Mental Accounting and Sunspot Equilibria *

Aditya GOENKA†
(University of Birmingham)

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Abstract: This paper considers a model of mental accounting where some consumers treat money as non-fungible. The budget is exogenously broken up into different expenditure groups. Given the amount of resources allocated to a given expenditure group, boundedly rational consumers optimize within the expenditure group. We study the general equilibrium effects of these 'mental accounting systems'. We show that mental accounting can lead to sunspot equilibria or self-fulfilling fluctuations.

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†Correspondence to A. Goenka, Department of Economics, University of Birmingham, Email: a.goenka@bham.ac.uk

1 Introduction

A key underlying assumption in economics is that money is *fungible*, that is, it has no labels, is undifferentiated, and is spent according to its best use. However, behavioral economists have argued that in practice, consumers do not treat money as fungible ([17], [18], [26], [27], [28], [29]). Empirical evidence (see below) also shows that in many contexts, consumers do not treat money as fungible.[1] This paper takes the view that if indeed consumers do not treat money as fungible then what are the general equilibrium consequences of this departure from full rationality at the individual level. Non-fungibility is equivalent to mental accounting where consumers have different expenditure groups, or mental accounts, and make consumption choices within these groups. While the choices within a mental account may be optimal, the mental accounts themselves are fixed and are not responsive to price and income changes. The consequence of mental accounting is that consumers act *as if* they face different budget constraints, when in practice these are only self imposed constraints. The paper shows that the presence of these self-imposed multiple constraints can make the general equilibrium outcomes exhibit endogenous fluctuations or make outcomes dependent on sunspots. There is a similarity of the model to other models that generate multiple budget constraints for some consumers, in particular the restricted market participation model of [8]. However, the mental accounting economy has a special structure which differentiates it economy from the canonical restricted market participation economy.

The interest in mental accounting and non-fungibility of money arose from Kahneman and Tversky's [18] classic experiment: Subjects were asked to imagine that they have decided to see a play, admission to which is \$10 per ticket. As they enter the theater they discover they have lost a \$10 note. They were asked whether they would still pay \$10 for the ticket to the play? 88% of the people said yes, while 12% said no. The same individuals were then confronted with a different situation. They were asked to imagine that they had decided to see a play and paid admission price of \$10 for ticket. As they enter the theater they discover that they have lost the ticket. The seat was not marked and the ticket cannot be recovered. Would they pay \$10 for another ticket? The wealth effect of both the situations are the same if money is non-fungible. However, now only 46% said they would purchase a new ticket while, 54% said they would not purchase a new ticket.[2] There is recent empirical evidence showing the presence of mental accounting: [15] find evidence of mental accounting in gasoline purchases, [1] finds evidence in a Dutch consumer data, and [20] finds evidence in online shopping. (Other examples of mental accounting are given in the papers by [5], [23]).

[1] Evidence also suggests that consumers use different sources of income as non-fungible (see [11]) though we do not model this.

[2] The results of this experiment were confirmed by [16] in a sample of MBA students.

Sociologists also cite widespread use of mental accounting in household behavior ([30]).[3] Another stand of theoretical literature argues that as a response to the complexity and cost of decision making, consumer not fully optimize (see [22], and [25]). More recently, [12] argue that mental accounting, in particular, can be generated in decision problems due to computational complexity.

The situation we model is similar to the one raised by Kahneman and Tversky where consumers use mental accounting systems to construct different expenditure groups e.g., one for housing, heating and transportation; for food; for entertainment; for savings and insurance; etc., and optimize within each expenditure group but not across the expenditure groups. Thus money is treated as non-fungible and has a label. The consumers after assigning expenditure levels, perhaps using a rule of thumb, then optimize given the levels in the expenditure groups. Different consumers may have different expenditure groups and different weights for each group. In the paper the weights for the expenditure groups are taken as given. The crucial behavioral postulate is that the allocation of expenditures are predetermined and are not responsive to changes in the prices and income. In this paper the different expenditure *shares* are fixed. An alternative model would be where the expenditure *amounts* are fixed as in Kahneman and Tversky's experiment. However, the latter is difficult to interpret in a general equilibrium set-up where prices and hence, wealth is endogenous.

This segmentation of decision making can prevent full against extrinsic risk and thus, makes sunspot equilibria possible. Thus, the use of simple decision rules can lead to endogenous fluctuations. This implies with mental accounting not only will there be loss of efficiency at the individual level - due to the additional mental constraints - but there will also be an additional loss at the aggregate level due to introduction of stochastic outcomes when all agents are risk averse. [7] and [19] examine the macroeconomic consequences of rule of thumb consumers who consume their entire income. In their model there is a single commodity thus, a single expenditure group. Our model can be considered as a generalization with multiple expenditure groups and multiple commodities, and thus captures the phenomenon of endogenous fluctuations that is not captured by them.

As bounded rationality can give rise to non-trivial sunspot equilibria, a relevant question is to what extent is the Walrasian model robust to introduction of mental accounting. In

[3] Zelizer's monograph deals with the sociology of mental accounting. The following quote captures the essence of mental accounting: "In their everyday existence, people understand that money is not really *fungible*, that despite the anonymity of dollar bills, not all dollars are equal or interchangeable. We routinely assign different meanings and separate uses to particular monies [p.5] ...Within their homes, families worked hard at earmarking their money. They bought the account ledgers and budget books recommended by experts to carefully register their expenses, or else invented all sorts of strategies to differentiate the household's multiple monies. Take, for instance, Mrs. M's system as she told it to *Woman's Home Companion* in the early 1920s: "I collected eight little cans, all the same size, and pasted on them the following words, in big letters: groceries, carfare, gas, laundry, rent, tithe, savings, miscellaneous. [p.39]"

particular, how large does the number of consumers with mental accounting have to be before qualitatively new equilibria can emerge? We show that equilibria in the Walrasian economy are generically robust to introduction of a small number of mental accounting consumers consumers. Thus, the Walrasian model is robust to small number of consumers deviating from full rationality.

Expenditures shares for different expenditure groups can also be generated through two-stage budgeting procedures. The paper maintains the assumption of additive separability of utility function across commodities in each expenditure group. This is necessary and sufficient for optimality of the second stage of two-stage budgeting. However, the expenditure weights are fixed and the assumption of additive separability is not sufficient for optimality of two-stage budgeting ([6]). Cobb-Douglas preferences also generate fixed expenditures which do not vary with income and prices. We distinguish prediction on demand for the model of mental accounting and one where consumers have Cobb-Douglas preferences. It should be noted that if consumers have Cobb-Douglas preferences then equilibria will not be affected by sunspots.

The plan of the paper is as follows. In section 1 the economy is described. Section 2 examines the effect of mental accounting on sunspot equilibrium. Section 3 considers the equilibrium outcomes when 'nearly all' consumers do not exhibit mental accounting. Section 4 distinguishes the implications of mental accounting from Cobb Douglas preferenes, and section 5 concludes.

2 The Economy

Consider a pure exchange economy with two intrinsically identical, equiprobable states of nature, $s = \alpha, \beta$. There are two groups of consumers: the boundedly rational consumers, $j = 1, \ldots, J$, and the fully rational consumers, $i = 1, \ldots, I$. In each state there are L commodities, $x^{\ell}(s)$, $\ell = 1, \ldots, L$. The consumption set for each consumer is $\Re^L \times \Re^L$. For both types of consumers, the utility functions are separable and symmetric (with respect to the states of nature). $U_h : \Re^{2L} \to \Re$ is given by $\sum_s u_h(x_h(s))$ for $h = i, j$. The von Neumann - Morgenstern specification, $U_h = \sum_s \pi(s) v_h(x_h(s))$ where $\pi(s)$ is the probability of occurrence of state s is a special case. The utility functions, $u_h(\bullet)$, are C^{∞}, strictly increasing, and strictly concave. The indifference surfaces are bounded from below. The absence of intrinsic uncertainty implies that endowments are state symmetric as well, $\omega_h(\alpha) = \omega_h(\beta) = \omega_h^*$, $h = i, j$. Thus, we have $\omega_h = (\omega_h^*, \omega_h^*)$. The price vector is $p = (p(\alpha), p(\beta))$.

The fully rational consumers treat the income (wealth) to be fungible and have a single budget constraint. They are equivalent to the usual Walrasian consumers. Their maximization problem is given as:

$$\max \quad \sum_{s}(u_i(x_i^1(s),\ldots,x_i^L(s)), \quad i=1,\ldots,I, \tag{1}$$

$$\text{s.t.} \quad \sum_{s=\alpha,\beta}\sum_{l=1}^{L}(p^l(s)x_i^l(s)) = \sum_{s=\alpha,\beta}\sum_{l=1}^{L}(p^l(s)\omega_i^{l*})$$

The consumers who do mental accounting treat their income (wealth) as non-fungible. Thus, they assign "mental accounts" whereby money assigned to a specific account is used only for specified purposes. This, theoretically, has the effect of partitioning the commodities into different groups each with its own budget constraint. For modeling purposes, for each j consumer partition the L commodities into two different groups, θ_{j1} and θ_{j2}.[4] Without any loss of generality the first $L_j < L$ commodities are in the first group and the remaining commodities in the second group. The expenditure groups thus are not common across the boundedly rational consumers. The consumers are modeled as assigning a fixed share of wealth $\lambda_j \in (0,1)$ to the first expenditure group, and the remaining to the second expenditure group. The share in each mental account does not vary with prices or endowments.[5]

There can be two different forms of mental accounting: a 'weak' form where the a fixed share of ex-ante wealth is assigned to each expenditure group and a 'strong' form where a fixed share of spot wealth is assigned to each expenditure group. In the 'weak' form, there is a mental account for *both* the states and consumers can transfer funds across the two states within a mental account. For the 'strong' form of mental accounting, there is a separate account for each state, and the consumer does not transfer income across the two states. The behavioral literature does not give a clear guide on which of the two is predominant. Thaler [29] discusses that it appears that poorer families have stricter implementation of mental accounting than richer families. The strict forms will be close to the 'strong' form discussed above. Moreover, in another paper [28] Thaler says that mental accounting can make consumers act *as if* they are credit constrained when they are in fact *unwilling* to borrow. This clearly corresponds to the 'strong' form. In the paper we discuss implications of both types of mental accounting. The 'strong' form of mental accounting results in restrictions in participation in insurance (against sunspots) markets and similar to the restrictions in[8]. Note, that the motivation is different, and there is a difference in analytics of the problem.

The paper does not model the choice of expenditure weights across the two expenditure groups but treats it as given. One could view this as being chosen according to some criteria,

[4] Multiple expenditure groups can be analyzed at the cost of extra notation.

[5] Cobb-Douglas preferences also generate the same behavior and in Section 5 we show the difference of mental accounting from this preference structure.

but is then held fixed. This is consistent with the view that consumers who perform mental accounting re-optimize only infrequently (see the discussion in [29]). The expenditure share of the first group of commodities in each state is $\lambda_j \in (0,1)$ and the share of the second group is $(1-\lambda_j)$.

For the the weak form of boundedly rational consumers the choice problem is given by:

$$\begin{aligned} \max \quad & \sum_s (u_j(x_j^1(s),\ldots,x_j^{L_j}(s),x_j^{L_j+1}(s),\ldots,x_j^L(s)), \quad j=1,\ldots,J, \\ \text{s.t.} \quad & \sum_{s=\alpha,\,\beta}\sum_{l\in\theta_{j1}}(p^l(s)x_j^l(s)) = \lambda_j \sum_{s=\alpha,\,\beta}(p(s)\omega_j^*), \\ \text{and} \quad & \sum_{s=\alpha,\,\beta}\sum_{l\in\theta_{j2}}(p^l(s)x_j^l(s)) = (1-\lambda_j) \sum_{s=\alpha,\,\beta}(p(s)\omega_j^*). \end{aligned} \tag{2}$$

For the the strict form of boundedly rational consumers the choice problem is given by:

$$\begin{aligned} \max \quad & \sum_s (u_j(x_j^1(s),\ldots,x_j^{L_j}(s),x_j^{L_j+1}(s),\ldots,x_j^L(s)), \quad j=1,\ldots,J, \\ \text{s.t.} \quad & \sum_{l\in\theta_{j1}}(p^l(s)x_j^l(s)) = \lambda_j(p(s)\omega_j^*), \quad s=\alpha,\ \beta, \\ \text{and} \quad & \sum_{l\in\theta_{j2}}(p^l(s)x_j^l(s)) = (1-\lambda_j)(p(s)\omega_j^*), \quad s=\alpha,\ \beta. \end{aligned} \tag{3}$$

The necessary and sufficient condition for optimization within each group is that the preferences are weakly separable, see [9].[6] We assume the stronger condition that preferences are block additive across the groups, $u_j : \Re^{2L} \to \Re$ is given by $\sum_s [u_{j1}(x_j^1(s),\ldots,x_j^{L_j}(s)) + u_{j2}(x_j^{L_j+1}(s),\ldots,x_j^L(s))]$. If preferences are additively separable across expenditure groups, then we canit "break up" the optimization problem into sub-problems of maximizing an objective function subject to a single constraint. Thus, for problem (2) we can represent each boundedly rational consumer j into two "quasi-rational" consumers who maximize a utility function subject to a single budget constraint, and for problem (3) into four quasi-rational consumers. Each of these quasi-rational consumers consumes a strict subset of the commodities and has 'endowments' which are proportional to the expenditure weights (see [3], [4], [13], [14]). Thus, for problem (2) with each consumer j associate consumers $k1$ and $k2$ and for problem (3) associate consumers $j1\alpha, j2\alpha, j1\beta, j2\beta$. Consumer $k1$ consumes only goods in group θ_{j1} in both the states with preferences represented by $\sum_{s=\alpha,\,\beta}(u_{j1}(x_j^1(s),\ldots,x_j^{L_j}(s))$ and has endowments $(\lambda\omega_j^*, \lambda\omega_j^*)$. Consumer $k2$ consumes only goods in group θ_{j2} in both

[6]However, this is not suffiicient of optimality of a two-stage budgeting procedure, see [6].

state, has preferences represented by $\sum_{s=\alpha,\,\beta}(u_{j2}(x_j^{L_j+1}(s),\ldots,x_j^L(s))$ and has endowments $((1-\lambda)\omega_j^*,(1-\lambda)\omega_j^*)$.

Consumer $j1s$ consumes only goods in group θ_{j1} in state $s, s=\alpha,\beta$ with preferences represented by $(u_{j1}(x_j^1(s),\ldots,x_j^{L_j}(s))$ and has endowments $\lambda\omega_j^*$. Similarly consumer $j2s$ consumes only goods in group θ_{j2} in state s, has preferences represented by $u_{j2}(x_j^{L_j+1}(s),\ldots,x_j^L(s))$ and has endowments $(1-\lambda)\omega_j^*$ in state s only, with zero endowments in state $s'\neq s, s, s'=\alpha,\beta$. The economy with either $I+2J$ or $I+4J$ consumers is called the "expanded model." This is an Arrow-Debreu economy with a special endowment structure and with some consumers consuming only subsets of the commodity. We will also refer to a "certainty economy" or a "reduced model." This is the economy with only one state (no extrinsic uncertainty) and either $I+2J$ or $I+4J$ consumers derived naturally from the original model.

3 Sunspot Equilibrium

The economy has only extrinsic uncertainty and the notion of equilibrium is sunspot equilibrium. There are two questions on their existence. First, do they exist, and second, are the effects of sunspots non-trivial, i.e. are allocations dependent of the realization of the sunspot state. It is easy to show the equilibria will exist using a standard fixed point argument. The idea of the proof is that economy with quasi-rational consumers is a standard Arrow-Debreu economy where some consumers consume strict subsets of all the commodities but where resource relatedness is satisfied (via the rational consumers). The second question is more subtle and is the focus of this paper.

Definition 1. *(p,ω) is a Sunspot Equilibrium for to the distribution of the expenditure weights $\lambda=(\lambda_1,\ldots,\lambda_J)$ if*

$$\sum_i f_i+\sum_j f_j=\sum_i\omega_i+\sum_j\omega_j, \tag{4}$$

where $(f_h),\ h=i,j$ are solutions to the maximization problems (1) and (2) or (3).
Let the set of Sunspot Equilibria relative to the distribution of expenditure weights, λ, (for variable ω) be denoted as

$$E(\omega,\lambda)=\{(p,\omega):\sum_i f_i+\sum_j f_j=\sum_i\omega_i+\sum_j\omega_j\}. \tag{5}$$

Definition 2. *Extrinsic uncertainty (sunspots) does not matter or sunspots have a trivial* *effect if:*

$$x_h(\alpha) = x_h(\beta) \ \ \forall h = i, j.$$

First consider the case of 'weak' mental accounting. In this case, the equilibrium allocations will not be Pareto efficient as the expenditure weights are arbitrary, and in general, by changing the expenditure weights, it will be possible to have a welfare improvement. However, a weaker form of constrained efficiency holds. In particular, all agents will fully insure against the sunspot risk, and thus, sunspots will not matter.

Proposition 1. *If there is only 'weak' mental accounting, then sunspots do not matter.*

Proof. The proof parallels the argument in [8], Proposition 3. In the expanded economy with $I + 2J$ consumers, each of the consumers are unrestricted consumers, and given the strict convexity of preferences, any equilibrium allocation will exhibit full insurance against sunspots. Hence, allocations will be state symmetric. □

The model with 'strong' mental accounting is similar to the case of restricted market participation in [8]. However, the incomes of the quasi-rational consumers are related as they are derived from a model where there is mental accounting, and thus, one needs to see if the non-trivial effect of sunspots still holds with interrelated incomes. This follows from the fact that the 'endowments' and hence 'incomes' given to the two quasi-rational consumers in a state depend on the expenditure weight. This factor of proportionality will be the same for the corresponding quasi-rational consumers in the other state. This relationship is given below:

$$w_{j1s} = \frac{\lambda_j}{(1 - \lambda_j)} w_{j2s} \ \ s = \alpha, \beta, \tag{6}$$

where $w_{j1s} = \sum_{l \in \theta_{j1}} p^l(s) x_j^l(s)$ and $w_{j2s} = \sum_{l \in \theta_{j2}} p^l(s) x_j^l(s)$, the expenditure on the two groups in the two states.

Instead of trying to analyze the allocations directly, we study the Price and Income Equilibria (see [3], [13], [14]). The strategy is to work with the expanded model with $I + 4J$ consumers. It is sufficient to examine whether prices and incomes of the consumers (rational and quasi-rational) are state symmetric or not, and as demand is a diffeomorphism (the economy is Walrasian) the allocations will be state symmetric if, and only if, the prices and incomes are state symmetric. We start with the space of all potential prices and incomes and then impose restrictions so as to be able to focus on a restricted set which will be consistent with the special structure of the model. The set of all potential prices and incomes, B, will be a set of dimension $2L + I + 4J$: $2L$ prices and $I + 4J$ incomes. As there is only extrinsic uncertainty, the aggregate resources have to be state symmetric, thus imposing L restrictions and giving the set of price and income equilibria consistent with symmetric resources, B_s. As the incomes of the two quasi-rational consumers derived from each boundedly rational consumer have to be proportional *within* each state, there are $2J$ additional restrictions. Thus, the set of equilibrium prices and incomes consistent with symmetric resources and proportional resources, $B_s(\lambda)$, should have dimension $L - 1 + I + 2J$ (One dimension is lost due to price normalization). The set of equilibria where sunspots do not matter, $\overline{B_s(\lambda)}$, will be where the income of the quasi-rational consumers are symmetric *across* states. This set has J additional restrictions and thus is a lower dimensional subset of the set of all equilibria (should be of dimension $L - 1 + I + J$). This is in terms of prices and incomes. To show existence of non-trivial effects firstly, asymmetric price and income equilibria should be consistent with symmetric endowments, and these actually exist. This is detailed below.

Definition 3. $(p, w) \in \Re^{2L}_{++} \times \Re^{I+4J}$ *is a price and income equilibria for fixed resources,* r*, if*

$$\sum_{h=i,\, j1\alpha,\, j2\alpha\, j1\beta,\, j2\beta} f_h(p, w_h) = r. \tag{7}$$

Let the set of price and income equilibrium be

$$B = \{b = (p, w_h) : \sum_{h=i,\, j1\alpha,\, j2\alpha\, j1\beta,\, j2\beta} f_h(p, w_h) = r\} \tag{8}$$

The absence of aggregate uncertainty requires that we restrict our attention to the set of price and income equilibria with symmetric resources, B_s.

Definition 4. *The set of price and income equilibria with symmetric resources is:*

$$B_s = \{b = (p, w_h) \in B : r \text{ is symmetric}\}. \tag{9}$$

As mentioned above, the incomes of the quasi-rational consumers are related in order for the equilibria to be consistent with the expenditure weights.

Definition 5. *The set of price and income equilibria with symmetric aggregate demand consistent with the distribution of expenditure weights,* $\lambda = (\lambda_1, \ldots, \lambda_J)$*, is:*

$$B_s(\lambda) = \{b = (p, w_h) \in B_s : \; w_{j1s} = \frac{\lambda_j}{(1 - \lambda_j)} w_{j2s} \; s = \alpha, \beta, \; j = 1, \ldots, J\}. \tag{10}$$

The set of equilibria where extrinsic uncertainty does not matter will be the subset where the prices and incomes are state symmetric.

Definition 6. *The set of price and income equilibria consistent with the distribution of expenditure weights,* λ*, where extrinsic uncertainty does not matter is:*

$$\overline{B_s(\lambda)} = \{b \in B_s(\lambda) : (p, w_h) \text{ is symmetric}\} \tag{11}$$

The analysis of the effect of extrinsic uncertainty involves studying properties of these sets, and relating them to the underlying endowments. Define the following map, $\xi : E(\omega, \lambda) \to B$:

$$\xi(p, (\omega_i)_i, (\omega_{j1s})_{j1s}, (\omega_{j2s})_{j2s}) = (p, \; (p \cdot \omega_i)_i, (p \cdot \omega_{j1s})_{j1s}, (p \cdot \omega_{j2s})_{j2s}) \tag{12}$$

The set $E(\omega, \lambda)$ is the set of sunspot equilibrium consistent with the expenditure weights λ. Now, $\xi(E(\omega, \lambda)) \subset B_s(\lambda)$. Consider, $\xi : \overline{E(\omega, \lambda)} \to B$, where the set $\overline{E(\omega, \lambda)} = \{(p, \omega_h) \in E(\lambda) : p(\alpha) = p(\beta)\}$. Note that we have $\xi(\overline{E(\omega, \lambda)}) \subset \overline{B_s(\lambda)}$. The strategy is to show that there exist points in $B_s(\lambda)$ that belong to image of $E(\omega, \lambda)$ but *not* to the image of $\overline{E(\omega, \lambda)}$ through the map ξ. The map ξ it should be noted is neither onto nor one-to-one. The sufficient condition for the asymmetric price and income equilibria to be consistent with symmetric endowments is given below.

Proposition 2. *If* $L \geq 2$*, and* $p(\alpha) \neq \nu p(\beta)$*,* $\nu \in \Re_{++}$*, then the asymmetric price and income equilibria are consistent with symmetric endowments.*

Proof. First, "endowments" for $j1\alpha, j1\beta, j2\alpha$ and $j2\beta$, $j = 1, \ldots, J$ are constructed and then used to find the symmetric endowments for the consumers j. The endowments of $jk\alpha$ and $jk\beta$, $k = 1, 2$ should be equal. These will be a solution to the linear equations

$$
\begin{aligned}
p(\alpha)\omega^*_{jk} &= w_{jk\alpha} \\
p(\beta)\omega^*_{jk} &= w_{jk\beta}
\end{aligned}
$$

A solution exists if and only if $P(p) = [(p(\alpha), (p(\beta)]^T$ and $R(p) = [P(p) \quad w_j]$ have the same rank (Kronecker-Capelli Theorem). A sufficient condition is that $L \geq 2$, and rank $P(p) = 2$.

Now let $\omega_j = (\omega^*_{j1} + \omega^*_{j2},\ \omega^*_{j1} + \omega^*_{j2})$. The essential thing to notice is that the constraints on the expenditures have been subsumed in the definition of "income".

For the $i = 2, \ldots, I$ rational consumers, the endowments are derived as follows. First solve

$$
(p(\alpha) + p(\beta))\omega^*_i = w_i \tag{13}
$$

and then set $\omega_i = (\omega^*_i, \omega^*_i)$. Define, $\omega_1 = (\omega^*_1, \omega^*_1) = r - \sum_{i \neq 1} \omega_i - \sum_j \omega_j$. For the distribution of the endowments to be consistent with the price and income equilibria, it must be the case that

$$
\begin{aligned}
&(p(\alpha), p(\beta))\omega_1 = \\
&\quad (p(\alpha), p(\beta))(r - \sum_{i \neq 1} \omega_i - \sum_j \omega_j) \\
&\quad = w_1
\end{aligned}
$$

which is true by Walras' Law. □

To study the existence of non-trivial sunspot equilibria, first one shows that $\overline{B_s(\lambda)}$ is a lower dimension subset of $B_s(\lambda)$. Then one shows that the complement of $\overline{B_s(\lambda)}$ in $B_s(\lambda)$ is non-empty. The intuition behind the dimensionality of the equilibria is as follows. One starts with $2L$ prices and $I + 4J$ incomes. There are L restrictions on the symmetry of aggregate demand, and $2J$ restrictions on proportionality of the incomes of the quasi-rational consumers. Once these are imposed, one obtains $B_s(\lambda)$ which has dimension $L - 1 + I + 2J$. In looking at $\overline{B_s(\lambda)}$ there are J additional restrictions on the symmetry of the incomes of the quasi-rational consumers, thus it has dimension $L - 1 + I + J$.

Proposition 3. *For a given distribution of expenditure weights, λ, $B_s(\lambda)$ is a smooth manifold of dimension $(L - 1 + I + 2J)$, and $\overline{B_s(\lambda)}$ is a smooth submanifold of dimension $(L - 1 + I + J)$ embedded in $B_s(\lambda)$.*

Proof. A sketch of the proof is given. First show that the aggregate demand map in the expanded economy, $F : \Re^{2L}_{++} \times \Re^{I+4J} \rightarrow \Re^{2L}$, is a submersion. It is sufficient to show that the Jacobian has maximal rank, $DF = [A = \frac{DF}{Dp}, B = \frac{DF}{Dw_i}, C = \frac{DF}{Dw_{jms}}]$ for $m = 1, 2$. Manipulating the matrix it can be shown to take the form $[S, B, C]$ where S is the sum of individual Slutsky matrices of the rational and quasi-rational consumers. If $I \neq \emptyset$, then this can be further simplified to

$$\begin{pmatrix} & * & & \\ S^* & \vdots & B^* & C^* \\ & * & & \\ 0 \ldots 0 & & 1 \ldots 1 & * \ldots * \end{pmatrix},$$

where S^* is symmetric negative definite with rank $2L - 1$. Completing the matrix with any column of $\begin{pmatrix} B^* \\ 1 \end{pmatrix}$ will show that the entire matrix has full rank (see [3], Lemma 5 for details). This will imply that F is transverse to Δ, $\Delta = \{(r(\alpha), r(\beta)) \ : r(\alpha) = r(\beta)\}$. This gives us the property that B_s is a smooth submanifold of $\Re^{2L}_{++} \times \Re^{I+4J}$ of dimension $(L + K + I + 4J)$ as $codim\ F^{-1}(\Delta) = codim\ (\Delta)$, or $2L + I + 4J - dim\ F^{-1}(\Delta) = L$. Now, $F^{-1}(\Delta) = B_s$. Normalization of prices reduces one degree of freedom and hence the dimension is $(L - 1 + I + 2J)$.

Next consider $B_s \cap B(\lambda)$, where $B(\lambda)$ is the set of prices and incomes which satisfy restriction (6). That is,

$$B(\lambda) = \{b = (p, w_h) \in B : \ w_{j1s} = \frac{\lambda_j}{(1 - \lambda_j)} w_{j2s}\ s = \alpha, \beta,\ j = 1, \ldots, J\}.$$

The set of interest is $B_s(\lambda)$. The intersection if transverse is a smooth submanifold of dimension B_s+ dimension $B(\lambda)$ - dimension B. The intersection is transverse if the tangent spaces to the two manifolds together span the tangent space to B (see [10]). If this is the case the dimension of $B_s(\lambda)$ can be calculated to be $(L-1+I+2J)$. To see that the intersection is transverse note that F is a submersion, and B_S has co-dimension L. Moreover, as $B(\lambda)$ is defined by the restriction (6) which are $2J$ independent restrictions on incomes of the quasi-rational consumers, it has co-dimension $2J$ or $codim\ B_s(\lambda) = L + 2J$ as desired.

To study $\overline{B_s(\lambda)}$ define the following maps, $\chi \ : \overline{B_s(\lambda)} \to B^*(\lambda)$, where $B^*(\lambda)$ is the set of price and income equilibria consistent with the distribution of expenditure weights in the reduced economy. $\chi(p, w_i, w_{j1}) = (p^*, (w_i^*), (w_{j1}^*))$, where $p^* = p(\alpha) = p(\beta)$, $w_i^* = w_i$, and $w_{j1}^* = w_{j1\alpha} = w_{j1\beta}$. The map χ is bijective. The inverse map ψ is defined as follows: $\psi \ : B^*(\lambda) \to \overline{B_s(\lambda)}$, with $\psi(p^*, w_i^*, w_{j1}^*) = (p, w_i, w_{j1})$, where $p = (p^*, p^*)$, $w_i = w_i^*$, $w_{j1\alpha} = w_{j1\beta} = w_{j1}^*$. This map is proper, injective and an immersion, and takes its values in $\overline{B_s(\lambda)}$. It is an embedding, thus showing the required property. The dimensions can now be easily calculated. $\square$

Corollary 1. *The set of asymmetric price and income equilibria $B_s(\lambda)\backslash\overline{B_s(\lambda)}$ is an open subset of $B_s(\lambda)$.*

One would like to know if this set is non-empty, i.e., there exist asymmetric price and income equilibria. A *sufficient* condition is that there are multiple equilibria in the certainty economy (the argument parallels the one in [4]). As we know the conditions for uniqueness of certainty equilibrium are very strong (gross substitutability in the expanded economy), in a robust class of environments the above theorem would be true. One can alternatively follow the construction of a non-trivial equilibrium in [13] which involves showing that we can find preferences such that the price vector in the states are not collinear. Given Propositions 1 and, we would have proved the desired result.

Theorem 1. *In the economy with some rational and some boundedly rational consumers, if $L \geq 2$, and there are multiple equilibria in the certainty economy, then sunspots matter.*

Note that in the absence of boundedly rational consumers, even if there were multiple equilibria in the certainty economy, sunspots would not matter.

4 Near-Rationality

As mental accounting can induce non-trivial sunspot equilibria, a natural question is whether the Walrasian model is robust to small perturbations to include bounded rationality. We want to take a process where in the limit the economy is Walrasian with all consumers fully rational. Thus, first, hold the endowments and expenditure levels, λ_j, of the different boundedly rational consumers fixed, and then change the *proportion* of these consumers. In particular, let the proportion go to zero, so that nearly everyone in the economy is rational. Generically, the the economy is robust against sunspots if the proportion of boundedly rational consumers is small enough.

To carry out this perturbation, let there be M types of consumers, with each type consisting of a continuum of non-atomic agents with unit mass. Let $\gamma_h, h = 1, \ldots, M$ proportion of consumers of each type being bounded rational with some fixed expenditure weights λ_h. If $\gamma = 0$, there are no boundedly rational consumers and the economy corresponds to a Walrasian economy. Then one can show that there will be a neighborhood of $\mathbf{0}$, i.e., the Walrasian economy, where sunspots do not matter. The key intuition is that the equilibria are locally constant if the economy is regular and the equilibrium will vary smoothly with small variations in the parameter ([2]). In particular, no new equilibria will be introduced in the neighborhood of regular economies. Generically (in endowment space), the economies are indeed regular. Thus, if we were to perturb the Walrasian economy by introducing small fractions of boundedly rational consumers, the equilibria will not be affected by sunspots. Denote the equilibrium set with γ_h proportion of boundedly rational consumers with each of the boundedly rational consumers of type h allocating λ_h of their income on first group as $E(\omega, \lambda, \gamma)$ where $\lambda = (\lambda_1, \ldots, \lambda_J)$ and $\gamma = (\gamma_1, \ldots, \gamma_J)$. We will hold ω, λ constant and perturb γ in the neigbourhood of $\mathbf{0}$.

Theorem 2:

Let ω^* be a regular economy (of the certainty model), and λ be a vector of expenditure weights. Then for this economy there exists an open neighborhood V of $\mathbf{0} \in \Re^M$, such that for $\gamma \in V$, sunspots do not matter.

Proof:

The proof follows in three steps. First, show that $E(\omega, \lambda, 0) = E(\omega^*) \times E(\omega^*) \cap \Delta$, where Δ is the diagonal in the of the Cartesian product of the space of endowments of the certainty economy. This implies that the set of equilibria when $\gamma = 0$ is the same as that of the Walrasian economy with the same parameters but with no boundedly rational consumers. Second, $E(\omega, \lambda, 0) \subset E(\omega, \lambda, \gamma)$. This implies that the symmetric Walrasian equilibrium allocations are also equilibria in the economy where the boundedly rational (j) consumers spend λ_j of their income in each state on the commodities in group θ_{j1} are also equilibria in the economy with boundedly rational consumers. Then apply an adapted version of Theorem 2.4, [4] which establishes that there are no other branches in the equilibrium set $E(\omega, \lambda, \gamma)$ other than the constant branches emanating from $E(\omega, \lambda, 0)$ for a regular economy in the certainty economy. The trick to use this result is that in the limit economy, it does not matter what is the level of λ. *Q.E.D.*

Note that the economies in Theorem 1 and 2 are different: in the first case there are a finite number of consumers, and in the second there are a continuum of consumers. Hence, there is no contradiction between the two results.

5 Implications for Demand Functions

In this section the implications of mental accounting on demand functions are discussed. The restrictions are stronger than those placed by the block additivity assumption on the preferences. In particular, mental accounting has definite restrictions on the sign and magnitude of some partial derivatives. To make the discussion self-contained, some definitions and results are re-collected here [7] (see [21] for details).

Definition 7

A utility function $u(x^1, \ldots, x^L)$ is said to be block additive if there exists a partition of

[7] In the subsequent discussion the subscript pertaining to the consumer is suppressed.

commodities into m subsets, m functions $u_r(x^r)$, and a function F with $F' > 0$ such that

$$F[u(x)] = \sum_{r=1}^{m} u_r(x^r), \quad m \geq 2.$$

For block additive utility function, the demand functions can be written as a function of prices of commodities in that block and the expenditure on that block. For simplicity suppose the partition consists of two sets θ and θ^*. In this case

$$f^{ri}(p,Y) = g^{ri.\theta^*}(p_\theta, \kappa^\theta(p,Y)), \;\; r \neq s$$

where f^{ri} is the demand for the ith commodity in the rth block, $g^{ri.\theta^*}$ is the conditional demand for the same commodity given the consumption levels in the other blocks $s \neq r$, Y is the total income, p_θ is the vector of prices in the rth block, and $\kappa^\theta(p,Y)$ is given by $\kappa^\theta(p,Y) = Y - \sum_{k \in \theta^*} p^k f^k(p,Y))$. The following equations can be derived in a straightforward way:

$$\begin{aligned} \frac{\partial f^{ri}}{\partial p^{sj}} &= \frac{\partial g^{ri.\theta^*}}{\partial A^\theta} \cdot \frac{\partial \kappa^\theta}{\partial p^j}, \; r \neq s \\ \frac{\partial f^{ri}}{\partial Y} &= \frac{\partial g^{ri.\theta^*}}{\partial A^\theta} \cdot \frac{\partial \kappa^\theta}{\partial Y} \end{aligned}$$

where $A^\theta = \sum_{i \in \theta} p^i x^i$. If $\dfrac{\partial \kappa^\theta}{\partial Y} \neq 0$, then by eliminating $\dfrac{\partial g^{ri.\theta^*}}{\partial A^\theta}$ between the two equations we obtain:

$$\begin{aligned} \frac{\partial f^{ri}}{\partial p^{sj}} &= \frac{\partial \kappa^\theta / \partial p^{sj}}{\partial \kappa^\theta / \partial Y} \cdot \frac{\partial f^{ri}}{\partial Y} \quad r \neq s \\ &= \mu^{sj} \frac{\partial f^{ri}}{\partial Y} \end{aligned}$$

where $\mu^{sj} = \dfrac{\partial \kappa^\theta / \partial p^{sj}}{\partial \kappa^\theta / \partial Y}$. From this it follows that

$$\frac{\partial f^{ri} / \partial p^{sj}}{\partial f^{tk} / \partial p^{sj}} = \frac{\partial f^{ri} / \partial Y}{\partial f^{tk} / \partial Y} \;\; r \neq s, \; t \neq s.$$

These are the restrictions for block additive preferences. Once the constraint of bounded rationality is added we have in addition, $\kappa^\theta = \lambda \sum_{l=1}^{L} p^l \omega^l$. Thus, we have $\dfrac{\partial \kappa^\theta}{\partial p^{sj}} = \lambda \omega^j$, and $\dfrac{\partial \kappa^\theta}{\partial Y} = \lambda$. It then follows:

$$\frac{\partial f^{ri}}{\partial p^{sj}} = \omega^j \frac{\partial f^{ri}}{\partial Y}, \quad r \neq s.$$

The cross price effects are still proportional to the income effect, but the factor of proportionality is the endowment of the commodity whose price has changed.

This restriction is, however, the same that will be generated if the consumer had a Cobb-

Douglas utility function. In this case, as $f^j(p,Y) = \frac{\kappa_k}{\sum_{i=1}^{L}\kappa^i}\frac{Y}{p^j}$, the above restriction falls out. Thus, it seems that once we have block additivity and bounded rationality, the restrictions on cross-partial derivatives are the same as in the Cobb-Douglas case. The Cobb-Douglas case, however, places a stronger restriction on the income effect – it is linear in income. In the model with mental accounting non-linear income effects are necessary for sunspots to have a role (see proof of Theorem 1).

The similarity can be pushed further to understand the model. If the maximization problem had allowed consumers to choose their expenditure share, instead of having these constant, and each of the consumers had Cobb-Douglas preferences, we would be in the first best situation, and sunspots could only have a trivial effect. In general, for the preferences to be consistent with two stage budgeting, we need a very particular structure for the preferences. Either the sub-utility functions are homothetic, or the sub-utility functions have a particular structure which includes the fact that the generated expenditure function for the groups are additive and each is homogeneous of degree one in the prices of the commodities in that group ([6]). While the price aggregation (demand for a group depending only on an index of prices for each of the other groups) is not important for our purposes, this stage is important as it generates the optimal expenditure weight for each portfolio.

6 Conclusion

This paper considers the general equilibrium effects of mental accounting. The effect of mental accounting is that consumers act as if they face multiple budget constraints. This implies that consumers who do mental accounting do not fully use insurance markets even if they are available. This makes self fulfilling fluctuations or non-trivial sunspot equilibria possible. While most of the evidence on mental accounting is either experimental or in partial equilibrium settings, the empirical study of Dutch data ([1]) points that at least in Netherlands, mental accounting is widespread. The poorer households are more likely to use mental accounting to control restrictive budgets. These households would also be less likely to use financial assets to hedge against uncertainty. As the fraction of such households is non-trivial, this departure from full optimizing behavior can contribute to endogenous fluctuations.

This paper considers the effect of one departure from fully optimizing behavior on general equilibrium outcomes. If indeed we take the experimental and other evidence of other deviations from fully optimizing behavior, then one should model them in a general equilibrium framework to see whether these have wider consequences on economic outcomes than just at the individual level.

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