+1}K_{t+1})}{\lambda_{t+1} - (1-\mu_{t+1})(A_{t+1} - B_{t+1}K_{t+1})} \\ &= g_L(K_t, K_{t+1}) \end{aligned} \tag{33}$$

where it can be shown that

$$\frac{\partial g_L}{\partial K_t} \geq 0, \frac{\partial g_L}{\partial K_{t+1}} \leq 0.$$

Then, the difference equation with restricted funding is obtained by equating 30 and 33:

$$g_L(K_t, K_{t+1}) = h_L(K_t) \tag{34}$$

By Implicit function theorem,

$$\begin{aligned} \frac{dK_{t+1}}{dK_t} &= -\frac{\frac{\partial g_L}{\partial K_t} - \frac{\partial h_L}{\partial K_t}}{\frac{\partial g_L}{\partial K_{t+1}}} \\ &\geq 0 \end{aligned}$$

Thus, the difference equation 34 expresses K_{t+1} as an increasing function of K_t. This ensures the existence of a fixed point of the equation 34 at the full funding level, denoted by:

$$\mu_{t+1}(L - \frac{1}{\lambda_{t+1}}) + \frac{\left(\mu_{t+1}(L - \frac{1}{\lambda_{t+1}}) - \overline{K}\right)(A_{t+1} - B_{t+1}\overline{K})}{\lambda_{t+1} - (1-\mu_{t+1})(A_{t+1} - B_{t+1}\overline{K})} = \frac{\left(\mu_t(L - \frac{1}{\lambda_t}) - \overline{K}\right)(A_t - B_t\overline{K})}{\delta(1-\mu_t)\left[\lambda_t - (1-\mu_t)(A_t - B_t\overline{K})\right]}$$

which can be succinctly rewritten as the quadratic equation:

$$\Phi_L(r_t, s_t) = \gamma_{L1} r_t^2 + \gamma_{L2} s_t^2 + \gamma_{L3} r_t s_t + \gamma_{L1} = 0$$

This denotes the switching point. Δ_F is the area below the switching point:

$$\Delta_F := \{(r_t, s_t) \in \mathbb{K}_{\Delta_{[0,1]}} \backslash \Delta_S^C | \Phi_L(r_t, s_t) \leq 0\}$$

Next lemma establishes that the switching point given by 29 lies above the stopping threshold, i.e., $\Delta_S^C \subset \Delta_F$, and also at the switching point the project is liquidates after obtaining patent, i.e., $\Delta_F \cap \Delta_D = \phi$.

LEMMA 9. *The switching point locus always lies above the optimal stopping threshold, i. e.* $\Delta_S^C \subset \Delta_F$.

There exists a δ_L *such that, if* $\frac{\lambda_0 L - 2}{\lambda_0\left(\frac{L}{2} + \overline{K}\right) - 1} \leq \delta \leq \delta_L$*, at the switching point the project is liquidated after obtaining patent, i. e.* $\Delta_F \cap \Delta_D = \phi$.

Proof. we show that at the last period, the posterior belief lies below the switching point, which will show that $\Delta_S^C \subset \Delta_F$.

At $t = T$, $L = \frac{2}{\lambda_T}$. Plugging this in 29, it is shown that, if the sufficiency condition does not hold,

$$\begin{aligned} \Phi_L(r_T, s_T) &= \frac{\left(\frac{2}{L}(1+\overline{K}) - 2\right)\left(A_T - B_T\overline{K}\right)}{\frac{2}{L} - (1 - \frac{2\overline{K}}{L})(A_T - B_T\overline{K})} - \frac{\left(A_{T-1} - B_{T-1}\overline{K}\right)}{\delta(1 - \frac{2\overline{K}}{L})\left[\frac{2}{L} - (1 - \frac{2\overline{K}}{L})(A_{T-1} - B_{T-1}\overline{K})\right]} \\ &< 0 \end{aligned}$$

Similarly, the boundary of Δ_D is given by the locus where CF is indifferent between developing the product and liquidating after being granted a patent (say, at time $t = t_D$):

$$p_t \left[R - \frac{1}{p_t - vq_t}\left(I + \frac{1}{\lambda_t}\right)\right] = L - \frac{1}{\lambda_t}$$

plugging it in 29, we can show that

$$\Phi_L(r_{t_D}, s_{t_D}) \geq 0$$

if

$$\begin{aligned} \delta &\leq \delta_L \\ &= \frac{\left(A_{t_D} - B_{t_D}\overline{K}\right)}{\lambda_{t_D}\left[\lambda_{t_D} - (1 - \lambda_{t_D}\overline{K})(A_{t_D+1} - B_{t_D+1}\overline{K})\right]}. \end{aligned}$$

Now, we consider the second case: where at the switching point the project will be developed after being granted a patent. ∎

LEMMA 10. *If* $\Delta_F \cap \Delta_D \neq \phi$*, then the switching point can be given as a quadratic* *equation in* (r_t, s_t) :

$$\Phi_D(r_t, s_t) = \gamma_{D1} r_t^2 + \gamma_{D2} s_t^2 + \gamma_{D3} r_t s_t + \gamma_{D1} = 0 \tag{35}$$

and

$$\begin{aligned}\Delta_F \quad : \quad &= \{(r_t, s_t) | \Phi_D(r_t, s_t) < 0\} \\ &= \quad \textit{the region of posteriors where the project does not receive full funding.}\end{aligned}$$

The expected value of RL along the equilibrium path can be represented as:

$$EV_t(r_{t-1}, s_{t-1}) = \mu_t p_t \left[R - \frac{1}{p_t - vq_t} \left(I + \frac{1}{\lambda_t} \right) \right] + \delta(1 - \mu_t) EV_{t+1}(r_t, s_t)$$

Now, with restricted funding, IC_t^{RL} binds on the equilibrium path, so:

$$\mu_t p_t \left[R - \frac{1}{p_t - vq_t} \left(I + \frac{1}{\lambda_t} \right) \right] - K_t = \delta \left[\frac{\lambda_t p_t}{\lambda_{t+1} p_{t+1}} - (1 - \mu_t) \right] EV_{t+1}(r_t, s_t) \tag{36}$$

Using the expression for $\lambda_{t+1} p_{t+1}$:

$$\begin{aligned}\lambda_{t+1} p_{t+1} &= \frac{\lambda_G - \lambda_U}{\lambda_G + \lambda_B - 2\lambda_U} \lambda_t + \left(\lambda_U - 2 \frac{\lambda_G - \lambda_U}{\lambda_G + \lambda_B - 2\lambda_U} \right) p_t \\ &= F\lambda_t + G p_t\end{aligned}$$

we can rewrite 36 as:

$$\begin{aligned}EV_{t+1}(r_t, s_t) &= \frac{\mu_t p_t \left[R - \frac{1}{p_t - vq_t} \left(I + \frac{1}{\lambda_t} \right) \right] - K_t}{\delta \left[\frac{\lambda_t p_t}{F\lambda_t + G p_t} - (1 - \mu_t) \right]} \\ &= h_D(K_t)\end{aligned}$$

with

$$\frac{\partial h_D}{\partial K_t} \leq 0$$

Using the similar technique as in the case of deriving Δ_F, we obtain the difference equation with restricted funding as:

$$g_D(K_t, K_{t+1}) = h_D(K_t) \tag{37}$$

where

$$\begin{aligned}
EV_{t+1}(r_t, s_t) &= \mu_{t+1} p_{t+1} \left[R - \frac{1}{p_{t+1} - v q_{t+1}} \left(I + \frac{1}{\lambda_{t+1}} \right) \right] \\
&\quad + \frac{\mu_{t+1} p_{t+1} \left[R - \frac{1}{p_{t+1} - v q_{t+1}} \left(I + \frac{1}{\lambda_{t+1}} \right) \right] - K_{t+1}}{\delta \left[\frac{\lambda_{t+1} p_{t+1}}{F \lambda_{t+1} + G p_{t+1}} - (1 - \mu_{t+1}) \right]} \\
&= g_D(K_t, K_{t+1}) \qquad (38)
\end{aligned}$$

with

$$\frac{\partial g_D}{\partial K_t} \geq 0, \frac{\partial g_D}{\partial K_{t+1}} \leq 0.$$

Then, by Implicit function theorem,

$$\begin{aligned}
\frac{dK_{t+1}}{dK_t} &= -\frac{\frac{\partial g_D}{\partial K_t} - \frac{\partial h_D}{\partial K_t}}{\frac{\partial g_D}{\partial K_{t+1}}} \\
&\geq 0
\end{aligned}$$

Thus, the difference equation 37 expresses K_{t+1} as an increasing function of K_t. The fixed point can be written as the quadratic equation:

$$\Phi_D(r_t, s_t) = \gamma_{D1} r_t^2 + \gamma_{D2} s_t^2 + \gamma_{D3} r_t s_t + \gamma_{D1} = 0 \qquad (39)$$

This denotes the switching point. Also, denote the area below the switching point as:

$$\Delta_F := \{(r_t, s_t) \in \mathbb{K}_{\Delta_{[0,1]}} \backslash \Delta_S^C | \Phi_D(r_t, s_t) \leq 0\}$$

For $\delta > \delta_L, \Delta_S^C \subset \Delta_F$, and at the switching point the project is developed after obtaining patent.

Next lemma shows that $\Delta_D \backslash \Delta_F$ shrinks as v increases, i. e. as CF becomes more ambiguity averse, the project receives full funding for longer horizon under the case where at the switching point the project would be developed if granted a patent.

Lemma 11. *If $\Delta_F \cap \Delta_D \neq \phi$, then $\Delta_D \backslash \Delta_F$ shrinks as v increases.*

Proof. The switching point 39 is given as:

$$\Phi_D(r_t, s_t) = g_D(K_t, K_{t+1}) - h_D(K_t) = 0$$

Taking derivative with respect to v, it can be shown that

$$\begin{aligned}\frac{\partial \Phi_D}{\partial v} &= \frac{\mu_t p_t \left[\frac{q_t}{p_t - v q_t}\left(I + \frac{1}{\lambda_t}\right)\right]}{\delta\left[\frac{\lambda_t p_t}{F\lambda_t + G p_t} - (1-\mu_t)\right]} \\ &\quad -\mu_{t+1} p_{t+1}\left[-\frac{q_{t+1}}{p_{t+1} - v q_{t+1}}\left(I + \frac{1}{\lambda_{t+1}}\right)\right]\left[1 + \frac{1}{\delta\left[\frac{\lambda_{t+1} p_{t+1}}{F\lambda_{t+1} + G p_{t+1}} - (1-\mu_{t+1})\right]}\right] \\ &> 0\end{aligned}$$

Thus, as v increases, the project receives full funding for a longer time if $\delta > \delta_L$. ∎

Now, as v increases, the dynamic moral hazard decreases in the region where the project will be developed if patented. Thus, in the region $\Delta_D \backslash \Delta_F$, the project always receives full funding, and in the region Δ_F, investment gradually declines. This completes the proof of the proposition 4.

Proof of Proposition 5. Since $\Delta_L \neq \phi$, the project is liquidated even after being patented in that region.

The optimal stopping region for the Policymaker is:

$$\Delta_S = \{(r_t, s_t) | \beta_{S1} r_t + \beta_{S2} s_t < \beta_{S3}\}$$

where:

$$\begin{aligned}\beta_{S1} &= \delta[R(2\lambda_G - \lambda_U) - 2\overline{K}(I+L)(\lambda_G - \lambda_U)] \\ \beta_{S2} &= \delta[R\lambda_U - 2\overline{K}(I+L)(\lambda_U - \lambda_B)] \\ \beta_{S3} &= 2L(1-\delta) + 2\overline{K}\delta\lambda_B(I+L)\end{aligned}$$

For the partnership, the analogous region is:

$$\Delta_S^C = \{(r_t, s_t) | \lambda_t < \frac{2}{L}\}$$

At $r_t = s_t$, we can see the point on $\beta_{S1} r_t + \beta_{S2} s_t = \beta_{S3}$ is $r_S = s_S = \frac{L(1-\delta) + \overline{K}(I+L)\delta\lambda_B}{\delta R \lambda_G - \delta \overline{K}(I+L)(\lambda_G - \lambda_B)}$ and the point on $\lambda_t = \frac{2}{L}$ is $r_S^C = s_S^C = \frac{\frac{2}{L} - \lambda_B}{\lambda_G - \lambda_B}$. Even for $\delta = 1$, since $R > I$, it is always the case that (r_S^C, s_S^C) lies to the right of (r_S, s_S). Thus, $\Delta_S \subset \Delta_S^C$.

Proof. Also, we have already established in Proposition 4 that the project may not obtain full funding till the end, unlike the case with the Policymaker. ∎

∎

8.2. Appendix B: Ambiguity Framework

Denote the space of consequences as $\mathcal{X}$, which is a separable metric space with a topology that can be given by a metric making it complete. Let $C_b(\mathcal{X})$ denote the set of bounded, continuous functions on $\mathcal{X}$ with the supnorm topology, and $\Delta(\mathcal{X})$ be a weak* closed and separable, convex subset of the dual space of $C_b(\mathcal{X})$. Let $\mathbb{K}_{\Delta(\mathcal{X})}$ be the set of non-empty, compact, convex subsets of $\Delta(\mathcal{X})$ with the Hausdorff metric.

Then, a weak* continuous rational preference relation on $\mathbb{K}_{\Delta(\mathcal{X})}$ is a complete, transitive relation, $\succeq$, such that for all $B \in \mathbb{K}_{\Delta(\mathcal{X})}$, the sets $\{A : A \succ B\}$ and $\{B : B \succ A\}$ are open. The continuous linear preferences satisfy the Independence axiom given below.

AXIOM 1. *(Independence) For all $A, B, C \in \boldsymbol{K}_{\Delta(\mathcal{X})}$, and all $\beta \in (0, 1)$, $A \succeq B$ if and only if $\beta A + (1-\beta)C \succeq \beta B + (1-\beta)C$.*

Then, the representation theorem shows that a continuous rational preference relation on $\mathbb{K}_{\Delta(\mathcal{X})}$ satisfies Axiom 1 if and only if it can be represented by a continuous linear functional.

THEOREM 1 (Representation Theorem: Dumav and Stinchcombe, 2013). *A continuous rational preference relation on $\mathbb{K}_{\Delta(\mathcal{X})}$ satisfies Axiom 1 if and only if it can be represented by a continuous linear functional $L : \mathbb{K}_{\Delta(\mathcal{X})} \to \mathbb{R}$.*

Using this representation theorem, we can define the value of ambiguous information analogous to the risky case.

In a risky case, for an expected utility maximizing decision maker, the information they will have when making a decision can be encoded in a posterior distribution, $\beta \in \Delta(\mathcal{X})$. The value of β is

$$V_u(\beta) = \max_{a \in A} \int u(a, x) d\beta(x), \quad where \;\; u : A \times X \to \mathbb{R}.$$

In risky case, a prior is a point $p \in \Delta(\mathcal{X})$, and an information structure is a dilation of p, that is, a distribution, $Q \in \Delta(\Delta(X))$, such that

$$\int \beta dQ(\beta) = p.$$

The value of the information structure is given by

$$V_u(Q) := \int_{\Delta(\mathcal{X})} V_u(\beta) dQ(\beta)$$

An information structure Q dominates Q' if for all u, $V_u(Q) \geq V_u(Q')$.

Analogously, for vNM utility maximizing decision maker facing an ambiguous problem, the information they will have when making a decision can be encoded in a set of posterior distributions, $B \in \mathbb{K}_{\Delta(\mathcal{X})}$.

The value of B is

$$V_U(B) = \max_{a \in A} U(\delta_a \times B)$$

where $U : A \times \mathbb{K}_{\Delta(\mathcal{X})} \to \mathbb{R}$ is a continuous linear functional on compact convex subsets of $\Delta(A \times \mathcal{X})$ of the form $\delta_a \times B$ (where δ_a is point mass on a).

A set-valued prior is a set $A \in \mathbb{K}_{\Delta(\mathcal{X})}$,and an information structure is a distribution, $Q \in \Delta(\mathbb{K}_{\Delta(\mathcal{X})})$, such that

$$\int_{\mathbb{K}_{\Delta(\mathcal{X})}} BdQ(B) = A.$$

Then, the value of the information structure Q is given by

$$V_U(Q) := \int_{\mathbb{K}_{\Delta(\mathcal{X})}} V_U(B)dQ(B).$$

As above, an information structure Q dominates Q' if for all U, $V_U(Q) \geq V_U(Q')$.

This framework follows the standard Bayesian approach and models information structures as dilations. By contrast, previous work has limited the class of priors, A, and then studied a special class of dilations of each $p \in A$. The set of A for which this can be done is non-generic in both the measure theoretic and the topological sense, and the problems that one can consider are limited to ones in which the decision maker will learn only that the true value belong to some $E \subset \mathcal{X}$.

In this approach, A is expressed as a convex combination of/integral of B's in $\mathbb{K}_{\Delta(\mathcal{X})}$, and this is what makes the problem tractable and brings about dynamic consistency.

In a two-consequence case which will be considered in this paper, this approach simplifies to representing preferences as linear functionals in a simplex. If $\mathcal{X} = \{Good, Bad\}$, then $\mathbb{K}_{\Delta(\mathcal{X})}$ is the class of non-empty closed, convex subsets of the probabilities represented as a simplex:

$$\mathbb{K}_{\Delta(\mathcal{X})} = \{[p - r, p + r] : 0 \leq p - r \leq p + r \leq 1\}.$$

In this case, continuous linear functionals on the convex sets of probabilities must be of the form

$$U([a, b]) = u_1 a + u_2 b$$

for $u_{1,} u_2 \in \mathbb{R}$.

Rewriting $[a,b]$ as $[p-r,p+r]$, where $p=\frac{a+b}{2}$ and $q=\frac{b-a}{2}$ yields

$$U([p-r,p+r])=(u_1+u_2)p-(u_1-u_2)r=p-vr$$

with $v=u_1-u_2$ measuring the trade-off between risk and ambiguity, $v>0$ represents ambiguity averse attitude.

Graphically, a set-valued prior $[a,b]$ can be represented as a point in the simplex T with three vertices, $(0,0)$ representing *Bad* state, $(1,1)$ representing *Good* state and the new epistemic state "*Unknowable*" represented by the vertex $(0,1)$. Each $[a,b]$ has a unique representation as

$$(a,b)=w_{1,1}(1,1)+w_{0,1}(0,1)+(1-w_{1,1}-w_{0,1})(0,0)$$

solving,

$$w_{1,1}=a, w_{0,1}=b-a, w_{0,0}=1-b.$$

Thus, the prior $[a,b]$ assigns weight a on $(1,1)$, $1-b$ on $(0,0)$ and $b-a$ on the state $(0,1)$, i. e. , according to the decision maker, the evidence is thoroughly inconclusive with probability $(b-a)$.

In this setting, a signal is a dilation of the prior which enables Bayesian updating of the weights on each vertex of T. For example, if a binary signal $s\in\{s_1,s_2\}$, $\Pr(s=s_1|Good)=\eta_{1,1}$; $\Pr(s=s_1|Bad)=\eta_{0,0}$ and $\Pr(s=s_1|Unknowable)=\eta_{0,1}$, then the decision maker with prior $[a,b]$ updates his prior after observing s_1 as follows:

$$\begin{aligned}
\Pr(Good|s_1) &= \frac{\eta_{1,1}a}{\eta_{1,1}a+\eta_{0,0}(1-b)+\eta_{0,1}(b-a)}\\
\Pr(Bad|s_1) &= \frac{\eta_{0,0}(1-b)}{\eta_{1,1}a+\eta_{0,0}(1-b)+\eta_{0,1}(b-a)}\\
\Pr(Unknowable|s_1) &= \frac{\eta_{0,1}(b-a)}{\eta_{1,1}a+\eta_{0,0}(1-b)+\eta_{0,1}(b-a)}
\end{aligned}$$

Hence, posterior

$$[a',b']|_{s=s_1}=\left[\frac{\eta_{1,1}a}{\eta_{1,1}a+\eta_{0,0}(1-b)+\eta_{0,1}(b-a)},1-\frac{\eta_{0,0}(1-b)}{\eta_{1,1}a+\eta_{0,0}(1-b)+\eta_{0,1}(b-a)}\right]$$

In this paper we use this framework to model ambiguous decision making in the innovation process.

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