Business cycles in presence of exogenous financial exclusion with productivity shocks

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Abstract

Recent literature on macroeconomic fluctuations in emerging and developed countries has focussed on the role of various frictions and shocks rather than the differences in the ability of the households in these two types of countries to absorb these shocks. Emerging and developed countries differ in proportion of agents who have access to financial services. The paper highlights the contribution of such heterogeneity to the volatility of aggregate consumption. I compute the correlation generated by the model between financial inclusion and ratio of volatility of aggregate consumption to volatility of aggregate income for emerging and developed countries and compare with the data counterpart to justify the model.

1. INTRODUCTION

For decades in economics, theoretical and empirical arguments have been given to prove that consumption is smoother than income especially in the U.S.A. (Campbell and Deaton 1989 [7]). Recent literature in macroeconomics (Aguiar and Gopinath 2007 [1] henceforth referred as "AG 2007"), has highlighted that consumption is more volatile than income in emerging countries at business cycle frequencies. On the other hand, consumption is less volatile than income in developed countries. Literature has delved into identifying shocks which can help to match those business cycle features but neglected the role of heterogeneity in participation of households in financial markets as a potential mechanism to explain the phenomenon.

Wide disparity exists between economies in the proportion of households with access to formal financial services (commercial banks, microfinance institutions, etc.). Developed economies have higher proportion of agents with access to such services than emerging countries. 2014 Global Findex database defines account ownership as having an account at a financial institution or through a mobile money provider. In high-income economies account ownership is 91.5% on average among adults and 40% in low and middle income economies. The 1st and 3rd quantile are 87% and 98% for high-income economies and 19% and 54% for low and middle income economies respectively. Access to financial markets affects the ability of the households

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to smooth consumption effectively. Not only does it affect the ability to borrow from formal institutions but also the ability to channel their savings effectively. Savings are exposed to inflation rather than earning interest and being utilized to build capital or earning equity returns or precautionary purposes. Households have to rely on informal markets (friends and family or employer to borrow and friends and family or informal savings club to save) to overcome this formal market non-participation problem. An attempt has been made in this paper to understand fluctuations in consumption to productivity shocks in an economy which includes both financially included and excluded households. Assuming that there exists a given proportion of financially included and excluded households, the paper tries to explain the response of each type of agent to productivity shocks and the cumulative effect on the economy. The ability to borrow allows financially included agents to make consumption more volatile than income. The volatility of income is equal to volatility of consumption for a financial excluded agent in the absence of saving and borrowing. Thus, increasing the proportion of financially included agents increases the ratio of volatility of consumption to volatility of income in the economy.

There are plenty of reasons of why people do not open accounts at a financial institution. Respondents in the 2014 Global Findex Database were asked to self-report reasons for not using an account at a financial institution. Some of the sole reasons cited are "Not enough money" (16% respondents), "Family member already has an account" (6%), "Do not need an account" (4%), "Financial institutions too far away" (2%) and "Accounts too expensive" (1%)[1]. "Not enough money" was also the most cited reason along with other reasons. Using these reasons, the world can be pictured as 2 regions, say rural and urban where banks are located near urban region, reflecting the geographical distance hindrance that "Financial institutions are too far away". At the same time a combination of factors such as fixed cost of opening and maintaining an account and mode of operation and low returns to production could lead to the other reasons reported by the respondents. A high fixed cost of opening and maintaining an account could lead to respondents in the rural region to claim "Accounts are too expensive". At the same time, the production technology and mode of operation might evolve in such a way for the respondents to report "Not have money", "Do not need an account" and "Family member already has an account". The production technology of the rural region good might be characterised by a high labour intensive technology which leads to low income and "Not having money" to open an account. This is not difficult to fathom as the rural sector and informal sector, especially in emerging countries, are characterized by disguised unemployment and lack of technological innovation. At the same time, lack of financial services could lead or create a cash based operating structure which requires little starting capital and owners pay cash for the resources they employ after selling the good. This leads to respondents reporting "Do not need an account" and "Family member already has an account".

All of the above factors combine to force the rural region to produce a different good than the urban region. This creates a both geographical and sectoral divide. Financial services do not exist for the rural region to arrange the necessary capital required to produce the urban region good. The urban region evolves into producing a good which requires capital and working capital which necessitates the need to be near a financial institution. They do not produce the same good as the rural region since their good can generate higher returns for them by making use of the capital. The above interpretation is not the sole way to think about the various reasons of

[1]These numbers were taken from Demirguc-Kunt et al. 2012 [11]. Some of the other reaons are "Religious Reasons", "Lack of mistrust", "Cannot get an account" and "Lack of necessary documentation".

why households do not own a financial account but it can be one plausible story. Not all of the features of the story get translated into the model directly, since the focus of the paper is not to understand how segmentation occurred but rather understand implications of a given segmentation. Taking the possible reasons and consequences of segmentation into account helps to create a model environment which takes into account as many aspects of reality as possible.

The model will be characterised by 2 types of agents with proportion of each type being fixed. Both types will work in their own backyard to produce different goods. Both production technologies will be constant returns to scale so that the two types do not differ in returns to production but which factors of production are employed. An attempt will be made in the model to bring in as little differences between the agents as possible so as to highlight the role of segmentation in consumption smoothing and volatility of aggregate consumption. Financial inclusion is defined as the ability to save/borrow in international one time period bonds and invest in capital markets. One type will be financially excluded who will produce a good with their own labour and without capital and not participate in either bond or capital market. The other type will be defined as financially included who will produce a good with their own labour and capital and who can participate in both markets. The 2 production process are not made different in modes of operation i.e. cash based or working capital based, since it brings more frictions to the environment, complicating the other mechanisms. To ensure that the financially included agents do not produce the good produced by the financially excluded agents, at steady state financially included agents earn more than financially excluded agents. I do not have an analytical condition imposed in the model which will ensure the above holds. We check this condition for the parameters chosen for the calibration exercise.

Given the above set-up of the agents, when a positive transitory shock hits the economy, financially included agents save in bonds/invest in capital today instead of increasing consumption by a large magnitude. When a positive trend shock hits the economy, financially included agents smooth consumption by increasing both investment and consumption. A positive trend shock increases output and future output more and thus, the need to sacrifice consumption today for investment does not arise. This makes consumption more volatile than income for financially included agents for a trend shock than a transitory shock. Financially excluded agents increase income and consumption by the same magnitude in the absence of any saving or borrowing instrument. Thus, to reconcile the business cycle feature of consumption being more volatile than income for emerging countries than developed countries, emerging countries need a profile of productivity shocks where trend shocks display higher variance than transitory shocks. The calibration exercise confirms this intuition with 0.5 and 1 being the variances of transitory and trend shocks for emerging countries and 0.5 and 0.1 are the variances of transitory and trend shocks for developed countries. In a log-linearised model, the ratio of variances matter and not the actual numbers. To justify the model, the data and model correlation of financial inclusion and the ratio of volatility of consumption and volatility of output is compared. For emerging countries, the model and data correlation is close but that is not true for developed countries. This is attributed to both the data having more variation than the model can account for and the model discontinuity when no financially excluded agents exist.

The paper is connected with 2 areas of research. Firstly, Business cycle models which deal with volatility and correlation of economy aggregates such as consumption volatility against output volatility, current account cyclicality and volatile and persistent real exchange rate movements and secondly, Financial segmentation models where the key implication is that open market operations reduce the nominal interest rate and generate a liquidity effect.

Aguiar and Gopinath 2007 [1] was one of the first papers to highlight the differences between the business cycle features of emerging and developed countries. Emerging countries exhibit consumption volatility that exceeds income volatility and strongly countercyclical current accounts. Their conclusion was that trend shocks rather than transitory shocks play an important role for emerging countries to match their business cycle features. Most of the recent literature has added different frictions to the basic dynamic shochastic model of Mendoza 1991 [19]. Informational frictions (Boz et al. 2011 [5]), search frictions (Boz et al. 2015 [6]) and financial frictions (Chang and Fernandez 2013 [8] have been the most common discussed.

Informational frictions have been interpreted as agents not being fully informed about the nature of the shocks i.e. whether they are transitory or trend. The authors argue that emerging countries suffer from more severe informational frictions than developed countries. Chang and Fernandez claim that interest rate shocks exemplified by Neumeyer and Perri [20] rather than trend shocks are more empirically consistent with emerging countries. Boz et al. rely on search frictions and interest rate shocks to explain the consumption, wage and employment volatility with respect to output for emerging countries. All of the above papers have a representative agent set-up with the mechanism being a friction or shock to or a combination of them to explain the business cycle feature. Rather than focus on shocks which are more difficult to understand, an attempt is made to focus attention on the role played by heterogeneity in financial inclusion, income and wealth. Financial exclusion goes hand-in-hand with income and wealth inequality. In the model, financially excluded agents earn less than financially included agents at steady state and do not have any capital wealth. Krueger et al. 2017 [17] show the role of income, wealth and preference heterogeneity in amplifying and propagating a macroeconomic shock. They show that households across various segments of the wealth expenditure changed their propensity to consume out of income and wealth during the Great Recession.

Some very recent papers Gao, Hnatkovska and Marmer 2014 [14] and Epstein et al. 2017 [12] have incorporated financially segmented agents to analyse different business cycle features. Gao et al. argue that incorporating financial segmentation significantly improves the statistical performance of standard international business cycle models to solve the "international comovement" puzzle, "quantity" puzzle and overall performance in terms of variances and correlations of economic aggregates such as output, consumption, hours, investment, etc. Epstein et al use financial segmentation to show that increase in global financial risk leads to smaller contractions in output, employment and investment in emerging countries than advanced economies. My paper builds the case further for incorporating financial segmentation in business cycle models. Even though I consider hand-to-mouth agents, the conclusions for consumption volatility against output volatility are non-trivial. The interaction of the consumption volatility of the two types of agents to an output shock is non-monotonic which is a contribution from a theoretical viewpoint. Empirically, the model tries to account for the correlation between financial inclusion and volatility of consumption to volatility of income.

The exogenous segmentation models consider either a fixed proportion of agents who can trade between bonds and money and agents who only hold money (Alvarez et al. 2001 [4], Lahiri et al. 2007 [18]) or a fixed duration between which agents can rebalance their portfolios (Rotemberg 1984 [21], Alvarez el al. 2009 [2]). Lahiri et al. discuss the role of asset market segmentation in choosing the optimal exchange rate regime. The conclusions are that flexible exchange rates are optimal under monetary shocks and fixed exchange rates are optimal under real shocks in an environment with no nominal rigidity and the only friction being exogenous asset market segmentation. Alvarez et al. 2009 show that prices and inflation respond sluggishly to an exogenous open market operations and exogenous increase in interest rates respectively with flexible price setting. Since, certain agents every period are able to transfer resources between their brokerage and bank account which then they use to buy goods, so aggregate velocity of money falls partially. All of the above papers have an endowment economy set-up with the focus being to analyse monetary policy.

The endogenous financial segmentation have looked at either endogenizing the timing of the transfers between bonds and money (Chiu 2007 [10]) or the pool of agents who choose to trade between bonds and money any period thus, creating a segment of agents who are active traders any period ((Chatterjee and Corbae 1992 [9], Alvarez et al. 2002 [3], Khan and Thomas 2015 [16])). Chiu tries to understand the timing of the rebalancing between money and bonds and the short run and long run effects of monetary shocks. Agents get a chance to transfer funds between their bank account (equivalent to holding money) and brokerage account (save or borrow in bonds) and there is a fixed cost associated with it. The conclusion of the paper is that for a small money shock, agents do not adjust their transfer frequencies but for a large money shock, it is optimal for agents to adjust their transfer frequencies. Chatterjee and Corbae were the first to examine endogenous financial segmentation in a general equilibrium set-up to examine how different constant inflation rates affect the steady state. Alvarez et al. 2002 analyse the dynamic effects on interest rates and exchange rates. Open market operations lead to a negative relation between expected inflation and real interest rates as real interest rates are now determined by the segment of the population who choose to trade between money and bonds.

Khan and Thomas develop a monetary model which tries to deliver a correlation between aggregate consumption growth and short-term real interest rates consistent with U.S. data. Agents incur fixed transactions costs when exchanging bonds and money and, as a result, carry money balances in excess of current spending to limit the frequency of such trades which leads to a nontrivial distribution of money holdings across time periods. The fixed transaction costs are independently and identically distributed across time and agents have access to a complete set of state contingent claims in their brokerage account. The key conclusions are that they can attain a negative correlation between aggregate consumption growth and real interest rates observed in post-war U.S. which cannot be attained by a representative household model. Fixed time-invariant exogenous market segmentation will give a positive correlation but exogenous financial market segmentation which changes with aggregate conditions can deliver a negative correlation.

Most of the financial segmentation models above, deal with an endowment economy or in some cases a production economy but there is no role of capital accumulation in these models. Access to equity market is another type of segmentation which is not considered in this literature. I believe it is sufficient to restrict myself to time-invariant exogenous segmentation since time-varying exogenous segmentation or endogenous segmentation will help to make the pool of included and excluded agents non-trivial but not change the nature of their response in consumption to a shock.

An attempt has been to made to connect the above two literature since the business cycle models deals with volatility of consumption but in an environment where all agents are allowed to insure against shocks (even if imperfectly) and financial segmentation models which have not considered the implications of the differences in the ability of the two types of agents to insure against shocks.

The rest of the paper has been organized as follows. Section 2 presents the data on some facts about financial inclusion and business cycle features of emerging and developed economies. Section 3 presents our model and impulse response functions. I discuss calibration, results and data based justification of model in Section 4. The Appendix deals with data sources, breakdown of aggregate data into each country and log-linearised equations of the model.

2. DATA

To answer the question well, we would have liked to have microdata on consumption, income, demographics and access to financial services. But due to the unavailability of such data, I focus on macroeconomic aggregates which can help answer the question. In this section I document some facts about financial inclusion and business cycle features of emerging and developed economies. The countries included in the analysis are; Emerging: Botswana, Brazil, Colombia, Costa Rica, Ecuador, India, Indonesia, Malaysia, Mexico, Peru, Philippines, South Africa, Thailand and Turkey ; Developed: Australia, Austria, Belgium, Finland, Ireland, Israel, Italy, Luxembourg, Netherlands, Norway, Singapore, Spain, Sweden and Switzerland. There are 14 countries in each category to have a balanced sample. To make the distinction between emerging and developed economies I considered those economies as "Emerging" which were always classified by the World Bank as either Lower or Lower-middle or Upper-middle economies from 1987 - 2016 and those economies as "Developed" which were always classified as High income economies by the World Bank from 1987 - 2016. AG 2007 argue that emerging and developed countries receive different productivity shocks so I want to consider those economies which might be experiencing the same underlying productivity processes within the same group. The reason to not use the same sample of countries as AG 2007 is because countries like Chile, Korea are considered High income countries now. Also, countries like Canada and New Zealand which were in the AG 2007 sample of developed countries, have not been chosen so as to get some variation in the level of financial inclusion among the group of developed countries which will be useful for later analysis.

Table 1 shows the level of financial inclusion across two measures and income classes. "Account" has been defined by the 2014 Global Findex Database as the proportion of respondents (above 15 years) having an account (themselves or with someone) at a bank or another type of financial institution; having a debit card in their own name; receiving wages, government transfers, or payments for agricultural products into an account at a financial institution or through a mobile phone in the past 12 months; paying utility bills or school fees from an account at a financial institution in the past 12 months; receiving wages or government transfers into a card in the past 12 months; or personally using a mobile phone to pay bills or to send or receive money through a

GSM Association (GSMA) Mobile Money for the Unbanked (MMU) service in the past 12 months. "Account at financial institution" excludes mobile banking from the definition used for "Account". "Account, income poorest 40%" denotes the percentage of respondents in the poorest 40% of households.

Developed countries have a significantly higher level of account penetration across both measures and income classes than emerging countries. On average, account penetration is 96.63% and 53.16% for developed and emerging countries respectively with little variation among the developed countries unlike the emerging countries. The range of account penetration for emerging countries in our sample is 29% (Peru) - 80% (Malaysia) with 1st and 3rd quantiles at 39% and 68% respectively. The highest level of financial inclusion across any measure for emerging countries is less than the minimum level for the corresponding measure for developed countries. Comparing between "Account" and "Account at a financial institution", mobile banking increases the average account penetration by 1% which is not very significant so I will use "Account at a financial institution" as the measure of financial inclusion for the 2 groups of countries. The poorest 40% of the income distribution, are also more unbanked than the richest 60% of the income distribution. On average, for emerging countries account penetration is 42% and 59% for the poorest 40% households and richest 60% households respectively, whereas in the developed countries banking facilities are available to 95% and 97.6% on average to poorest 40% households and richest 60% households respectively.

The reason financial inclusion is important to policy-makers because it is a good proxy of the resilience of the households against a negative shock to their income. A lower level of financial inclusion in a country possibly reflects the inability of the majority households to smooth consumption when hit with income shocks. Figure 2 shows the average across the developed and emerging countries when respondents were asked about the possibility of coming up with emergency funds within the next month [2]. Emergency funds refer to an amount equivalent to 1/20 of GNI per capita in local currency. On average 52% and 17% of the respondents among developed and emerging countries, said it was "very possible" that they could come up with such funds. 23.5% and 31% said it was " somewhat possible" and 10% and 18% said it was "not very possible" to come up with such funds among the developed and emerging countries respectively. 10% and 18.5% said it was "Not at all possible" to come up with such funds among developed and emerging countries respectively on average.

This gap exists across different income categories in the ability to face income shocks between the 2 countries. On average, 79% and 54% respondents among the poorest 40% households and 91% and 75% among the richest 60% households believe they have any possibility of coming up with emergency funds among developed and emerging countries respectively. 20% and 43% among the poorest 40% households and 9% and 25% among the richest 60% households believe that it is not possible for them to come up with the required funds among developed and emerging countries respectively. The answers to some degree are subjective and depend on the interpretation of the respondent to be able to put themselves into either of the 4 categories since the poorest 40% households also record 3% of non-answers. But the differences in the responses of households in developed and emerging countries are not small, suggesting that financial inclusion can be interpreted as a proxy of the resilience of the agents to sustain income shocks. I make this resilience extreme in the paper for simplification, that financially included agents can smooth consumption and financially excluded agents can-

[2]Table showing averages and quantiles of the figure is in **Appendix I**

Table 1

Financial inclusion 2014

	Developed		Emerging	
Account	96.63	(96.17, 99.29)	53.16	(39.00, 68.12)
Account, income poorest 40%	95.13	(93.82, 98.67)	42.93	(24.40, 58.47)
Account, income richest 60%	97.66	(97.28, 99.76)	60.10	(45.87, 74.62)
Account at financial institution	96.63	(96.17, 99.30)	52.50	(38.35, 68.12)
Account at financial institution, income poorest 40%	95.13	(93.82, 98.67)	42.24	(23.4, 58.47)
Account at financial institution, income richest 60%	97.66	(97.28, 99.76)	59.46	(45.59, 74.62)

Note: Average level of account penetration among group of developed (14) and emerging (14) countries. 1st and 3rd quantile reported in parentheses.

not, but the data does suggest that on average, households in countries with higher financial inclusion feel much better equipped to deal with shocks [3].

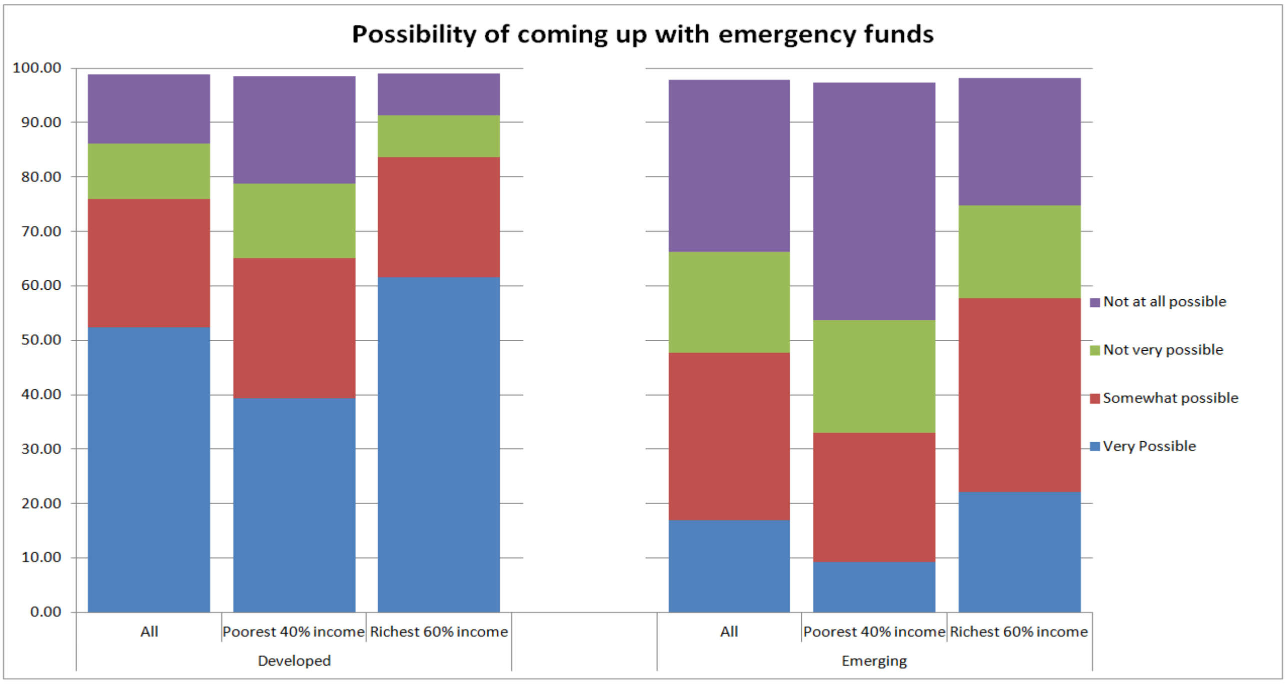

Figure 1: Possibility of coming up with emergency funds on average across developed (14) and emerging (14) countries. Emergency funds refer to the ability of coming up with an amount equivalent to 1/20 of GNI per capita in local currency within the next month. The categories do not sum to 100 because of "don't know" and "refuse" answers to the question.

[3]On average, the main source of emergency funds among the respondents who declared any possibility to come up with funds are savings, financial institution, family and friends, employer loan and informal lender with respondents reporting 60%, 6%, 18%, 11% and 0.1% among developed countries and 27%, 3%, 46%, 16% and 2.3% among emerging countries. This fact also reflects inability of households in emerging countries to channel and use savings in an effective manner. They depend on informal agreements to smooth consumption which work well maybe for individual shocks but exposes themselves to aggregate shocks.

Table 2

Emerging vs. Developed markets

	Developed				Emerging			
	Quarterly		Annual		Quarterly		Annual	
$\sigma(C)/\sigma(Y)$	0.88	(0.74, 1.01)	0.93	(0.79, 0.99)	1.21	(1.04, 1.35)	1.32	(1.14, 1.43)
$\sigma(Y)$	1.64	(1.17, 2.07)	1.47	(1.04, 1.97)	2.61	(1.63, 3.27)	2.21	(1.85, 2.55)
$\sigma(\triangle Y)$	1.13	(0.71, 1.43)	2.56	(1.82, 3.12)	1.78	(1.28, 2.35)	3.70	(2.98, 4.12)
$\rho(Y)$	0.79	(0.79, 0.86)	0.25	(0.14, 0.39)	0.79	(0.75, 0.85)	0.21	(0.10, 0.30)
$\rho(\triangle Y)$	0.14	(-0.02, 0.27)	0.36	(0.25, 0.45)	0.20	(0.04, 0.34)	0.26	(0.14, 0.45)
$\sigma(I)/\sigma(Y)$	3.10	(2.52, 3.67)	3.05	(2.53, 3.31)	3.39	(2.81, 3.90)	3.32	(2.71, 3.84)
$\sigma(NX/Y)$	1.47	(0.70, 2.01)	1.02	(0.64, 1.24)	3.16	(1.50, 2.35)	2.09	(1.25, 2.53)
$\rho(NX/Y,\ Y)$	0.00	(-0.04, 0.22)	-0.11	(-0.23, 0.06)	-0.35	(-0.44, -0.26)	-0.33	(-0.58, -0.13)
$\rho(C,\ Y)$	0.59	(0.47, 0.67)	0.62	(0.55, 0.74)	0.68	(0.55, 0.82)	0.63	(0.52, 0.83)
$\rho(I,\ Y)$	0.64	(0.52, 0.80)	0.69	(0.59, 0.82)	0.74	(0.70, 0.83)	0.68	(0.56, 0.78)

Note: The table lists the average values of the moments for the group of emerging(14) and developed(14) economies. The 1st and 3rd quantile of each moment are reported in parentheses.

Table 2 shows some key moments of the business cycle averaged over emerging and developed economies. The breakdown of all the indicators for all countries is in **Appendix I**. To obtain the business cycle averages, Output (Gross Domestic Product), Consumption (Household Consumption Expenditure including Non-Profit Institutions), Investment (Gross Fixed Capital Formation) and Net exports (Exports - Imports) at constant prices was obtained from either IFS or OECD database. For emerging countries, the average (minimum) length of the time period in the sample is 114 quarters (83 quarters) for quarterly data and 48 years (38 years) for annual data. For developed countries, the length of the time period in the sample is 148 quarters for quarterly data for each country and average (minimum) length of annual data is 49 years (46 years). The detailed list of the length of the time period and source of data for each country is in **Appendix I**. Output, Consumption and Investment were logged first and along with Net exports were HP-filtered to calculate volatility and autocorrelation. First differenced output was not filtered. A smoothing parameter of 1600 for quarterly data and 6.25 for annual data was employed. The 1st and 3rd quantile of each moment are reported in parentheses.

One would expect that less financially included countries to display a higher ratio of standard deviation of consumption to standard deviation of output as they cannot smooth consumption as well as they would wish. This turns out to be true, as Emerging countries have a much higher ratio than the Developed countries. Emerging countries have a ratio of 1.32 (1.21) for annual (quarterly) data whereas Developed countries have a ratio of 0.93 (0.88). Across both frequency levels, the third quantile of the ratio is less for developed countries than the first quantile for emerging countries for the corresponding frequency. This suggests that the difference in the ability to smooth consumption for the two groups of countries is quite pronounced. I obtain this ratio by HP-filtering the log series of consumption and output.

The 2nd and 3rd row show the standard deviation of filtered log output and standard deviation of first difference of unfiltered log output. Output is more volatile for emerging countries than developed countries across both measures. The 4th and 5th row show the autocorrelation of filtered log output and autocorrelation of first difference of unfiltered log output. Output shows similar autocorrelation across the 2 groups with quarterly frequency having a higher autocorrelation than annual for filtered log output and otherwise for first difference of unfiltered log output.

The 6th and 7th row show the ratio of standard deviation of filtered log investment to standard deviation of filtered log output and standard deviation of filtered net exports to output. The ratio of standard deviation of investment to standard deviation of output is similar across both groups of countries but the standard deviation of net exports to output is much higher for emerging countries than developed countries. Rows 8, 9 and 10 show the autocorrelation of filtered log consumption and investment with filtered log output and filtered net exports to output with filtered output. The autocorrelation of consumption and investment with output is similar on average across the 2 groups of countries with similar quantiles but net exports is on average countercyclically related to output for emerging countries but for developed countries this correlation is weak.

3. MODEL

The model will be characterised by 2 types of agents. Financial inclusion is defined as the ability to save/borrow in international one time period bonds and invest in capital markets. I will assume for now, that this segmentation is exogenous. λ proportion of agents are financially excluded and rest $1 - \lambda$ proportion of agents are financially included. Each financially excluded type (referred with superscript e) will produce good "n" with their own labour and without capital. They do not participate in either the bond or capital market. Each financially included type (referred with superscript i) will produce good "m" with their own labour and capital. Good "m" will be the numeraire good. There is no heterogeneity within each group of agents. All agents are price-takers.

There will be productivity shocks of 2 types which will be the same for each agent in the economy across both types so as to understand the responses and interaction of the agents to the same shock. The focus is to understand role of the agents in contributing to consumption volatility and so, introducing productivity shocks is sufficient for our purpose. The structure of the shocks will be explained below but these shocks can be thought of as a combination of various demand and supply shocks, labour market frictions, informational frictions and political economy frictions. The structure of the financially included agents and nature of shocks follows closely AG 2007. Below, the model is presented in detail. The superscript appears for variable where heterogeneity exists. No superscript appears for variables where either no heterogeneity exists or is irrelevant to a particular group of agents.

Production

The production function is the following:

$$y_t^i = Z_t \left(\Gamma_t l_t^i\right)^\alpha k_t^{1-\alpha} \text{ for the financially included agents} \tag{1}$$

$$y_t^e = Z_t \Gamma_t l_t^e \text{ for the financially excluded agents} \tag{2}$$

$$ln\left(Z_t\right) = z_t = \rho_z z_{t-1} + \epsilon_t^z \ ; \ \epsilon_t^z \sim N\left(0, \sigma_z^2\right) \tag{3}$$

$$\Gamma_t = G_t \Gamma_{t-1} \tag{4}$$

$$ln\left(G_t\right) = g_t = (1-\rho_g) ln\left(\mu_g\right) + \rho_g g_{t-1} + \epsilon_t^g \ ; \ \epsilon_t^g \sim N\left(0, \sigma_g^2\right) \tag{5}$$

Thus, z_t and $g_t$follows an A.R.(1) process. z_t will be referred to as transitory shocks and g_t as trend shocks.

Utility

The infinitely lived agents will maximise the following utility function (credited to Jaimovich and Rebelo [15]):

$$U^j = \mathbb{E}_0 \left(\sum_{t=0}^{\infty} \beta^t \frac{\left(u_t^j - \tau \Gamma_{t-1} \left(l_t^j\right)^\theta\right)^{1-\gamma} - 1}{1-\gamma} \right) \tag{6}$$

$$u_t^j = \left[\psi_m \left(m_t^j\right)^{\frac{s-1}{s}} + (1-\psi_m)\left(n_t^j\right)^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}} \tag{7}$$

where I assume that, $0 < \beta < 1$, $\theta > 1$, $\tau > 0$, $\gamma > 0$, $0 < \psi_m < 1$, $s \geq 0$ and $j = i, e$.

Financially included agents

Financially included agents maximise utility subject to a budget constraint where they choose their consumption, labour, capital and bond portfolio. Bonds are one-time period, risk-free and internationally traded. Each such agent produces the good m $\left(y_t^i\right)$ with the help of his own labour $\left(l_t^i\right)$ and capital input (k_t) today which was decided by them yesterday. With this output, the agent can either consume good m $\left(m_t^i\right)$ or good n $\left(n_t^i\right)$ at a relative price in terms of good m (p_t) or invest capital (k_{t+1}) or pay his debt (b_t) and get new debt (b_{t+1}) at price q_t.

Investment is defined as:

$$x_t = k_{t+1} - (1-\delta)\,k_t + \frac{\psi_k}{2}\left(\frac{k_{t+1}}{k_t} - \mu_g\right)^2 k_t \tag{8}$$

The equation contains an adjustment factor so that adjusting investment is costly. This is to prevent all the adjustment in capital rather than consumption by an agent when hit by a shock.

The budget constraint of the household is:

$$y_t^i + (1-\delta)\,k_t - \frac{\psi_k}{2}\left(\frac{k_{t+1}}{k_t} - \mu_g\right)^2 k_t - b_t + q_t b_{t+1} = m_t^i + p_t n_t^i + k_{t+1} \tag{9}$$

Also, internationally, the price of the bonds satisfies:

$$1/q_t = 1 + r^* + \psi_b\left(\exp\!\left(\frac{b_{t+1}}{\Gamma_t} - \overline{b}\right) - 1\right) \tag{10}$$

The interest paid on the debt is a function of the quantity of debt that is held by agents and $\psi_k > 0$ is introduced to ensure stationarity in asset holdings just as shown in Schmitt-Grohe and Uribe 2003 [22] and $\overline{b}$ is the normalised steady state level of debt.

The realization of g affects Γ permanently and output is non-stationary with a stochastic trend. So, for any variable x, let $\hat{x}$ denotes its detrended counterpart.

The detrended problem such agents will solve is the following:

$$V\left(Z, G, \hat{k}, \hat{b}\right) = \max_{\left\{\hat{m^i}, \hat{n^i}, l^i, \hat{k\prime}, \hat{b\prime}\right\}} \frac{\left(\hat{u^i} - \tau\left(l^i\right)^\theta\right)^{1-\gamma} - 1}{1-\gamma} + \beta G^{(1-\gamma)} \mathbb{E} V\left(Z\prime, G\prime, \hat{k}\prime, \hat{b}\prime\right) \tag{11}$$

subject to

$$\hat{u^i} = \left[\psi_m\left(\hat{m^i}\right)^{\frac{s-1}{s}} + (1-\psi_m)\left(\hat{n^i}\right)^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}} \tag{12}$$

$$\hat{y^i} + (1-\delta)\,\hat{k} - \frac{\psi_k}{2}\left(\frac{G\hat{k}\prime}{\hat{k}} - \mu_g\right)^2 \hat{k} - \hat{b} + Gq\hat{b}\prime = \hat{m^i} + p\hat{n^i} + G\hat{k}\prime \tag{13}$$

$$\hat{x} = G\hat{k}\prime - (1-\delta)\hat{k} + \frac{\psi_k}{2}\left(\frac{G\hat{k}\prime}{\hat{k}} - \mu_g\right)^2 \hat{k} \tag{14}$$

$$\hat{y^i} = ZG^\alpha\left(l^i\right)^\alpha \hat{k}^{1-\alpha} \tag{15}$$

The total consumption of agent i is:

$$\hat{c^i} = \hat{m^i} + p\hat{n^i} \tag{16}$$

Financially excluded agents

Financially excluded agents do not have access to any financial institution and so cannot participate in either the bond or capital market. Each financially excluded agent produces good n (y_t^e) with their own labour input (l_t^e). With this output, the agent can either consume good m (m_t^e) or good n (n_t^e).

The budget constraint of the household is:

$$m_t^e + p_t n_t^e = p_t y_t^e \tag{17}$$

Thus, while solving the detrended problem, the household maximises:

$$V(Z,G) = \max_{\{\hat{m^e},\hat{n^e},l^e\}} \frac{\left(\hat{u^e} - \tau\left(l^e\right)^{\theta}\right)^{1-\gamma} - 1}{1-\gamma} + \beta G^{(1-\gamma)}\mathbb{E}V\left(Z\prime, G\prime\right) \tag{18}$$

subject to:

$$\hat{u^e} = \left[\psi_m\left(\hat{m^e}\right)^{\frac{s-1}{s}} + (1-\psi_m)\left(\hat{n^e}\right)^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}} \tag{19}$$

and

$$\hat{m^e} + p\hat{n^e} = p\hat{y^e} \tag{20}$$

$$\hat{y^e} = ZGl^e \tag{21}$$

The total consumption of agent i is:

$$\hat{c^e} = \hat{m^e} + p\hat{n^e} \tag{22}$$

Market clearing for good m requires that the following holds:

$$\hat{m^i} + \hat{x} + \hat{b} - qG\hat{b}\prime + \frac{\lambda}{1-\lambda}\hat{m^e} = \hat{y^i} \tag{23}$$

By Walras Law, if the bond market and market for good m clears then the market for good n will clear as well. The following equation will hold:

$$\frac{1-\lambda}{\lambda}\hat{n^i} + \hat{n^e} = \hat{y^e} \tag{24}$$

Aggregate consumption

$$C_t = \lambda c_t^e + (1-\lambda)\, c_t^i \tag{25}$$

Aggregate output

$$Y_t = \lambda p_t y_t^e + (1-\lambda)\, y_t^i \tag{26}$$

Net exports

$$nx_t = \frac{NX_t}{Y_t} = \frac{y_t^i - m_t^i - p_t n_t^i - x_t}{Y_t} \tag{27}$$

Impulse Response Functions

Figure 2 shows the impulse responses to a 1% transitory and trend shock[4]. With a positive transitory shock, total income increases with both financially included and excluded agents increasing their output by increasing their labour supply and also, investment. Financially included agents see a positive transitory shock as an opportunity to increase their investment but the adjustment cost prevents them from increasing it too much at any given point of time. So, they borrow a little today and increase consumption and capital stock today. Thus financial inclusion helps them in two aspects: being able to borrow today and also, invest today, together both of which help them to increase output today and in the future. They increase their labour supply today but let it decrease from the next period onwards. Consumption and investment of the included agents increases for a while and so does their borrowing, though investment continues to rise for longer than consumption. The financially excluded agents also increase labour supply but their consumption rises less than output, because the price of the good that they produce has fallen. The relative price of good n falls today because total demand (consumption, investment and bonds) for good m has increased more than supply and so, good m is more expensive now. Financially included agents demand good m for investment and output of m does not keep pace with demand so relative price of good n decreases.

The effect of a positive trend shock is different as total income increases today with both the types of agents increasing their production. The response of the agents is much different to a trend shock than transitory shock. The financially included agents borrow today to increase investment and consumption. The top right panel in Figure 2 which plots the ratio of deviations of total consumption to total income. With a positive trend shock, total consumption increases more than total income today than with a positive transitory shock. Financially included agents borrow more to consume when a trend shock hits them. Output today and future output will increase forever with a higher trend with everything else constant. Thus, financially included agents do not sacrifice consumption to increase investment, just as they do for a transitory shock. They also increase their labour supply today to enjoy the positive trend shock. Then they reduce their labour supply to enjoy leisure as the positive trend ensures that reducing the labour supply will hurt their output less as this positive trend is not mean-reverting and will always persist. The financially excluded agent also increase their output and consumption today. Thus, in an economy which experiences greater variance of trend shocks then consumption becomes more volatile than output. If transitory shocks are more common in an economy then consumption will be less volatile than output. The fraction of financially included agents in the economy is also key as they are responsible to generate this higher volatility by being able to borrow. I will discuss about this in detail in the results section.

[4]The benchmark parameter values for Specification 1 were used to generate the impulse responses except $\rho_z = 0.98$ instead of $\rho_z = 0.95$.

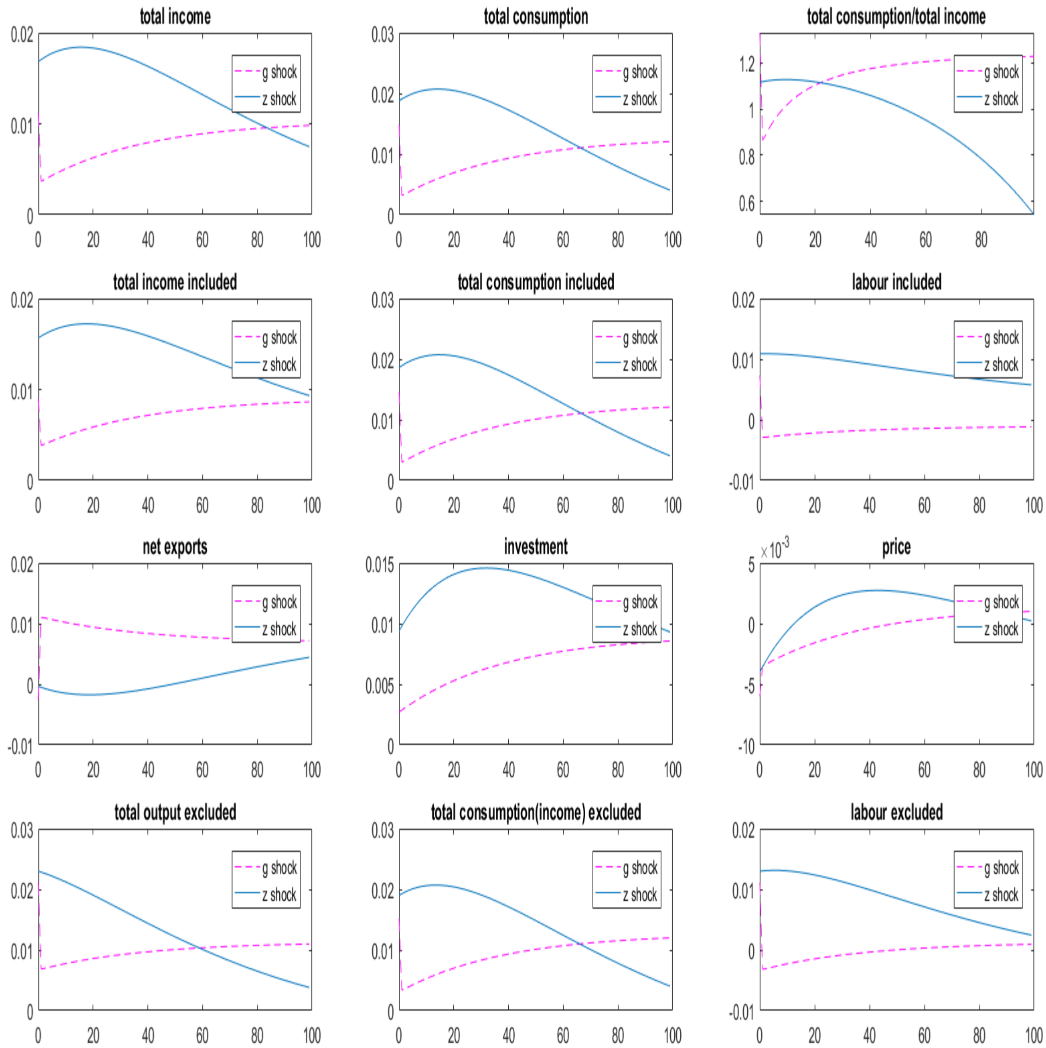

Figure 2: Impulse Response to 1% transitory and trend shock. The y-axis represents the% deviations from the steady state for each variable, and x-axis represents the time horizon.

4. SOLUTION

I solve the model by log-linearising the equilibrium conditions around the steady state. The parameters chosen are either widely used in the literature or taken from the data or calibrated to the steady state of the model. I explain them in more detail in the next section. I obtained the steady state of the model by expressing all endogenous variables (except the price) in terms of the steady state price level and then I minimized the distance between the analytical price level and the guess. The log-linearised equations have been derived in **Appendix II**. Harald Uhlig's toolkit [24] was used to derive the policy functions. Then, theoretical moments were calculated after HP-filtering the relevant variables with a smoothing parameter of 6.25. I use the same methodology as AG 2007.

Calibration parameters

A time period in the model corresponds to a year. Specification 1 corresponds to an average emerging country whereas Specification 2 corresponds to an average developed country. λ is set equal to the average financial exclusion over the countries in our dataset. ψ_m is set equal to $1 - \lambda$ as it represents the importance of the good produced by the financially included agents. As countries get more financially included, then more sectors get mechanised and become technologically advanced. Agents left financially excluded are those whose goods are valued less resulting in them not having enough money to open an account. β is the discount factor that is implied by the Euler equation with an average risk-free rate R^* of 1.1 over all countries. γ is the coefficient of relative risk-aversion which is set equal to 2. This value is widely used in the literature. s is the coefficient of elasticity of substitution between the 2 goods which is set to 1.1. I do not have a strong prior on it so I can check the sensitivity of my results to changing this parameter. τ has been set at 1.95 and 6 to achieve a steady-state of working hours to non-working hours of 0.54 and 0.43 for the emerging and developed countries respectively[5]. The Frisch labour supply elasticity is $\frac{1}{\theta-1}$. θ is set at 1.6 to get an elasticity of 1.7 which is widely used in the literature. The productivity process parameters ρ_z, μ_g, ρ_g have been set at 0.95, 1.04 and 0.01 and 0.985, 1.04 and 0.29 respectively across the 2 specifications. Ideally, I want to calibrate all the productivity process parameters but for the purpose of explaining the dynamics of the model, I have chosen the above parameter values. α has been set equal to 0.52 and 0.6 which is the average share of labour income in total income in the data. ψ_b is the interest rate premium parameter which has been set at a very small value of 0.00001 as in Schmitt-Grohé and Uribe (2003). ψ_k has been set at 25 which is common in the literature. δ is equal to 0.05 which is the average depreciation rate over all countries. $\overline{b}/\overline{Y}$ has been set at 0.1 which is used in the literature.

[5]To derive steady state of labour hours I found the average of working to non working hours annually in a year for each group of countries. To derive average labour income, I averaged the share of labour compensation in total income. Depreciation rate is the annual average depreciation rate. The data for depreciation rate, annual working hours per capita, share of labour compensation in total income are taken from Penn World Tables version 9.

Table 3

Benchmark Parameter values

Parameter	Specification 1	Specification 2	Source
λ	0.47	0.04	Average financial exclusion
ψ_m	0.53	0.96	$1-\lambda$
β	0.98	0.98	Euler equation
γ	2	2	Literature
s	1.1	1.1	Set
τ	1.95	6.1	Steady state labour hours
θ	1.6	1.6	Literature
ρ_z	0.95	0.98	AG 2007 use 0.95 and 0.97
μ_g	1.04	1.04	Set
ρ_g	0.005	0.29	AG 2007
α	0.52	0.6	Share of labour income in total income
ψ_b	0.00001	0.00001	Fernandez and Meza 2015 [13]
ψ_k	25	25	Altug and Kabaca 2017 [23]
δ	0.05	0.05	Average depreciation rate
$\overline{b}/\overline{Y}$	0.1	0.1	Literature

Results

I am able to match the ratio of standard deviation of cyclical aggregate consumption to standard deviation of cyclical aggregate output for emerging and developed countries. A financially included agent shows higher volatility in consumption than income to a trend shock. But such an agent shows lower volatility in consumption than income to a transitory shock. When trend shocks have higher variance than transitory shocks then consumption of financially included agents becomes more volatile than their income. Financially excluded agents are hand-to-mouth consumers so their volatility of consumption is equal to their volatility of income. In a log-linearised model, the ratio of $\sigma(g)/\sigma(z)$ matters, not the actual magnitude of the two shocks. If $\sigma(g)/\sigma(z) < 1$ then financially excluded agents will higher volatility in consumption w.r.t. income than financially included agents. But, there has to exist a sufficiently high number of financially excluded agents to get $\sigma(C)/\sigma(Y) > 1$. We will discuss in more details later, what happens by just changing the levels of financial inclusion and keeping the same profile of shocks. If $\sigma(g)/\sigma(z) > 1$ then any proportion of financially included agents is sufficient to get $\sigma(C)/\sigma(Y) > 1$. Thus, to match the business cycle feature for emerging countries higher variance of trend than transitory shocks is required given the proportion of financially excluded agents and persistence of shocks. And similarly, in order to get aggregate consumption less volatile than aggregate income, lower variance of trend than transitory shocks is required given relatively more financially included than excluded agents and persistence of shocks.

Table 4

Theoretical Moments

	Specification 1 $\sigma_z = 0.5$, $\sigma_g = 1$	Specification 2 $\sigma_z = 0.5$, $\sigma_g = 0.1$
Var (C) / Var (Y)	1.33	0.94
Var $\left(c^i\right)$/ Var $\left(y^i\right)$	1.59	0.94
Var (c^e)/ Var (y^e)	1	1
Var (Y)	0.95	0.50
Var (C)	1.26	0.47
Var $\left(y^i\right)$	0.79	0.50
Var $\left(c^i\right)$	1.24	0.46
Var (y^e)	1.28	0.48
Var(c^e)	1.28	0.48

Discussion of results

Relevance of capital and adjustment cost of capital

Capital has an important role to play to achieve the above results. Financially included agents increase consumption less today when hit by a positive trend or transitory shock is because they view a good shock as an opportunity to participate in the capital market. If the capital market is removed and the rest of the structure is the same as before then qualitatively, results do not change. Trend shocks still generate higher volatility in consumption to volatility in income, but quantitatively the desired results are not obtained with the above set of shocks.

If I consider the same model without any adjustment cost of capital then I will have a lower ratio of cyclical standard deviation of aggregate consumption to cyclical standard deviation of aggregate output for emerging countries than before and much higher for developed countries than before. With a positive transitory shock today, and no adjustment cost of capital, investment increases today much higher than output by sacrificing consumption today. Consumption then increases a lot which makes consumption more volatile than a model with adjustment cost. With a positive trend shock today, the increases in consumption is less and increase is investment is more than a model with adjustment cost. This makes consumption less volatile than a model with adjustment cost.

Relevance of financial inclusion parameter $1 - \lambda$

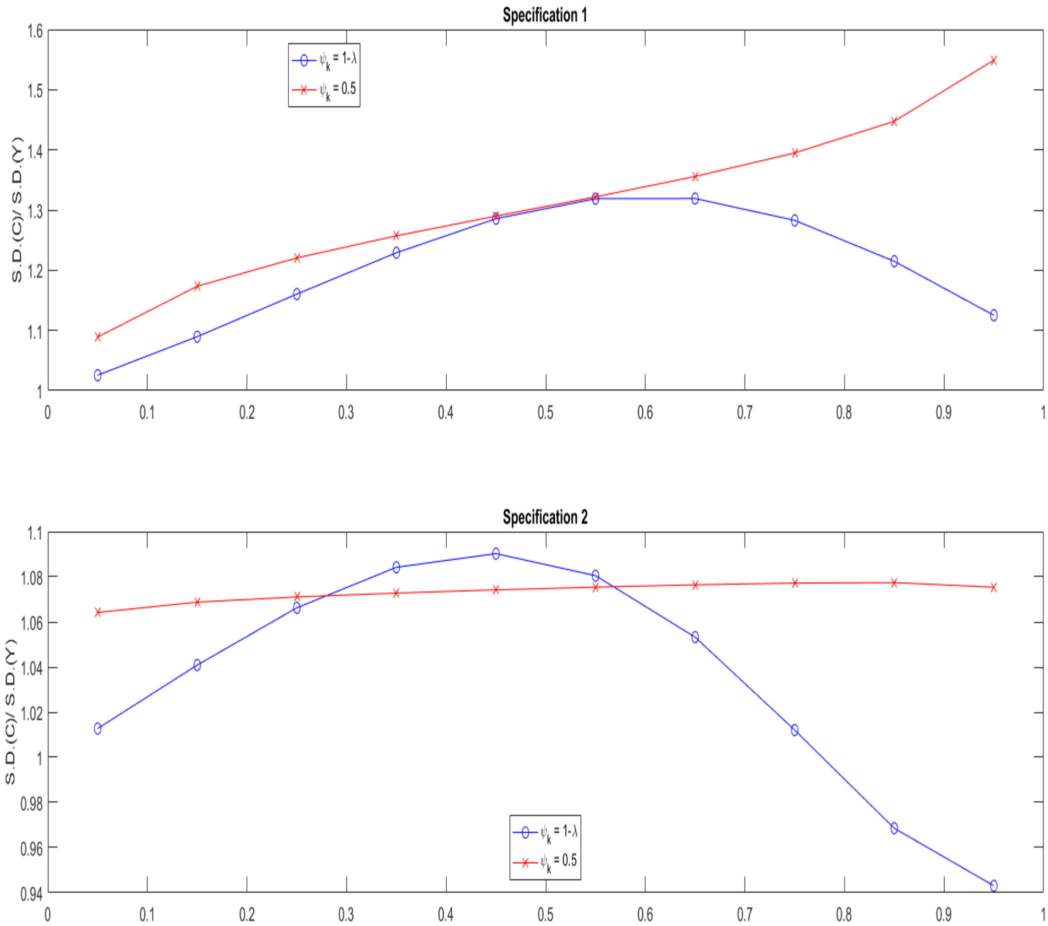

Figure 3: Ratio of standard deviation of cyclical total consumption to standard deviation of cyclical total output across the 2 specifications with different levels of financial inclusion.

The financial inclusion parameter $1 - \lambda$ affects two things, the proportion of financial included agents and the share parameter of good m vis-a-vis good n in the utility function. The ratio of standard deviation of total consumption to standard deviation of total income depends on two things, the magnitudes of the standard deviations of consumption and income of each type of agent and the proportion of the 2 types of agents. Having more financially included agents reduces the aggregate volatility in consumption to output if the variance of transitory shocks is higher w.r.t. to trend shocks and increases the aggregate volatility in consumption to output otherwise. But, increasing $1 - \lambda$ also increases the the importance of good m relative to good n in the utility function of an agent. When say $1 - \lambda = 0.95$ then good m is valued more in the utility function of each agent. When income increases, the consumption of good m will increase more relative to good n and so, the price of good n will change less. Thus, there is less price volatility.

The difference between the two panels of Figure 3 is that the top panel corresponds to the 1st Specification used in the calibration whereas the bottom panel refers to the 2nd Specification. If, trend shocks have higher variance than transitory shocks, then increasing financial inclusion will increase the volatility of total consumption to total income with a constant share parameter because of 2 reasons. One reason is that as the financially included agents show higher volatility and increasing their proportion increases the ratio. The difference between a share parameter of 0.5 and 0.95 is that with share parameter of 0.5, good m is as important to an agent as good n. To increase utility, an agent will increase consumption of both and so with an income shock, demand of both goods is similar. When there are few excluded agents, then demand for their good is more than supply so price changes a lot with an income shock. This increases the volatility in consumption for both sets of agents. This is why the ratio of volatility of consumption to volatility of income is much higher with a share parameter of 0.5 than 0.95 when $1 - \lambda = 0.95$. Similarly when transitory shocks have higher variance than trend shocks then, the financially included agents display less consumption volatility when good m occupies a larger share in their consumption basket and financially included agents are in greater proportion than financially excluded agents.

Justification of model

The model predicts a different profile of productivity shocks for each group of countries. Now suppose with the same set of shocks as in the calibration exercises, how well does the model do in attaining the correlation between financial inclusion and ratio of standard deviation of consumption to standard deviation of income across countries within the same group of countries. Table 5 shows data on financial inclusion and the ratio of standard deviation of filtered log consumption to standard deviation of filtered log income for each emerging and developed economy at annual frequency. The correlation between financial inclusion and the ratio of standard deviation of consumption to standard deviation of income is 0.52 for emerging countries and -0.33 for developed economies. Now, using the Specification 1 for emerging countries and Specification 2 for developed countries, I will find the data corresponding correlation in the model.

Figure 4 shows the performance of the model against the data. For emerging countries, the data correlation is 0.52 whereas the model correlation is 0.58. For developed countries, the data correlation is -0.32 and the model gives a correlation of 0.21. The model does better in capturing the relation between financial inclusion and volatility of consumption to volatility of income for emerging countries but less so for developed countries. For emerging countries, changing the share parameter is important to get such a close match between the data and the model. For developed countries, the data shows more variation for each country than what the model can account for. Most of the countries are in the range of 94-100 percent level of financial inclusion. The model generates similar ratio for financial inclusion between 94-100 percent. There is a model discontinuity at 100% as there is only one good in the economy. This contributes to getting a high ratio at 100% level of financial inclusion.

Table 5

Financial inclusion and relative volatility of consumption

	Financial inclusion	$\sigma(C)/\sigma(Y)$
	Emerging	
Philippines	28.1	0.72
Peru	29	0.99
Indonesia	35.9	1.39
Colombia	38.4	1.32
Mexico	38.7	1.19
Ecuador	46.2	1.14
Botswana	49.2	1.43
India	52.8	1.15
Turkey	56.5	1.39
Costa Rica	64.6	1.80
Brazil	68.1	2.07
South Africa	68.8	1.33
Thailand	78.1	1.03
Malaysia	80.7	1.54
Correlation		0.52
	Developed	
Italy	87.3	0.99
Israel	90	1.59
Ireland	94.7	0.86
Luxembourg	96.2	0.56
Singapore	96.4	0.98
Austria	96.7	0.86
Spain	97.6	1.09
Switzerland	98	0.58
Belgium	98.1	0.80
Australia	98.9	0.88
Netherlands	99.3	0.96
Sweden	99.7	0.78
Finland	100	0.79
Norway	100	1.41
Correlation		-0.33

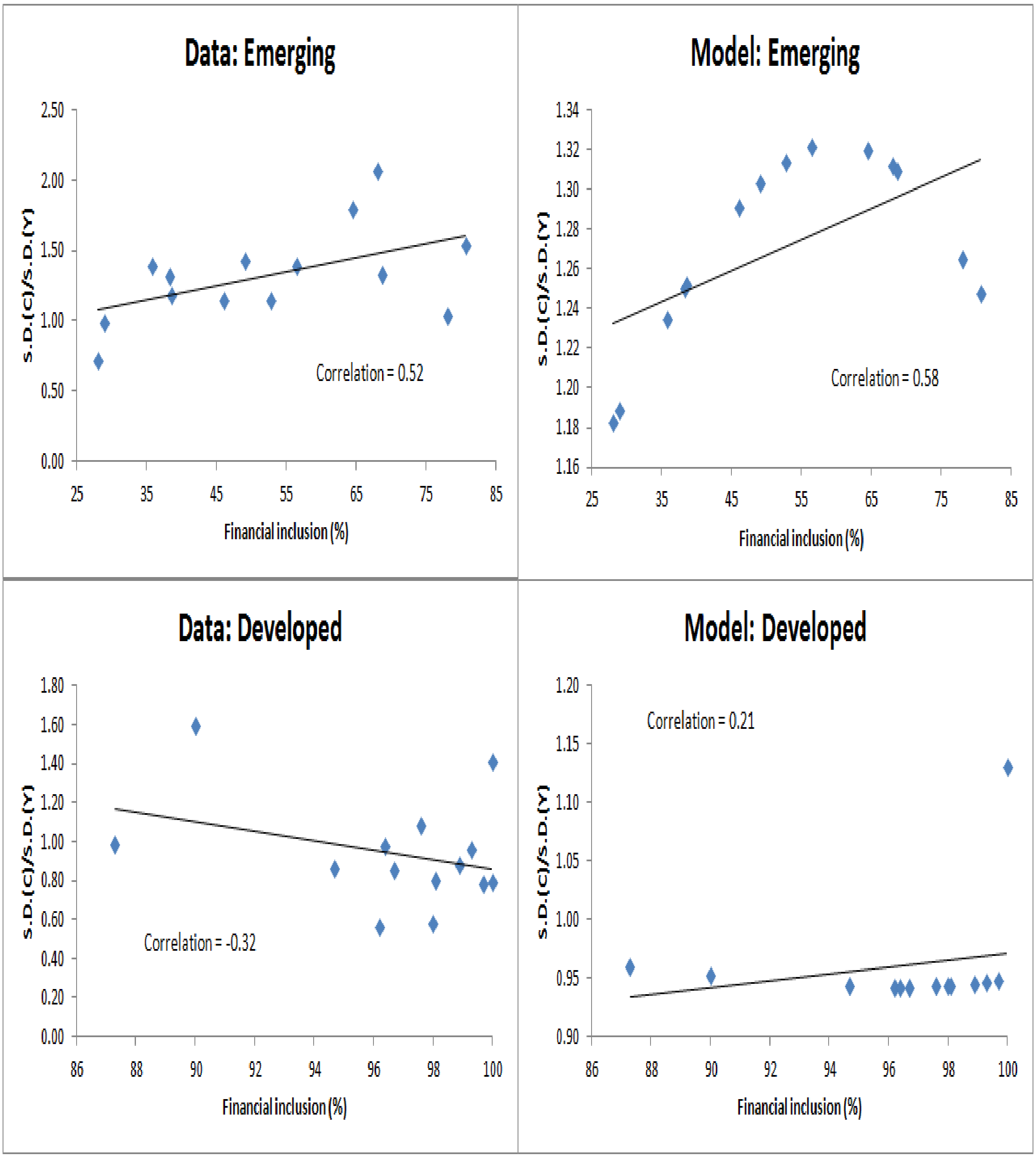

Figure 4: Correlation between financial inclusion and ratio of standard deviation of consumption to standard deviation of output for model and data among each group of economies.

Conclusion

The focus of the paper has been on the heterogeneity in agents in their ability to access financial institutions. Agents differing in their access to financial institutions react differently to income shocks. Agents with access to financial institutions have more volatile consumption than income with trend shocks whereas they display less volatility in consumption than income with transitory shocks. Agents without access to financial institutions display the same income and consumption volatility. With trend shocks showing more volatility than transitory shocks and a lower proportion of financially excluded agents, emerging countries display high consumption volatility than income volatility. Developed countries display low consumption volatility than income volatility due to much higher proportion of financially included agents and higher variance of transitory shocks than trend shocks. An attempt has been made to justify the model by checking the correlation between financial inclusion and variance of consumption to variance of income between the model and the data for the two groups of countries.

The focus has been on the consequences of such financial segmentation rather than the reason for such a segmentation. This is not to say that the reasons for segmentation are not important for fiscal and monetary policy considerations but rather more work is required to be performed in this regard. Since, the paper talks about differences in productivity shocks being faced by countries with varying levels of financial inclusion, further research can look into the aspect of how endogenously choosing to be included or excluded from the financial market leads to volatility in output and resulting volatility in consumption. Since we define financial inclusion as the ability to participate in capital markets, output becomes volatile if agent choose to invest sometime and let their capital depreciate otherwise. This kind of endogeneity between financial inclusion and output volatility and subsequent consumption volatility has not been discussed in the literature and can be an area of future research.

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Appendix

I: Data

Table corresponding to Figure 2

Possibility of coming up with emergency funds

	Developed		Emerging	
		Overall		
Very Possible	52.40	(48.06, 59.68)	16.87	(13.78, 20.21)
Somewhat Possible	23.57	(19.29, 30.65)	30.86	(27.17, 34.63)
Not very Possible	10.18	(7.93, 11.73)	18.50	(14.24, 23.09)
Not at all Possible	12.68	(10.44, 16.02)	31.63	(24.78, 38.10)
		Poorest 40% income		
Very Possible	39.26	(31.41, 44.43)	9.26	(5.48, 13.41)
Somewhat Possible	25.77	(22.52, 28.08)	23.74	(21,52, 26.19)
Not very Possible	13.83	(11.20, 15.02)	20.76	(13.36, 28.32)
Not at all Possible	19.60	(17.04, 26.65)	43.63	(37.78, 52.47)
		Richest 60% income		
Very Possible	61.54	(54.21, 69.66)	22.05	(18.34, 24.99)
Somewhat Possible	22.08	(17.06, 27.89)	35.70	(29.70, 42.40)
Not very Possible	7.65	(5.48, 9.45)	16.99	(14.85, 19.91)
Not at all Possible	7.82	(4.69, 10.20)	23.45	(16.26, 29.15)

Note: Average level of responses among group of developed (14) and emerging (14) countries. 1st and 3rd quantile reported in parentheses.

Table

Data Sources

	Quarterly		Annual	
	Period	Source	Period	Source
		Emerging		
Botswana	Q1-1994: Q4-2016	IFS	1974: 2016	IFS
Brazil	Q1-1996: Q4-2016	OECD	1965: 2011	IFS
Colombia	Q1-1994: Q1-2016	IFS	1968: 2015	IFS
Costa Rica	Q1-1991: Q4-2016	OECD	1960: 2016	IFS
Ecuador	Q1-1991: Q3-2016	IFS	1965: 2015	IFS
India	Q1-1996: Q4-2016	OECD	1960: 2016	IFS
Indonesia	Q1-1990: Q4-2016	OECD	1978: 2016	IFS
Malaysia	Q1-1991: Q4-2016	IFS	1970: 2016	IFS
Mexico	Q1-1980: Q4-2016	OECD	1970: 2015	IFS
Peru	Q1-1980: Q3-2016	IFS	1979: 2015	IFS
Philippines	Q1-1981: Q4-2016	IFS	1960: 2016	IFS
South Africa	Q1-1980: Q4-2016	OECD	1960: 2014	OECD
Thailand	Q1-1993: Q4-2016	IFS	1960: 2016	IFS
Turkey	Q1-1980: Q4-2016	OECD	1970: 2015	OECD
		Developed		
Australia	Q1-1980: Q4-2016	OECD	1960: 2015	OECD
Austria	Q1-1980: Q4-2016	OECD	1970: 2016	IFS
Belgium	Q1-1980: Q4-2016	OECD	1970: 2016	OECD
Finland	Q1-1980: Q4-2016	OECD	1970: 2016	OECD
Ireland	Q1-1980: Q4-2016	OECD	1970: 2016	OECD
Israel	Q1-1980: Q4-2016	IFS	1970: 2016	IFS
Italy	Q1-1980: Q4-2016	OECD	1970: 2016	OECD
Luxembourg	Q1-1980: Q4-2016	OECD	1970: 2016	OECD
Netherlands	Q1-1980: Q4-2016	OECD	1969: 2016	IFS
Norway	Q1-1980: Q4-2016	OECD	1970: 2016	OECD
Singapore	Q1-1980: Q4-2016	IFS	1960: 2016	IFS
Spain	Q1-1980: Q4-2016	OECD	1970: 2016	OECD
Sweden	Q1-1980: Q4-2016	OECD	1970: 2016	OECD
Switzerland	Q1-1980: Q4-2016	OECD	1960: 2015	OECD

Table: Quarterly Moments

A. Volatility and Autocorrelation of filtered income and growth rates

	$\sigma(Y)$	$\sigma(\triangle Y)$	$\rho(Y)$	$\rho(\triangle Y)$
		Emerging		
Botswana	3.361	3.613	0.432	-0.326
Brazil	1.626	1.277	0.745	0.259
Colombia	3.718	2.992	0.686	-0.047
Costa Rica	1.599	1.120	0.771	-0.111
Ecuador	2.072	1.281	0.827	0.286
India	1.389	1.039	0.748	0.043
Indonesia	2.813	1.674	0.852	0.337
Malaysia	2.470	1.675	0.793	0.297
Mexico	2.248	1.316	0.838	0.320
Peru	4.367	2.599	0.847	0.406
Philippines	2.417	1.416	0.862	0.215
South Africa	1.510	0.806	0.882	0.532
Thailand	3.275	2.151	0.801	0.133
Turkey	3.267	2.355	0.753	0.036
Mean	2.61	1.78	0.79	0.20
		Developed		
Australia	1.163	0.734	0.819	0.221
Austria	1.074	0.696	0.814	0.209
Belgium	0.989	0.620	0.832	0.326
Finland	2.157	1.274	0.859	0.176
Ireland	2.878	2.241	0.748	-0.034
Israel	1.782	1.732	0.545	-0.307
Italy	1.170	0.706	0.860	0.453
Luxembourg	2.071	1.432	0.792	0.069
Netherlands	1.284	0.852	0.816	0.114
Norway	1.439	1.296	0.612	-0.267
Singapore	2.774	1.716	0.831	0.266
Spain	1.300	0.789	0.882	0.266
Sweden	1.622	1.108	0.786	-0.019

Switzerland	1.232	0.645	0.883	0.431
Mean	1.64	1.13	0.79	0.14

B. Financial inclusion and relative volatility of consumption, investment and net exports

	Financial inclusion	$\sigma(C)/\sigma(Y)$	$\sigma(I)/\sigma(Y)$	$\sigma(NX/Y)$
		Emerging		
Botswana	49.2	2.148	2.540	20.585
Brazil	68.1	1.132	2.923	0.688
Colombia	38.4	1.345	2.809	1.418
Costa Rica	64.6	0.983	4.216	1.618
Ecuador	46.2	1.667	3.903	3.547
India	52.8	1.284	2.346	1.463
Indonesia	35.9	1.069	3.903	1.457
Malaysia	80.7	1.549	4.278	3.972
Mexico	38.7	1.181	3.785	1.568
Peru	29	1.045	2.203	2.253
Philippines	28.1	0.521	4.040	2.350
South Africa	68.8	1.348	3.585	1.500
Thailand	78.1	1.020	3.265	3.932
Turkey	56.5	1.154	3.196	1.622
Mean	52.51	1.21	3.39	3.16
		Developed		
Australia	98.9	0.822	3.790	0.848
Austria	96.7	0.815	2.168	0.704
Belgium	98.1	0.698	3.671	0.680
Finland	100	0.827	2.587	1.201
Ireland	94.7	0.737	3.173	3.074
Israel	90	1.485	3.338	2.012
Italy	87.3	1.008	2.516	0.758
Luxembourg	96.2	0.639	4.037	2.384
Netherlands	99.3	0.864	3.110	0.613
Norway	100	1.120	4.066	1.419
Singapore	96.4	0.960	2.275	3.453
Spain	97.6	1.132	3.291	0.766
Sweden	99.7	0.828	3.020	0.690

Switzerland	98	0.437	2.336	2.014
Mean	96.63	0.88	3.10	1.47

C. Contemporaneous Correlation with output

	$\rho(C, Y)$	$\rho(I, Y)$	$\rho(NX/Y, Y)$
		Emerging	
Botswana	0.145	0.155	0.204
Brazil	0.749	0.877	-0.610
Colombia	0.934	0.701	-0.255
Costa Rica	0.808	0.809	-0.373
Ecuador	0.549	0.795	-0.327
India	0.331	0.763	0.024
Indonesia	0.569	0.684	-0.419
Malaysia	0.621	0.756	-0.441
Mexico	0.915	0.876	-0.577
Peru	0.769	0.739	-0.401
Philippines	0.470	0.748	-0.229
South Africa	0.823	0.694	-0.383
Thailand	0.865	0.825	-0.292
Turkey	0.778	0.851	-0.501
Mean	0.68	0.74	-0.35
		Developed	
Australia	0.343	0.803	-0.423
Austria	0.540	0.571	0.362
Belgium	0.580	0.651	-0.010
Finland	0.823	0.868	-0.044
Ireland	0.541	0.528	0.223
Israel	0.466	0.516	0.062
Italy	0.671	0.804	-0.023
Luxembourg	0.313	0.347	0.221
Netherlands	0.732	0.745	0.032
Norway	0.673	0.253	-0.086
Singapore	0.440	0.418	0.085
Spain	0.886	0.849	-0.615
Sweden	0.596	0.820	-0.039

Switzerland	0.587	0.741	0.324	
Mean	0.59	0.64	0.005	

Table: Annual Moments

A. Volatility and Autocorrelation of filtered income and growth rates

	$\sigma(Y)$	$\sigma(\triangle Y)$	$\rho(Y)$	$\rho(\triangle Y)$
		Emerging		
Botswana	3.037	5.465	-0.015	0.088
Brazil	2.066	3.758	0.191	0.346
Colombia	2.551	4.070	-0.049	-0.234
Costa Rica	1.832	2.975	0.255	0.324
Ecuador	2.300	4.254	0.099	0.259
India	1.707	2.962	0.083	0.140
Indonesia	2.232	3.570	0.266	0.266
Malaysia	2.057	3.562	0.110	0.167
Mexico	2.136	3.443	0.265	0.267
Peru	3.886	5.720	0.438	0.452
Philippines	1.853	2.956	0.447	0.504
South Africa	1.357	2.388	0.308	0.480
Thailand	2.058	3.800	0.298	0.464
Turkey	2.724	4.123	0.203	0.003
Mean	2.21	3.70	0.21	0.26
		Developed		
Australia	1.001	1.713	0.136	0.251
Austria	1.041	1.823	0.104	0.195
Belgium	1.009	1.759	0.066	0.077
Finland	1.981	3.123	0.389	0.455
Ireland	2.428	4.406	0.276	0.425
Israel	1.565	2.646	0.243	0.329
Italy	1.216	2.324	0.068	0.323
Luxembourg	1.967	3.308	0.182	0.224
Netherlands	1.101	1.906	0.291	0.406
Norway	1.016	1.794	0.491	0.650

Singapore	2.343	4.055	0.216	0.299
Spain	1.191	2.382	0.463	0.691
Sweden	1.332	2.195	0.221	0.308
Switzerland	1.409	2.319	0.402	0.475
Mean	1.47	2.56	0.25	0.36

B. Financial inclusion and relative volatility of consumption, investment and net exports

	Financial inclusion	$\sigma(C)/\sigma(Y)$	$\sigma(I)/\sigma(Y)$	$\sigma(NX/Y)$
		Emerging		
Botswana	49.2	1.425	3.268	6.270
Brazil	68.1	2.070	3.841	1.038
Colombia	38.4	1.324	2.818	1.489
Costa Rica	64.6	1.804	3.435	2.526
Ecuador	46.2	1.145	2.711	2.935
India	52.8	1.149	2.173	0.562
Indonesia	35.9	1.393	2.493	1.622
Malaysia	80.7	1.538	4.412	3.307
Mexico	38.7	1.186	3.630	1.125
Peru	29	0.993	2.283	2.214
Philippines	28.1	0.716	4.016	1.582
South Africa	68.8	1.328	3.902	1.519
Thailand	78.1	1.034	3.465	2.424
Turkey	56.5	1.390	3.701	1.254
Mean	52.51	1.32	3.32	2.09
		Developed		
Australia	98.9	0.875	3.311	0.655
Austria	96.7	0.855	2.527	0.500
Belgium	98.1	0.804	3.311	0.436
Finland	100	0.788	2.509	0.847
Ireland	94.7	0.861	2.676	1.766
Israel	90	1.594	3.499	1.239
Italy	87.3	0.986	2.293	0.679
Luxembourg	96.2	0.559	3.014	1.554
Netherlands	99.3	0.955	3.128	0.496
Norway	100	1.409	4.236	1.152

Singapore	96.4	0.979	2.431	2.476
Spain	97.6	1.086	3.263	0.639
Sweden	99.7	0.780	2.724	0.671
Switzerland	98	0.581	3.793	1.058
Mean	96.63	0.93	3.05	1.02

C. Contemporaneous Correlation with output

	$\rho(C, Y)$	$\rho(I, Y)$	$\rho(NX/Y, Y)$
		Emerging	
Botswana	0.183	0.505	0.246
Brazil	0.293	0.543	-0.346
Colombia	0.925	0.702	-0.201
Costa Rica	0.664	0.735	-0.130
Ecuador	0.517	0.176	0.079
India	0.604	0.564	-0.007
Indonesia	0.310	0.733	-0.646
Malaysia	0.609	0.716	-0.421
Mexico	0.959	0.933	-0.708
Peru	0.858	0.818	-0.576
Philippines	0.546	0.779	-0.291
South Africa	0.831	0.715	-0.564
Thailand	0.787	0.870	-0.665
Turkey	0.667	0.745	-0.580
Mean	0.63	0.68	-0.33
		Developed	
Australia	0.385	0.722	-0.479
Austria	0.635	0.688	0.222
Belgium	0.583	0.757	-0.187
Finland	0.871	0.841	-0.139
Ireland	0.576	0.593	0.257
Israel	0.562	0.732	-0.042
Italy	0.737	0.822	-0.193
Luxembourg	0.278	0.468	0.371
Netherlands	0.713	0.766	-0.035
Norway	0.793	0.404	-0.337

Singapore	0.552	0.397	0.063
Spain	0.916	0.882	-0.650
Sweden	0.495	0.723	-0.103
Switzerland	0.717	0.864	-0.226
Mean	0.62	0.69	-0.11

II: Log-linearised model

A financially included agent will solve the following detrended problem:

$$V\left(Z,G,\hat{k},\hat{b}\right)=\max_{\left\{\hat{m^i},\hat{n^i},l^i,\hat{k\prime},\hat{b\prime}\right\}}\frac{\left(\hat{u^i}-\tau\left(l^i\right)^{\theta}\right)^{1-\gamma}-1}{1-\gamma}+\beta G^{(1-\gamma)}\mathbb{E}V\left(Z\prime,G\prime,\hat{k}\prime,\hat{b}\prime\right)$$

subject to

$$\hat{u^i}=\left[\psi_m\left(\hat{m^i}\right)^{\frac{s-1}{s}}+\left(1-\psi_m\right)\left(\hat{n^i}\right)^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}}$$

$$\hat{y^i}+\left(1-\delta_k\right)\hat{k}-\frac{\psi_k}{2}\left(\frac{G\hat{k}\prime}{\hat{k}}-\mu_g\right)^2\hat{k}-\hat{b}+Gq\hat{b}\prime=\hat{m^i}+p\hat{n^i}+G\hat{k}\prime$$

$$\hat{y^i}=ZG^{\alpha}\left(l^i\right)^{\alpha}\hat{k}^{1-\alpha}$$

F.O.C's

$$\left(\frac{\psi_m}{1-\psi_m}\right)^{s}p^{s}\hat{n^i}=\hat{m^i}$$

$$\left(\frac{\psi_m}{\tau\theta}\right)^{s}\left(\frac{\hat{u^i}}{\hat{m^i}}\right)\left(\alpha\hat{y^i}\right)^{s}=\left(l^i\right)^{s\theta}$$

$$\left(\hat{u^i}-\tau\left(l^i\right)^{\theta}\right)^{-\gamma}\psi_m\left(\frac{\hat{u^i}}{\hat{m^i}}\right)^{1/s}\left(G+\psi_k\left(\frac{G\hat{k}\prime}{\hat{k}}-\mu_g\right)G\right)=\beta G^{1-\gamma}\mathbb{E}V_{\hat{k}\prime}\left(Z\prime,G\prime,\hat{k}\prime,\hat{b}\prime\right)$$

$$-\left(\hat{u^i}-\tau\left(l^i\right)^{\theta}\right)^{-\gamma}\psi_m\left(\frac{\hat{u^i}}{\hat{m^i}}\right)^{1/s}qG=\beta G^{1-\gamma}\mathbb{E}V_{\hat{b}\prime}\left(Z\prime,G\prime,\hat{k}\prime,\hat{b}\prime\right)$$

Envelope Conditions

$$V_{\hat{k}\prime}=\left(\hat{u^i\prime}-\tau\left(l^i\prime\right)^{\theta}\right)^{-\gamma}\psi_m\left(\frac{\hat{u^i\prime}}{\hat{m^i\prime}}\right)^{1/s}\left\{\left(1-\delta\right)+\left(1-\alpha\right)\frac{\hat{y^i\prime}}{\hat{k}\prime}+\frac{\psi_k}{2}\left(\left(\frac{G\prime\hat{k}\prime\prime}{\hat{k}\prime}\right)^2-\mu_g^2\right)\right\}$$

$$V_{\hat{b}\prime}=-\left(\hat{u^i\prime}-\tau\left(l^i\prime\right)^{\theta}\right)^{-\gamma}\psi_m\left(\frac{\hat{u^i\prime}}{\hat{m^i\prime}}\right)^{1/s}$$

A financially excluded agent will solve the following detrended problem:

$$V(Z,G,) = \max_{\{\hat{m^e},\hat{n^e},l^e\}} \frac{\left(\hat{u^e} - \tau (l^e)^\theta\right)^{1-\gamma} - 1}{1-\gamma} + \beta G^{(1-\gamma)} \mathbb{E} V(Z\prime, G\prime)$$

subject to:

$$\hat{u^e} = \left[\psi_m \left(\hat{m^e}\right)^{\frac{s-1}{s}} + (1-\psi_m)\left(\hat{n^e}\right)^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}}$$

and

$$\hat{m^e} + p\hat{n^e} = p\hat{y^e}$$

$$\hat{y^e} = ZGl^e$$

F.O.C's

$$\left(\frac{\psi_m}{1-\psi_m}\right)^s p^s \hat{n^e} = \hat{m^e}$$

$$\left(\frac{\psi_m}{\tau\theta}\right)^s \left(\frac{\hat{u^e}}{\hat{m^e}}\right) \left(p\hat{y^e}\right)^s = (l^e)^{s\theta}$$

Other equations

$$ln(Z_t) = \rho_z ln(Z_{t-1}) + \epsilon_t^z$$

$$ln(G_t) = (1-\rho_g) ln(\mu_g) + \rho_g ln(G_{t-1}) + \epsilon_t^g$$

$$1/q = 1 + r^* + \psi_b \left(\exp\left(\hat{b}\prime - \overline{b}\right) - 1\right)$$

$$\hat{x} = \hat{k}\prime - (1-\delta)\hat{k} + \frac{\psi_k}{2}\left(\frac{G\hat{k}\prime}{\hat{k}} - \mu_g\right)^2 \hat{k}$$

$$\hat{m^i} + \hat{x} + \hat{b} - qG\hat{b}\prime + \frac{\lambda}{1-\lambda}\hat{m^e} = \hat{y^i}$$

$$\hat{c^i} = \hat{m^i} + p\hat{n^i}$$

$$\hat{c^e} = \hat{m^e} + p\hat{n^e}$$

$$\hat{C} = \lambda \hat{c^e} + (1-\lambda)\hat{c^i}$$

$$\hat{Y} = \lambda p\hat{y^e} + (1-\lambda)\,\hat{y^i}$$

$$\hat{nx} = \frac{\hat{NX}}{\hat{Y}} = \frac{\hat{y^i} - p\hat{n^i} - \hat{m^i} - \hat{x}}{\hat{Y}}$$

For any variable x, $\tilde{x}$ denotes log-deviations of the variable and $\overline{x}$ denotes steady state. Log-linearizing the F.O.C.'s and other equations, I get the following equations.

Steady state relationships

I will express all variables as a function of $\overline{p}$ and the parameters of the model. Then I obtain an analytical expression of $\overline{p}$ in terms of the other endogenous variables and parameters. I will solve for $\overline{p}$ which minimises the distance between the initial guess and the analytical expression. Assume $\overline{b}/\overline{Y} = 0.1$.

$r^* = \frac{1}{\beta}\mu_g^{\gamma} - 1$

$\overline{q} = \frac{1}{1+r^*} = \beta\mu_g^{-\gamma}$

$\frac{\overline{y^i}}{\overline{k}} = \left(\frac{\delta-1+1/\overline{q}}{1-\alpha}\right)$

$\frac{\overline{k}}{\overline{l^i}} = \mu_g\left(\frac{\overline{y^i}}{\overline{k}}\right)^{-1/\alpha}$

$\frac{\overline{y^i}}{\overline{l^i}} = \mu_g^{\alpha}\left(\frac{\overline{k}}{\overline{l^i}}\right)^{1-\alpha}$

$\frac{\overline{m^i}}{\overline{n^i}} = \left(\frac{\psi_m}{1-\psi_m}\right)^s \overline{p}^s$

$\frac{\overline{u^i}}{\overline{m^i}} = \left[\psi_m + (1-\psi_m)\left(\frac{\overline{m^i}}{\overline{n^i}}\right)^{\frac{1-s}{s}}\right]^{\frac{s}{s-1}}$

$\overline{l^i} = \left[\left(\frac{\psi_m}{\tau\theta}\right)^s \frac{\overline{u^i}}{\overline{m^i}}\left(\frac{\alpha\overline{y^i}}{\overline{l^i}}\right)^s\right]^{1/s(\theta-1)}$

$\overline{y^i} = \frac{\overline{y^i}}{\overline{l^i}}\overline{l^i}$

$\overline{k} = \frac{\overline{k}}{\overline{l^i}}\overline{l^i}$

$\overline{x} = \overline{k}\,(\mu_g - 1 + \delta)$

$\frac{\overline{m^e}}{\overline{n^e}} = \left(\frac{\psi_m}{1-\psi_m}\right)^s \overline{p}^s$

$\frac{\overline{u^e}}{\overline{m^e}} = \left[\psi_m + (1-\psi_m)\left(\frac{\overline{m^e}}{\overline{n^e}}\right)^{\frac{1-s}{s}}\right]^{\frac{s}{s-1}}$

$\overline{l^e} = \left[\left(\frac{\psi_m}{\tau\theta}\right)^s \frac{\overline{u^e}}{\overline{m^e}}\left(\overline{p}\mu_g\right)^s\right]^{1/s(\theta-1)}$

$\overline{y^e} = \mu_g l^e$

$\overline{Y} = \lambda\overline{p}\overline{y^e} + (1-\lambda)\,\overline{y^i}$

$\overline{b} = \frac{\overline{b}}{\overline{Y}}\overline{Y}$

$\overline{m^i} = \frac{\overline{y^i}-\overline{x}+\overline{b}(q\mu_g-1)}{1+\overline{p}\left(\frac{\overline{m^i}}{\overline{n^i}}\right)^{-1}}$

$\overline{n^i} = \overline{m^i}\left(\frac{\overline{m^i}}{\overline{n^i}}\right)^{-1}$

$\overline{u^i} = \left[\psi_m\left(\overline{m^i}\right)^{\frac{s-1}{s}} + (1-\psi_m)\left(\overline{n^i}\right)^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}}$

$\overline{c^i} = \overline{m^i} + \overline{p}\overline{n^i}$

$\overline{m^e} = \frac{\overline{p}\overline{y^e}}{1+\overline{p}\left(\frac{\overline{m^e}}{\overline{n^e}}\right)^{-1}}$

$\overline{n^e} = \overline{m^e}\left(\frac{\overline{m^e}}{\overline{n^e}}\right)^{-1}$

$\overline{u^e} = \left[\psi_m\left(\overline{m^e}\right)^{\frac{s-1}{s}} + (1-\psi_m)\left(\overline{n^e}\right)^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}}$

$\overline{c^e} = \overline{m^e} + \overline{p}\overline{n^e}$

$\overline{C} = \lambda\overline{p}\overline{c^e} + (1-\lambda)\,\overline{c^i}$

$\overline{nx} = \frac{\overline{y^i}-\overline{x}-\overline{m^i}-\overline{p}\overline{n^i}}{\overline{Y}}$

Log-linearised equations

Euler equation between m and capital:

L.H.S.:

$$\left(\hat{u^i} - \tau\left(l^i\right)^{\theta}\right)^{-\gamma}\psi_m\left(\frac{\hat{u^i}}{\hat{m^i}}\right)^{1/s}G^{\gamma}\left(1+\psi_k\left(\frac{G\hat{k\prime}}{\hat{k}}-\mu_g\right)\right) \approx$$

$$\left(\frac{\overline{u^i}}{\overline{m^i}}\right)^{1/s}\mu_g^{\gamma}\left(\overline{u^i}-\tau\left(\overline{l^i}\right)^{\theta}\right)^{-\gamma}\left(\frac{\gamma\tau\theta\left(\overline{l^i}\right)^{\theta}\tilde{l^i}-\gamma\overline{u^i}\tilde{u^i}}{\overline{u^i}-\tau\left(\overline{l^i}\right)^{\theta}}+\frac{1}{s}\left(\tilde{u^i}-\tilde{m^i}\right)+(\psi_k+\gamma)\,\tilde{g}+\psi_k\mu_g\left(\tilde{k\prime}-\tilde{k}\right)\right)$$

R.H.S.:

$$\beta\left(\hat{u^i\prime}-\tau\left(l^i\prime\right)^{\theta}\right)^{-\gamma}\psi_m\left(\frac{\hat{u^i\prime}}{\hat{m^i\prime}}\right)^{1/s}\left\{(1-\delta)+(1-\alpha)\frac{\hat{y^i\prime}}{\hat{k\prime}}+\frac{\psi_k}{2}\left(\left(\frac{G\prime\hat{k\prime\prime}}{\hat{k\prime}}\right)^2-\mu_g^2\right)\right\} \approx$$

$$\beta\left(\frac{\overline{u^i}}{\overline{m^i}}\right)^{1/s}\left(\overline{u^i}-\tau\left(l^i\right)^{\theta}\right)^{-\gamma}\left(1-\delta+(1-\alpha)\frac{\overline{y^i}}{\overline{k}}\right)\left(\frac{\gamma\tau\theta\left(\overline{l^i}\right)^{\theta}\tilde{l^i\prime}-\gamma\overline{u^i}\tilde{u^i\prime}}{\overline{u^i}-\tau\left(\overline{l^i}\right)^{\theta}}+\frac{1}{s}\left(\tilde{u^i\prime}-\tilde{m^i\prime}\right)\right) +$$

$$\beta\left(\frac{\overline{u^i}}{\overline{m^i}}\right)^{1/s}\left(\overline{u^i}-\tau\left(l^i\right)^{\theta}\right)^{-\gamma}\left((1-\alpha)\frac{\overline{y^i}}{\overline{k}}\left(\tilde{y^i\prime}-\tilde{k^i\prime}\right)+\mu_g^2\psi_k\tilde{g}+\mu_g^2\psi_k\left(\tilde{k\prime\prime}-\tilde{k\prime}\right)\right)$$

Equating the L.H.S. with the expectation of R.H.S. and using the steady state relationship $\frac{\overline{y^i}}{\overline{k}} = \left(\frac{\delta - 1 + \frac{1}{\beta}\mu_g^{\gamma}}{1-\alpha}\right)$

$$\implies \left(\frac{1}{s} - \frac{\gamma\overline{u^i}}{\overline{u^i} - \tau\left(\overline{l}^i\right)^{\theta}}\right)\left(\tilde{u^i} - \mathbb{E}\tilde{u^i\prime}\right) + \frac{\gamma\tau\theta\left(\overline{l}^i\right)^{\theta}}{\overline{u^i} - \tau\left(\overline{l}^i\right)^{\theta}}\left(\tilde{l^i} - \mathbb{E}\tilde{l^i\prime}\right) - \frac{1}{s}\left(\tilde{m^i} - \mathbb{E}\tilde{m^i\prime}\right) \quad +$$

$$(\psi_k + \gamma)\tilde{g} + \left(\psi_k\mu_g + \beta\mu_g^{-\gamma}\left((1-\alpha)\frac{\overline{y^i}}{\overline{k}} + \mu_g^2\psi_k\right)\right)\tilde{k\prime} - \psi_k\mu_g\tilde{k} \quad =$$

$$\beta\mu_g^{-\gamma}\mathbb{E}\left((1-\alpha)\frac{\overline{y^i}}{\overline{k}}\tilde{y^i\prime} + \psi_k\mu_g^2\tilde{g\prime} + \psi_k\mu_g^2\tilde{k\prime\prime}\right)$$

Euler equation between m and bonds:

$$\left(\hat{u^i} - \tau\left(l^i\right)^{\theta}\right)^{-\gamma}\psi_m\left(\frac{\hat{u^i}}{\hat{m^i}}\right)^{1/s} qG^{\gamma} = \beta\mathbb{E}\left(\hat{u^i\prime} - \tau\left(l^i\prime\right)^{\theta}\right)^{-\gamma}\psi_m\left(\frac{\hat{u^i\prime}}{\hat{m^i\prime}}\right)^{1/s}$$

$$\left(\frac{1}{s} - \frac{\gamma\overline{u^i}}{\overline{u^i} - \tau\left(\overline{l}^i\right)^{\theta}}\right)\left(\tilde{u^i} - \mathbb{E}\tilde{u^i\prime}\right) + \frac{\gamma\tau\theta\left(\overline{l}^i\right)^{\theta}}{\overline{u^i} - \tau\left(\overline{l}^i\right)^{\theta}}\left(\tilde{l^i} - \mathbb{E}\tilde{l^i\prime}\right) - \frac{1}{s}\left(\tilde{m^i} - \mathbb{E}\tilde{m^i\prime}\right) + \tilde{q} + \gamma\tilde{g} = 0$$

Relationship between m and n for included agent:

$$\left(\frac{\psi_m}{1-\psi_m}\right)^{s} p^s\hat{n^i} = \hat{m^i}$$
$$\tilde{m^i} - s\tilde{p} - \tilde{n^i} = 0$$

Relationship between m and labour for included agent:

$$\left(\frac{\psi_m}{\tau\theta}\right)^{s}\left(\frac{\hat{u^i}}{\hat{m^i}}\right)\left(\alpha\hat{y^i}\right)^{s} = \left(l^i\right)^{s\theta}$$
$$\tilde{m^i} + s\theta\tilde{l^i} - \tilde{u^i} - s\tilde{y^i} = 0$$

Utility function between m and n for included agent:

$$\hat{u^i} = \left[\psi_m\left(\hat{m^i}\right)^{\frac{s-1}{s}} + (1-\psi_m)\left(\hat{n^i}\right)^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}}$$
$$\left(\overline{u}^i\right)^{\frac{s-1}{s}}\tilde{u^i} - \psi_m\left(\overline{m}^i\right)^{\frac{s-1}{s}}\tilde{m^i} - (1-\psi_m)\left(\overline{n}^i\right)^{\frac{s-1}{s}}\tilde{n^i} = 0$$

Production function for included agent:

$$\hat{y^i} = ZG^{\alpha}\left(l^i\right)^{\alpha}\hat{k}^{1-\alpha}$$
$$\tilde{y^i} - z - \alpha g - \alpha\tilde{l^i} - (1-\alpha)\tilde{k} = 0$$

Budget constraint for included agent:

$$\hat{y^i} = \hat{m^i} + p\hat{n^i} + \hat{b} - Gq\hat{b\prime} + \hat{x}$$
$$\overline{y^i}\tilde{y^i} - \overline{x}\tilde{x} - \overline{m^i}\tilde{m^i} - \overline{p}\overline{n^i}\left(\tilde{p} + \tilde{n^i}\right) - \overline{b^i}\tilde{b^i} + \overline{q}\overline{b}\mu_g\left(\tilde{q} + \tilde{b\prime} + \tilde{g}\right) = 0$$
$$\text{where } \overline{x}\tilde{x} = \overline{k}\left(\mu_g\tilde{k\prime} - (1-\delta)\tilde{k} + \mu_g\tilde{g}\right)$$

Total consumption of included agent:

$$\hat{c^i} = \hat{m^i} + p\hat{n^i}$$
$$\overline{c^i}\tilde{c^i} - \overline{m^i}\tilde{m^i} - \overline{p}\overline{n^i}\left(\tilde{p} + \tilde{n^i}\right) = 0$$

Prices of bonds:

$$1/q = 1 + r^* + \psi_b\left(exp^{\left(\hat{b\prime} - \bar{b}\right)} - 1\right)$$
$$\tilde{q} + \psi_b\overline{b}\overline{q}\tilde{b\prime} = 0$$

Productivity shocks:

$$ln\left(Z_t\right) = \rho_z ln\left(Z_{t-1}\right) + \epsilon_t^z$$
$$ln(\tilde{z\prime}) = \rho_z ln(\tilde{z}) + \varepsilon^z\prime$$
$$ln\left(G_t\right) = (1-\rho_g)ln\left(\mu_g\right) + \rho_g ln\left(G_{t-1}\right) + \epsilon_t^g$$
$$ln(\tilde{g\prime}) = \rho_g ln(\tilde{g}) + \epsilon^g\prime$$

Relationship between m and n for excluded agent:

$$\left(\frac{\psi_m}{1-\psi_m}\right)^s p^s \hat{n^e} = \hat{m^e}$$
$$\tilde{m^e} - s\tilde{p} - \tilde{n^e} = 0$$

Relationship between m and labour for excluded agent:

$$\left(\frac{\psi_m}{\tau\theta}\right)^s \left(\frac{\hat{u^e}}{\hat{m^e}}\right) \left(p\hat{y^e}\right)^s = \left(l^e\right)^{s\theta}$$
$$\tilde{m^e} + s\theta\tilde{l^e} - \tilde{u^e} - s\left(\tilde{y^e} + \tilde{p}\right) = 0$$

Utility function between m and n for excluded agent:

$$\hat{u^e} = \left[\psi_m \left(\hat{m^e}\right)^{\frac{s-1}{s}} + \left(1-\psi_m\right)\left(\hat{n^e}\right)^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}}$$
$$\left(\overline{u}^e\right)^{\frac{s-1}{s}} \tilde{u^e} - \psi_m \left(\overline{m}^e\right)^{\frac{s-1}{s}} \tilde{m^e} - \left(1-\psi_m\right)\left(\overline{n}^e\right)^{\frac{s-1}{s}} \tilde{n^e} = 0$$

Production function for included agent:

$$\hat{y^e} = ZGl^e$$
$$\tilde{y^i} - z - g - \tilde{l^e} = 0$$

Budget constraint of excluded agent:

$$p\hat{y^e} = \hat{m^e} + p\hat{n^e}$$
$$\overline{p}\overline{y^e}\left(\tilde{p} + \tilde{y^e}\right) - \overline{m^e}\tilde{m^e} - \overline{p}\overline{n^e}\left(\tilde{p} + \tilde{n^e}\right) = 0$$

Total consumption of excluded agent:

$$\hat{c^e} = \hat{m^e} + p\hat{n^e}$$
$$\overline{c^e}\tilde{c^e} - \overline{m^e}\tilde{m^e} - \overline{p}\overline{n^e}\left(\tilde{p} + \tilde{n^e}\right) = 0$$

Market clearing:

$$\hat{m^i} + \hat{x} + \hat{b} - qG\hat{b\prime} + \frac{\lambda}{1-\lambda}\hat{m^e} = \hat{y^i}$$

$$\overline{m^i}\tilde{m^i} + \overline{x^i}\tilde{x^i} + \overline{b^i}\tilde{b^i} - \overline{q}\overline{b}\mu_g\left(\tilde{q} + \tilde{b\prime} + \tilde{g}\right) + \frac{\lambda}{1-\lambda}\overline{m^e}\tilde{m^e} - \overline{y^i}\tilde{y^i} = 0$$

Aggregate Consumption:

$$\hat{C} = \lambda\hat{c^e} + (1-\lambda)\hat{c^i}$$

$$\overline{C}\tilde{C} - \lambda\overline{c^e}\tilde{c^e} - (1-\lambda)\overline{c^i}\tilde{c^i} = 0$$

Aggregate output:

$$\hat{Y} = \lambda p\hat{y^e} + (1-\lambda)\hat{y^i}$$

$$\overline{Y}\tilde{Y} - \lambda\overline{y^e}\overline{p}\left(\tilde{y^e} + \tilde{p}\right) - (1-\lambda)\overline{y^i}\tilde{y^i} = 0$$

Net exports:

$$\hat{nx} = \frac{\hat{y^i} - p\hat{n^i} - \hat{m^i} - \hat{x}}{\hat{Y}}$$

$$-\Delta nx - \overline{nx}\tilde{Y} + \frac{1}{\overline{Y}}\left(\overline{y^i}\tilde{y^i} - \overline{x^i}\tilde{x^i} - \overline{m^i}\tilde{m^i} - \overline{p}\overline{n^i}\left(\tilde{p} + \tilde{n^i}\right)\right) = 0$$