# Contribution to a public good under subjective uncertainty*

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**Abstract**

This paper examines how voluntary contributions to a public good are affected by the contributors' heterogeneity in beliefs about the uncertain impact that their contribution may have on the public good. Assuming that contributors have Savagian preferences that are represented by a state-dependant expected utility function and different beliefs about the effect on their contribution, the paper identifies additional conditions on those preferences that are necessary and sufficient for an increase in consensus among contributors to increase the overall level of contributions in the two-state case.

"The debate's over. The people who dispute the international consensus on global warming are in the same category now with the people who think the moon landing was staged on a movie lot in Arizona." (Al Gore)

"Whether Global Warming or Climate change. The fact is: We didn't cause it. We cannot change it." (Donald Trump)

## 1 Introduction

There are many circumstances where agents contribute to a public good about which they are *uncertain.* The fight against global warming through carbon emission reduction is an example of such a situation. As shown on Figure 1, there is considerable scientific uncertainty about the impact of carbon accumulation on the Earth temperature at, say, 2050 horizon. Moreover, as is also apparent on the picture, the probability distributions over the increase in the Earth temperature brought about by a specific scenario of carbon accumulation (the absence of any effort of reduction under a "business as usual" benchmark on Figure 1) produced with the best available models differ markedly among scientific teams.[1] This *heterogeneity of beliefs* about the environmental impact

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[1] See also the latest IPCC report TF, D, G.K., M., and S.K. (2013), and the the survey profided by Doran and Zimmerman (2009).

of carbon emission is also reflected in the variety of opinions on this matter found in the public debate, and illustrated by the polarized views of two leading American political figures quoted above.[2] There is little doubt that a person's belief on the impact of carbon emission on global warming will affect this person's propensity to make costly efforts in preventing climate change (see for example Roser-Renouf, Maibach, Leiserowitz, and Zhao (2014)). After all, had he be US president, Al Gore would have certainly not taken the same decision *vis-à-vis* the Paris agreement on global climate change than that taken last June by Donald J. Trump. Other examples of situations involving uncertainty on the impact of individuals' contributions to the public good are contributions to charities or institutions by agents who are uncertain about their reliability or effectiveness.

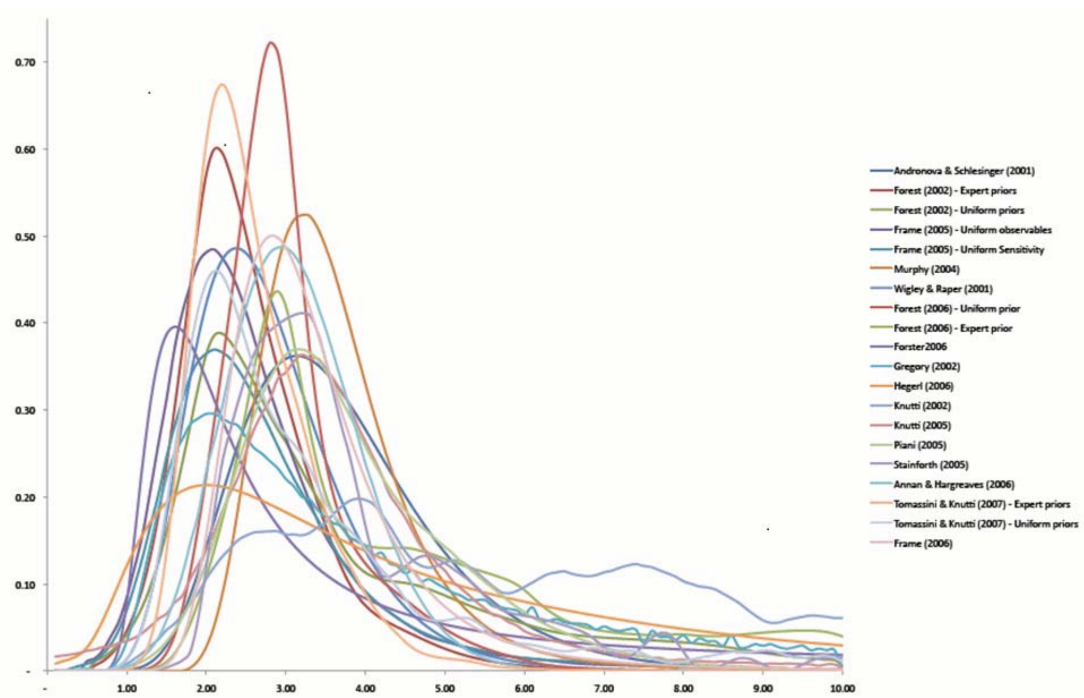


Figure 1: Estimated distributions of the increase in the Earth temperature in the next 50 years (source: Meinshausen, Meinshausen, Hare, Raper, Frieler, Knutti, Frame, and Allen (2009))

In this paper, we examine the impact of beliefs heterogeneity on the agents' contribution to a public good in a setting *à la* Bergstrom, Blume, and Varian (1986). We specifically ask two questions in such a setting:

1) Does the increase in some (or all) contributors' *optimism* about the impact of individuals' contributions to a public good increase the total amount

[2] See also. Pelham (2009) for a survey on individuals opinion about global warming in 127 countries. An intesting analysis of beliefs polarization in the context of choice under ambiguity is provided by Dixit and Weibull (2007).

of individual contributions ? That is, would the US make more global effort in reducing carbon emissions if some, or all, of the US citizens who currently share Donald Trump's beliefs about human contribution to global warming switch to Hal Gore's view on this matter ?

2) Does an increase in the existing level of consensus in a community about the impact of individual contributions to a public good increase the overall level of provision ? That is, would Donald Trump and Al Gore contribute together more to the fight against global warming if they could get their different beliefs closer to each other ?

We address this question in a simple model in which the valuation of the benefit of any given sum of individual contributions is uncertain, and where individuals differ with respect to this uncertainty. Uncertainty concerns two possible states of the world: an optimistic one, in which a given sum of individuals' efforts yields a significant benefit in terms of public good provision, and a pessimistic one in which the benefit of this sum is subjectively perceived to be lower. Contributors differ by the probability that they attach to these two states. Optimistic people like Hal Gore would attach probability close to 1 to the first state. At the other extreme, pessimistic (or skeptical) individuals like Donald Trump would attach almost zero probability to the state where individuals efforts affect public good provision. Between these two extremes, possible contributors may attach various probabilities to either of these two states. We assume that every contributor evaluates the benefit of his/her contribution by the expectation - taken over his/her beliefs - of the same state-dependant expected utility that depends, just like in standards models of voluntary public good provision *à la* Bergstrom, Blume, and Varian (1986), upon two variables: individual effort and public good provision. In either the optimistic or the pessimistic state, the state-dependant utility function is decreasing with effort, increasing with the public good, and concave with respect to both goods. For reason of tractability, we also assume that the state-dependant utility is additively separable with respect to the two variables. In such a setting, and whatever is the distribution of beliefs among contributors, there will be a unique Nash equilibrium level of contribution. The main contribution of this paper is to identify the impact of specific changes in the distribution of beliefs on the Nash equilibrium aggregate level of contributions. We first establish easily that every individual's equilibrium level of contribution is increasing with respect to his/her own belief. This entails that individuals' Nash equilibrium levels of contributions will be ordered by their beliefs. We then show that an increase in optimism in the population in the sense of first-order dominance (see e.g. Hadar and Russell (1974)) leads to an increase in the overall level of contribution to the public good. The most important result of the paper concerns the impact of an *increase in the consensus* about the probability of being in the good state on the overall level of contribution. In the context of prevention of global warming, would a homogenization of individuals' beliefs about the impact of human activities on the Earth temperature increase the individuals' overall propensity to make carbon emission reduction efforts ? Answering this question requires of course a definition of what it means for a distribution of beliefs to be "more homogenous" than another. Borrowing here again from the stochastic dominance literature, and exploiting the two-state feature of our framework, we define a distribution of beliefs to be more homogenous than another when the dominating distribution has the same average belief than the dominated one and when the dominating distribution has been obtained from the other by a finite sequence of Pigou-Dalton transfers of probabilities attached by individuals to the optimistic state. We observe that the generalization of this plausible notion of homogenization to more than two-state is not immediate. We show that the decrease in the concavity of the state-dependant utility with respect to individual effort with the level of effort is necessary and sufficient - given the other assumptions imposed on preference - for the homogenization of beliefs in this sense to increase the overall amount of contributions.

Our paper contributes to two strands of literature. Firstly, we add to the literature on uncertainty in public goods, with the analysis for heterogeneous agents. Secondly, we contribute to the literature on distributional comparative statics for aggregative games and games with strategic substitutes.

The literature on voluntary contributions to a public good, initiated largely by Bergstrom, Blume, and Varian (1986), is by now well-established (see Cornes and Sandler (1996) for a textbook survey). Yet there are relatively few contributions to this literature that have analyzed the impact of uncertainty on public good provision. Some of them, like Austen-Smith (1980) or Sandler, Sterbenz, and Posnett (1987) have considered uncertainty regarding the actions of others. Our paper does not have much to say on this matter. Among the papers who have examined the impact of uncertainty on the quantity of public good provided as a result of individuals' contribution, one finds Gradstein, Nitzan, and Slutsky (1992) who examined the impact of price uncertainty on public good provision. One finds also a sizable literature that address the issue of bargaining and negotiation on public good provision under uncertainty. For example, Kolstad (2007) studies coalition formation under systematic or common uncertainty, while Bramoulle and Boucher (2010) extends the treaty formation model of Barrett (1994) to the case of uncertainty for both a public good and a public bad. Yet, these treaty formation models assume a great deal of cooperation from the agents as they assume that these agent wants to maximize a sum of utility. Moreover, these papers assume risk neutrality from the part of the negotiators. Bramoulle and Treich (September 2009) have examined the impact of uncertainty regarding the benefit of collective action with risk-averse agents. Their results indicate that the introduction of uncertainty can, under some conditions, lower the amount of a public bad or increase the amount of a public good. However their paper assumes that all contributors face the same uncertainty and, therefore, does not address the issue of contributors' heterogeneity in their perception of uncertainty. The only paper that we are aware that allow for heterogeneity of beliefs about the public good provision resulting from contributors' efforts is Sakamoto (2014). Yet this paper is concerned with the effect of an increase in the ambiguity of the beliefs on equilibrium provision and the role played by the information acquisition on this ambiguity. It does not examine the impact of belief heterogeneity on aggregate public good provision.

As for the literature on distributional comparative statics in aggregative games, the literature that grows in the tradition of Topkis (1978) have established quite general results for games in which the actions of the players are strategic complements. A good summary of these results is provided by Milgrom and Shannon (1994). However, general results for games in which the players' strategies are strategic substitutes - like games of voluntary contributions to a public good - are much more sparse. Corchòn (1994) provides some powerful comparative static results in that setting for the case where players have strongly concave payoff functions. Corchòn (1994) results have generalized significantly by Acemoglu and Jensen (2013). However, these papers only consider the impact on the equilibrium of monotonic changes in the individual exogenous parameters of the models (for instance their beliefs) and do not explore the impact, on the equilibrium choice of strategies by the players, of changes in the distribution of those parameters among players. Jensen (June 2012.) provides some comparative static results on the effect of specific changes in the distribution of individual parameters in the context of Bayesian games. Nocetti and Smith (2015) analyzes the impact of an increase in the risk faced by a player in an aggregative game with uncertainty, but does not provide any examination of the effect that changes in the distribution of these risks among player have on the aggregate outcome of the game. ANWESHA WILL DETAIL A BIT THIS LITERATURE REVIEW. FOR EXAMPLE IT IS UNCLEAR WHAT Nocetti and Smith (2015) DO.

The rest of the paper is organized as follows. The next section introduces our model of contribution to a public good with uncertainty and heterogeneous beliefs. The main comparative static results are provided and discussed in Section 3 and Section 4 concludes.

# 2 The Model

We consider a community made of a set $N = \{1, 2, ..., n\}$ of $n$ individuals (with $n \geq 2$)). Any individual $i \in N$ must choose a level $e_i \in [0, \overline{e}]$ of effort (say in carbon emission reduction), where $\overline{e}$ is some strictly positive number, interpreted to be the maximal amount of effort that any individual can provide. It does not matter for the analysis that this number be finite. However, it must be the same for all individuals. Any given profile $(e_1, ..., e_n) \in [0, \overline{e}]^n$ of efforts made by the members of this community generates an aggregate public good $G = \sum_{i=1}^{n} e_i$ that they all value. There is, however, uncertainty regarding the subjective valuation made by an individual of any combination $(e, G) \in [0, \overline{e}] \times [0, n\overline{e}]$ of his/her effort and the aggregate public good produced by the sum of those efforts. We formulate this uncertainty as concerning two possible states of "optimism" (Hal Gore) or skepticism (Donald Trump) about the effect of individuals efforts on public good provision. If the optimistic state $o$ materializes, then a given combination $(e, G) \in [0, \overline{e}] \times [0, n\overline{e}]$ of effort and aggregate public good yields a utility of $U^o(e, G)$. On the other hand, if the pessimistic state $p$ happens, then the utility provided by this very same combination is $U^p(e, G)$. Individuals differ in terms of their *beliefs* about the likelihood of the optimistic state. Individual $i$ believes the true state to be optimistic with probability $\pi_i \in [0, 1]$. Individuals are expected (state dependant) utility maximizers. An individual with belief $\pi_i$ about the likelihood of the optimistic state thus evaluates the combination $(e, G) \in [0, \overline{e}] \times [0, n\overline{e}]$ of effort and aggregate public good by the expected state dependant utility $EU(\pi_i; e, G)$ defined by:

$$EU(\pi_i; e, G) = \pi_i U^o(e, G) + (1 - \pi_i) U^p(e, G) \tag{1}$$

We assume throughout that the functions $U^o$ and $U^p$ are at least thrice differentiable[3] with respect to their two arguments and are both decreasing

[3] The (partial) derivative of a function $g$ with respect to its $j$th argument is denoted by $g_j$.

with respect to effort, increasing with respect to aggregate public good and strictly concave.[4] We also assume that $U^p(e,G) \leq U^o(e,G)$ for any combination of effort and aggregate public good $(e,G) \in [0,\overline{e}] \times [0,n\overline{e}]$ (given effort and current aggregate public good, optimism is weakly preferable to pessimism). A degenerate case of this model happens of course when $U^o$ and $U^p$ are the same functions and when, as a result, there is no uncertainty about the benefit of contributing and, as a result, no heterogeneity among individuals. We avoid this degeneracy by assuming that the functions $U^o$ and $U^p$ are different. We actually make the stronger assumption that $U^o$ and $U^p$ satisfy, for any $(\widetilde{e},\widetilde{G}) \in [0,\overline{e}] \times [0,n\overline{e}]$, the property that $U^o_j(\widetilde{e},\widetilde{G}) \geq U^p_j(\widetilde{e},\widetilde{G})$ for $j = e, G$, with at least one of the two inequalities being strict. That is, we assume that the additional benefit an individual gets from an additional aggregate public good quantity - given effort - is (weakly) stronger in the optimistic state than in the pessimistic one. This assumption also entails that the (subjective) marginal cost of effort - given public good provision - is lower in the optimistic state than in the pessimistic one. This assumption will be largely responsible for the comparative static result that increasing one's optimism will lead to an overall increase in the (Nash) equilibrium level of public good provision. Inverting the sign of these inequalities will naturally lead to inverting the direction of this comparative static effect. Of course the assumption that the ordering of the partial derivatives of the functions $U^o$ and $U^p$ is invariant to the choice of the particular combination of effort and public good at which the derivatives are evaluated is strong. The last assumption made on the functions $U^j$ (for $s = o,p$) is to satisfy $U^s_{eG}(\widetilde{e},\widetilde{G}) = U^s_{Ge}(\widetilde{e},\widetilde{G}) \leq 0$ for any $(\widetilde{e},\widetilde{G}) \in [0,\overline{e}] \times [0,n\overline{e}]$. This assumption rules out the possibility for the (subjective) marginal cost of effort - given public good provision and irrespective of the state - to be strictly increasing with respect to public good quantity. The weak formulation of this condition makes it compatible with the possibility that either (or both) the functions $U^o$ and $U^p$ be additively separable with respect to their two arguments. We finally assume that $U^s_e(0,0) + U^s_G(0,0) > 0 > U^s_e(\overline{e},G) + U^s_G(\overline{e},G)$ for any $G \in [0,n\overline{e}]$ and $s = o,p$. We denote by $\mathcal{U}$ the class of all pairs of functions $U^o$ and $U^p$ that satisfy all those properties.

There are many problems of voluntary contribution to a public good under uncertainty that would fit in this framework. A simple one would consist in assuming, for $s = o,p$, that $U^s(e,G) = -C(e) + \Phi^s(G)$ for some state independent increasing and convex cost function $C$ and some increasing and concave state dependant $\Phi^s$. In the context of preventing global warming, such a specification would be quite natural (ADD SOME REFERENCE HERE). The cost of preventing global warming by devoting costly immediate effort in carbon emission could plausibly be independent from the subjective appraisal of the impact of aggregate carbon emissions on global warming, The state dependant function $\Phi^s$ would measure, on the other hand, the monetary benefit of global warming reduction in state $s$ of belief about the impact of aggregate human efforts - as measured by $G$ - on that reduction. It is quite natural to consider that this monetary benefit would be an increasing and concave function of the total effort in carbon emission reduction. The only additional assumption imposed by the general requirement that $U^o$ and $U^p$ belong to $\mathcal{U}$ is the that $\Phi^o_G(\widetilde{G}) > \Phi^p_G(\widetilde{G})$ for

[4] That is, the function $U^j$ (for $j = o,p$) satisfies $U^j(\lambda e + (1-\lambda)e', \lambda G + (1-\lambda)G') > \lambda U^j(e,G) + (1-\lambda)U^j(e',G')$ for every $\lambda \in ]0,1[$ and every distinct combinations $(e,G)$ and $(e',G')$ of effort and aggregate public good.

any aggregate effort $\widetilde{G} \in [0, n\overline{e}]$. But this seems to be somewhat plausible in this context. After all, the extra monetary benefit of a reduction in human carbon emission has all the chance to be larger in states where human contribution to global warming is believed to be high than in state where it is believe to be low.

But there are many other contexts where the set of assumptions on individual preferences associated to the requirement for the functions $U^o$ and $U^p$ to belong to $\mathcal{U}$.

Any distribution of beliefs $(\pi_1, ..., \pi_n) \in [0,1]^n$ in the population generates the game in strategic (or normal) form $G(\pi_1, ..., \pi_n)$ in which $N$ is the set of players, $[0, \overline{e}]$ is the strategy set of any such player, and $v^i(\pi_i, e_1, ..., e_n) = EU(\pi_i; e_i, e_i + \sum_{j \neq i} e_j)$ is the payoff received by player $i$ at the strategy profile $(e_1, ..., e_n) \in [0, \overline{e}]^n$ when he/she holds belief $\pi_i$. It is easy to see that the game $G(\pi_1, ..., \pi_n)$ is what has been called by **?** an *aggregative game* (see also Dubey, Mas-Colell, and Shubik (1980) and Shubik (1984)). A (pure strategy) Nash equilibrium for the game $G(\pi_1, ..., \pi_n)$ is an effort profile $(e_1^*, ..., e_n^*) \in [0, \overline{e}]^n$ such that, for every individual $i \in N$ and every effort level $e_i \in [0, \overline{e}]$ for this individuals, one has:

$$EU(\pi_i; e_i^*, e_i^* + \sum_{j \neq i} e_j^*) \geq EU(\pi_i; e_i, e_i + \sum_{j \neq i} e_j^*) \tag{2}$$

We first establish, in the next proposition, that for any distribution of beliefs $(\pi_1, ..., \pi_n) \in [0,1]^n$, the game $G(\pi_1, ..., \pi_n)$ admits a unique Nash equilibrium. Such a result is obviously an important preliminary step for identifying the effect of specific changes in the distribution of beliefs on the Nash equilibrium of the game. Such an endeavour can obviously not be achieved if Nash equilibria do not exist for some specification of the beliefs. Moreover, if there are many different Nash equilibria that can result from a particular distribution of beliefs, it is difficult to predict which of them would be achieved in such a case.

We start the formal analysis, whose methodology is very much inspired by **?**, with the following technical lemma. This lemma establishes some monotonicity properties of function $T : [0,1] \times [0, \overline{e}] \times [0, n\overline{e}] \to \mathbb{R}$ defined, for any $(\widetilde{\pi}, \widetilde{e}, \widetilde{G}) \in [0,1] \times [0, \overline{e}] \times [0, n\overline{e}]$, by:

$$T(\widetilde{\pi}, \widetilde{e}, \widetilde{G}) := \widetilde{\pi}[U_e^o(\widetilde{e}, \widetilde{G}) + U_G^o(\widetilde{e}, \widetilde{G})] + (1 - \widetilde{\pi})[U_e^p(\widetilde{e}, \widetilde{G}) + U_G^p(\widetilde{e}, \widetilde{G})] \tag{3}$$

This function $T$ is the derivative of the expected state-dependent utility function of Expression (1) with respect to effort when considering the effect of effort on total public good provision given the efforts by others. This derivative, that is zero for any individual who contributes a positive amount at a Nash equilibrium of the game $G(\pi_1, ..., \pi_n)$, plays for this reason a key role in the characterization of such Nash equilibria. The lemma that is (immediately) proved is the following.

**Lemma 1** *Let $(\pi_1, ..., \pi_n) \in [0,1]^n$ be a distribution of beliefs and let $G(\pi_1, ..., \pi_n)$ be the associated game in strategic form. Then, if any player i's payoff of this game writes $v^i(\pi_i, e_1, ..., e_n) = \pi_i U^o(e_i, e_i + \sum_{j \neq i} e_j) + (1 - \pi_i) U^p(e_i, e_i + \sum_{j \neq i} e_j)$ for a pair of functions $U^o$ and $U^p$ in the set $\mathcal{U}$, the function $T$ defined by (3) is strictly increasing with respect to $\pi$ and strictly decreasing with respect to both $e$ and $G$.*

***Proof.*** *Since $U^o$ and $U^p$ are in $\mathcal{U}$, they are at least thrice differentiable. Hence, the function $T$ defined by (3) is twice differentiable. Proving the result amounts therefore to verifying that:*

$$T_\pi(\widetilde{\pi}, \widetilde{e}, \widetilde{G}) = U^o_e(\widetilde{e}, \widetilde{G}) - U^p_e(\widetilde{e}, \widetilde{G}) + U^o_G(\widetilde{e}, \widetilde{G}) - U^p_G(\widetilde{e}, \widetilde{G}) > 0 \tag{4}$$

$$\begin{aligned} T_e(\widetilde{\pi}, \widetilde{e}, \widetilde{G}) &= \pi[U^o_{ee}(\widetilde{e}, \widetilde{G}) + 2U^o_{eG}(\widetilde{e}, \widetilde{G}) + U^o_{GG}(\widetilde{e}, \widetilde{G})] \\ &\quad +(1-\pi)[U^p_{ee}(\widetilde{e}, \widetilde{G}) + 2U^p_{eG}(\widetilde{e}, \widetilde{G}) + U^p_{GG}(\widetilde{e}, \widetilde{G}) \\ &< 0 \end{aligned} \tag{5}$$

*and,*

$$\begin{aligned} T_G(\widetilde{\pi}, \widetilde{e}, \widetilde{G}) &= \pi[U^o_{eG}(\widetilde{e}, \widetilde{G}) + U^o_{GG}(\widetilde{e}, \widetilde{G})] \\ &\quad +(1-\pi)[U^p_{eG}(\widetilde{e}, \widetilde{G}) + U^p_{GG}(\widetilde{e}, \widetilde{G}) \\ &< 0 \end{aligned} \tag{6}$$

*But Inequality (4) follows from the fact that functions $U^o$ and $U^p$ in $\mathcal{U}$ satisfy $U^o_j(\widetilde{e}, \widetilde{G}) \geq U^p_j(\widetilde{e}, \widetilde{G})$ for $j = e, G$ with at least one of the two inequalities being strict. Inequalities (5) and (6) on the other hands are implied by the concavities of the functions $U^o$ and $U^p$ and the fact that they satisfy $U^j_{eG}(\widetilde{e}, \widetilde{G}) = U^j_{Ge}(\widetilde{e}, \widetilde{G}) \leq 0$ for $j = o, p$.* ■

Equipped with this Lemma, we first establish the existence and uniqueness of a Nash equilibrium for the game $G(\pi_1, ..., \pi_n)$ for any distribution $(\pi_1, ..., \pi_n)$ of beliefs.

**Proposition 1** *Let $(\pi_1, ..., \pi_n) \in [0, 1]^n$ be a distribution of beliefs and let $G(\pi_1, ..., \pi_n)$ be the associated game in strategic form. Then, if any player $i$'s payoff of this game writes $v^i(\pi_i, e_1, ..., e_n) = \pi_i U^o(e_i, e_i + \sum_{j \neq i} e_j) + (1 - \pi_i)U^p(e_i, e_i + \sum_{j \neq i} e_j)$ for a pair of functions $U^o$ and $U^p$ in the set $\mathcal{U}$, then the game $G(\pi_1, ..., \pi_n)$ admits a unique Nash equilibrium.*

***Proof.*** *We first observe that by strict concavity of the functions $U^o$ and $U^p$, the program:*

$$\max_{e_i \in [0, \overline{e}]} \pi_i U^o(e_i, e_i + G_{-i}) + (1 - \pi_i)U^p(e_i, e_i + G_{-i}) \tag{7}$$

*admits a unique solution for any $\pi_i$ and any given real number $G \in [0, (n-1)\overline{e}]$. In effect, for any such $\pi_i$ and $G$, the function $\Phi^{\pi_i G} : [0, \overline{e}] \longrightarrow \mathbb{R}$ defined by*

$$\Phi^{\pi_i G}(e) = \pi_i U^o(e, e + G) + (1 - \pi_i)U^p(e, e + G)$$

*is continuous. By Weirstrass theorem, the maximization of a continuous function over a compact set (as is $[0, \overline{e}]$) admits a solution. The strict concavity of both $U^o$ and $U^p$ ensures the strict concavity of the function $\Phi^{\pi_i G}$ and, therefore, the uniqueness of the maximizer of this function for any $\pi_i$ and $G$. Let $e^*(\pi_i, G)$ denote the value of this unique maximizer of $\Phi^{\pi_i G}$. If follows* *from Berge (1959) maximum theorem that* $e^*$ *is a continuous function from* $[0,1] \times [0,(n-1)\overline{e}]$ *to* $[0,\overline{e}]$*. It thus follows that, given the distribution of beliefs* $(\pi_1,...,\pi_n)$*, the function* $\widetilde{e}^* : [0,\overline{e}]^n \to [0,\overline{e}]^n$ *defined, for any* $(e_1,...,e_n) \in [0,\overline{e}]^n$*, by* $\widetilde{e}^*(e_1,...,e_n) = (e^*(\pi_1, \sum_{j\neq 1} e_j), e^*(\pi_2, \sum_{j\neq 2} e_j), ..., e^*(\pi_n, \sum_{j\neq n} e_j))$ *is continuous. Since the domain of* $\widetilde{e}^*$ *is compact and convex, the function* $\widetilde{e}^*$ *admits a fixed point by by Brouwer theorem. Any fixed point of* $\widetilde{e}^*$ *is clearly a Nash equilibrium. Hence a Nash equilibrium of the game* $G(\pi_1,...,\pi_n)$ *exists for any distribution of beliefs* $(\pi_1,...,\pi_n)$*. We now show that this equilibrium is unique. By contradiction suppose* $(\pi_1,...,\pi_n) \in [0,1]^n$ *is a distribution of beliefs for which there are two distinct combinations of efforts* $(e_1^*,...,e_n^*)$ *and* $(\widehat{e}_1,...,\widehat{e}_n)$ *that are Nash equilibria for the game* $G(\pi_1,...,\pi_n)$*. Since* $(e_1^*,...,e_n^*)$ *and* $(\widehat{e}_1,...,\widehat{e}_n)$ *are distinct, there exists some* $i \in N$ *for which* $e_i^* \neq \widehat{e}_i$*. Without loss of generality (up to a change in the role of* $(e_1^*,...,e_n^*)$ *and* $(\widehat{e}_1,...,\widehat{e}_n)$ *in the argument), we assume* $0 \leq e_i^* < \widehat{e}_i$*. We consider two mutually exclusive cases:*

*(i)* $\sum_{j\in N} e_j^* \geq \sum_{j\in N} \widehat{e}_j$ *and,*

*(ii)* $\sum_{j\in N} e_j^* < \sum_{j\in N} \widehat{e}_j$*.*

*If case (i) holds, then, since* $0 \leq e_i^* < \widehat{e}_i$ *for some individual* $i$*, there must be some individual* $h$ *for which one has* $0 \leq \widehat{e}_h < e_h^*$*. Since* $0 > U_e^s(\overline{e},G) + U_G^s(\overline{e},G)$ *for any* $G \in [0,n\overline{e}]$ *and* $s = o,p$*, one has* $e_h^* < \overline{e}$*. Hence* $e_h^*$ *is in the interior of the interval* $[0,\overline{e}]$ *and, as a Nash equilibrium, must satisfy the first order condition of Program (7). Similarly,* $\widehat{e}_h \geq 0$ *is by assumption a Nash equilibrium which may, or may not, be interior. One must thus have:*

$$T(\pi_h, e_h^*, \sum_{j\in N} e_j^*) = 0 \geq T(\pi_h, \widehat{e}_h, \sum_{j\in N} \widehat{e}_j)$$

*But since* $\widehat{e}_h < e_h^*$*, this inequality is incompatible with the properties, established in Lemma 1, that* $T$ *is strictly decreasing with respect to both* $e$ *and* $G$*.*

*If case (ii) holds, then we have* $0 \leq e_i^* < \widehat{e}_i$ *for some individual* $i$ *and* $\sum_{j\in N} e_j^* < \sum_{j\in N} \widehat{e}_j$*. For the same reason than before,*$\widehat{e}_i$ *is interior to the interval* $[0,\overline{e}]$ *while* $e_i^*$ *is either zero or in the interior of that same interval. Since by assumption both* $e_i^*$ *and* $\widehat{e}_i$ *are part of a Nash equilibrium, they satisfy (using the first order conditions of Program (7)):*

$$T(\pi_i, \widehat{e}_i, \sum_{j\in N} \widehat{e}_j) = 0 \geq T(\pi_i, e_i^*, \sum_{j\in N} e_j^*)$$

*But again, since both* $e_i^* < \widehat{e}_i$ *and* $\sum_{j\in N} e_j^* < \sum_{j\in N} \widehat{e}_j$*, this inequality incompatible with the fact, established in Lemma 1, that* $T$ *is strictly decreasing with respect to* $e$ *and* $G$*.* ■

Because of proposition 1, we denote, for every distribution of beliefs $(\pi_1,...,\pi_n) \in [0,1]^n$, by $\mathbf{e}^*(\pi_1,...,\pi_n) = (e_1^*(\pi_1,...,\pi_n),...,e_n^*(\pi_1,...,\pi_n))$ the unique Nash equilibrium of the game $G(\pi_1,...,\pi_n)$ associated to $(\pi_1,...,\pi_n)$. In the next proposition, we establish that for any such distribution of beliefs $(\pi_1, ..., \pi_n) \in [0,1]^n$, the individuals' levels of contributions at the (unique) Nash equilibrium of the associated game will be weakly ordered by their beliefs. We also establish that, for those individuals who contribute positively to the public good, their levels of contribution will be strictly increasing with respect to their belief. This result, which is of interest in its own right, will also play an important role in the two additional comparative static results of this paper. For one thing, it implies that any Nash equilibrium combination of efforts will be entirely determined by the individuals' beliefs in the following sense that up to a belief threshold, nobody will contribute while everyone with belief above the threshold will contribute a strictly positive amount. Moreover, those positive contributors, who will always exist thanks to the assumption that $U_e^s(0,0) + U_G^s(0,0) > 0$ for $s = o, p$, will be strictly ordered by their beliefs. The proved result is the following.

**Proposition 2** *Let $(\pi_1, ..., \pi_n) \in [0,1]^n$ be a distribution of beliefs and let $\mathbf{e}^*(\pi_1, ..., \pi_n)$ be the unique Nash equilibrium of the associated game in normal form $G(\pi_1, ..., \pi_n)$. Then, if any player $i$'s payoff of this game writes $v^i(\pi_i, e_1, ..., e_n) = \pi_i U^o(e_i, e_i + \sum_{j \neq i} e_j) + (1-\pi_i)U^p(e_i, e_i + \sum_{j \neq i} e_j)$ for a pair of functions $U^o$ and $U^p$ in the set $\mathcal{U}$, then one has $\pi_i \geq \pi_h \Longrightarrow e_i^*(\pi_1, ..., \pi_n) \geq e_h^*(\pi_1, ..., \pi_n)$. Moreover, for any individuals $h$ and $i$ such that $e_i^*(\pi_1, ..., \pi_n) > 0$ and $e_h^*(\pi_1, ..., \pi_n)] > 0$, one has $\pi_i > \pi_h \Longrightarrow e_i^*(\pi_1, ..., \pi_n) > e_h^*(\pi_1, ..., \pi_n)$.*

***Proof.*** *In order to prove the first statement of the Proposition, let $(\pi_1, ..., \pi_n) \in [0,1]^n$ be a distribution of beliefs and $\mathbf{e}^*(\pi_1, ..., \pi_n)$ be the unique Nash equilibrium of the associated game and assume by contradiction that there are two individuals $h$ and $i$ such that $\pi_i \geq \pi_h$ and $e_i^*(\pi_1, ..., \pi_n) < e_h^*(\pi_1, ..., \pi_n)$. This entails that $e_h^*(\pi_1, ..., \pi_n) > 0$. Since $0 > U_e^s(\overline{e}, \sum_{j \in N} e_j^*(\pi_1, ..., \pi_n)) + U_G^s(\overline{e}, \sum_{j \in N} e_j^*(\pi_1, ..., \pi_n))$ for $s = o, p$, $e_h^* < \overline{e}$. Hence $e_h^*(\pi_1, ..., \pi_n)$ is in the interior of the interval $[0, \overline{e}]$ and satisfies therefore the first order condition of Program (7):*

$$T(\pi_h, e_h^*(\pi_1, ..., \pi_n), \sum_{j \in N} e_j^*(\pi_1, ..., \pi_n)) = 0$$

*while $e_i^*(\pi_1, ..., \pi_n)$ satisfies:*

$$T(\pi_i, e_i^*(\pi_1, ..., \pi_n), \sum_{j \in N} e_j^*(\pi_1, ..., \pi_n)) \leq 0$$

*but, when combined with the assumption that $e_i^*(\pi_1, ..., \pi_n) < e_h^*(\pi_1, ..., \pi_n)$, these two inequalities are clearly incompatible with the increasingness of $T$ with respect to $\pi$ and the strict decreasingness of $T$ with respect to $e$ established in Lemma 1. As for the second statement of the lemma, let again $(\pi_1, ..., \pi_n) \in [0,1]^n$ be a distribution of beliefs and $\mathbf{e}^*(\pi_1, ..., \pi_n)$ be the unique Nash equilibrium of the associated game. Assume that $h$ and $i$ are two individuals such that $e_i^*(\pi_1, ..., \pi_n) > 0$, $e_h^*(\pi_1, ..., \pi_n)] > 0$ and $\pi_i > \pi_h$. By contradiction, assume that $e_i^*(\pi_1, ..., \pi_n) \leq e_h^*(\pi_1, ..., \pi_n)$. For the same reason as before, both levels of contributions $e_h^*(\pi_1, ..., \pi_n)$ and $e_h^*(\pi_1, ..., \pi_n)$ are in the interior of the interval*

$[0,\overline{e}]$ *and satisfy therefore the first order condition of Program.(7):*

$$T(\pi_h, e_h^*(\pi_1, ..., \pi_n), \sum_{j \in N} e_j^*(\pi_1, ..., \pi_n)) = 0 = T(\pi_i, e_i^*(\pi_1, ..., \pi_n), \sum_{j \in N} e_j^*(\pi_1, ..., \pi_n))$$

*But, when combined with* $e_i^*(\pi_1, ..., \pi_n) \leq e_h^*(\pi_1, ..., \pi_n)$ *and* $\pi_i > \pi_h$*, this equality is incompatible with the strict increasingness of* $T$ *with respect to* $\pi$ *and the decreasingness of* $T$ *with respect to* $e$ *established in Lemma 1.* ■

An obvious consequence of Proposition 2 is that individuals' contributions at a Nash equilibrium are (weakly increasing) function of their beliefs *only*. In particular, any permutation of a distribution of beliefs $(\pi_1, ..., \pi_1) \in [0,1]$ will have no effect on the total sum of efforts provided at equilibrium and will only lead to the very same permutation of the individuals' contributions. Because of this, we can restrict attention, in what follows, to distributions of beliefs such that $\pi_1 \leq \pi_2 \leq ... \leq \pi_n$. For such ordered distribution of beliefs, we now establish our first comparative static result. Specifically, we show that the total amount of contribution to the public good at a Nash equilibrium will never diminish when there is an improvement in optimism in the sense of first order stochastic dominance. Recall that an (ordered) distribution of beliefs $(\pi_1, ..., \pi_1)$ first order stochastically dominates the (ordered) distribution $(\pi_1', ..., \pi_1')$ if and only if it is the case that $\pi_i \geq \pi_i'$ for every individual $i$. We then establish the following result.

**Proposition 3** *Let* $(\pi_1, ..., \pi_n)$ *and* $(\pi_1', ..., \pi_n')$ *be two distributions of beliefs in* $[0,1]^n$ *satisfying* $\pi_1 \leq \pi_2 \leq ... \leq \pi_n$ *and* $\pi_1' \leq \pi_2' \leq ... \leq \pi_n'$*. Then, if* $\pi_i' \geq \pi_i$ *for all* $i$ *and the payoff functions of the associated games in strategic form both result result from functions* $U^o$ *and* $U^p$ *in the set* $\mathcal{U}$*, one has* $\sum_i e_i^*(\pi_1', ..., \pi_n') \geq \sum_i e_i^*(\pi_1, ..., \pi_n))$*. Moreover, if the Nash equilibria of the two games are such that* $e_i^*(\pi_1', ..., \pi_n') > 0$ *and* $e_i^*(\pi_1, ..., \pi_n) > 0$ *for all* $i$ *and the two distributions of beliefs are distinct, then one has* $\sum_i e_i^*(\pi_1', ..., \pi_n') > \sum_i e_i^*(\pi_1, ..., \pi_n))$*.*

***Proof.*** *For the first statement of the Proposition, let* $(\pi_1, ..., \pi_n)$ *and* $(\pi_1', ..., \pi_n')$ *be two distributions of beliefs satisfying* $\pi_1 \leq \pi_2 \leq ... \leq \pi_n$*,* $\pi_1' \leq \pi_2' \leq ... \leq \pi_n'$ *and* $\pi_i' \geq \pi_i$ *for all* $i$ *and assume by contradiction that* $\sum_i e_i^*(\pi_1', ..., \pi_n') < \sum_i e_i^*(\pi_1, ..., \pi_n)$*. For this inequality to hold, there must be some individual* $h$ *such that* $0 \leq e_h^*(\pi_1', ..., \pi_n') < e_h^*(\pi_1, ..., \pi_n)$*. Since* $e_h^*(\pi_1, ..., \pi_n)$ *is interior of* $[0,\overline{e}]$*, it follows from the first order condition of Program (7) that:*

$$T(\pi_h, e_h^*(\pi_1, ..., \pi_n), \sum_{j \in N} e_j^*(\pi_1, ..., \pi_n)) = 0 \geq T(\pi_h^{'}, e_h^*(\pi_1', ..., \pi_n'), \sum_{j \in N} e_j^*(\pi_1', ..., \pi_n'))$$

*But this inequality is incompatible with the strict increasingness of* $T$ *with respect to* $\pi$ *and its strict decreasingness with respect to both* $e$ *and* $G$ *established in Lemma 1. For the second statement of the proposition, let* $(\pi_1, ..., \pi_n)$ *and* $(\pi_1', ..., \pi_n')$ *be two distinct distributions of beliefs satisfying* $\pi_1 \leq \pi_2 \leq ... \leq \pi_n$*,*

*$\pi'_1 \leq \pi'_2 \leq ... \leq \pi'_n$ and $\pi'_i \geq \pi_i$ for all $i$ and assume that $e^*_i(\pi'_1, ..., \pi'_n) > 0$ and $e^*_i(\pi_1, ..., \pi_n) > 0$ for all $i$. Suppose by contradiction that $\sum_i e^*_i(\pi'_1, ..., \pi'_n) \leq \sum_i e^*_i(\pi_1, ..., \pi_n)$. Since.$(\pi_1, ..., \pi_n)$ and $(\pi'_1, ..., \pi'_n)$ are distinct, there is an individual $h$ such that $\pi'_h > \pi_h$. Since $e^*_h(\pi'_1, ..., \pi'_n) > 0$ and $e^*_h(\pi_1, ..., \pi_n) > 0$, both $e^*_h(\pi'_1, ..., \pi'_n)$ and $e^*_h(\pi_1, ..., \pi_n)$ are interior to $[0, \overline{e}]$ and satisfy, for this reason, the first order conditions of Program (7):*

$$T(\pi_h, e^*_h(\pi_1, ..., \pi_n), \sum_{j \in N} e^*_j(\pi_1, ..., \pi_n)) = 0 = T(\pi'_h, e^*_h(\pi'_1, ..., \pi'_n), \sum_{j \in N} e^*_j(\pi'_1, ..., \pi'_n))$$

*Since, by Lemma 1, $T$ is strictly increasing in $\pi$ and strictly decreasing with respect to both $e$ and $G$, the only way to make this equality compatible with $\pi'_h > \pi_h$ and $\sum_i e^*_i(\pi'_1, ..., \pi'_n) \leq \sum_i e^*_i(\pi_1, ..., \pi_n)$ is to have:*

$$e^*_h(\pi_1, ..., \pi_n) < e^*_h(\pi'_1, ..., \pi'_n) \tag{8}$$

*Moreover, since $e^*_i(\pi'_1, ..., \pi'_n) > 0$ and $e^*_i(\pi_1, ..., \pi_n) > 0$ for all $i$, all such individual levels of contributions are interior and satisfy the first order conditions:*

$$T(\pi_i, e^*_i(\pi_1, ..., \pi_n), \sum_{j \in N} e^*_j(\pi_1, ..., \pi_n)) = 0 = T(\pi'_i, e^*_i(\pi'_1, ..., \pi'_n), \sum_{j \in N} e^*_j(\pi'_1, ..., \pi'_n))$$

*for $i = 1, ..., n$. Since $\pi'_i \geq \pi_i$ for all $i$ and $T$ is strictly increasing in $\pi$ and strictly decreasing with respect to both $e$ and $G$ by Lemma 1, this equality can only hold if*

$$e^*_i(\pi_1, ..., \pi_n) \leq e^*_i(\pi'_1, ..., \pi'_n) \tag{9}$$

*holds for all $i$. But combining the strict inequality (3) with the $n$ weak inequalities (9) lead to the conclusion that:*

$$\sum_i e^*_i(\pi_1, ..., \pi_n) < \sum_i e^*_i(\pi'_1, ..., \pi'_n)$$

*a contradiction.* ■

We now examine the impact, on the overall Nash contributive effort, of an increase in the *consensus* that may exist in the community as to the likelihood that individual efforts can have significant impact on public good provision. In the example of global warming discussed earlier, former vice-president Al Gore was referring to the emergence of a consensus about the human origin of the currently observed carbon accumulation in the atmosphere. Debate and discussion among individuals are indeed likely to increase the existing consensus on that matter. Of course a consensus can *a priori* be reached around any "average" level of optimism. But suppose we take this average level of optimism as given. What is the effect - on the total contribution to the public good - of bringing everybody in the society closer to this average level of consensus ? This is the question that we now address.

Answering this question requires of course a definition of what it means for a distribution of beliefs to be "more consensual" than another. To motivate our proposed definition of that notion, consider again the case of global warming and imagine that Donald Trump and Al Gore are forming a community. Assume that Donald Trump initially assigns a zero probability to the (optimistic) scenario in which human efforts in carbon emission reduction prevents global warming while Al Gore assigns the opposite polar probability of 1 to that same scenario. Observe that the average probability assigned to the optimistic scenario in this two-individuals community is 1/2. We could now imagine D. Trump and A. Gore trying to convince each other of the validity of their respective beliefs. One could of course be more convincing than the other and therefore more successful in bringing the other closer to his view. But suppose that the two guys are of equal convincing power, and manage to get their belief closer. For example, at the end of the discussion, Donald Trump belief could be 1/4, while Al Gore one could be 3/4. The average probability assigned in the population to the optimistic scenario would still be 1/2, but the two members of the community would be closer to each other (and to this average). In such a case, we would say that the consensus in the society has increased.

Specifically, the definition of "increase in consensus" is based on the notion of Lorenz domination of one distribution of beliefs over another or, equivalently (see for example Dasgupta, Sen, and Starrett (1973)) on the idea that one distribution of beliefs has been obtained from the other by a finite sequence of "Pigou-Dalton" transfer of beliefs. We specifically consider the following definition of one distribution of beliefs to be more consensual than another.

**Definition** Let $(\pi_1, ..., \pi_n)$ and $(\pi'_1, ..., \pi'_n)$ be two distributions of beliefs in $[0, 1]^n$ such that $\pi_1 \leq \pi_2 \leq ... \leq \pi_n$, $\pi'_1 \leq \pi'_2 \leq ... \leq \pi'_n$ and $\sum_{i \in N} \pi_i = \sum_{i \in N} \pi'_i$. We say that $(\pi_1, ..., \pi_n)$ is more consensual than $(\pi'_1, ..., \pi'_n)$ if and only if, for any $k \in N$, it is the case that $\sum_{i=1}^{k} \pi_i \geq \sum_{i=1}^{k} \pi'_i$.

As is well-known from the inequality measurement literature, and in particular the Hardy-Littlewood-Polya theorem (see e.g. Berge (1959)), there is an equivalent definition of "more consensual than" that can be expressed in terms of bilateral Pigou-Dalton transfers. As it turns out, this equivalent definition will be easier to use for establishing the last comparative static result of this paper.

We start by the following definition of what it means for a distribution of beliefs to be obtained from another by a *bilateral Pigou-Dalton transfer*.

**Definition** Let $(\pi_1, ..., \pi_n)$ and $(\pi'_1, ..., \pi'_n)$ be two distributions of beliefs in $[0, 1]^n$ such that $\pi_1 \leq \pi_2 \leq ... \leq \pi_n$, $\pi'_1 \leq \pi'_2 \leq ... \leq \pi'_n$. We say that $(\pi_1, ..., \pi_n)$ has been obtained from $(\pi'_1, ..., \pi'_n)$ by a bilateral Pigou-Dalton transfer if there exists two individuals $g$ and $h$ and a non-negative number $\delta$ such that $\pi_i = \pi'_i$ for all $i \notin \{h, i\}$ and $\pi_g = \pi'_g + \delta \leq \pi'_h - \delta = \pi_g$

In words, a Pigou-Dalton transfer is the formal description of a balanced "debate" between optimistic individual $h$ (Gore) and pessimistic individual $g$ (Trump). At the end of this balanced debate, Trump has gained $\delta$ of optimism but this gain has been exactly counterbalanced by the loss of optimism by Gore by exactly that same $\delta$.

The well-known Hardy-Littlewood-Polya theorem elegantly demonstrated by Berge (1959) establishes an equivalence between the fact for one distribution of beliefs to be more consensual than another as per Definition 2 and the possibility of going from the less to the more consensual distribution by a finite sequence of bilateral Pigou-Dalton transfer. For later use, we state formally this theorem as follows.

**Theorem 1** *(Hardy-Littlewood-Polya) Let* $(\pi_1, ..., \pi_n)$ *and* $(\pi'_1, ..., \pi'_n)$ *be two distributions of beliefs in* $[0,1]^n$ *such that* $\pi_1 \leq \pi_2 \leq ... \leq \pi_n$, $\pi'_1 \leq \pi'_2 \leq ... \leq \pi'_n$. *Then* $(\pi_1, ..., \pi_n)$ *is more consensual than* $(\pi'_1, ..., \pi'_n)$ *as per Definition 2 if and only if there exists a sequence of* $t \in \mathbb{N}_+$ *distributions of beliefs (with* $t \geq 2$*)* $(\pi_1^k, ..., \pi_n^k) \in [0,1]^n$, *for* $k = 1, ..., t$ *such that*
*(i)* $(\pi_1^1, ..., \pi_n^1) = (\pi_1, ..., \pi_n)$,
*(ii)* $(\pi_1^t, ..., \pi_n^t) = (\pi'_1, ..., \pi'_n)$ *and*
*(iii)* $(\pi_1^k, ..., \pi_n^k)$ *has been obtained from* $(\pi_1^{k+1}, ..., \pi_n^{k+1})$ *by a bilateral Pigou-Dalton transfer as per Definition 2 for all* $k = 1, ..., t-1$.

We now examine the impact that an increase in the consensus in a given community in the sense of Definition 2 can have on the aggregate contributive effort at the non-cooperative Nash equilibrium. As it turns out, the set of assumptions made thus far on the utility functions - namely that $U^o$ and $U^p$ belong to $\mathcal{U}$ - does not suffice in obtaining clear cut conclusions on that matter. Intuitively, if an individual "transfers" part of his/her optimism to someone else, this has two conflicting effects. On the one hand, the "giver" of optimism will tend to reduce his/her contribution while the "receiver" of optimism will increase his/her. The two forces are clearly playing in opposite direction. Hence, some additional condition on the utility functions are required to predict the relative strength at which these two opposite forces will play. As it turns out, the following set of conditions on $U^o$ and $U^p$ are sufficient, when applied to functions that belong to $\mathcal{U}$, for establishing the result that an increase in consensus - in the sense of Definition 2 - will increase the aggregate amount of contributions.

**Condition 1** *For any* $G \in [0, n\overline{e}]$, *and any* $e$ *and* $e^{'}$ *such that* $e \geq e'$, *one has:*
*(i)* $U_j^o(e', G) - U_j^p(e', G) \geq U_j^o(e, G) - U_j^p(e, G)$ *for* $j = e, G$ *and*
*(ii)* $U_{kl}^s(e', G) \geq U_{kl}^s(e, G)$ *for* $s = o, p$, $k = e, G$ *and* $l = e, G$

In plain English the part $(i)$ of this condition requires that the increase (decrease) of the marginal benefit (cost) of increasing (decreasing) global (individual) effort be non-increasing with respect to individual effort. Hence, as individual increase their individual effort for a given total quantity of effort, they feel less difference in the marginal benefit (or cost) of additional effort between the two states. The part (ii) of the condition requires the concavity of the state dependant function $U^s$ (for $s = o$,$p$) to be stronger, for any given aggregate quantity of efforts, when the individual perform a low effort than when he or she performs a large one. This condition is certainly demanding. It is actually not necessary for the comparative static result to come. But it may be useful to assess its strength in the somewhat specific but plausible context of the additively separable monetary evaluation of the benefit to global warming prevention discussed above. In this context, we had $U^s(e, G) = -C(e) + \Phi^s(G)$. In such a setting, Condition (i) would holds trivially. And Condition (ii) would be equivalent to requiring that the function $C$ had a negative third derivative. In words, this amount to requiring that the increase in marginal cost be decreasing with effort (or that the marginal cost curves be increasing and concave). This is clearly a restrictive condition. But it does not strike us as being unreasonable.

We now turn to the last comparative static result of this paper. Contrary to what was obtained for the previous results (for example Proposition 8), the result we are about to state - namely that aggregate effort will increase with consensus - holds only when all community members are strict contributors. There is an obvious reason for this. Suppose in effect that, at some Nash equilibrium, someone with very optimistic belief is contributing while another person, endowed with more pessimistic belief, is not contributing at all. Imagine then that a small Pigou-Dalton transfer of belief take place between these two individuals, everything else remaining the same. Suppose that the increase in optimism of the non-contributor brought about by the transfer is not sufficient for making him/her a contributor. Then the transfer will only end up reducing the optimism of the set of active contributors. As shown in Proposition 8, this will lead to a reduction in the total amount of contributions by those contributors. Hence, in a case like this, a bilateral Pigou-Dalton transfer will actually lead to a reduction in the total contributive effort of the community.

However, if everybody contributes both before and after the increase in consensus, then the latter will cause the total contributive effort to increase. The formal statement of this result is as follows.

**Proposition 4** *Let* $(\pi_1, ..., \pi_n)$ *and* $(\pi'_1, ..., \pi'_n)$ *be two distinct distributions of beliefs in* $[0,1]^n$ *satisfying* $\pi_1 \leq \pi_2 \leq ... \leq \pi_n$ *and* $\pi'_1 \leq \pi'_2 \leq ... \leq \pi'_n$*. Suppose that* $(\pi_1, ..., \pi_n)$ *is more consensual than* $(\pi'_1, ..., \pi'_n)$ *as per Definition 2 and the payoff functions of the associated games in strategic form both result result from functions* $U^o$ *and* $U^p$ *in the set* $\mathcal{U}$ *that satisfy Condition 1. Suppose also that* $e^*_i(\pi'_1, ..., \pi'_n) > 0$ *and* $e^*_i(\pi_1, ..., \pi_n) > 0$ *for all* $i$*. Then one has* $\sum_i e^*_i(\pi'_1, ..., \pi'_n) > \sum_i e^*_i(\pi_1, ..., \pi_n))$*.*

***Proof.*** *By Theorem 1,* $(\pi_1, ..., \pi_n)$ *is more consensual than* $(\pi'_1, ..., \pi'_n)$ *if and only if* $(\pi_1, ..., \pi_n)$ *has been obtained from* $(\pi'_1, ..., \pi'_n)$ *by a finite sequence of bilateral Pigou-Dalton transfers. Hence, in order to prove each statement of the Proposition, it is clearly sufficient to prove that a bilateral Pigou-Dalton performed between a pair of individuals (say* $h$ *and* $i$*) in a community of strict contributors lead to an increase in the sum of individuals" contributions. We actually provide the proof for a "small" Pigou-Dalton transfer of* $\delta$*, for which the approximation provided by calculus is adequate (A MORE GENERAL PROOF SHALL BE PROVIDED LATER). Specifically, we consider* $(\pi_1, ..., \pi_n)$ *and* $(\pi'_1, ..., \pi'_n)$ *such that*
$\pi_j = \pi'_j$ *for all* $j \notin \{h, i\}$ *and*
$\pi_i = \pi'_i - \delta \geq \pi'_h + \delta = \pi_h$
*with* $\delta$ *suitably small. We can approximate the impact on (interior) equilibrium of such a small Pigou-Dalton transfer by differentiating totally the first order conditions of Program 7 above and evaluating the differential at the Nash equilibrium associated to the distribution of beliefs* $(\pi'_1, ..., \pi'_n)$*. Denote by* $(e^*_1, ..., e^*_n)$ *and* $G^* = \sum_{j=1}^{n}$ *the distribution of efforts and aggregate effort (respectively associated to this Nash equilibrium. Doing the total differentiation yields, for individuals* $h$ *and* $i$

$$T_e(\pi'_h, e^*_h, G^*)de_h + T_G(\pi'_h, e^*_h, G^*)dG \equiv -T_\pi(\pi'_h, e^*_h, G^*)d\pi_h$$
$$T_e(\pi'_i, e^*_i, G^*)de_i + T_G(\pi'_i, e^*_i, G^*)dG \equiv -T_\pi(\pi'_i, e^*_i, G^*)d\pi_i$$

*while for the other individuals $j$ (if any), whose beliefs have not changed, the impact on the first condition is simply:*

$$T_e(\pi'_j, e^*_j, G^*)de_j + T_G(\pi'_j, e^*_j, G^*)dG \equiv 0$$

*Exploiting the fact that $d\pi_i = -d\pi_h = \delta$ and the strict monotonicity of $T$ established in Lemma 1, one can write these equations as:*

$$de_h \equiv \delta \frac{T_\pi(\pi'_h, e^*_h, G^*)}{T_e(\pi'_h, e^*_h, G^*)} - \frac{T_G(\pi'_h, e^*_h, G^*)}{T_e(\pi'_h, e^*_h, G^*)} dG$$

$$de_i \equiv -\delta \frac{T_\pi(\pi'_i, e^*_i, G^*)}{T_e(\pi'_i, e^*_i, G^*)} - \frac{T_G(\pi'_i, e^*_i, G^*)}{T_e(\pi'_i, e^*_i, G^*)} dG$$

*and:*

$$de_j \equiv -\frac{T_G(\pi'_j, e^*_j, G^*)}{T_e(\pi'_j, e^*_j, G^*)} dG$$

*for all $j \notin \{h, i\}$.Summing these equations over all individuals and rearranging yields, after exploiting the fact that $\sum_{j=1}^{n} de_j = dG$:*

$$dG = \frac{\delta\left[\frac{T_\pi(\pi'_h, e^*_h, G^*)}{T_e(\pi'_h, e^*_h, G^*)} - \frac{T_\pi(\pi'_i, e^*_i, G^*)}{T_e(\pi'_i, e^*_i, G^*)}\right]}{1 + \sum_{j=1}^{n} \frac{T_G(\pi'_j, e^*_j, G^*)}{T_e(\pi'_j, e^*_j, G^*)}}$$

*The denominator of this expression is strictly positive for all functions $U^o$ and $U^p$ in the set $\mathcal{U}$. Hence, the sign of $dG$ - that is the fact that aggregate effort increases or decreases - depends upon the sign of the expression:*

$$\frac{T_\pi(\pi'_h, e^*_h, G^*)}{T_e(\pi'_h, e^*_h, G^*)} - \frac{T_\pi(\pi'_i, e^*_i, G^*)}{T_e(\pi'_i, e^*_i, G^*)}$$

*This expression is weakly positive if and only if:*

$$\frac{T_\pi(\pi'_h, e^*_h, G^*)}{T_\pi(\pi'_i, e^*_i, G^*)} \geq \frac{T_e(\pi'_h, e^*_h, G^*)}{T_e(\pi'_i, e^*_i, G^*)} \tag{10}$$

*Thanks to Proposition 2, one has $e^*_h(\pi'_1, ..., \pi'_n) \leq e^*_i(\pi'_1, ..., \pi'_n)$. Hence it follows from part (i) of Condition 1 that:*

$$\begin{aligned} T_\pi(\pi'_h, e^*_h, G^*) &= U^o_e(e^*_h, G^*) - U^p_e(e^*_h, G^*) + U^o_G(e^*_h, G^*) - U^p_G(e^*_h, G^*) \\ &\geq U^o_e(e^*_i, G^*) - U^p_e(e^*_i, G^*) + U^o_G(e^*_i, G^*) - U^p_G(e^*_i, G^*) \\ &= T_\pi(\pi'_i, e^*_i, G^*) \end{aligned}$$

*Moreover, it also follows from part (ii) of Condition 1 that:*

$$\begin{aligned} T_e(\pi_h, e_h^*, G^*) &= \pi_h'[U_{ee}^o(e_h^*, G^*) + 2U_{eG}^o(e_h^*, G^*) + U_{GG}^o(e_h^*, G^*)] \\ &\quad +(1-\pi_h^{'})[U_{ee}^p(e_h^*, G^*) + 2U_{eG}^p(e_h^*, G^*) + U_{GG}^p(e_h^*, G^*) \\ &\leq \pi_i'[U_{ee}^o(e_i^*, G^*) + 2U_{eG}^o(e_i^*, G^*) + U_{GG}^o(e_i^*, G^*)] \\ &\quad +(1-\pi_i')[U_{ee}^p(e_i^*, G^*) + 2U_{eG}^p(e_i^*, G^*) + U_{GG}^p(e_i^*, G^*)] \end{aligned}$$

*Hence one has :*
$$\frac{T_\pi(\pi_h', e_h^*, G^*)}{T_\pi(\pi_i', e_i^*, G^*)} \geq 1 \geq \frac{T_e(\pi_h', e_h^*, G^*)}{T_e(\pi_i', e_i^*, G^*)}$$

*and Inequality (10) follows.* ■

Proposition 3 shows that increasing optimism in the community in the sense of 1st order stochastic dominance increases the total effort that individuals are willing to devote to the production of a public good when they behave non-cooperatively. Proposition 4 shows, under the additional condition 1, that increasing the consensus among the community members for a given average level of optimism also increases the total effort that the community members are willing to make for providing the public good. An obvious corollary to these two propositions is the favorable impact, on global effort, of a combination of an increase in optimism - in the sense of first order dominance - and an increase in consensus, in the form of a sequence of Pigou-Dalton transfer. Specifically consider two ordered distributions of beliefs $(\pi_1, ..., \pi_n)$ and $(\pi_1', ..., \pi_n')$ such that, for any $k = 1, ...n$, one has

$$\sum_{j=1}^{k} \pi_j \geq \sum_{j=1}^{k} \pi_j' \tag{11}$$

Observe that, contrary to what was the case for the definition of an increase in consensus, we do not require the average optimism to be the same. It is, for instance, possible to have $\sum_{j=1}^{n} \pi_j > \sum_{j=1}^{n} \pi_j'$ so that the community with belief $(\pi_1, ..., \pi_n)$ is more optimistic in average (or in total if the population size is the same) than $(\pi_1', ..., \pi_n')$. The requirement that Inequality (11) holds for all $k = 1, ..., n$ between two distributions $(\pi_1, ..., \pi_n)$ and $(\pi_1', ..., \pi_n')$ is usually referred to as Generalized Lorenz dominance (see e.g. Shorrocks (1983)) It is then an immediate corollary of Propositions 3 and 4 that the following holds.

**Corollary 1** *Let $(\pi_1, ..., \pi_n)$ and $(\pi_1', ..., \pi_n')$ be two distinct distributions of beliefs in $[0, 1]^n$ satisfying $\pi_1 \leq \pi_2 \leq ... \leq \pi_n$, $\pi_1' \leq \pi_2' \leq ... \leq \pi_n'$ and $\sum_{j=1}^{k} \pi_j \geq \sum_{j=1}^{k} \pi_j'$ for every $k = 1, ...n$. Suppose that the payoff functions of the associated games in strategic form both result result from functions $U^o$ and $U^p$ in the set $\mathcal{U}$ that satisfy Condition 1. Suppose also that $e_i^*(\pi_1', ..., \pi_n') > 0$ and $e_i^*(\pi_1, ..., \pi_n) > 0$ for all $i$. Then one has $\sum\limits_i e_i^*(\pi_1', ..., \pi_n') > \sum\limits_i e_i^*(\pi_1, ..., \pi_n))$*

# 3 Conclusion

This paper has examined the problem, for a community of individuals, of contributing to a public good when there is subjective uncertainty about the impact of the aggregate contribution on the considered public good. The problem of making individual efforts for reducing carbon emission with the aim of preventing global warming is a good example of this kind of situations. Other examples are provided by individual decisions to resort to vaccination to prevent the global risk of getting disease, or, in developing countries, by decisions to defecate in toilet rather than openly. When individuals are subjectively uncertain about the impact of the sum of their individual contributions on the benefit that they receive from the public good and differ in terms of this uncertainty, they may try to modify the beliefs held by others about this through debate or, perhaps, activism. This paper has examined to what extent an increase in the average belief that the sum of efforts impacts favorably the benefit of the public good lead a community of individuals to increase their effort. It has also examined to what extent an increase in the consensus about these beliefs - in the sense of Lorenz dominance - leads to such an increase in aggregate effort. It has shown this to be the case under somewhat strong, but not unreasonable, conditions on contributors' subjective evaluation of the benefit of the public good and the cost of their private effort.

The analysis performed in this paper is yet incomplete in many respects. One of its limitation is the two-state setting in which it is framed. As modeled in this paper, an individual contributor is facing indeed only two states of the world: an "optimistic" one in which the sum of the community members is impacting significantly the benefit that she receives from the public good, and a "pessimistic" one where the impact of the community's aggregate contribution on the benefit of the public good is less favorable. It would obviously be interesting to generalize the analysis to more than two states. But doing so is not as straightforward as it may seem. For one thing, it leaves open the question of defining what it means for the consensus about the beliefs in the occurrence of the various state to increase in the society. When there are only two states, each assigned with some probability, it is natural to define an increase in consensus by Lorenz dominance over one of the two probabilities (that sum to one). If there are, say, three states, then each state comes with a (possibly) different probability, and making the probabilities assigned to a state by two different individuals closer in the sense of a Pigou-Dalton transfer does not mean making the probabilities assigned by these two individuals closer. There seems therefore to be the need of developing a theory of what it means for two probability distributions to be closer to each other.

# References


Acemoglu, D., and M. K. Jensen (2013): "Aggregate Comparative Statics," *Games and Economic Behavior*, 81, 27–49.

Austen-Smith, D. (1980): "INDIVIDUAL CONTRIBUTION TO PUBLIC GOODS," *Economic Letters*, 5, 359–361.

Barrett, S. (1994): "Self-enforcing international environmental agreements," *Oxford Econ. Papers*, 46, 878âĂŞ894.

Berge, C. (1959): *Espaces topologiques et fonctions multivoques.* Dunod, Paris.

Bergstrom, T., L. Blume, and H. Varian (1986): "On the Private Provision of Public Goods," *Journal of Public Economics*, 29, 25–49.

Bramoulle, Y., and V. Boucher (2010): "Providing global public goods under uncertainty," *Journal of Public Economics*, 94, 591–603.

Bramoulle, Y., and N. Treich (September 2009): "CAN UNCERTAINTY ALLEVIATE THE COMMONS PROBLEM?," *Journal of the European Economic Association*, 7(5), 1042âĂŞ1067.

Corchòn, L. C. (1994): "Comparative statics for aggregative games. The strong concavity case.," *Mathematical Social Sciences*, 28, 151–165.

Cornes, R., and T. Sandler (1996): *The Theory of Externalities, Public Goods, and Club Goods*, Cambridge Books. Cambridge University Press.

Dasgupta, P., A. K. Sen, and D. Starrett (1973): "Notes on the Measurement of Inequality," *Journal of Economic Theory*, 6, 180–187.

Dixit, A. K., and J. W. Weibull (2007): "Political polarization.," *Proceedings of the National Academy of Sciences*, 104(18), 7351–7356.

Doran, P., and M. Zimmerman (2009): "Examining the scientific consensus on climate change," *Eos Trans Am Geophys Union*, 90, 21âĂŞ22.

Dubey, P., A. Mas-Colell, and M. Shubik (1980): "Efficiency Properties of Strategic Market Games," *Journal of Economic Theory*, 22, 339–362.

Gradstein, M., S. Nitzan, and S. Slutsky (1992): "The effect of uncertainty on interactive behaviour," *Economics Journal*, 102(412), 554–561.

Hadar, J., and W. Russell (1974): "Stochastic Dominance in Choice under Uncertainty," in *Essays on Economics Behavior under Uncertainty*, ed. by M. S. Balch, D. L. McFadden, and S. Wu. North Holland, Amsterdam, UK.

Jensen, M. K. (June 2012.): "Distributional comparitive statics," Department of Economics Discussion paper, University of Birmingham.

Kolstad, C. D. (2007): "Systematic uncertainty in self-enforcing international environmental agreements," *Journal of Environmental Economics and Management*, 53, 68âĂŞ79.

Meinshausen, M., N. Meinshausen, W. Hare, S. C. B. Raper, K. Frieler, R. Knutti, D. J. Frame, and M. R. Allen (2009): "Greenhouse-gas emission targets for limiting global warming to 2ř c," *Nature*, 458, 1158–1163.

Milgrom, P., and C. Shannon (1994): "Monotone comparative statics," *Econometrica*, 62, 157–180.

Nocetti, D., and W. T. Smith (2015): "Changes in risk and strategic interaction," *Journal of Mathematical Economics*, 56, 37–46.

Pelham, B. W. (2009): "Awareness, opinions about global warming vary worldwide," Gallup Poll.

Roser-Renouf, C., E. W. Maibach, A. Leiserowitz, and X. Zhao (2014): "The genesis of climate change activism: from key beliefs to political action," *Climatic Change*, 125(2), 163–178.

Sakamoto, H. (2014): "Public Bads, Heterogeneous Beliefs, and the Value of Information," Discussion papers e-13-009, Graduate School of Economics Project Center, Kyoto University.

Sandler, T., F. P. Sterbenz, and J. Posnett (1987): "FREE RIDING AND UNCERTAINTY," *European Economic Review*, 31, 1605–1617.

Shorrocks, A. F. (1983): "Ranking Income Distributions," *Economica*, 50, 3–17.

Shubik, M. (1984): *Game Theory in the Social Sciences: concepts and solutions*. MIT Press, Cambridge, MA.

TF, S., Q. D, P. G.K., T. M., and A. S.K. (eds.) (2013): *Intergovernmental Panel on Climate Change (2013)Climate Change 2013: The Physical Science Basis*. Cambridge University Press, Cambridge, United Kingdom and NY, USA.

Topkis, D. (1978): "Minimizing a submodular function on a lattice.," *Operational Research*, 26, 305–321.