Informality, congestion and public capital efficiency: A case for optimal taxation and maintenance allocation

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Abstract

The informal sector is a typical feature of many developing countries. This sector comprises of unincorporated production units that do not fall under the ambit of tax administration. Although the sector provides income and employment to a large fraction of the labor force, it introduces new complexities into the economy. One such complexity stems from the fact that both formal and the informal sectors use public capital (like roads, railways, and power) in their production processes. But new investments in public capital and maintenance of existing public capital are financed by taxes levied only on the formal sector. This free rider problem has important implications on public capital efficiency. This paper sets up a two-sector endogenous growth model incorporating the informal sector, and examines the implications of efficiency of public capital on the optimal tax rate, and on the optimal allocation of tax revenues toward maintenance of public capital. Results show that in presence of the informal sector, the growth-maximizing tax rate in the decentralized economy is lower than Barro's tax rate. The welfare-maximizing optimal share of maintenance expenditure is shown to be positively related to the informal to formal output ratio. Thus countries having large informal sectors would benefit by allocating a greater amount of resources toward maintenance of public capital.

Keywords: Informal sector, public capital, efficiency, tax rate, maintenance.

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1 Introduction

The beneficial role of public infrastructure on economic activity has been well debated in the literature. Aschauer [1989] provided an initial estimate of 0.39 for the output elasticity of public capital. Later studies however, criticized this apparently high elasticity on several grounds. Gramlich [1994] argued that a 0.39 elasticity estimate implies a more than 100 percent marginal product of public capital. Since in theory, it is hard to refute the hypothesis that public investment fosters economic growth, studies have tried to incorporate different factors in order to bring down the large effect to a more plausible figure. Bom and Ligthart [2014] provides an excellent literature review and performing a meta analysis, they find a conditional short run elasticity estimate of 0.085 when controlled for heterogeneity across studies. The theoretical literature, on the other hand, was mainly propounded by Barro [1990] that formalized the role of public investment in an endogenous growth framework. Evidently, Aschauer's empirical study, coupled with Barro's theoretical approach, opened a new avenue of research in the growth literature. Other notable mentions are: Glomm and Ravikumar [1997], Devarajan et al. [1996] and, Futagami et al. [1993]. The treatment of public capital in these models has also evolved over the years. While most studies adhere to Barro's specification of a public capital flow that enters the production function as an input, studies like Futagami et al. [1993], Turnovsky [1997], and more recently Turnovsky [2004] and Chatterjee and Turnovsky [2007] formalize the concept of public capital stock that has its own accumulation process. The conceptual notion of public capital stock envisages an important role to maintenance of the existing stock owing to depreciation. The role of maintenance of public infrastructure was highlighted by the World Bank [1994] study, which said *"timely maintenance expenditure of \$12 billion would have saved road constructions costs of \$45 billion in Africa"*. The study also claimed that curbing maintenance expenditure in times of budgetary austerity is a wrong policy as high costs have to be incurred later for rehabilitation and reconstruction. Data from different countries also provide evidence that maintenance expenditures comprise a major part of the total spending on infrastructure.

Figure 1 shows the shares of maintenance expenditures and new investment in total spending on road infrastructure for select countries. It is evident that countries that spend nearly 1 percent of their GDP on an average on road infrastructure, allocate nearly 35 percent of the total spending on average to road maintenance and the rest 65 percent to building new road infrastructure. Interestingly, countries like Denmark and India devote more than 50 percent of the total spending on roads towards maintenance of existing roads.

The role of maintenance expenditure on economic growth, however, lacked a formal theoretical exposition until very recently. Rioja [2003] formally introduced the concept of maintenance by endogenizing the depreciation rate (as a function of maintenance expenditure) and showed that reallocating funds from new investment to maintenance has a positive effect on economic growth. However, in Rioja's model maintenance expenditure is funded by tax revenues while new investment in infrastructure is funded by foreign aid. This assumption precluded a discussion on optimal allocation of tax revenues between maintenance and new investment. Kalaitzidakis and Kalyvitis [2004] take into account government budgetary constraint and derive optimal allocations toward maintenance and new investments. But their model departs from household optimizing behavior ignoring the possibility of welfare loss through higher taxation. Dioikitopoulos and Kalyvitis [2008] expound on the implication of public capital on the trade-off between maintenance expenditure and new investments. They use an endogenous growth model to show that countries facing low congestion in public infrastructure would require a threshold level of maintenance expenditure to experience a balanced growth in output. On the other hand, countries facing high congestion would require a threshold level of new investments for balanced growth. Agenor [2009] departs from the previous studies and models maintenance expenditure through an additional efficiency parameter keeping intact the concept of endogenous depreciation rate as in Rioja [2003]. He shows that the growth-maximizing tax rate in a decentralized economy is equal to the output elasticity of public capital as in the Barro model. The welfare-maximizing share of spending on maintenance is shown to be identical to the growth-maximizing share when the tax rate is set at the level implied by the Barro rule. There are only a handful of empirical studies that test the role of maintenance expenditures on economic activity. Owing to unavailability of cross country data on maintenance expenditures, Kalyvitis and Vella [2015] asses the productivity effects of infrastructure's operations and maintenance spending by 48 U.S states over the period 1978-2000. They find a positive and significant output elasticity of maintenance expenditures when controlling for cross-state spillovers. Kalaitzidakis and Kalyvitis [2005] use Canadian data and show that the Canadian economy would benefit by altering the allocation between maintenance expenditures and new investments in public infrastructure.

The studies cited above however fail to provide a perspective on developing countries. An important feature of a developing country is the existence of a large informal sector and its equally large contribution to the GDP. In India for example, the unregistered and unincorporated small production units that usually do not fall under the tax administration, contribute almost 55 percent to the GDP. Figure 2 shows how the informal sector in India has evolved during the period 2004-2011, along with the spending patterns on road infrastructure during the same period. As can be seen from the figure, there has been a slight decrease in the contribution of the informal sector to the GDP, while the share of maintenance spending in total spending on road infrastructure has increased during the period 2008-2011. This paper tries to incorporate the informal sector into a two-sector endogenous growth model and examine it's implication on optimal fiscal policies. The objective of this paper is to draw some inference regarding the optimal tax rate, and the optimal allocation of tax revenues toward maintenance and new investments for a country characterized by a large informal sector. This is the main contribution of this study.

But what role does the informal sector play in determining the optimal tax rate and the optimal allocation of spending toward maintenance of public capital? The mechanism through which the informal sector affects the provision of infrastructure and maintenance spending is through congestion and efficiency. Since the informal sector firms usually do not fall under the tax administration, the government's main source of tax revenues is the formal sector. Tax revenues that are collected from the formal sector are then spent on infrastructure (like roads, railways, and power). Spending on infrastructure has two components: new investments and maintenance. Congestion of public infrastructure stems from the non-excludable nature of public infrastructure that can be used by the formal sector and informal sector in their production processes. For example, the benefits of building a new road (or the benefits of maintaining a good condition of roads) accrue to both formal and informal productions units as none of them can be excluded from using it. But financing new investments and maintenance of existing infrastructure are however borne out of the tax revenues that are collected from the formal sector. Put differently, the informal sector poses a free-rider problem by using public infrastructure that is financed by taxes levied on the formal sector. Figure 3 provides cross-country evidence of a positive correlation between the size of the informal sector (using the share of informal sector employment in the total labor force as a proxy) and power outages faced by a firm in a typical month. Figure 4 shows a similar correlation between the size of the informal sector and the value lost due to electrical outages. Both these figures insinuate to some degree of congestion faced by economies with large informal sectors. This paper contributes to the literature on public capital maintenance and economic growth by incorporating congestion effects posed by the informal sector in developing countries.

An implication of such a congestion effect is the loss of efficiency of public infrastructure which in turn has a negative effect on overall production. In addition to this, corrupt practices, inefficient appraisal processes, and fund disbursal systems also make public investment inefficient in developing countries. So, to contend that higher public investment has a positive impact on output and productivity would be wrong if not controlled for inefficiency of public investments. An interesting study by Hulten [1996] showed that a large portion of differential growth rate between Africa and East Asia can be explained by the difference in effective use of infrastructure capital in the two regions. Pritchett [2000] argued on methodological grounds that not all of public investments translate into capital which is quite pertinent to developing countries. Dabla-Norris et al. [2012] expounded on this idea and created the Public Investment Management Index (PIMI). Using the PIMI, Gupta et al. [2014] produced cross country estimates for efficiency adjusted public capital. The estimated factor share of efficiency adjusted public capital was in the range of 0.143 to 0.158. Figure 5 shows a negative correlation between the size of the informal sector and the PIMI scores that underscores the fact that large informal sectors reduce the efficiency of public infrastructure. Although these studies prove in a way that inefficiency in public capital is a major hindrance to output and productivity, few studies have looked into the theoretical aspects of efficiency of public capital. Chakraborty and Dabla-Norris [2009] made a serious effort in formalizing a model with the efficiency parameter. They argue that simply increasing public investment could be highly inefficient in low income countries where effort must be put in to maintain quality of public investment. Agenor [2009] also models the maintenance expenditure through an additional efficiency parameter keeping intact the concept of endogenous depreciation rate as in Rioja [2003].[1] This paper extends Agenor [2009] framework by defining the efficiency parameter as a function of the share of maintenance expenditure in total spending, taking into account the congestion effect of public capital due to the informal sector. In my model, the efficiency of public infrastructure increases with the share of maintenance expenditure and decreases with the size of the informal sector relative to formal sector. This is the second major contribution of this paper.

Results in the paper show that the growth-maximizing tax rate is a function of the output elasticities of public infrastructure (for formal and the informal sectors), and the efficiency parameter which is exogenously given in the decentralized economy. This tax rate is however lower than the Barro (1990) tax rate which is equal to the output elasticity of public

[1] Loayza [1996]incorporated the informal sector into the Barro type model and showed that the optimal taxation should be less than that in the Barro model in the presence of the informal sector, but fails to recognize the efficiency and maintenance effects of the informal sector. Penalosa and Turnovsky [2005] analyzed the burden of taxation on labor and capital when one of the sectors in the economy cannot be taxed but does not take into account the effects of over utilization of public capital on efficiency and maintenance expenditure.

infrastructure in a one-sector growth model. The growth-maximizing share of maintenance expenditure is shown to be a function of the production elasticities, the efficiency elasticity of maintenance expenditure, and the responsiveness of depreciation rate to maintenance spending. The welfare maximizing tax rate and the share of maintenance are not separately identified in a centrally planned economy. But if the social planner imposes a tax rate that maximizes the decentralized growth rate, the welfare-maximizing share of maintenance share is shown to be positively related to the ratio of informal to formal sector output. Thus economies with large informal sectors would benefit by devoting more resources toward maintenance of existing public infrastructure. This paper, by deriving the optimal tax rate and optimal share of maintenance would help the policy makers in developing countries to optimally spend resources on public infrastructure.

The rest of the paper is organized as follows. Section 2 lays out the analytical framework. Section 3 derives the welfare-maximizing equilibrium for a centrally planned economy. Section 4 describes the results for a decentralized economy. Section 5 discusses an optimal fiscal policy possibility for the social planner. Section 6 concludes.

2 Analytical Framework

We consider a closed economy with two sectors: formal and informal. The formal sector produces relatively more capital intensive goods as compared to the informal sector. We would assume the two sectors are populated by different set of individuals who consume either the formal sector good or the informal sector good but not both. In other words, we are assuming that the two markets function independently of each other. Although it seems a restrictive assumption, but it is useful to think the informal sector being populated by low-income individuals who cannot afford to buy high-quality capital intensive goods that are relatively expensive than the informal sector goods. The high income individuals, however, can afford to buy informal sector goods. But owing to the low-quality of these goods, the representative agent in the formal sector has a strict preference for the formal sector good over the informal sector good. The formal sector representative agent consumes C_F of the formal sector good. The informal sector representative agent consumes C_I of the informal sector good. Henceforth, formal and informal sectors are indexed by F and I respectively. The utility functions for the formal and informal representative agents are given by:

$$U_F = \int_0^\infty [\frac{1}{\gamma} C_F^\gamma] \, e^{-\rho t} dt, \ -\infty < \gamma \leq 1 \tag{1}$$

$$U_I = \int_0^\infty [\frac{1}{\gamma} C_I^\gamma] \, e^{-\rho t} dt, \ -\infty < \gamma \leq 1 \tag{2}$$

Each agent in the formal sector produces a private good whose output is given by Y_F, using a Cobb-Douglas production function. Private capital (K_F) and the economy-wide stock of effective public capital (eK_G) serve as the factors of production. The actual stock of public capital is adjusted with an efficiency parameter e (discussed below) to arrive at an effective stock of public capital. The informal sector is characterized by a similar technology. The production functions of the formal and informal sectors are given by:

$$Y_F = (eK_G)^\alpha K_F^{1-\alpha} \tag{3}$$

$$Y_I = (eK_G)^\beta K_I^{1-\beta} \tag{4}$$

Note that the input elasticities (α and β) are different in the formal and informal sectors. If we assume that formal sector firms are relatively more capital intensive than the informal sector firms then $1-\alpha > 1-\beta$. But since this assumption does not have any consequence on the analytical framework, we abstain from making such *a priori* assumption. It is important however, to contend different output elasticities of public capital for the formal and informal sectors. Since public capital is indivisible and non-excludable in nature, it is available to both the sectors. But the rival nature of public capital makes the degree of accessibility of the public capital different for the two sectors as captured by α and β, where $\alpha \neq \beta$. This is an important distinction of this study from the specification used by Loayza [1996] where it is assumed that the informal sector uses a fraction of public capital. It is this fact that drives congestion in public goods. Private capital depreciates at the rate $\delta_i \in (0, 1)$. We would assume the depreciation rate as exogenously given for the two sectors. If I_i is investment, then the evolution of private capital is given by:

$$\dot{K_i} = I_i - \delta_i K_i, \; i \in (F, \, I). \tag{5}$$

Government

The government provides public capital, the evolution of which is given by :

$$\dot{K_G} = I_G - \delta_G K_G \tag{6a}$$

where I_G is investment in public capital and δ_G is the depreciation rate. As in Rioja [2003], we assume the depreciation to be negatively related to maintenance expenditure. Additionally, we also assume that depreciation is directly proportional to the stock of public capital (which is used as a scaling factor as in Agenor [2009]. For simplicity we assume the functional form to be linear given by:

$$\delta_G = \bar{\delta_G} - \theta_G.(\frac{M}{K_G}) \, , \, \theta_G \in (0, 1) \tag{6b}$$

Equation (6b) implies that when $M = 0$, $\delta_G = \bar{\delta_G}$, where $\bar{\delta_G}$ denotes the maximum depreciation rate. Each period the government invests I_G in public capital and allocates some amount M on maintenance which is funded from tax revenues collected from the formal sector. The informal sector does not pay taxes. Let τ be the output tax rate. Let v_g and v_m be the proportion of taxes that are allocated to new investment in public capital and maintenance respectively. Thus we have,

$$I_G = \upsilon_g(\tau Y_F) \tag{6c}$$

$$M = \upsilon_m(\tau Y_F) \tag{6d}$$

The government budget constraint is given by :

$$I_G + M = \tau Y_F \Rightarrow \upsilon_g + \upsilon_m = 1 \tag{6e}$$

Combining (6a)-(6e) we have,

$$\frac{\dot{K_G}}{K_G} = \upsilon_g(\tau Y_F/K_g) + \theta_G(\frac{\upsilon_m \tau Y_F}{K_G}) - \bar{\delta_G} \tag{6f}$$

Effective public capital stock

The public capital stock may be non-excludable but its efficiency is a function of maintenance expenditure and the size of the informal sector. Specifically, the efficiency of public capital increases with higher maintenance expenditure and decreases with the size of the informal sector. The negative relationship between public capital efficiency and the size of the informal sector stems from the fact that both maintenance and new investment on public capital are financed by taxes levied on the formal sector. The informal sector thus poses a free rider problem that negatively affects the efficiency of public infrastructure. Because of this over utilization, efficiency of public capital goes down. We consider a concave function for the efficiency parameter given by:

$$e = (\frac{M}{Y_I})^{\chi},\ \chi \in (0,1) \tag{7a}$$

where χ measures the elasticity of maintenance with respect to efficiency. Substituting for $M = \upsilon_m(\tau Y_F)$ in (7a), the efficiency parameter can be characterized by a function:

$$e = f(\chi, \tau, \upsilon_m, s_F)\,;\; s_F = Y_F/Y_I. \tag{7b}$$

where $e_\chi < 0$; $e_\tau > 0$; $e_{\upsilon_m} > 0$; 0; $e_{s_F} > 0$. Thus an increase in tax rate increases the efficiency of the existing public capital as more resources become available for maintenance. A reallocation of resources toward maintenance as captured by υ_m, increases efficiency of public capital by increasing overall maintenance expenditure. As the size of the informal sector increases relative to the formal sector, efficiency of public capital decreases. Lastly, efficiency decreases with an increase in the efficiency elasticity of maintenance (as $0 < \frac{M}{Y_I} < 1$). A closed form solution for the efficiency parameter (as a function of the inputs to the production functions) is derived in the Appendix. Substituting for the efficiency parameter, the production functions can be represented as:

$$Y_F = (\upsilon_m \tau)^{\chi\alpha/\eta}(K_G)^{\epsilon_1}(K_F)^{\epsilon_2}(K_I)^{\epsilon_3} \tag{8}$$

$$Y_I = (\upsilon_m \tau)^{\chi\beta/\eta}(K_G)^{\epsilon_4}(K_F)^{\epsilon_5}(K_I)^{\epsilon_6} \tag{9}$$

3 A Centrally Planned Economy

The social planner in the centrally planned economy internalizes the congestion *ex ante* by optimally choosing the tax rate and the share of maintenance expenditure in total spending. In other words, the planner optimizes the efficiency of public capital by choosing the policy instruments at his disposal. Additionally, the planner faces the market clearing condition given by equation (10a). He also faces the capital accumulation constraint for the economy as a whole given by (10b).

Market clearing condition:

$$Y_F + Y_I = C_F + C_I + I_F + I_I + I_G + M \tag{10a}$$

$$\Rightarrow (1 - \tau v_m) Y_F + Y_I - C_F - C_I = I_F + I_I + I_G$$

Capital accumulation constraint:

$$\dot{K_F} + \dot{K_I} + \dot{K_G} = (1 - \tau v_m) Y_F + Y_I - C_F - C_I - \delta_F K_F - \delta_I K_I - \bar{\delta}_G K_G + \tau v_m \theta_G Y_F \tag{10b}$$

The planner's utility function takes the form:

$$\underset{C_I, C_F}{max} U = \int_0^\infty \frac{1}{\gamma} [C_F^\gamma + C_I^\gamma] \, e^{-\rho t} dt \tag{10c}$$

The planner makes the resource allocation decision for the representative agents by choosing consumption (C_F, C_I), tax rate (τ), share of maintenance expenditure in total spending (v_m), and the accumulation of private capital in the two sectors (K_F, K_I) and public capital (K_G) by maximizing (10c) subject to the resource constraint given by (10b). The optimality conditions are given by the following equations.

$$C_F^{\gamma-1} - \lambda = 0 \tag{11a}$$

$$C_I^{\gamma-1} - \lambda = 0 \tag{11b}$$

$$[-v_m + \theta_G v_m] Y_F \tau + \frac{\alpha \chi}{\eta} (1 - \tau v_m + \tau v_m \theta_G) Y_F + \frac{\beta \chi}{\eta} Y_I = 0 \tag{11c}$$

$$[-\tau + \theta_G \tau] Y_F v_m + \frac{\alpha\chi}{\eta}(1 - \tau v_m + \tau v_m \theta_G) Y_F + \frac{\beta\chi}{\eta} Y_I = 0 \tag{11d}$$

$$(1 - \tau v_m + \tau v_m \theta_G)\epsilon_2 \frac{Y_F}{K_F} + \epsilon_5 \frac{Y_I}{K_F} - \delta_F = \rho - \frac{\dot{\lambda}}{\lambda} \tag{11e}$$

$$(1 - \tau v_m + \tau v_m \theta_G)\epsilon_3 \frac{Y_F}{K_I} + \epsilon_6 \frac{Y_I}{K_I} - \delta_I = \rho - \frac{\dot{\lambda}}{\lambda} \tag{11f}$$

$$(1 - \tau v_m + \tau v_m \theta_G)\epsilon_1 \frac{Y_F}{K_G} + \epsilon_4 \frac{Y_I}{K_G} - \bar{\delta_G} = \rho - \frac{\dot{\lambda}}{\lambda} \tag{11g}$$

where $\epsilon_1,\ \epsilon_2,\ \epsilon_3,\ \epsilon_4,\ \epsilon_5,\ \epsilon_6$ are the parameters from the production functions.[2] The optimality conditions (11a)-(11g) can be interpreted as follows. Equations 11a and 11b equate the marginal utility of consumption for the formal and informal sectors to the shadow price of private capital. Equations (11c) and (11d) are the optimality conditions with respect to tax rate and the share of maintenance expenditure respectively. Equations (11e) and (11f) equate the rate of return of private capital in the formal and informal sectors to the corresponding return on consumption in the two sectors. Equation (11g) equates the rate of return on public capital to the corresponding return on consumption. The equilibrium growth rate ($\frac{\dot{C_F}}{C_F} = \frac{\dot{C_I}}{C_I} = \frac{\dot{K_F}}{K_F} = \frac{\dot{K_I}}{K_I} = \frac{\dot{K_G}}{K_G}$), which has many equivalent forms, can be written as:

$$\psi^* = \frac{(1 - \tau v_m + \tau v_m \theta_G)\epsilon_2 \frac{Y_F}{K_F} + \epsilon_5 \frac{Y_I}{K_F} - \delta_F - \rho}{1 - \gamma} \tag{12}$$

We solve equations (11c) and (11d) to find the optimal tax rate and the share of maintenance. The social planner however cannot identify the two instruments separately. So, the product of optimal maintenance and tax rate is given by:

[2] See Appendix A

$$(\tau v_m)^{**} = \frac{\beta\chi(Y_I/Y_F)+}{(1-\theta_G)(1+\beta\chi)} + \frac{\alpha\chi}{(1-\theta_G)(1+\beta\chi)} \tag{13}$$

Proposition 1. *Under a centrally planned economy, the optimal tax rate and the share of maintenance expenditure in total spending are not separately identified. The government can arbitrary set either the tax rate or the share of maintenance spending such that (13) always satisfies. The product τv_m is however identified which is a function of: (i) the ratio of informal to formal sector output (Y_I/Y_F), (ii) the output elasticities of public capital in the two sectors (α, β),(iii) the elasticity of maintenance expenditure with respect to public capital efficiency to the formal sector output (χ) and, (iv) the effectiveness of maintenance expenditure to depreciation rate (θ_G).*[3]

4 A Decentralized Economy

In this section we consider a decentralized economy where the government assumes a passive role whereas the representative agents in the two sectors make their own resource allocation decisions. In this set up, the government provides the entire stock of public capital using the policy instruments at its disposal: the tax rate and the share of maintenance expenditure in total spending. The representative agents take the stock of public capital as exogenously given when making their own resource allocation decisions with respect to consumption and accumulation of private capital stock. Additionally, the representative agents do not internalize the sources of congestion. They assume efficiency of public capital as exogenously given and they have no influence over it. Since there are two production sectors we have two different optimization problems: one for the formal and one for the

[3]The comparative statics discussion will be part of the optimal fiscal policy discussion in Section 5 where we derive a specific solution for the welfare-maximizing v_m.

informal sector. The utility functions are given as in (1) and (2).

Formal sector

Household problem is characterized by :

$$\underset{C_F}{max} U = \int_0^\infty \frac{1}{\gamma} C_F^\gamma e^{-\rho t} dt$$

subject to the budget constraint,

$$C_F + I_F = (1-\tau) Y_F$$

and

$$\dot{K_F} = I_F - \delta_F K_F$$

The household solves the optimization problem taking as given the depreciation rate of the private capital, the tax rate, the discount rate and the effective stock of public capital. Equations (14a) and (14b) describe the optimality conditions for the formal sector:

$$C_F^{\gamma-1} - \lambda = 0 \tag{14a}$$

$$\lambda[(1-\tau)(1-\alpha)(eK_{GF})^\alpha - \delta_F] = \rho\lambda - \dot{\lambda} \tag{14b}$$

Equation (14a) equates the marginal utility of consumption to the shadow price of private capital. Equation (14b) equates the rate of return on private capital in the formal sector to the corresponding return on consumption.

Combining the first order conditions and using (8), we find the balanced growth path ($\frac{\dot{C_F}}{C_F} = \frac{\dot{K_F}}{K_F}$) as:

$$\psi_F = \frac{(1-\tau)(1-\alpha)(\upsilon_m \tau)^{\chi\alpha/\eta}(K_G)^{\epsilon_1}(K_F)^{\epsilon_2 - 1}(K_I)^{\epsilon_3} - \delta_F - \rho}{1-\gamma} \tag{15}$$

The growth path ($\frac{\dot{C_F}}{C_F} = \frac{\dot{K_F}}{K_F}$) can also be represented by the growth path of the stationary variable $c_F = \frac{C_F}{K_F}$, where

$$\frac{\dot{c_F}}{c_F} = \frac{(\gamma - \alpha)(1-\tau)(\upsilon_m \tau)^{\chi\alpha/\eta}(K_G)^{\epsilon_1}(K_F)^{\epsilon_2 - 1}(K_I)^{\epsilon_3} - \gamma\delta_F + (1-\gamma)c_F - \rho}{1-\gamma} \tag{16}$$

where $c_F = \frac{C_F}{K_F}$.

It is interesting to note that output growth rate in the formal sector is a function of public capital (K_G), the informal sector private capital (K_I) and its own private capital (K_F).

Informal sector

We proceed with the informal sector optimization problem in the same way as the formal sector. The only difference here is that the informal sector does not pay taxes, so the budget constraint looks different. Assuming the same utility function, the optimization problem for the informal sector is given by:

$$\underset{C_I}{max} U = \int_0^\infty \frac{1}{\gamma} C_I^\gamma e^{-\rho t} dt$$

subject to the budget constraint,

$$C_I + I_I = Y_I$$

and

$$\dot{K_I} = I_I - \delta_I K_I$$

The informal sector household solves the optimization problem taking as given the depreciation rate of the private capital, the discount rate and the effective stock of public capital. Equations (17a) and (17b) describe the optimality conditions for the formal sector:

$$C_I^{\gamma-1} - \lambda = 0 \tag{17a}$$

$$\lambda[(1-\beta)(eK_{GI})^{\beta}-\delta_I]=\rho\lambda-\dot{\lambda} \tag{17b}$$

The optimality conditions (17a) and (17b) have the same interpretations as above. Combining the first order conditions and using (9), we find the balanced growth path ($\frac{\dot{C}_I}{C_I}=\frac{\dot{K}_I}{K_I}$) as:

$$\psi_I=\frac{(1-\beta)(v_m\tau)^{\chi\beta/\eta}(K_G)^{\epsilon_4}(K_F)^{\epsilon_5}(K_I)^{\epsilon_6-1}-\delta_I-\rho}{1-\gamma} \tag{18}$$

It is interesting to note the endogeneity of the growth process for both the formal and the informal sectors. Both sector's growth rates are functions of the stock of formal and informal private capital, and the stock of effective public capital. The informal sector growth is also a function of the tax rate and the fraction of tax revenue allocated to maintenance. This may seem paradoxical at first but it actually makes intuitive sense. Since, maintenance expenditure increases efficiency of public capital, the informal sector output increases for any given level of public capital stock. Basically, the informal sector also reaps the benefits of higher maintenance expenditure on public capital. The growth path ($\frac{\dot{C}_I}{C_I}=\frac{\dot{K}_I}{K_I}$) can also be represented by the growth path of the stationary variable $c_I=\frac{C_I}{K_I}$. Thus combining equations (17b) and (18) we have,

$$\frac{\dot{c}_I}{c_I}=\frac{(-\beta)(v_m\tau)^{\chi\beta/\eta}(K_G)^{\epsilon_4}(K_F)^{\epsilon_5}(K_I)^{\epsilon_6-1}-\gamma\delta_I+(1-\gamma)c_I-\rho}{1-\gamma} \tag{19}$$

Using (6f), (8) and (9) we also have,

$$\frac{\dot{K}_G}{K_G}=\tau(v_G+\theta_G v_m)(v_m\tau)^{\chi\alpha/\eta}(K_G)^{\epsilon_1-1}(K_F)^{\epsilon_2}(K_I)^{\epsilon_3} \tag{20}$$

4.1 Growth maximizing tax rate: decentralized economy

The objective is to find a tax rate that maximizes the growth rate in the decentralized economy given by:

$$\psi_F = \frac{(1-\tau)(1-\alpha)(v_m\tau)^{\chi\alpha/\eta}(K_G)^{\epsilon_1}(K_F)^{\epsilon_2-1}(K_I)^{\epsilon_3} - \delta_F - \rho}{1-\gamma} \tag{15}$$

which can also be characterized by,

$$\psi_F = \tau(v_G + \theta_G v_m)(v_m\tau)^{\chi\alpha/\eta}(K_G)^{\epsilon_1-1}(K_F)^{\epsilon_2}(K_I)^{\epsilon_3} \tag{20}$$

Differentiating (15) w.r.t τ and setting it to 0 we have,

$$\tau^* = \frac{\alpha\chi}{1+\beta\chi} \tag{21}$$

The growth maximizing tax rate in the decentralized economy is thus a function of the public capital elasticities for the formal and the informal sectors and the exogenously given efficiency elasticity of maintenance expenditure (χ). It can be readily seen that when there is no informal sector (or to be less restrictive: if the informal sector does not use public capital in its production process), then the parameter $\beta = 0$. Imposing this condition the optimal tax rate reduces to $\alpha\chi$, which is similar to the Barro model prediction ($\tau = \alpha$). On the other hand, as public capital becomes more productive in the informal sector it is optimal for the government to lower the tax rate. Intuitively, public capital is financed by taxing the formal sector which is freely used by the informal sector. The formal sector would get discouraged by higher taxes levied on them, which would have a negative effect on the output and growth.

4.2 Growth maximizing maintenance: decentralized economy

In this section we look into the optimal allocation of tax revenues to maintenance in the presence of an informal sector. The objective is to find that allocation of maintenance out of tax revenues that maximizes the decentralized growth rate. Differentiating (20) (which is the optimal growth path for the economy) with respect to v_m and imposing the condition that $dv_m = -dv_g$ and $d\tau = 0$,which means the government balances its budget, we have,

$$v_m^* = (\frac{\alpha\chi}{1-\theta_G})(\frac{1}{1+\beta\chi}) \tag{22}$$

Agenor [2009] has only one production sector in the model, so the optimal maintenance is just the first expression in the parentheses which can be readily derived by setting $\beta = 0$. But in the presence of the informal sector, the allocation towards maintenance is smaller because $1 + \beta\chi > 1$. This makes intuitive sense because the presence of the informal sector means greater utilization of public capital which is actually financed by the taxes levied on the formal sector. Because of the illegal utilization of public capital by the informal sector, the government is tempted to spend less on maintenance and spend higher in new investments instead. The growth-maximizing share of maintenance is also positively related to the responsiveness of depreciation rate to maintenance (θ_G). Thus greater is the response of maintenance spending on the depreciation rate, more resources must be allocated to maintenance. The maintenance share is also negatively related to the exogenously given efficiency elasticity of maintenance (χ).

5 Optimal fiscal policy

In section 3, we showed that the social planner cannot separately implement welfare maximizing tax rate and the share of maintenance expenditure on total spending. This means that the planner can arbitrarily choose any one of the policy instruments at his disposal such that equation (13) holds. It is instructive to ask however, which arbitrary level of τ or v_m the planner should choose. Since, taxes are imposed on the formal sector output, let us say that the one objective of the social planner is to maximize the formal sector output in the decentralized economy. Under such an objective, the planner can implement the decentralized growth maximizing tax rate and choose v_m such that equation (13) holds. Under such a condition, the planner achieves the welfare-maximizing share of maintenance expenditure in total spending.

Proposition 2. *If the social planner arbitrary sets the tax rate that maximizes the formal sector growth, the welfare-maximizing share of maintenance expenditure (v_m^{**}) in total spending is given by:*

$$v_m^{**} = \frac{1}{(1-\theta_G)}\left[Y_I/Y_F\left(\beta/\alpha\right)+1\right] \tag{23}$$

Some important predictions that emerge from this result are as follows. First, it is interesting to note that the welfare-maximizing share of maintenance is positively related to the ratio of informal to formal sector output. Thus as the size of the informal sector increases relative to the formal sector, the social planner should increase the share of maintenance in order to maximize overall welfare. The mechanism through which the informal-formal output ratio affects the optimal share of maintenance is through the efficiency parameter. An increase in the Y_I/Y_F decreases the efficiency of public capital. Since the social planner internalizes the efficiency parameter, he tries to maintain the same level of efficiency by devoting more resources toward maintenance. This is a major departure from the decentralized solution since in that case the grow-maximizing share of maintenance is independent of the relative size of the informal sector. Second, higher the response of depreciation rate to maintenance expenditure (higher θ_G), more resources should to be devoted to maintenance in raising the stock of public capital. The optimal share of maintenance is independent of the efficiency elasticity of maintenance (χ) because the social planner internalizes the efficiency parameter in the decision making process.

6 Conclusion

Efficient public infrastructure is an important part of the growth process for all advanced and developing economies. However, a fundamental difference between an advanced country and a developing country is the existence of a large informal sector in the latter. In developing countries, the informal sector contributes to almost half of the country's GDP. This sector is characterized by low-productivity unincorporated firms that operate at a low level of capital intensity relative to the formal sector firms. A large fraction of the population is dependent on the informal sector as it produces cheap low quality goods. The existence of such a sector has important implications on fiscal policies. First, the informal sector does not pay taxes which imply less amount of resources that can be devoted to the provision of public goods. Second, public infrastructure which is a non-excludable good, is used by the formal and informal sectors as an input in their production processes. Since public infrastructure is funded by taxes imposed on the formal sector, informal sector's use of public infrastructure introduces some degree of congestion with regards to public infrastructure. We set up a two-sector endogenous growth model to examine the implication of the informal sector on optimal fiscal policies.

Since congestion affects the efficiency of public infrastructure, in order to make public infrastructure efficient, the government must devote some resources to maintain the existing stock of public infrastructure. This paper throws some light on how tax revenues should be spent on investments on new infrastructure and maintenance of existing public infrastructure. We find that under a centrally planned economy, the social planner may choose an arbitrary level of tax rate and implement a welfare maximizing share of maintenance expenditure in total spending. The tax rate however may be chosen to maximize the decentralized growth rate. The results shown in this paper have important policy implications. First, it is shown that the welfare maximizing share of maintenance spending in total spending on infrastructure is positively related to the ratio of informal to formal sector output. Thus economies with large informal sectors must devote more resources toward maintaining public infrastructure. Second, we find that both welfare maximizing and decentralized fiscal policies are functions of the parameters of the formal and informal production processes. Overall, these results show a non-trivial effect of the informal sector on the dynamics of fiscal policies. Thus, policy makers in developing countries must take these into consideration when implementing fiscal polices in order to achieve their desired outcomes.

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Figures

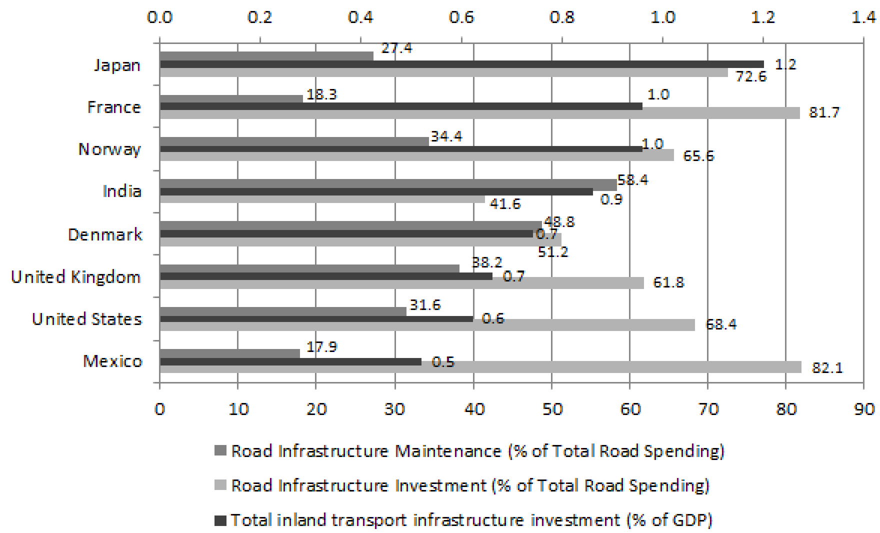
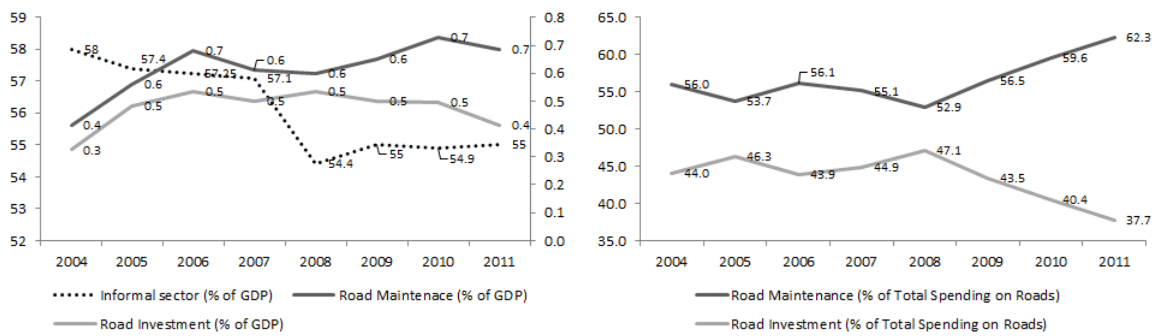


Figure 1: Road Infrastructure Investment and Maintenance (% of Total spending), Average 2006-2010

Source: International Transport Forum (ITF) Outlook, 2013



(a) Informal sector, Road Investment and, Maintenance (% of GDP)

(b) Road Investment and Maintenance (% of Total Spending)

Figure 2: Road Investment, Maintenance and Informal Sector, India, 2004-2011

Source: ITF Outlook, 2013 and National Accounts Statistics Reports (India)

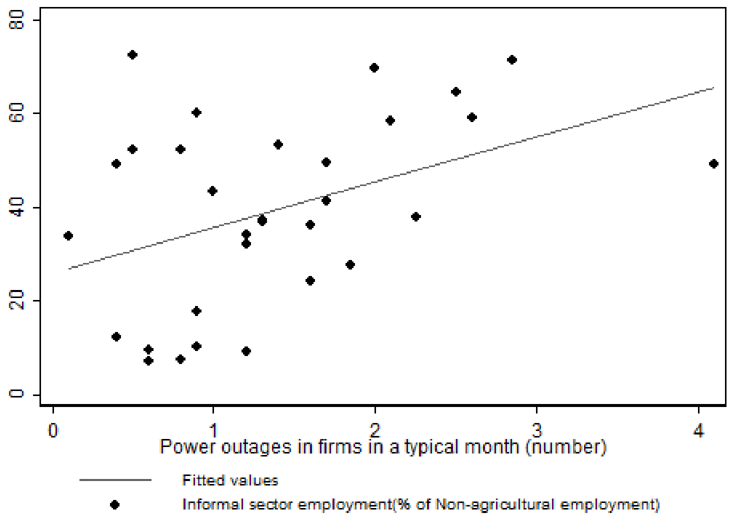


Figure 3: Informal Sector Employment (% of Non-Agricultural Employment) vs No. of Power Outages in a Firm in a Typical Month
Source: ILO [2012] and World Development Indicators

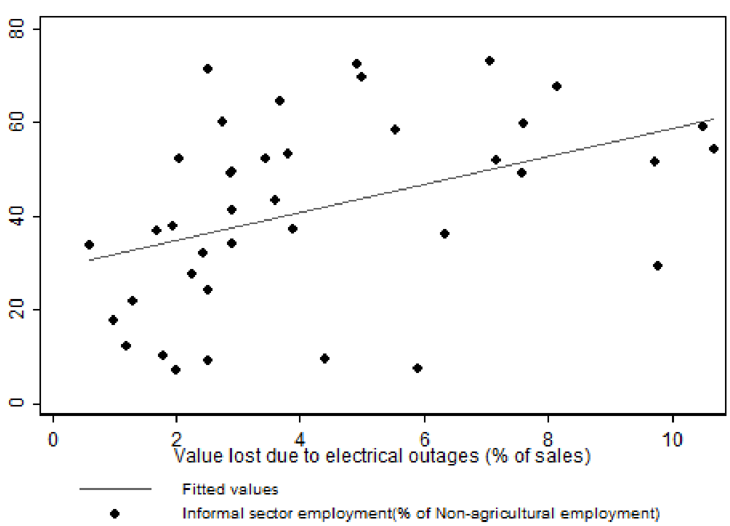


Figure 4: Informal Sector Employment (% of Non-Agricultural Employment) vs Value Lost Due to Electrical Outages (% of Sales)
Source: ILO [2012] and World Development Indicators

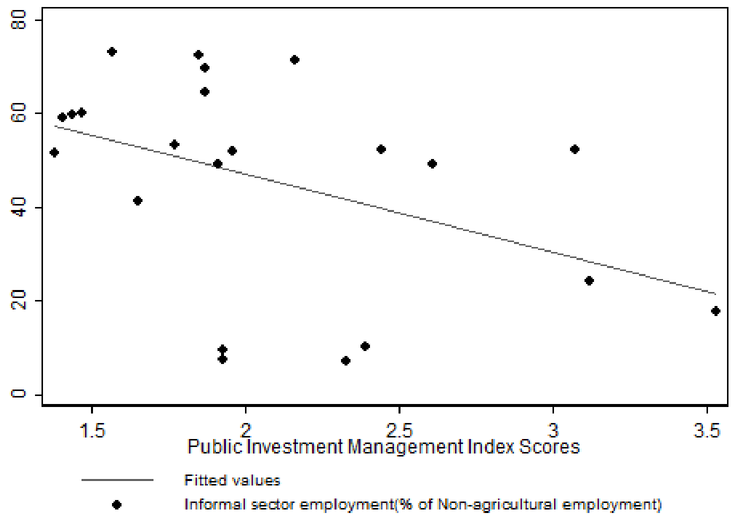


Figure 5: Informal Sector Employment (% of Non-Agricultural Employment) vs PIMI scores, cross country

Source: ILO [2012] and World Development Indicators

A Appendix

A.1 Characterizing the efficiency parameter

From (7a) and (6d) we note that,

$$e = (\frac{\upsilon_m \tau Y_F}{Y_I})^{\chi}$$

$$\rightarrow e = (\frac{\upsilon_m \tau Y_F / K_G}{Y_I / K_G})^{\chi} \tag{A.1}$$

From the production functions (3) and (4) we have,

$$\frac{Y_F}{K_G} = e^{\alpha} k_{GF}^{\alpha-1}$$

and

$$\frac{Y_I}{K_G} = e^{\beta} k_{GI}^{\beta-1}$$

where $k_{GF} = \frac{K_G}{K_F}$ and $k_{GI} = \frac{K_G}{K_I}$. Substituting these into (A.1) we have,

$$e = (\upsilon_m \tau)^{\chi} (e^{\alpha} k_{GF}^{\alpha-1})^{\chi} (e^{-\beta} k_{GI}^{1-\beta})^{\chi}$$

Solving for e we have,

$$e = (\upsilon_m \tau)^{\chi/\eta} (k_{GF})^{\chi(\alpha-1)/\eta} (k_{GI})^{\chi(1-\beta)/\eta} \tag{A.2}$$

$$e^{\alpha} = (\upsilon_m \tau)^{\chi\alpha/\eta} (K_G)^{\chi\alpha(\alpha-\beta)/\eta} (K_F)^{\chi\alpha(1-\alpha)/\eta} (K_I)^{\chi\alpha(\beta-1)/\eta} \tag{A.3}$$

where $\eta \equiv 1 - \chi(\alpha - \beta)$.

Substituting for e, the production functions can be rewritten as:

$$Y_F = (\upsilon_m \tau)^{\chi\alpha/\eta} (K_G)^{\epsilon_1} (K_F)^{\epsilon_2} (K_I)^{\epsilon_3} \tag{A.4}$$

$$Y_I = (\upsilon_m \tau)^{\chi\beta/\eta} (K_G)^{\epsilon_4} (K_F)^{\epsilon_5} (K_I)^{\epsilon_6} \tag{A.5}$$

where $\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5, \epsilon_6$are given by the following expressions:

$$\epsilon_1 = \alpha[1 + (\alpha - \beta)\chi/\eta]$$

$$\epsilon_2 = \alpha\chi/\eta(1-\alpha) + (1-\alpha)$$

$$\epsilon_3 = \alpha\chi(\beta-1)/\eta$$

$$\epsilon_4 = \beta[1 + (\alpha-\beta)\chi/\eta]$$

$$\epsilon_5 = \beta\chi(1-\alpha)/\eta$$

$$\epsilon_6 = \beta\chi/\eta(\beta-1) + (1-\beta)$$

A.2 Comparative statics for welfare-maximizing share of maintenance (v_m^{**}) when $\tau = \frac{\alpha\chi}{(1+\beta\chi)}$

$$\frac{\partial v_m^{**}}{\partial(Y_I/Y_F)} = \frac{1}{(1-\theta_G)}[(\beta/\alpha)] > 0 \tag{A.6}$$

$$\frac{\partial v_m^{**}}{\partial(\theta_G)} = \frac{1}{(1-\theta_G)^2}[Y_I/Y_F\,(\beta/\alpha)] > 0 \tag{A.7}$$

A.3 Comparative statics for growth-maximizing share of maintenance (v_m)

$$\frac{\partial v_m}{\partial(\chi)} = \frac{1}{(1-\theta_G)}[\frac{\alpha}{(1+\beta\chi)^2}] > 0 \tag{A.8}$$

$$\frac{\partial v_m}{\partial(\theta_G)} = \frac{1}{(1-\theta_G)^2}[\frac{\alpha\chi}{(1+\beta\chi)}] > 0 \tag{A.9}$$