

A Monetary Business Cycle Model for India*

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Abstract

This paper builds a New Keynesian monetary business cycle model with specific features of the Indian economy to understand *why* the aggregate demand channel of monetary transmission is weak. Our model allows us to identify the propagation mechanism and quantify the variance decomposition of a variety of real shocks (TFP, investment specific technological change, and fiscal policy) and nominal shocks (liquidity shocks, interest rate shocks) on the economy. In a specific environment, we show that fiscal dominance (in the form of a statutory liquidity ratio and administered interest rates) does not weaken monetary transmission. This is contrary to the consensus view in policy discussions in Indian monetary policy. We also show that a larger borrowing-lending spread, more aggressive inflation and output targeting by the Central Bank *weakens* the transmission from the policy rate to output.

Keywords : Monetary Business Cycles, Fiscal Dominance, Monetary Transmission, Inflation Targeting, Indian Macroeconomics

JEL Codes : [JEL Codes here]

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1 Introduction

With the formal adoption of inflation targeting by the Reserve Bank of India, monetary policy in India has undergone a major overhaul. With clearly defined objectives, operating procedures, and a nominal anchor, the transmission mechanism of monetary policy has become more transparent. India is now a flexible inflation targeter, where a newly convened monetary policy committee (as of September 2016) is tasked to maintain a medium term CPI-headline inflation at 4%, within a floor of 2% and a ceiling of 6%.

Despite major changes in monetary policy however, monetary transmission has found to be partial and asymmetric as well as slow (Das (2015); Mishra, Montiel and Sengupta (2016); and Mohanty and Rishab (2016)). The lack of complete pass-through to bank lending and deposit rates has also been a bugbear in recent monetary policy statements. These reports routinely mention that the pass through of past policy rate cuts by the banking system should be a pre-requisite for further rate cuts. Decomposing monetary transmission through the bank lending channel in two steps - from policy rates to bank lending rates - and then from lending rates to aggregate demand, Mishra, Montiel and Sengupta (2016) find that not only is pass through from policy rate to banking lending rates incomplete, but there is little empirical support for any effect of monetary policy shocks on aggregate demand.¹ Consistent with this, the "Report of the Expert Committee to Strengthen the Monetary Policy Framework (2014)"², also known as the Urjit Patel Committee Report, highlights several structural factors that hinder monetary transmission in India, i.e., the role of fiscal dominance in the form of SLR³, small savings schemes (or administered interest rates), and the presence of a large informal sector to name a few. The Urjit Patel Committee Report (2014) also notes that "... the conduct of liquidity management (is) often mutually inconsistent and conflicting. Often, increases in policy rate have been followed up with discretionary measures to ease liquidity conditions (page 36)". Shocks to autonomous drivers of liquidity, such as currency demand, bank reserves (required plus excess), government deposits with the Reserve Bank of India, and net foreign market operations, complicate the alignment of the policy repo rate - the short term signalling rate - with the overnight weighted average call rate (WACR) under the liquidity adjustment facility.⁴

¹Both Mishra, Montiel and Sengupta (2016) and Mohanty and Rishab (2016) provide excellent surveys of monetary transmission in India and emerging market developing economies (EMDEs) respectively.

²https://rbi.org.in/scripts/BS_PressReleaseDisplay.aspx?prid=30446

³The SLR, or the statutory liquidity ratio, provides a captive market for government securities and helps to artificially suppress the cost of borrowing for the Government, dampening the transmission of interest rate changes across the term structure. See the Urjit Patel Committee Report (2014).

⁴Since 2001, the Reserve Bank of India has conducted monetary policy through a corridor system called the LAF (liquidity adjustment facility). The LAF essentially allows banks to undertake collateralized lending and borrowing to meet short term asset-liability mismatches. The Repo rate is the rate at which

This paper develops a New Keynesian monetary business cycle model of the Indian economy to understand *why* the aggregate demand channel of monetary transmission is weak. As in Mishra, Montiel and Sengupta (2016), we define the aggregate demand channel in two steps: from policy rates to bank lending rates – and then from lending rates to GDP (including its components, consumption and investment) and inflation. We think that the aggregate demand channel is important because consumption and investment constitute roughly 87% of Indian GDP.

Our goal is two-fold. First, while there are a large number of empirical papers and policy reports that study the strength of monetary transmission channels in the Indian context, there are very few studies of monetary transmission in India using a DSGE model.⁵ Our paper fills this gap. Second, motivated by the role of fiscal dominance in hindering monetary transmission, our model embeds fiscal dominance in monetary policy making in India by incorporating two key features endemic to the Indian financial sector (i) SLR requirements by banks, and (ii) administered interest rate setting by the government. Allowing for such frictions in the banking sector helps us understand the role of banking intermediation in the transmission of monetary impulses.

Our calibrated baseline model yields several results. First, we identify the propagation mechanism of autonomous liquidity shocks, or autonomous base money shocks, to the rest of the economy. We show that an expansionary base money shock stimulates consumption, investment, hours worked, capital accumulation, and opens up a positive output gap on impact. An increase in base money is also inflationary. The rise in inflation and the output gap leads to a rise in the policy rate via the Taylor rule, which is equal to the short term interest rate on government bonds. On the other hand, a fall in the government bond rate also has similar expansionary effects on the economy.

Second, our baseline model shows that about half of the fluctuation (variance) in output are explained by TFP shocks and one third is explained by fiscal shocks. Monetary policy in terms of interest rate shocks and base money shocks explain a negligible fraction of output variation exemplifying the weak transmission channel of monetary policy.

Third, our sensitivity experiments with respect to structural and policy parameters indicate that household's preference for commercial bank deposits vs postal deposits (or deposits with administered interest rates), the statutory liquidity ratio, and administered interest rates have negligible effects on the monetary transmission channel measured by the forecast

banks borrow money from the RBI by selling short term government securities to the RBI, and then "re-purchasing" them back. A reverse repo operation takes place when the RBI borrows money from banks by lending securities. See Mishra, Montiel and Sengupta (2016, pages 73-74).

⁵See Levine et al. (2012) for an early attempt. Banerjee and Basu (2017) develop a small open DSGE model for India but do not study the monetary transmission mechanism.

error variance of output due to autonomous monetary shocks or the money-output correlation. While this result is specific to the way we have modelled the banking sector’s problem, this observation itself can be treated in the context of policy discussion on monetary transmission. In this specific environment, neither administered interest rate nor fiscal dominance (in the form of SLR) cause weak monetary transmission in the economy as it is widely believed. On the other hand, a larger borrowing-lending spread and less aggressive inflation and output targeting makes the monetary policy transmission resulting from a policy rate shock weaker and the effect of autonomous shocks to monetary base stronger.

Fourth, our sensitivity analysis suggests that a higher long term interest rate on government bond weakens monetary transmission and raises the importance of fiscal spending shocks. This happens because a higher long term policy rate necessitates higher taxes to retire outstanding public debt. This highlights fiscal dominance in our model.

The paper is organized as follows. In the next section, we lay out our model. Section 3 is devoted to a quantitative analysis of the model. Section 4 concludes.

2 The Model

2.1 Environment

Our core model is a monetary RBC model with sticky prices. On the household and production side, the model economy is similar to Gerali et al. (2010) but with important differences. The economy is populated by households and entrepreneurs, each group having unit mass. Households consume, work, and accumulate savings in risk-free bank deposits as well as postal deposits with a fixed government set interest rate.⁶ We assume that households own the banks. On the production side, wholesale entrepreneurs produce homogenous intermediate goods using capital, bought from capital goods producers, loans obtained from banks, and hired labor from households. As in Gerali et al (2010), capital goods producers are introduced to derive a market price for capital. A monopolistically competitive retail sector buys intermediate goods from wholesale entrepreneurs, and produces a single final good. Retail prices are sticky and indexed to steady state inflation. This allows monetary policy to have real effects. Retailers also face a quadratic price adjustment cost *a la* Rotemberg.

Unlike Gerali et al (2010), banks in the current set-up are assumed to be perfectly competitive.⁷ Banks maximize cash flows in every period, offer savings deposits to households and loans to wholesale entrepreneurs, subject to the constraints that a fixed fraction of

⁶In Gerali et al. (2010), there are no administered postal accounts.

⁷Market power in the banking industry in Gerali et al. (2010) is modelled using a Dixit-Stiglitz framework for retail and credit deposit markets.

deposits in every period are set aside for (i) a statutory liquidity requirement (SLR) and (ii) reserve requirements. We allow for the stochastic withdrawal of deposits in each period as in Chang et al. (2014). At date t , if the withdrawal, exceeds bank reserve (cash in vault), banks fall back on the Central Bank for emergency loans at a penalty rate mandated by the central bank. The presence of SLR requirements and administered interest rates capture the essence of fiscal dominance in the Indian economy.

We assume that the central bank is a flexible inflation targeter, as in India. There is no currency in the model, and so the supply of reserves equals the monetary base. The central bank lets the monetary base, or the supply of reserves increase by a simple rule that is perturbed in every period by base-money shocks, or autonomous liquidity shocks. In addition to these shocks, the economy is also hit in every period by total factor productivity shocks, fiscal policy shocks, investment specific technology (IST) shocks, and interest rate shocks. To deal with the inflationary consequences of autonomous liquidity shocks, the central bank has one instrument at its disposal: the short term interest rate on government bonds, which we interpret as the policy rate, and which is governed by a conventional Taylor rule. The economy is hit periodically with autonomous liquidity, or base money shocks, which are inflationary, and therefore warrant a monetary policy response using the Taylor rule.

On the fiscal side, the government spends stochastically and issues public debt held by banks to cover the difference between government spending and lump sum taxes. Administered postal deposits, which attract a government set interest rate in the Indian economy, are directly assumed to augment government revenues in every period.

We calibrate the model and provide a baseline parameterization that can replicate the regularities of the Indian business cycles broadly. Then, we focus on impulse response properties and variance decomposition results to highlight and explain the role of shocks and frictions on monetary transmission.

2.2 Households optimization

The economy is populated by infinitely lived households of unit mass. The representative household maximizes expected utility

$$\max_{C_t, H_t, D_{t+1}, D_{t+1}^p} E_0 \sum_{t=0}^{\infty} \beta^t [U(C_t) - \Phi(H_t) + V(D_t/P_t, D_t^a/P_t)] \quad (1)$$

which depends on hours worked, H_t , consumption of the final good, C_t , and saving in the form of risk-free bank and postal deposits, D_t , and D_t^a respectively. Household choices must

obey the following budget constraint (in nominal terms)

$$P_t(C_t + T_t) + D_t + D_t^a \leq W_t H_t + (1 + i_t^D)D_{t-1} + (1 + i^a)D_{t-1}^a + \Pi_t^k + \Pi_t^r \quad (2)$$

The left hand side of equation (2) represents the flow of expenses which includes current consumption (where P_t is the aggregate price index and $T_t > 0$ denote lump-sum transfers), nominal bank deposits, D_t and postal deposits, D_t^a . Resources consist of wage earnings, $W_t H_t$, where W_t is the wage rate, payments on deposits made in the previous period, $t - 1$, where $i_t^D > 0$ is the rate on one-period deposits (or savings contracts) in the banking system, and $i^a > 0$ is the fixed government administered interest rate on postal deposits made by households. Π_t^k and Π_t^r denote nominal profits rebated back from the capital goods sector, and the retail goods sector, to households respectively.⁸

Using $D_t/P_t = d_t$ and $D_t^a/P_t = d_t^a$, and substituting out for $U'(C_t) = \lambda_t P_t$, we can re-write the household's optimality conditions as:

$$D_t : U'(C_t) = V'_1(d_t, d_t^a) + \beta U'(C_{t+1})(1 + i_{t+1}^D)(P_t/P_{t+1}), \quad (3)$$

$$D_t^p : U'(C_t) = V'_2(d_t, d_t^a) + \beta U'(C_{t+1})(1 + i^a)(P_t/P_{t+1}) \quad (4)$$

$$\Phi'(H_t) = (W_t/P_t) U'(C_t). \quad (5)$$

Equation (3) is the standard Euler equation for deposits. Equation (4) is the Euler equation for postal deposits which attract the administered interest rate, i^a . Equation (5) is the standard intra-temporal optimality equation for labor supply.

2.3 Capital good producing firms

Our description of the capital goods producing firms is standard. Perfectly competitive firms buy last period's undepreciated capital, $(1 - \delta_k)K_{t-1}$, at price Q_t from wholesale-entrepreneurs (who own the firms) and I_t units of the final good from retailers at price P_t . The transformation of the final good into new capital is subject to adjustment costs, S_t .⁹ Capital goods producing firms maximize

⁸Please refer to Appendix A for all derivations.

⁹ We assume that

$$S\left(\frac{I_t}{I_{t-1}}\right) = (\kappa/2) \left(\frac{I_t}{I_{t-1}} - 1\right)^2.$$

$$\max_{I_t} E_t \sum_{j=0}^{\infty} \Omega_{t,t+j} P_{t+j} \left[Q_{t+j} I_{t+j} - \left\{ 1 + S \left(\frac{I_{t+j}}{I_{t+j-1}} \right) \right\} I_{t+j} \right] \quad (6)$$

$$\text{s.t.} \quad K_t = (1 - \delta_k) K_{t-1} + Z_{x,t} I_t \quad (7)$$

where $\Omega_{t,t+j}$ is the stochastic discount factor and $Z_{x,t}$ is an investment specific technology (IST) shock that follows an AR(1) process.

The first order condition is

$$\frac{\partial (\cdot)}{\partial I_t} = \Omega_{t,t} P_t Q_t - \Omega_{t,t} P_t \left\{ 1 + S \left(\frac{I_t}{I_{t-1}} \right) \right\} - \Omega_{t,t} P_t S' \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} + \Omega_{t,t+1} P_{t+1} S' \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 = 0. \quad (8)$$

which yields the capital good pricing equation,

$$Q_t = 1 + S \left(\frac{I_t}{I_{t-1}} \right) + S' \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} - \beta E_t \frac{U'(C_{t+1})}{U'(C_t)} \left[S' \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right]. \quad (9)$$

2.4 Wholesale good producing firms

Wholesale, or intermediate goods firms are run by risk neutral entrepreneurs who produce intermediate goods for the final good producing retailers in a perfectly competitive environment. The entrepreneurs hire labor from households and purchase new capital from the capital good producing firms. They borrow an amount $L_t > 0$ of loans from the bank in order to meet the value of new capital, $Q_t K_t$, where K_t is the capital stock. We assume that all capital spending is debt financed. Used capital at date $t + 1$ is sold at the resale market at the price Q_{t+1} . The balance sheet condition of the wholesale firms is:

$$Q_t K_t = \left(\frac{L_t}{P_t} \right). \quad (10)$$

In the steady state $Q_t = 1$ which means $l_t = \frac{L_t}{P_t} = K_t$, i.e., all capital is intermediated. The production function for a representative wholesale goods producer is given by

$$Y_t^W = \xi_t^a K_{t-1}^\alpha H_t^{1-\alpha}$$

with $0 < \alpha < 1$. ξ_t^a denotes stochastic total factor productivity, and follows an AR(1) process. The (real) wage rate, W_t , is given by

$$W_t/P_t = (P_t^W/P_t)MPH_t = (1 - \alpha) \frac{(P_t^W/P_t)Y_t^W}{H_t} \quad (11)$$

This allows us to obtain the rate of rate of return from capital, $1 + r_{t+1}^k$, as

$$\begin{aligned} 1 + r_{t+1}^k &= \frac{(P_{t+1}^W/P_{t+1})Y_{t+1}^W - (W_{t+1}/P_{t+1})H_{t+1} + (1 - \delta_k) K_t Q_{t+1}}{Q_t K_t} \\ &= \frac{(P_{t+1}^W/P_{t+1}) \left(\frac{Y_{t+1}^W}{K_t} \right) - (1 - \alpha) \frac{(P_{t+1}^W/P_{t+1})Y_{t+1}^W}{H_{t+1}} \left(\frac{H_{t+1}}{K_t} \right) + (1 - \delta_k) Q_{t+1}}{Q_t} \\ &= \frac{(P_{t+1}^W/P_{t+1})MPK_{t+1} + (1 - \delta_k)Q_{t+1}}{Q_t} \end{aligned}$$

The optimality condition for a firms' demand for capital is given by the following arbitrage condition,

$$1 + r_{t+1}^k = (1 + i_{t+1}^L) \frac{P_t}{P_{t+1}}. \quad (12)$$

This yields,

$$\begin{aligned} (1 + i_{t+1}^L) &= \frac{P_{t+1}^W MPK_{t+1} + (1 - \delta_k) P_{t+1} Q_{t+1}}{P_t Q_t} \\ 1 + i_{t+1}^L &= \left[\left(\frac{P_{t+1}^W}{P_{t+1}} \right) \frac{MPK_{t+1}}{Q_{t+1}} + 1 - \delta_k \right] \left[\frac{P_{t+1} Q_{t+1}}{P_t Q_t} \right]. \end{aligned} \quad (13)$$

2.5 Final good retail firms

Retailers buy intermediate goods at price P_t^W and package them into final goods and operate in a monopolistically competitive environment as in Bernanke, Gertler, and Gilchrist (1999). They convert the i^{th} variety of the intermediate good, $y_t^W(i)$, into $y_t(i)$ one-to-one and differentiate the goods at zero cost. Each retailer sells his unique variety of final product after applying a markup over the wholesale price, and factoring in the market demand condition which is characterized by price elasticities (ε^Y). Retailer's prices are sticky and indexed to past and steady state inflation as in Gali et al. (2010). If retailers want to change their price over and above what indexation allows, they have to bear a quadratic adjustment cost given by ϕ_p .

Retailers choose $\{P_{t+j}(i)\}_{j=0}^{\infty}$ to the maximize present value of their expected profit.

$$\max_{P_t(i)} E_t \sum_{j=0}^{\infty} \Omega_{t,t+j} \{ \Pi_{t+j|t}^r \} \quad (14)$$

subject to the demand constraint, $y_{t+j|t}(i) = \left(\frac{P_{t+j}(i)}{P_{t+j}} \right)^{-\varepsilon^Y} y_{t+j}$, where the profit function of

the i^{th} retailer is given by,

$$\Pi_t^r(i) = P_t(i) y_t(i) - P_t^W(i) y_t^W(i) - \frac{\phi_p}{2} \left[\left\{ \frac{P_{t+j}(i)}{P_{t+j-1}(i)} - (1 + \pi_{t-1})^{\theta_p} (1 + \bar{\pi})^{1-\theta_p} \right\}^2 P_t y_t \right] \quad (15)$$

$\phi_p > 0$, $0 < \theta_p < 1$, and

$$y_t = \left[\int_0^1 y_t(i)^{\frac{\varepsilon^Y - 1}{\varepsilon^Y}} di \right]^{\frac{\varepsilon^Y}{\varepsilon^Y - 1}}; \quad \varepsilon^Y > 1.$$

Note that θ_p is an indexation parameter. This price adjustment cost specification is borrowed from Gerali et al (2010). The first order condition after imposing a symmetric equilibrium is standard:

$$1 - \varepsilon^Y + \varepsilon^Y \left(\frac{P_t}{P_t^W} \right)^{-1} - \phi_p \left\{ 1 + \pi_t - (1 + \pi_{t-1})^{\theta_p} (1 + \bar{\pi})^{1-\theta_p} \right\}. \quad (16)$$

In the steady state, when $\pi_{t+1} = \pi_t = \pi$, this implies that the steady state mark-up is,

$$\frac{P}{P^W} = \frac{\varepsilon^Y}{\varepsilon^Y - 1}. \quad (17)$$

2.6 Banks

The representative bank maximizes cash flows by offering savings contracts (deposits) and borrowing contracts (loans). Banks are also mandated to keep reserves with the central bank. In India, and many other emerging market economies (EMEs), they are also constrained to buy government debt from deposit inflows as mandated by a statutory liquidity ratio (SLR). In every period, following Chang et al. (2014), we assume that banks face a stochastic withdrawal of deposits at the end of each period, t . At date t , if the withdrawal (say \widetilde{W}_{t-1}) exceeds bank reserve (cash in vault), banks fall back on the Central Bank for emergency loans at a penalty rate i^p mandated by the Central Bank (CB). Banks pay back the emergency borrowing to the CB at the end of the period. This withdrawal uncertainty necessitates a demand for excess reserve by the banks.¹⁰

¹⁰ A practical application of the stochastic withdrawal is as follows. Suppose the withdrawal of deposits is expected at the end of the period. Suppose the bank anticipates that it will fall short of reserve by x rupees at the end of the day. charges a y percent penalty rate. Thus the bank's expected penalty is $x(1 + y)$ rupees which includes the principal and interest that the bank has to pay at the end of the period or the start of the next period. Taking this into consideration, the bank chooses its reserve holding optimally at the start of date t . Thus a higher expected penalty will make the bank hold more cash reserve.

Define i_t^L to be the interest rate on loans, L_{t-1} , i^R to be the interest rate on reserves, M_t^R , mandated by the central bank, and \widetilde{W}_t is the stochastic withdrawal. Since government bond and short term deposits are perfect substitutes, $i_t^D = i_t^G = i_t^s$ (say). D_t denotes deposits. We assume that bank has a SLR equal to $\alpha_s \in [0, 1]$.

The bank's cash flow at date t can be rewritten as:

$$\begin{aligned}
CF_t^b = & (1 + i_t^L)L_{t-1} + (1 + i^R)M_{t-1}^R + \underbrace{\alpha_s(1 + i_t^G)D_{t-1}}_{\text{SLR on last period's deposits}} - \underbrace{(1 + i_t^D)D_{t-1}}_{\text{Cost of Funds of Last period's Deposits}} \\
& - (1 + i^p)E \max(\widetilde{W}_{t-1} - M_{t-1}^R, 0) + \underbrace{D_t}_{\text{Current Deposits}} - \underbrace{\alpha_s D_t}_{\text{SLR this period}} - L_t - M_t^R
\end{aligned} \tag{18}$$

The first two terms on the right hand side correspond to the interest earned in time t on loans disbursed in time $t - 1$, and interest on reserves in the previous period, M_{t-1}^R . Since the bank is forced to hold government debt as a constant fraction, α_s , of incoming deposits, $\alpha_s(1 + i_t^G)D_{t-1}$, denotes the interest earnings on SLR debt holdings by banks. As described above, banks also face a penalty, at a constant penal rate, $i^p > 0$, for stochastic withdrawals over and above their bank reserves. The penalty amount is $(1 + i^p)E \max(\widetilde{W}_{t-1} - M_{t-1}^R, 0)$. We assume that banks offer a deposit rate, i_t^D , which is a mark-down of the interest rate that it receives on government bonds, i_t^G . In other words, $1 + i_t^D = \zeta(1 + i_t^G)$ where $0 < \zeta < 1$. We do not model the mark-down, ζ , but calibrate it. Rewrite the cash flow in equation (18) as

$$\begin{aligned}
CF_t^b = & (1 + i_t^L)L_{t-1} + (1 + i^R)M_{t-1}^R - (\zeta - \alpha_s)(1 + i_t^G)D_{t-1} \\
& - (1 + i^p)E \max(\widetilde{W}_{t-1} - M_{t-1}^R, 0) \\
& + (1 - \alpha_s)D_t - L_t - M_t^R.
\end{aligned} \tag{19}$$

The representative bank maximizes discounted cash flows in two stages. It first solves for its optimal demand for reserves, M_t^R . Next, it chooses the loan amount, L_t . Specifically, banks maximize

$$\underset{M_t^R, L_t}{Max} \quad E_t \sum_{s=0}^{\infty} \Omega_{t,t+s} CF_{t+s}^b$$

subject to the statutory reserve requirement:

$$M_t^R \geq \alpha_r D_t \quad (20)$$

where $\Omega_{t,t+s} = \frac{\beta^s U'(c_{t+s})}{U'(c_t)} \cdot \frac{P_t}{P_{t+s}}$ is the inflation adjusted stochastic discount factor.

The Euler equation is given by¹¹

$$E_t \Omega_{t,t+1} \left[(1 + i^R) + (1 + i^p) \int_{M_t^R}^{D_t} f(\widetilde{W}_t) d\widetilde{W}_t \right] + \lambda_t = 1 \quad (21)$$

The first term in the square bracket in equation (21) is the bank's interest income from reserves. The second term is the expected saving of penalty because of holding more reserves λ_t is the Lagrange multiplier associated with the reserve constraint (20). The Kuhn Tucker condition states that

$$\frac{M_t^R}{D_t} = \alpha_r \text{ if } \lambda_t > 0$$

Assume that the reserve requirement is not binding, which implies that $\lambda_t = 0$. Assuming a rectangular distribution for \widetilde{W}_t over $[0, D_t]$ ¹², (21) reduces to:

$$M_t^R : 1 = E_t \Omega_{t,t+1} \left[(1 + i^R) + (1 + i^p) \left(1 - \frac{M_t^R}{D_t}\right) \right]. \quad (22)$$

We solve $\frac{M_t^R}{D_t}$ as follows:

$$\frac{M_t^R}{D_t} = 1 - \frac{1 - (1 + i^R) E_t \Omega_{t,t+1}}{(1 + i^p) E_t \Omega_{t,t+1}} \quad (23)$$

which is the same as writing

$$\frac{x_t}{d_t} = 1 - \frac{1 - (1 + i^R) E_t \Omega_{t,t+1}}{(1 + i^p) E_t \Omega_{t,t+1}} \quad (24)$$

where $x_t = M_t^R / P_t$ and $d_t = D_t / P_t$. It is straightforward to verify that given the discount factor, $\Omega_{t,t+1}$, a higher i^R or i^p means a higher M_t^R as expected.

Once the bank's reserve demand problem is solved, we next turn to the holding of loans. Note that the bank solves a recursive problem of choosing L_t given L_{t-1} which were chosen in the previous period. This is a dynamic allocation problem. The first order condition with

¹¹See Technical Appendix A.

¹² Since \widetilde{W}_t follows a rectangular distribution, over $[0, D_t]$

$$\int_{M_t^R}^{D_t} f(\widetilde{W}_t) d\widetilde{W}_t = \frac{D_t - M_t^R}{D_t} = 1 - \frac{M_t^R}{D_t}.$$

respect to L_t is given by,

$$\Omega_{t,t}(-1) + E_t \Omega_{t,t+1}(1 + i_{t+1}^L) = 0.$$

This gives the loan Euler equation:

$$L_t : 1 = E_t \Omega_{t,t+1}(1 + i_{t+1}^L) \quad (25)$$

Substituting out for $E_t \Omega_{t,t+1}$ in equation (25) and putting it into equation (23), we see the following connection between the loan market premium and the reserve demand of bank:

$$\frac{x_t}{d_t} = 1 + \frac{1 + i^R}{1 + i^P} \left[1 - \frac{E_t \frac{1+i_{t+1}^L}{1+i^R}}{1 - \text{cov}_t(\Omega_{t,t+1}, (i_{t+1}^L - i^R))} \right] \quad (26)$$

The negative of the covariance term in the denominator picks up the risk premium associated with the risky loan of banks relative to the risk-free interest rate on reserves. If the bank loan is not risky, this covariance term is zero in which case a higher loan rate discourages the holding of bank reserves. However, if the loan is a bad hedge which makes the absolute value of the covariance bigger, it will encourage banks to hold more reserves which is reminiscent of the financial crisis.¹³

2.7 Monetary Policy

The Central Bank follows a simple money supply rule. It lets the monetary base (M_t^B), or the supply of reserves, M_t^R (since currency is zero), increase by the following rule:

$$\frac{M_t^B/M_{t-1}^B}{1 + \bar{\pi}} = \left(\frac{M_{t-1}^B/M_{t-2}^B}{1 + \bar{\pi}} \right)^{\rho_\mu} \exp(\xi_t^\mu) \quad (27)$$

¹³One may wonder whether there is any borrowing-lending spread because banks are not monopolistic. Curiously a steady state borrowing-lending spread still emerges in this model because deposit appears in the utility function and provides a liquidity service (convenience yield) to the household. Bank deposit provides some transaction utility to the household. Thus the household wishes that the banks do not loan out all their deposits and make them illiquid. This convenience yields (alternatively a liquidity premium) gives rise to a credit rationing which gives rise to a positive borrowing-lending spread in the steady state. To see it combine (3) and (25) to get the following steady state borrowing-lending spread.

$$i^L - i^D = \frac{(1 + \pi)}{\beta} \frac{V_1'(d, d^P)}{U'(c)} > 0$$

This convenience yield is akin to forward-spot spread in finance.

where ρ_μ is the policy smoothing coefficient and ξ_t^μ is the money supply shock, which follows an AR (1) process. We view a shock to the monetary base as an autonomous liquidity shock. Money market equilibrium implies that

$$M_t^R = M_t^B \text{ for all } t.$$

Such a money supply process imposes restriction on the short run growth rate of real reserve and inflation as follows:

$$\frac{(1 + \pi_t)(x_t/x_{t-1})}{1 + \pi} = \left(\frac{(1 + \pi_{t-1})(x_{t-1}/x_{t-2})}{1 + \pi} \right)^{\rho_\mu} \exp(\xi_t^\mu) \quad (28)$$

Since real reserves are proportional to deposits as shown in the bank's reserve demand function, this also imposes restriction on the dynamics of deposits, interest rate on loans and consumption.

2.8 Interest rate policy

The short term interest rate on government bonds (i_t^G) can be broadly interpreted as a policy rate. We give it an inflation targeting Taylor rule as follows:

$$\frac{(1 + i_t^G)}{(1 + \bar{i}^G)} = \left(\frac{(1 + i_{t-1}^G)}{(1 + \bar{i}^G)} \right)^{\rho_{iG}} \left[\left(\frac{1 + \pi_{t-1}}{1 + \pi} \right)^{\varphi_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\varphi_y} \right]^{(1 - \rho_{iG})} \exp(\xi_t^G) \quad (29)$$

The parameters $\phi_p > 0$, and $\phi_y > 0$ are the inflation, and output gap sensitivity parameters in the Taylor Rule. Y_t denotes GDP, and therefore $\frac{Y_t}{\bar{Y}}$ denotes the output gap. ρ_{iG} is the interest rate smoothing term and ξ_t^G is the policy rate shock.

We shall see later in the quantitative analysis section that the strength of monetary transmission of a money base shock is significantly influenced by the parameters of the Taylor rule.

2.9 Fiscal Policy

The government budget constraint (in nominal terms) is given by,

$$P_t G_t + (1 + i_t^G) B_{t-1} + (1 + i^R) M_{t-1}^R + (1 + i^a) D_{t-1}^a = P_t T_t + B_t + M_t^R + D_t^a + (1 + i^p) E \max(\widetilde{W}_t - M_t^R, 0) \quad (30)$$

where G_t corresponds to real government purchases, B_t and denotes the stock of public debt. The left hand side of equation (30) denotes total expenditures by the government (nominal

government purchases + interest payments on public debt + interest rates on reserves + interest payments on administered postal deposits).¹⁴ The right hand side of equation (30) denotes the total resources available to the government (nominal lump sum taxes + new debt + new reserves + administered deposits + interest payments from withdrawal penalties).

Government spending (or government purchases) evolves stochastically according to:

$$G_t - \bar{G} = \rho_G \left(G_{t-1} - \bar{G} \right) + \xi_t^G.$$

ξ_t^G denotes the shock to government spending, and follows an AR(1) process.

2.10 Steady State

In this section, we solve for the steady state values of the endogenous variables. Equation (13) in the steady state is given by,

$$1 + i^L = \left[\left(\frac{P^W}{P} \right) \frac{MPK}{Q} + 1 - \delta_K \right] (1 + \pi)$$

as $\frac{P_{t+1}}{P_t} = 1 + \pi_t$. Further, from equation (9) and (17) in the steady state, $Q = 1$ and $P^W = \frac{\varepsilon^Y - 1}{\varepsilon^Y} P$, respectively. Also, in the steady state, $Y^W = K^\alpha H^{1-\alpha}$ which implies that $MPK = \frac{\alpha Y^W}{K}$. The above equation thus reduces to,

$$1 + i^L = \left[\left(\frac{\varepsilon^Y - 1}{\varepsilon^Y} \right) \left(\frac{\alpha Y^W}{K} \right) + 1 - \delta_K \right] (1 + \pi) \quad (31)$$

Recalling that in the steady state, the stochastic discount factor is given by $\frac{\beta}{1+\pi}$, substituting this into the steady version of equation (25) yields, $1 + i^L = \frac{(1+\pi)}{\beta}$. From this expression, we can solve for the steady state capital-labor ratio, K/H , which is given by

$$\frac{K}{H} = \left\{ \alpha \left[\frac{\varepsilon^Y - 1}{\varepsilon^Y} \right] \left[\frac{1}{\frac{1}{\beta} - (1 - \delta_K)} \right] \right\}^{\frac{1}{1-\alpha}} \quad (32)$$

which we call Λ hereafter.

The national income identity is given by,

$$C + \delta_K K + G = K^\alpha H^{1-\alpha} \quad (33)$$

¹⁴We think of the government as a combined fiscal-monetary entity.

Assume the following functional forms: $\Phi(H_t) = H_t$, $U(C_t) = \ln(C_t)$ and $V(d_t, d_t^a) = \eta \ln d_t + (1 - \eta) \ln d_t^a$. Thus in steady state, $\Phi'(H) = 1$, $U'(C) = 1/C$, $V'_1(.,.) = \frac{\eta}{d}$ and $V'_2(.,.) = \frac{(1-\eta)}{d^a}$. Substituting for these values into equation (5), in the steady state, we get

$$C = W/P$$

Next note from (11) and (17), $W/P = (1 - \alpha) \left(\frac{\varepsilon^Y - 1}{\varepsilon^Y} \right) \left(\frac{K}{H} \right)^\alpha$. Therefore,

$$C = (1 - \alpha) \left(\frac{\varepsilon^Y - 1}{\varepsilon^Y} \right) (\Lambda)^\alpha. \quad (34)$$

Now, substituting $V'_1(.,.) = \frac{\eta}{d}$ in equation (3), we get,

$$\frac{1}{C} = V'_1(.,.) + \beta \frac{1}{C} \frac{(1 + i^D)}{(1 + \pi)}$$

The above can be re-written as,

$$1 + i^D = \frac{1 + \pi - \eta \frac{C}{d} (1 + \pi)}{\beta} \quad (35)$$

Similarly substituting $V'_2(.,.) = \frac{(1-\eta)}{d^a}$ in equation (4), we get,

$$1 + i^a = \frac{1 + \pi - (1 - \eta) \frac{C}{d^a} (1 + \pi)}{\beta} \quad (36)$$

Since $\frac{K}{H} = \Lambda$, equation (33) above thus reduces to,

$$C + G = [\Lambda^{-(1-\alpha)} - \delta_K] K \quad (37)$$

Recall, from equation (30) the government budget constraint is given by,

$$P_t G_t + (1 + i^G) B_{t-1} + (1 + i^R) M_{t-1}^R + (1 + i^a) D_{t-1}^a = P_t T_t + B_t + M_t^R + D_t^a + (1 + i^p) E \max(\widetilde{W}_t - M_t^R, 0)$$

Dividing throughout by P_t and noting that $\frac{P_{t+1}}{P_t} = 1 + \pi_t$, we get

$$G_t + (1 + i_t^G) \frac{b_{t-1}}{1 + \pi_t} + (1 + i_t^R) \frac{x_{t-1}}{1 + \pi_t} + (1 + i^a) \frac{d_{t-1}^a}{1 + \pi_t} = T_t + b_t + x_t + d_t^a + (1 + i_t^p) d_t E \max\left(\frac{\widetilde{W}_t}{D_t} - \frac{M_t^R}{D_t}, 0\right)$$

where $x_t = M_t^R/P_t$, $d_t = \frac{D_t}{P_t}$, and $b_t = B_t/P_t$.

In the steady state, the above equation becomes

$$G_t + (1 + i_t^G) \frac{b}{1 + \pi} + (1 + i_t^R) \frac{x}{1 + \pi} + (1 + i^a) \frac{d^a}{1 + \pi} = T + b + x + d^a + (1 + i_t^p) d E \max(\frac{\widetilde{W}_t}{D_t} - \frac{M_t^R}{D_t}, 0),$$

or,

$$G(1 + \pi) + (i^G - \pi) b + (i^R - \pi) x + (i^a - \pi) d^a = T + (1 + i^p) d E \max(\frac{\widetilde{W}_t}{D_t} - \frac{M_t^R}{D_t}, 0)$$

Dividing through the above expression by d , yields,

$$\frac{G(1 + \pi)}{d} + (i^G - \pi) \alpha_s + (i^R - \pi) \frac{x}{d} + (i^a - \pi) \frac{d^a}{d} = T + (1 + i^p) E \max(\frac{\widetilde{W}_t}{D_t} - \frac{M_t^R}{D_t}, 0) \quad (38)$$

since $B/D = \alpha_s$ (which implies $b/d = \alpha_s$) Also, $\frac{x}{d} = \frac{M^R/P}{D/P}$.

We can substitute out for $\frac{d^a}{d}$ in the above equation (38) from equation (35) and (36) nothing that.

$$\begin{aligned} d [1 + \pi - \beta (1 + i^D)] &= \eta C (1 + \pi) \\ d^a [1 + \pi - \beta (1 + i^a)] &= (1 - \eta) C (1 + \pi) \end{aligned}$$

or,

$$\frac{d}{d^a} = \frac{\eta}{1 - \eta} \left[\frac{1 + \pi - \beta (1 + i^a)}{1 + \pi - \beta (1 + i^D)} \right], \quad (39)$$

and

$$\frac{x}{d} = 1 - \frac{1 - (1 + i^R) \frac{\beta}{1 + \pi}}{(1 + i^p) \frac{\beta}{1 + \pi}}.$$

Finally, let us solve for $E \max(\frac{\widetilde{W}_t}{D_t} - \frac{M_t^R}{D_t}, 0)$ in the steady state. Assume $\frac{\widetilde{W}_t}{D_t} = Z_t$, and since D_t is given, Z_t follows an uniform distribution as \widetilde{W}_t but between $[0, 1]$. Thus,

$$E \max(Z_t - \frac{M_t^R}{D_t}, 0) = \int_{M_t^R/D_t}^1 \left(Z_t - \frac{M_t^R}{D_t} \right) h(Z_t) dZ_t$$

Since $h(Z_t) = 1$,

$$\begin{aligned}
E \max(Z_t - \frac{M_t^R}{D_t}, 0) &= \int_{M_t^R/D_t}^1 \left(Z_t - \frac{M_t^R}{D_t} \right) dZ_t \\
&= \frac{Z_t^2}{2} \Big|_{M_t^R/D_t}^1 - \frac{M_t^R}{D_t} Z_t \Big|_{M_t^R/D_t}^1 \\
&= \left(\frac{1 - \left(\frac{M_t^R}{D_t} \right)^2}{2} \right) - \frac{M_t^R}{D_t} \left(1 - \frac{M_t^R}{D_t} \right) \\
&= \frac{1}{2} + \frac{1}{2} \left[\frac{M_t^R}{D_t} \right]^2 - \frac{M_t^R}{D_t} \\
&= 0.5 \left(1 - \frac{M_t^R}{D_t} \right)^2
\end{aligned} \tag{40}$$

where $\frac{M_t^R}{D_t}$ is given by (23) evaluated at the steady state.

Continuing from the above government constraint (38) we get

$$\frac{G(1+\pi)}{d} + (i^G - \pi) \alpha_s + (i^R - \pi) \frac{x}{d} + (i^a - \pi) \frac{d^a}{d} = T + (1+i^p) 0.5 \left(1 - \left\{ 1 - \frac{1 - (1+i^R) \frac{\beta}{1+\pi}}{(1+i^p) \frac{\beta}{1+\pi}} \right\} \right)^2 \tag{41}$$

From the above equation, we can solve for steady state lump-sum taxes, T . In Technical Appendix B, we summarize the steady state equations in recursive form.

3 Quantitative Analysis

The objective of our quantitative analyses is to understand the monetary transmission mechanism for the baseline model. Monetary transmission implies how a monetary policy impacts the aggregate economy. As mentioned in the introduction, we define that the aggregate demand channel operates in the model economy via two layers: from policy rates to bank lending rates, and then, from lending rates to GDP (including its components of consumption and investment) and inflation. For this purpose, we focus on the standard instruments of monetary policy for an inflation targeting central bank (i) money base and (ii) short term interest rate (which is the government bond rate in our model). We examine the magnitude of transmission of the shocks from policy instruments to policy targets, primarily, using the results of a variance decomposition of key macroeconomic variables of the model economy. Parallel to this, we also consider the magnitude of cross correlations between the policy instruments and policy targets as the indicators of pass through of the policy shocks. To this

end, first, we specify the parameterization of the baseline model and then, validate the same with data by moment matching exercise.

We calibrate the model based on Indian macroeconomic data and evidence from the DSGE literature based on the Indian economy. After baseline configuration and validation with data, we explain the impulse response properties of the model followed by the variance decomposition results. Next, we present the variance decomposition results of counterfactual experiments which document the role of different structural and policy factors in determining the strength of monetary transmission. Finally, we present the results of our experiments on different degrees of fiscal dominance driven by different steady state policy rates which have some implications for weakening the monetary transmission channel.

3.1 Baseline Parameterization

Following the DSGE literature on India and using Indian data on the macroeconomic variables, we calibrate the parameters of our model. Baseline parameterization of the model is presented in Table 1. The share of capital in production process is set as 0.3 (Banerjee and Basu, 2017). The discount factor is taken as 0.98 (Gabriel et al., 2012). Household's preference for holding bank deposits is calibrated based on the share of commercial bank deposits to total deposits which is approximately 84%. Depreciation of physical capital is chosen as 2.5% on a quarterly basis. Investment adjustment cost parameter is set to 2 from Banerjee and Basu (2017). The mark down factor for the deposit interest rate is taken as 0.97 in order to match the savings account deposit rate at the steady state of 3.8%. Price adjustment cost parameter is taken as 118 from Anand et al. (2010) and indexation of past inflation is set to 58% following Sahu (2013). Policy parameters for the Taylor rule stabilizer are chosen from Banerjee and Basu (2017), where interest rate smoothing coefficient is 0.81, inflation stabilizing coefficient is 1.64, and output gap stabilizing coefficient is 0.50. In India, the statutory liquidity requirement of the commercial banks is 21.5%, and the value of α_s is set accordingly.

Table 1: Baseline Parameterization

Parameters	Description	Value	Source
α	Share of capital	0.3	Banerjee & Basu, 2017
β	Discount rate	0.98	Gabriel et al., 2011
η	Preference for holding bank deposit	0.84	RBI database
δ_k	Depreciation rate of capital	0.025	Banerjee & Basu, 2017
κ	Investment adjustment cost	2	Banerjee & Basu, 2017
ζ	Mark-down factor for Deposit rate	0.97	Set to match savings A/C rate
ε^Y	Price elasticity of demand	7	Gabriel et al., 2011
ϕ_p	Price adjustment cost	118	Anand et al., 2010
θ_p	Past inflation indexation	0.58	Shahu J. P., 2013
ρ_{i^G}	Interest rate smoothing parameter	0.81	Banerjee & Basu, 2017
φ_π	Inflation Stabilizing Coefficient	1.64	Banerjee & Basu, 2017
φ_y	Output Stabilizing Coefficient	0.50	Banerjee & Basu, 2017
α_s	Statutory Liquidity Ratio	21.5%	RBI Website
π	Long-run inflation target	4%	Urjit Patel Committee Report, 2013
i^G	Steady state policy rate	7%	RBI Database
i^a	Steady state administered rate	4%	Indian Postal Service Website
i^p	Steady state penalty rate	6.5%	RBI Database
ρ_a	Persistence coefficient of TFP shock	0.82	Anand et al., 2010
ρ_{Z_x}	Persistence coefficient of IST shock	0.59	Banerjee & Basu, 2017
ρ_G	Persistence coefficient of Fiscal shock	0.64	Anand et al., 2010
ρ_μ	Persistence coefficient of Money base shock	0.27	Estimated from Real reserve data (RBI)
σ_a	Standard error of TFP shock	0.06	Set by targeting output volatility
σ_{Z_x}	Standard error of IST shock	0.699	Banerjee & Basu, 2017
σ_G	Standard error of Fiscal shock	0.068	Anand et al., 2010
σ_μ	Standard error of Money base shock	0.065	Estimated from Real reserve data (RBI)
σ_{i^G}	Standard error of Interest rate shock	0.008	Anand et al., 2010

The long run inflation target is set to 4% as proposed by the Urjit Patel Committee Report of RBI (2014). The steady state value of the policy rate is set to 7% in line with the time average over the period of last five-years. The government administered postal interest rate is set to 4% as observed from the savings account in the Indian Postal Service. The steady state value of the penalty rate is set to 6.5% which roughly approximates the average of the marginal standing facility rate in the LAF corridor. The steady state value of productivity and policy shocks are normalized to one.

First order persistence and standard error of the TFP (0.82 and 0.06 respectively) and fiscal policy (0.64 and 0.068 respectively) shocks are in line with Anand et al. (2010). For the IST shock, the estimates for AR (1) coefficient and standard error are 0.59 and 0.699 respectively (Banerjee and Basu, 2017). In case of the autonomous shock to money base, we simply estimate an AR (1) process with the data on growth rate of real reserve. The persistence coefficient is found to be 0.27 while the standard error takes value of 0.065. Finally, the standard error of the policy rate shock is set to 0.008 following Anand et al. (2010).

3.2 Model Validation

We check the predictability of the baseline model by comparing model generated second moments and cross-correlations with the data counterpart. Due to availability of core CPI-inflation data from 2011 onwards, we target the second order moments of business cycle properties of the Indian macroeconomic variables and their inter-relationships with policy instruments for the sample period of 2011: Q2 to 2017: Q1. The quarterly data are taken from various RBI documents. In Table 2, the results are presented.

Table 2: Second Moments & Cross-correlations: Data and Model

List of	Std. Dev		Correlation with y		Correlation with (x_t/x_{t-1})		Correlation with i^G	
Variables	Data	Model	Data	Model	Data	Model	Data	Model
y	0.029	0.033	1	1	0.170	0.124	-0.059	-0.550
c	0.044	0.038	0.749***	0.360	0.268	0.191	-0.115	-0.756
i	0.034	0.126	0.729***	0.550	-0.210	0.035	-0.015	-0.544
π	2.067	0.019	-0.462**	-0.254	0.316	0.056	0.172	0.352
i^L	0.417	0.038	0.049	-0.072	-0.222	0.219	0.420*	-0.080
i^G	1.056	0.015	-0.059	-0.550	-0.029	-0.061	1	1
x	0.067	0.119	0.050	0.399	0.245	0.295	-0.289	-0.071
d	0.038	0.117	0.183	0.349	0.388*	0.292	-0.348*	0.028
d^a	0.072	0.608	0.139	0.501	0.156	0.063	-0.419**	-0.927
TD	0.040	0.147	0.187	0.588	0.377*	0.237	-0.365*	-0.644

We observe that the baseline model slightly over/under estimates the volatilities of relevant variables. However, it explains the key relationships among the variables reasonably well as observed from the cross-correlations. It predicts nearly all the statistically significant correlations pertaining to the Indian business cycle and the movements of policy instruments in the data.

3.3 Impulse Response Analysis of Monetary Transmission Mechanism

Following the reliability check of the baseline model with data, we study the propagation mechanism of the shocks to monetary policy instrument given that the Taylor-type stabilizer is in place. As mentioned earlier, there are two types of policy shocks in action. First, the shocks to base money growth which are akin to autonomous liquidity shocks and may be beyond the control of the central bank. Second, the shocks to short-term policy interest rate by the central bank as a conscious effort to impact the real and/or financial target variables according to their policy objectives. What is the propagation mechanism of such shocks to base money and policy interest rate in the model? We investigate that using the properties of impulse response plots given in Figure 1 and 2.

A positive shock to monetary base immediately translates into a positive inflation via the monetary base rule of equation (27). Higher inflation makes firms expand their supply of output along the standard New Keynesian channel and this raises the real marginal cost (P_w/P) which is similar to any staggered price adjustment cost model. As the real marginal cost rises, the nominal mark up (i.e. inverse of real marginal cost) falls. Higher real marginal cost translates into a higher implicit rental price of capital which promotes investment. Thus a Tobin type effect works for stimulating investment in response to an inflationary monetary shock. Real consumption rises because of higher output and real wage (a wealth effect). Output expansion of the wholesale firms increases the demand for labour, leads to a higher real wage, and encourages workers to supply more labour in the production process. Higher inflation raises the policy rate (i^G) via the Taylor rule which acts as a built in stabilizer for our policy experiment.

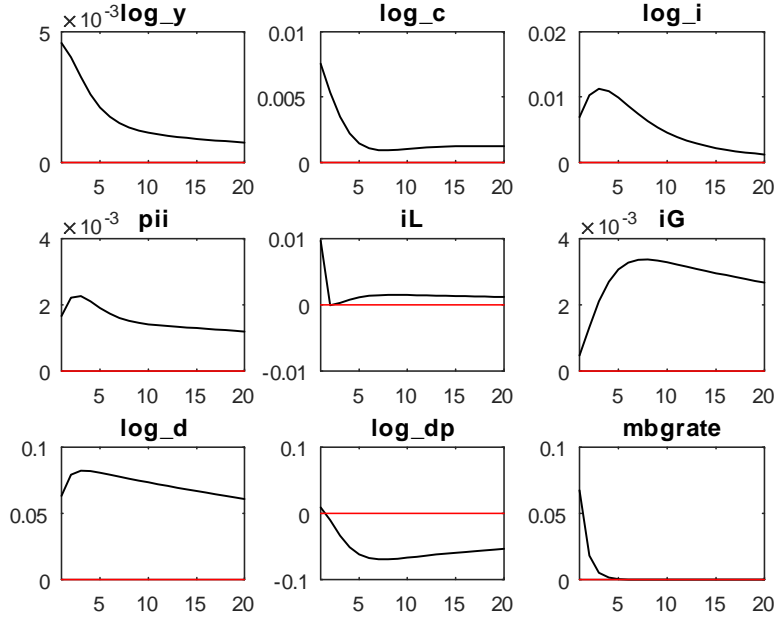


Figure 1: Effect of Shock to Money Base

In the banking sector, the interest rate on deposits rise (i^D) because it is set in proportion to the government bond rate i^G , which positively responds to inflation via the interest rate feedback rule. A higher deposit rate encourages depositors to hold more bank deposits which means real deposits in commercial banks (d) rise. Since, demand for real reserves by banks (x) is proportional to its deposits (d), as given in the bank's reserve demand equation (23), a higher money base raises real reserves although the ratio of money to deposit (x/d) falls. This facilitates more bank lending, more investment and greater accumulation of the capital stock which contributes further to a rise in the real wage. The nominal loan rate rises momentarily due to a higher inflation rate which is well known as Fisher effect. The higher interest rate on deposits due to the interest rate stabilizer makes bank deposits by households increase while deposits in administered postal deposits, i.e. d^a , fall. This happens because depositors substitute away from postal to bank deposits, although total deposits rise. Although the deposit rate rises due to the interest rate stabilizer, the borrowing-lending spread still widens, which means that the loan rate rises more than the deposit rate.

In a similar spirit to a positive shock to the monetary base, we examine a negative shock to the short-term policy interest rate to provide a comparable analysis for impulse responses. A negative shock to the policy interest rate generates an expansionary impact on output in the economy through the standard new Keynesian channel because inflation and

real marginal costs go up. A similar Tobin effect works which raises investment. Households reduce the holding of bank deposits due to a low deposit rate. They consume more and also switch to postal deposits. Shortage of loanable funds raises the interest rate on loan.

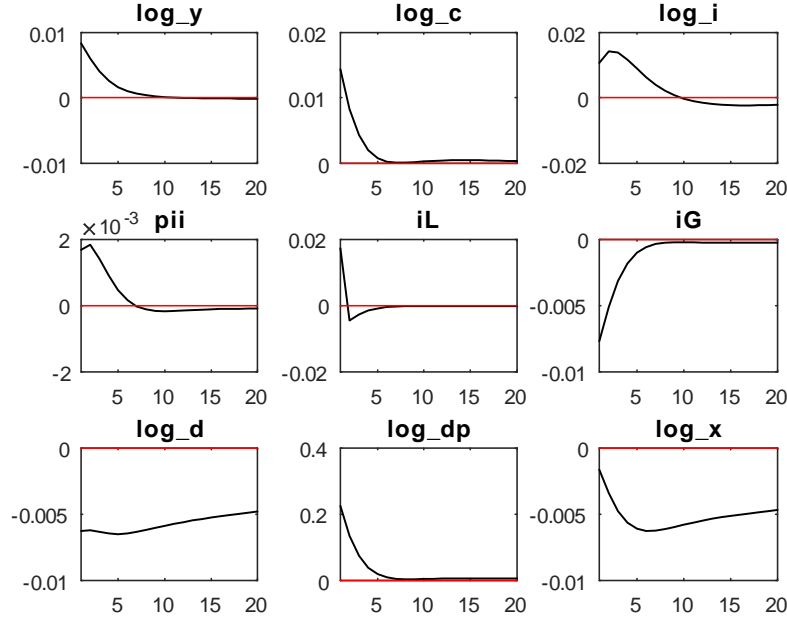


Figure 2: Effect of Shock to Short-term Policy Interest Rate

Due to inflationary pressure, the nominal interest rate on bank lending goes up sharply on impact. However, this rise of bank lending rate does not last for long, and comes down in subsequent periods, and follows the movement of the policy interest rate. Following the decline of the policy interest rate, the marked down interest rate for bank deposits also falls which motivates the households to reshuffle their deposit holding from the commercial banks to postal service for a relatively higher return from the administered rate. So, bank deposit declines and postal deposit increases. In consequence of declining bank deposit, real reserve falls.

3.4 Variance Decomposition Results from Baseline Model

While investigating the propagation mechanism of monetary transmission emanating from different types of monetary policy shocks, it is important to check the explanatory power of those shocks for the economic activities. In this regard, we look into the variance decomposition results of five exogenous shocks and document their contributions in business cycle fluctuations. It is found that monetary policy shocks, summing up both the autonomous

money base shock and shock to short term policy rate, can explain hardly 11.23% (8.43% + 2.80%) of output variations. Clearly, such result reveals the weak monetary transmission mechanism in India. The lion share (50.13%) of output fluctuations is explained by the shock to total factor productivity as argued in the literature (Banerjee and Basu, 2017). In addition, it is noticeable that government spending shock makes a significant contribution (36.26%) for cyclical variations in output. Inflation (95.12%) and the bank lending rate (60.23%) are also largely been driven by shocks to total factor productivity. Details of variance decomposition results are presented in Table 3.

Table 3: Variance Decomposition Results for Major Macroeconomic Variables

Variables	ξ^a	ξ^{Z_x}	ξ^G	ξ^μ	ξ^{i^G}
y	50.13	2.38	36.26	2.80	8.43
c	42.19	24.36	12.81	4.61	16.03
i	50.27	41.11	3.99	1.56	3.06
π	95.12	0.28	0.61	2.13	1.86
i^L	60.23	5.07	6.44	5.59	22.66
i^G	56.33	1.03	4.66	5.61	32.37
$(i^L - i^D)$	47.42	5.07	5.87	10.31	31.34
d	21.32	0.03	0.21	77.9	0.53
d^a	76.20	1.22	4.93	1.74	15.9
TD	61.53	0.54	2.09	29.12	6.72
x	24.67	0.03	0.14	74.82	0.34

3.5 Sensitivity Experiments of Monetary Transmission for Key Variables

In Table 4, we have presented the results of sensitivity experiments which are conducted for a variety of structural and policy parameters of the model. We decrease the baseline values of these parameters one at a time by 10%, and check how such a perturbation affects the transmission of autonomous shocks to monetary base and policy induced shock to the bond interest rate compared to the baseline values.

Table 4: Sensitivity Experiments for Monetary Transmission to Output

Sensitivity	Share of	Share of	Correlation	Correlation
Experiments	ξ^μ in FEVD in y	ξ^{i^G} in FEVD in y	between y and (x_t/x_{t-1})	between y and i^G
Baseline	2.80	8.43	0.124	-0.550
$\eta = 0.756$	2.80	8.43	0.124	-0.550
$i^a = 0.036$	2.80	8.43	0.124	-0.550
$\alpha_s = 0.194$	2.80	8.43	0.124	-0.550
$\zeta = 0.873$	26.51	3.11	0.364	-0.010
$\phi_p = 106$	2.58	7.76	0.121	-0.573
$\theta_p = 0.522$	2.84	8.83	0.124	-0.529
$\varphi_\pi = 1.476$	3.51	9.22	0.137	-0.524
$\varphi_y = 0.45$	2.90	8.47	0.126	-0.557
$\phi_p = 106$	2.08	1.90	0.058	0.356
$\theta_p = 0.522$	2.17	1.87	0.062	0.325
$\varphi_\pi = 1.476$	2.38	1.78	0.059	0.372
$\varphi_y = 0.45$	2.46	2.15	0.061	0.325

There are a few observations worth mentioning regarding these counterfactual experiments.

First, we perform a sensitivity experiment with respect to the preference parameter for commercial bank deposit holding (d), and find no change in the baseline values of the monetary transmission indicators.

Second, we examine whether the presence of fiscal dominance parameters like α_s (the SLR requirement) and i^a (the administered interest rate) can affect the central bank's ability to keep inflation at target, and close the output gap. Curiously, they do not as it is apparent from the table that these parameters have no effect on monetary transmission indicators.

Third, not surprisingly, with low price adjustment costs and higher degree of past inflation indexation in the retail sector, the monetary transmission becomes weaker. Lower values of the nominal friction and forward looking price setting behavior limits the real effects of a monetary policy shock via the expectation channel.

Fourth, the mark-down factor (ζ) for the deposit interest rate has a major implication for monetary transmission driven by the money base. The transmission of monetary base shock becomes conspicuously higher as seen by the error variance decomposition and money-output correlation while the transmission of interest rate shock is remarkably diminished. The intuition for this stems from the fact that a lower ζ marks down the interest rate on deposit which discourages the household to accumulate bank deposit. Since reserve demand is proportional to bank deposit (see eq 26), banks hold less reserve and extend more loans.

Thus the propagation of a shock to monetary base becomes stronger through the bank lending channel because banks hold less reserve. On the other hand, since a lower ζ widens the spread between borrowing and lending rates ($i_t^L - i_t^D$), the pass through from a policy rate shock to the bank lending rate (i_t^L) becomes weaker which explains why the policy rate accounts for less variation of output and also low correlation with output.

Finally, not surprisingly, less aggressive inflation targeting (lower φ_π) and less output stabilization (lower φ_y) raises the pass through of monetary base shock to output, inflation and the nominal loan rate.

3.6 Fiscal Dominance

We measure the dominance of fiscal policy shocks from its contribution to the business cycle variations of output. The variance decomposition result provides evidence for the same. Our counterfactual experiment reveals that fiscal policy receives more importance when the long run value of policy interest rate is elevated. The sensitivity experiment with respect to i^G suggests that there exists a monotonically inverse relationship between the long run interest rate on government bond and the strength of monetary policy transmission and direct relationship with the contribution of fiscal policy. Given an exogenous stream government spending, a higher interest rate on government bonds necessitates higher tax liability on the private sector which makes fiscal policy more important.

Table 5: Sensitivity Experiment on Fiscal Dominance

Output					
i^G	ξ^a	ξ^{Z_x}	ξ^G	ξ^μ	ξ^{i^G}
0.08	50.33	2.62	36.71	1.11	9.24
0.07	50.13	2.38	36.26	2.80	8.43
0.06	49.65	2.19	35.58	4.90	7.67
0.05	49.00	2.04	34.77	7.23	6.96

4 Conclusion and Policy Implications

The key research question of this paper is: *what explains weak monetary policy transmission mechanism in India?* We construct a monetary business cycle model with sticky prices calibrated to the Indian economy to address this question. Our baseline model shows that the major part of output fluctuations are explained by real shocks to the economy (TFP shocks) rather than nominal shocks (base money and interest rate shocks from the Taylor rule). Fiscal policy shocks have a fairly large role to play in explaining output variation, but a

lesser role in other macroeconomic aggregates. IST shocks have a negligible role in explaining output fluctuations in the economy. Our estimated baseline model is also consistent with empirical studies in the Indian context that show that while transmission to bank lending rates is incomplete, it is still stronger than policy rate transmission to GDP, and inflation.

Our paper also addresses a long standing hypotheses in the policy discussion on the impediments to monetary transmission . A prominent hypothesis is that the existence of a postal banking sector could undermine the role of monetary policy. A second hypothesis is fiscal dominance or financial repression. Since banks are asked to hold a fraction of deposits as government bonds, it could weaken the efficacy of monetary policy. Our estimated baseline model does not lend support to either of these two hypotheses. The impulse response and variance decompositions of monetary policy shock are robustly invariant to changes in the administered postal rate, allocation of deposits between these two savings institutions, and to changes in the statutory liquidity ratio.

The lesson is that we need to understand the instruments of monetary control better, as well as the nature of real-monetary interactions, before we make any serious predictions about monetary transmission. For future work, we plan to add an informal sector to the model to understand how the presence of this affects monetary transmission.

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5 Technical Appendix A

- The Lagrangian for the household problem is given by,

$$L_t = E_t \sum_{t=0}^{\infty} \beta^t [U(C_t) - \Phi(H_t) + V(D_t/P_t, D_t^a/P_t) - \lambda_t (P_t C_t + P_t T_t + D_t + D_t^a - W_t H_t - (1 + i_t^D) D_{t-1} - (1 + i^a) D_{t-1}^a - \Pi_t^k - \Pi_t^r - \Pi_t^b)] \quad (42)$$

The household's optimal choices are given by

$$\begin{aligned} \frac{\partial L_t}{\partial C_t} &= U'(C_t) - \lambda_t P_t = 0 \\ \frac{\partial L_t}{\partial H_t} &= \Phi'(H_t) - \lambda_t W_t = 0 \\ \frac{\partial L_t}{\partial D_t} &= \frac{V_1(D_t/P_t, D_t^a/P_t)}{P_t} - \lambda_t + \beta \lambda_{t+1} (1 + i_{t+1}^D) = 0 \\ \frac{\partial L_t}{\partial D_t^a} &= \frac{V_2(D_t/P_t, D_t^a/P_t)}{P_t} - \lambda_t + \beta \lambda_{t+1} (1 + i^a) = 0. \end{aligned}$$

- To obtain equation (9), set $\Omega_{t,t} = 1$ in (8), and solve for Q_t ,

$$\begin{aligned} 0 &= P_t Q_t - P_t \left\{ 1 + S \left(\frac{I_t}{I_{t-1}} \right) \right\} - P_t S' \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} + \Omega_{t,t+1} P_{t+1} S' \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \\ P_t Q_t &= P_t \left\{ 1 + S \left(\frac{I_t}{I_{t-1}} \right) \right\} + P_t S' \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} - \Omega_{t,t+1} P_{t+1} S' \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \\ Q_t &= \left\{ 1 + S \left(\frac{I_t}{I_{t-1}} \right) \right\} + S' \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} - \Omega_{t,t+1} \frac{P_{t+1}}{P_t} S' \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \end{aligned}$$

Note that $\Omega_{t,t+1} = \beta E_t [U'(C_{t+1})/U'(C_t)] [P_t/P_{t+1}]$. Substituting this, we get equation (9)

- To derive equation (23), the Lagrangian is given by

$$E_t \sum_{s=0}^{\infty} \Omega_{t,t+s} \left\{ \left[\begin{aligned} &(1 + i_t^L) L_{t-1} + (1 + i^R) M_{t-1}^R - (\zeta - \alpha_s)(1 + i_t^G) D_{t-1} \\ &\quad - (1 + i^P) E \max(\widehat{W}_{t-1} - M_{t-1}^R, 0) \\ &\quad + (1 - \alpha_s) D_t - L_t - M_t^R \\ &\quad + \lambda_t [M_t^R - \alpha_r D_t] \end{aligned} \right] \right\}$$

which is equivalent to

$$E_t \sum_{s=0}^{\infty} \Omega_{t,t+s} \left\{ \left[\begin{aligned} &(1 + i_t^L) L_{t-1} + (1 + i_t^R) M_{t-1}^R - (\zeta - \alpha_s)(1 + i_t^G) D_{t-1} \\ &-(1 + i^p) \int_{M_{t-1}^R}^{D_{t-1}} [\widetilde{W}_{t-1} - M_{t-1}^R] f(\widetilde{W}_{t-1}) dW_{t-1} \\ &+(1 - \alpha_s) D_t - L_t - M_t^R \\ &+\lambda_t [M_t^R - \alpha_r D_t] \end{aligned} \right] \right\}.$$

The first order condition with respect to M_t^R is given by¹⁵

$$(-1)\Omega_{t,t} + \Omega_{t,t}\lambda_t + E_t\Omega_{t,t+1}(1 + i^R) + E_t\Omega_{t,t+1}(1 + i^p) \int_{M_t^R}^{D_t} f(\widetilde{W}_t) d\widetilde{W}_t = 0. \quad (43)$$

Setting $\Omega_{t,t} = 1$, the Euler equation for M_t^R is given by equation (21)

¹⁵Note that $\frac{d}{dM_t^R} \int_{M_t^R}^{D_t} [\widetilde{W}_t - M_t^R] f(\widetilde{W}_t) dW_t = - \int_{M_t^R}^{D_t} f(\widetilde{W}_t) d\widetilde{W}_t$

6 Technical Appendix B

We have 18 steady state equations, which can be written as a recursive system. These are

1. $(1 + i^L) = (1 + \pi)/\beta$
2. $(1 + i^L) = \left[\left(\frac{\varepsilon^Y - 1}{\varepsilon^Y} \right) \alpha \left(\frac{K}{H} \right)^{\alpha-1} + 1 - \delta_K \right]$
3. $W/P = (1 - \alpha) \left(\frac{\varepsilon^Y - 1}{\varepsilon^Y} \right) (\Lambda)^\alpha$ where $\Lambda = K/H$ solved from the preceding equation
4. $C = W/P$
5. $G = \bar{G}$
6. Using $C + G = [\Lambda^{-(1-\alpha)} - \delta_K] K$, and steady state G , Solve K
7. Using $K/H = \Lambda$, solve H
8. Using $d [1 + \pi - \beta (1 + i^D)] = \eta C (1 + \pi)$, and (5) above solve for d .
9. $d^p [1 + \pi - \beta (1 + i^a)] = (1 - \eta) C (1 + \pi)$, solve for d^a
10. $\frac{x}{d} = 1 - \frac{1 - (1 + i^R) \Omega}{(1 + i^p) \Omega}$
11. $\frac{P_t}{P_t^W} = \frac{\varepsilon^Y}{\varepsilon^Y - 1}$.
12. $I = \delta K$
13. $\pi = \text{long run inflation target } (\bar{\pi})$ (Note that this is pinned down by the money supply rule (28))
14. T solved from the steady state government budget constraint
15. (Stochastic Discount Factor) $\Omega = \beta/(1 + \pi)$
16. $Y = AK^\alpha H^{1-\alpha}$
17. $A = \bar{A}$
18. $i^G = \bar{i}^G$
18. $1 + i^D = \zeta(1 + i^G)$

7 Technical Appendix C

The short run system has 18 endogenous variables:

$$\Omega_{t,t+1} \quad i_t^L \quad K_t \quad H_t \quad Y_t \quad C_t \quad I_t \quad d_t \quad d_t^p \quad x_t \quad i_t^p \quad T_t \quad \pi_t \quad Q_t \quad W_t/P_t \quad G_t \quad P_t/P_t^w \quad A_t \quad .$$

There are four interest rate parameters, i^R, i^a, i^G, i^D , and note that $i^a = i^G = i^D = \bar{i}^s$.

The 18 equations are given by.

$$1. \Omega_{t,t+1} = \frac{\beta u'(c_{t+1})}{u'(c_t)} (1 + \pi_{t+1})^{-1} \quad (44)$$

$$2. U'(C_t) = V'_1(d_t, d_t^a) + \beta U'(C_{t+1}) (1 + i_{t+1}^D) (1 + \pi_{t+1})^{-1} \quad (45)$$

$$3. U'(C_t) = V'_2(d_t, d_t^a) + \beta U'(C_{t+1}) (1 + i^a) (1 + \pi_{t+1})^{-1} \quad (46)$$

$$4. \Phi'(H_t) = (W_t/P_t) U'(C_t). \quad (47)$$

$$5. Q_t = 1 + S \left(\frac{I_t}{I_{t-1}} \right) + S' \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} - \beta E_t \frac{U'(C_{t+1})}{U'(C_t)} \left[S' \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right]. \quad (48)$$

$$6. (1 + i_t^L) = \frac{(P_t^W/P_t) MPK_t + (1 - \delta_k) (1 + \pi_t) Q_t}{Q_{t-1}} \quad (49)$$

$$7. \frac{W_t}{P_t} = (1 - \alpha) \frac{P_t^W Y_t}{P_t H_t} \quad (50)$$

$$8. \frac{P_t}{P_t^W} = \frac{\varepsilon^Y}{\varepsilon^Y - 1} \left[1 + \frac{\phi_p}{\varepsilon^Y - 1} \left(\frac{1 + \pi_t}{1 + \pi} \right) \left\{ \frac{(1 + \pi_t)}{(1 + \pi)} - 1 \right\} - \Omega_{t,t+1} \frac{\phi_p}{\varepsilon^Y - 1} \left\{ \frac{y_{t+1}}{y_t} \frac{(1 + \pi_{t+1})^2}{(1 + \pi)} \left[\frac{(1 + \pi_{t+1})}{(1 + \pi)} - 1 \right] \right\} \right] \quad (51)$$

$$9. Y_t = A K_{t-1}^\alpha H_t^{1-\alpha} \quad (52)$$

$$10. C_t + I_t + G_t + \underbrace{\frac{\phi_p}{2} \left[\left\{ \frac{1 + \pi_t}{1 + \pi} - 1 \right\}^2 y_t \right]}_{\text{price adj cost}} + S \left(\frac{I_t}{I_{t-1}} \right) I_t = A_t K_{t-1}^\alpha H_t^{1-\alpha} \quad (53)$$

$$11. G_t + (1 + i_t^G) \frac{b_{t-1}}{1 + \pi_t} + (1 + i_t^R) \frac{x_{t-1}}{1 + \pi_t} + (1 + i_t^a) \frac{d_{t-1}^a}{1 + \pi_t} = T_t + b_t + x_t + d_t^a + (1 + i_t^p) d_t E \max \left(\frac{\widetilde{W}_t}{D_t} - \frac{M_t^R}{D_t}, 0 \right) \quad (54)$$

$$12. \frac{x_t}{d_t} = 1 - \frac{1 - (1 + i^R)E_t\Omega_{t,t+1}}{(1 + i^p)E_t\Omega_{t,t+1}} \quad (55)$$

$$13. L_t : 1 = E_t\Omega_{t,t+1}(1 + i_{t+1}^L) \quad (56)$$

$$14. G_t - \bar{G} = \rho_G \left(G_{t-1} - \bar{G} \right) + \xi_t^G$$

$$15. A_t - \bar{A} = \rho_A \left(A_{t-1} - \bar{A} \right) + \xi_t^A$$

$$16. K_t = (1 - \delta)K_{t-1} + I_t$$

$$17. \frac{(1 + \pi_t)(x_t/x_{t-1})}{1 + \pi} = \left(\frac{(1 + \pi_{t-1})(x_{t-1}/x_{t-2})}{1 + \pi} \right)^{\rho_\mu}$$

$$18. 1 + i_t^D = \zeta(1 + i_t^G)$$

$$18. \frac{(1 + i_t^G)}{(1 + \bar{i}^G)} = \left(\frac{(1 + i_{t-1}^G)}{(1 + \bar{i}^G)} \right)^{\rho_{iG}} \left[\left(\frac{1 + \pi_{t-1}}{1 + \pi} \right)^{\varphi_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\varphi_Y} \right]^{(1 - \rho_{iG})}$$